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Read July 8, 1859.

IN bringing to a final stage the Reductions of the Greenwich Lunar Observations from 1750 to 1830, it was by no means my intention to stop at the last-mentioned year. The adoption of the year 1830 as a final limit had been, in fact, entirely accidental. The Reductions of the Planetary Observations, which were first effected, had been terminated at that year, because the work was commenced in 1831; and the Reductions of the Lunar Observations were terminated at the same year, because the methods and general system of calculations were similar. But there still existed cogent reasons for desiring a continuation of the reductions on the same general plan. In the year 1836 some changes were made in the system of observations, and very important changes in the system of daily reductions. Some of the elements concerned in these were varied from time to time, as the results of continued observations seemed to require; and the tabular comparisons were necessarily made with BURCKHARDT'S Tables, then employed in the computations of the *Nautical Almanac*. Thus there were inconsistencies of reduction which it was desirable to remove ; and tables of inferior character had been employed, which it was desirable to replace by tabular calculations made on the same theoretical elements as those of the preceding reductions. An occasion of recommencing the work, and a reason for fixing a definite limit, were alone wanting.

The occasion and the reason were supplied by Mr. Adams' discovery of the error in Burckhardt's Tables of the Moon's Parallax, and his corrections to that element, published in the *Nautical Almanac* for 1856. Astronomers cannot be too grateful to Mr. Adams for that investigation ; not only because it corrects a very important error in the construction and numerical values of the Parallax Tables, but also because it brings together the investigations founded on different theories and different observations, shows their precise agreement when correctly treated, and finally establishes the whole doctrine of Lunar Parallax on a firm basis. Nothing has ever been done which has aided so much to give a satisfactory character to the daily reductions of lunar observations.

The date of publication of Mr. Adams' researches enabled me to use the corrected parallax in the daily reductions from the beginning of 1852. This circumstance alone was sufficient to induce me to fix on the end of 1851 as the definitive limit of the proposed reductions. Another reason, however, co-operated with this : it was known that Professor Hansen's Tables were advancing, and would probably be ready for adoption in the *Nautical Almanac* at no distant time, and, even if not employed in the ephemeris for 1852, might with small effort be employed for the mere comparisons of a few years. These tables would, of course, supersede Plana's corrected theory (adopted in the reductions from 1750). For these reasons it was determined to close the new reductions with the end of the year 1851.

The proposal met with the approval of the Board of Visitors of the Royal Observatory, and of the Lords Commissioners of the Admiralty, by whom the necessary funds for computation were supplied. Occupation by other business, and the difficulty of finding a superintendent of calculations, delayed active proceedings for some time. The work, however, was taken up vigorously about the beginning of 1856, and was brought to a close in the summer of the present year. The results, up to the stage of " Annual

Sums of Groups for the different Inequalities," in the sense in which the phrase is explained in the former Memoir on the Correction of the Elements of the Moon's Orbit (*Mem. Roy. Ast. Soc.*, vol. xvii. pp. 24 and 25), are prepared for printing, in a volume intended to form the concluding part of the *Reduction of Greenwich Lunar Observations.*

For the method of treating these annual groups, I must refer in every point to the former Memoir. I have only to remark that, considering the present Reductions as part of the same system with the former Reductions, the limit of one of the terminal groups of years in the former Reductions was slightly altered. The last group is better terminated in 1829 than in 1830. The Extent of each Group in years, from the beginning, now stands as follows; it being remembered that the last group is derived solely from observations with the Altazimuth, and is to be considered as distinct from the others, which are derived entirely from Meridional Observations:—

Extent of each Group, in Years.	Range of Longitude of the Moon's Node.	Extent of each Group, in Years.	Range of Longitude of the Moon's Node.
1750 to 1759	311^g to 97^g	1802 to 1810	394^g to 201^g
1755 to 1764	604 to 389	1806 to 1815	308 to 93
1760 to 1768	497 to 303	1811 to 1819	201 to 7
1765 to 1773	389 to 196	1816 to 1824	493 to 300
1769 to 1778	303 to 88	1820 to 1829	407 to 192
1774 to 1782	196 to 2	1825 to 1833	300 to 106
1779 to 1787	488 to 295	1830 to 1838	592 to 399
1783 to 1791	402 to 209	1834 to 1842	506 to 313
1788 to 1796	295 to 101	1839 to 1847	399 to 205
1792 to 1801	609 to 394	1843 to 1851	313 to 119
1797 to 1805	501 to 308	1847 to 1851	219 to 119

For understanding of the equations which are placed below (and which are exactly similar in form to those of the preceding Memoir), it must be remarked that every comparison of an observed longitude with a tabular longitude gives an equation of the following form:—

Excess of observed longitude over theoretical longitude (expressed in sexagesimal seconds) =

R

+ S × number of centesimal grades in elliptic inequality,

+ T × number of grade-minutes in change of elliptic inequality for change of $0^{g}.1$ in argument, diminished by number of grade-minutes in change of evection for change of $0^{g}.1$ in argument,

+ V × number of grade-minutes in parallactic inequality,

+ W × number of grades in variation,

+ X × number of grade-minutes in annual equation,

+ Y × number of grades in evection.

And every comparison of an observed ecliptic N.P.D. with a tabular ecliptic N.P.D. gives an equation of the following form:—

Excess of observed ecliptic north polar distance over theoretical ecliptic north polar distance (expressed in sexagesimal seconds) =

w

+ x × number of centesimal grades in the effect of mean inclination on ecliptic north polar distance,

+ y × number of grade-minutes in change of effect of mean inclination for change of $0^{g}.1$ in argument,

+ z × number of grade-minutes in evection in ecliptic north polar distance.

Referring always to the former memoir for a description of the method by which the equations for groups are formed, I shall now exhibit the new equations and their solutions:—

Equations for Correction of Elements in Longitude.

1820 to 1829.

$$\begin{aligned}
-10.98 &= +10\times R - 0.73\times S - 0.24\times T - 2.48\times V - 0.68\times W + 9.4\times X + 0.39\times Y\\
-29.25 &= +87.25\times S + 0.12\times T - 3.17\times V + 0.00\times W + 19.2\times X + 0.60\times Y\\
-32.89 &= -0.95\times S + 14.37\times T + 1.56\times V - 0.04\times W + 23.0\times X - 2.14\times Y\\
+56.99 &= -7.34\times S + 0.96\times T + 54.69\times V + 7.58\times W + 21.6\times X - 0.47\times Y\\
+40.82 &= +1.44\times S + 0.40\times T + 34.63\times V + 10.53\times W + 7.9\times X + 0.67\times Y\\
+3.08 &= +5.39\times S + 1.00\times T + 4.43\times V + 0.33\times W + 263.3\times X - 0.95\times Y\\
+14.28 &= +2.91\times S - 2.00\times T - 1.71\times V + 0.36\times W - 17.6\times X + 18.34\times Y
\end{aligned}$$

Solution.

$R=-1.11$, $S=-0.302$, $T=-2.315$, $V=+0.987$, $W=+0.680$, $X=+0.006$, $Y=+0.609$.

1825 to 1833.

$$
\begin{aligned}
-14.02 &= +9\times R + 1.41\times S - 0.29\times T - 3.38\times V - 0.69\times W - 5.0\times X + 0.18\times Y \\
-25.06 &= +80.65\times S + 0.15\times T - 3.58\times V - 0.24\times W - 6.8\times X - 0.67\times Y \\
-28.10 &= +0.06\times S + 12.52\times T - 0.07\times V + 0.02\times W + 7.9\times X - 1.46\times Y \\
+54.16 &= -8.20\times S + 0.21\times T + 47.94\times V + 6.72\times W + 23.3\times X + 0.00\times Y \\
+42.06 &= -0.66\times S + 0.20\times T + 29.26\times V + 9.43\times W + 3.9\times X + 0.51\times Y \\
+12.29 &= -4.35\times S + 0.09\times T + 3.97\times V + 0.27\times W + 233.6\times X - 0.77\times Y \\
+18.33 &= -3.96\times S - 1.53\times T - 0.96\times V + 0.09\times W - 3.7\times X + 16.16\times Y
\end{aligned}
$$

Solution.

$R = -1.15,\ S = -0.273,\ T = -2.159,\ V = +0.821,\ W = +1.884,\ X = +0.036,\ Y = +0.906.$

1830 to 1838.

$$
\begin{aligned}
-17.36 &= +9\times R + 2.03\times S + 0.11\times T - 3.00\times V - 0.53\times W - 3.1\times X + 0.01\times Y \\
-40.00 &= +82.12\times S + 0.62\times T - 1.33\times V + 0.24\times W - 3.8\times X + 0.97\times Y \\
-14.05 &= +1.89\times S + 11.36\times T - 0.08\times V - 0.34\times W + 3.5\times X - 1.22\times Y \\
+31.89 &= -3.10\times S - 0.49\times T + 46.43\times V + 6.62\times W + 26.6\times X + 0.33\times Y \\
+24.03 &= +2.09\times S - 0.99\times T + 29.80\times V + 9.28\times W + 9.5\times X + 0.78\times Y \\
-5.39 &= -0.77\times S + 0.64\times T + 3.54\times V + 0.38\times W + 242.0\times X - 0.29\times Y \\
+15.46 &= +1.76\times S - 1.46\times T + 0.20\times V + 0.23\times W + 1.5\times X + 15.79\times Y
\end{aligned}
$$

Solution.

$R = -1.58,\ S = -0.487,\ T = -1.011,\ V = +0.506,\ W = +0.965,\ X = -0.032,\ Y = +0.948.$

1834 to 1842.

$$
\begin{aligned}
-10.39 &= +9\times R + 0.63\times S + 0.04\times T - 2.52\times V - 0.40\times W + 2.2\times X + 0.27\times Y \\
-46.34 &= +79.69\times S + 0.71\times T - 0.66\times V + 0.61\times W + 16.4\times X + 1.11\times Y \\
-23.52 &= +0.37\times S + 11.72\times T + 0.25\times V - 0.04\times W + 6.2\times X - 0.49\times Y \\
+22.48 &= -3.01\times S - 0.62\times T + 46.65\times V + 5.98\times W + 17.7\times X + 0.31\times Y \\
+10.84 &= +5.47\times S - 0.44\times T + 27.28\times Y + 9.29\times W + 16.5\times X + 1.33\times Y \\
-1.30 &= +5.86\times S + 0.34\times T + 2.66\times V + 0.46\times W + 243.6\times X + 0.10\times Y \\
+7.73 &= +3.28\times S - 0.53\times T + 0.54\times V + 0.44\times W - 12.8\times X + 15.54\times Y
\end{aligned}
$$

Solution.

$R = -1.01,\ S = -0.571,\ T = -1.978,\ V = +0.389,\ W = +0.185,\ X = +0.006,\ Y = +0.537.$

1839 to 1847.

$$
\begin{aligned}
+1.78 &= +9\times R + 0.04\times S - 0.02\times T - 2.53\times V - 0.57\times W + 5.3\times X + 0.53\times Y\\
-28.08 &= +80.23\times S + 0.28\times T - 2.29\times V - 0.01\times W + 10.7\times X + 0.23\times Y\\
-35.88 &= +0.28\times S + 11.68\times T + 1.25\times V + 0.53\times W + 16.9\times X - 0.40\times Y\\
+21.61 &= -8.07\times S + 0.14\times T + 48.13\times V + 5.36\times W + 26.2\times X + 0.15\times Y\\
+8.58 &= -0.23\times S + 0.78\times T + 23.61\times V + 9.32\times W + 23.3\times X + 1.00\times Y\\
+15.88 &= +1.86\times S + 0.60\times T + 4.66\times V + 0.38\times W + 247.8\times X - 0.75\times Y\\
+13.29 &= +1.48\times S - 0.55\times T - 0.11\times V + 0.34\times W - 22.6\times X + 15.99\times Y
\end{aligned}
$$

Solution.

$R=+0.15$, $S=-0.353$, $T=-3.205$, $V=+0.302$, $W=+0.010$, $X=+0.104$, $Y=+1.200$.

1843 to 1851.

$$
\begin{aligned}
+16.27 &= +9\times R + 1.35\times S - 0.24\times T - 1.71\times V - 0.41\times W + 6.0\times X + 0.14\times Y\\
-25.50 &= +80.91\times S - 0.19\times T - 3.02\times V - 0.12\times W + 3.6\times X - 0.29\times Y\\
-14.99 &= -0.91\times S + 11.44\times T + 0.91\times V + 0.33\times W + 11.7\times X - 0.65\times Y\\
+28.66 &= -9.19\times S + 0.47\times T + 47.48\times V + 5.38\times W + 30.4\times X + 0.47\times Y\\
+7.91 &= -2.87\times S + 0.45\times T + 23.36\times V + 9.05\times W + 32.1\times X + 0.85\times Y\\
+12.28 &= -0.32\times S + 0.76\times T + 4.05\times V + 0.30\times W + 247.6\times X - 1.44\times Y\\
+15.58 &= +1.13\times S - 0.57\times T - 1.05\times V + 0.30\times W - 7.4\times X + 16.26\times Y
\end{aligned}
$$

Solution.

$R=+1.87$, $S=-0.293$, $T=-1.326$, $V=+0.643$, $W=-1.056$, $X=+0.037$, $Y=+1.173$.

1847 to 1851 (Altazimuth).

$$
\begin{aligned}
+7.01 &= +5\times R + 0.43\times S + 0.19\times T - 0.20\times V - 0.05\times W + 11.7\times X + 0.28\times Y\\
-20.22 &= +44.36\times S - 0.33\times T + 0.20\times V + 0.21\times W - 23.0\times X + 1.21\times Y\\
-2.85 &= -1.57\times S + 7.27\times T + 1.42\times V + 0.23\times W + 4.7\times X - 0.42\times Y\\
+24.91 &= -0.64\times S + 0.59\times T + 26.35\times V + 1.70\times W + 13.7\times X + 0.04\times Y\\
+6.60 &= +2.47\times S + 0.09\times T + 6.13\times V + 4.75\times W + 0.7\times X + 0.55\times Y\\
+9.56 &= -6.61\times S + 0.63\times T + 3.09\times V + 0.05\times W + 128.9\times X + 0.93\times Y\\
+8.22 &= +6.05\times S - 0.40\times T - 0.54\times V + 0.38\times W + 17.3\times X + 8.83\times Y
\end{aligned}
$$

Solution.

$R=+1.43$, $S=-0.513$, $T=-0.601$, $V=+0.902$, $W=+0.563$, $X=-0.006$, $Y=+1.604$.

Equations for Correction of Elements in Ecliptic North Polar Distance.

1820 to 1829.

$$\begin{array}{rrrrr}
+17{\cdot}64 = +10\times w & -\ 4{\cdot}29\times x & +\ 0{\cdot}25\times y & +\ \ 0{\cdot}3\times z \\
-16{\cdot}03 = & +71{\cdot}05\times x & +\ 0{\cdot}00\times y & +\ \ 7{\cdot}1\times z \\
-38{\cdot}68 = & -\ 0{\cdot}94\times x & +11{\cdot}67\times y & -\ 30{\cdot}0\times z \\
+11{\cdot}57 = & +\ 2{\cdot}90\times x & -\ 1{\cdot}54\times y & +206{\cdot}0\times z
\end{array}$$

Solution.

$w=+1{\cdot}75,\quad x=-0{\cdot}229,\quad y=-3{\cdot}243,\quad z=+0{\cdot}035.$

1825 to 1833.

$$\begin{array}{rrrrr}
+15{\cdot}44 = +\ 9\times w & -\ 2{\cdot}42\times x & -\ 0{\cdot}04\times y & +\ \ 7{\cdot}5\times z \\
-17{\cdot}58 = & +63{\cdot}18\times x & -\ 0{\cdot}23\times y & +\ \ 0{\cdot}7\times z \\
-46{\cdot}01 = & +\ 0{\cdot}14\times x & +10{\cdot}49\times y & -\ 27{\cdot}6\times z \\
+22{\cdot}67 = & +\ 0{\cdot}05\times x & -\ 1{\cdot}73\times y & +182{\cdot}4\times z
\end{array}$$

Solution.

$w=+1{\cdot}55,\quad x=-0{\cdot}294,\quad y=-4{\cdot}158,\quad z=+0{\cdot}085.$

1830 to 1838.

$$\begin{array}{rrrrr}
+14{\cdot}43 = +\ 9\times w & +\ 0{\cdot}59\times x & -\ 0{\cdot}27\times y & +\ \ 2{\cdot}7\times z \\
-30{\cdot}91 = & +64{\cdot}36\times x & -\ 0{\cdot}85\times y & +\ \ 4{\cdot}3\times z \\
-32{\cdot}62 = & -\ 3{\cdot}41\times x & +10{\cdot}43\times y & -\ 21{\cdot}6\times z \\
+\ 9{\cdot}83 = & +\ 1{\cdot}12\times x & -\ 1{\cdot}37\times y & +183{\cdot}6\times z
\end{array}$$

Solution.

$w=+1{\cdot}53,\quad x=-0{\cdot}525,\quad y=-3{\cdot}233,\quad z=+0{\cdot}032.$

1834 to 1842.

$$\begin{array}{rrrrr}
+10{\cdot}06 = +\ 9\times w & -\ 0{\cdot}02\times x & -\ 0{\cdot}04\times y & -\ \ 0{\cdot}8\times z \\
-27{\cdot}85 = & +64{\cdot}61\times x & -\ 0{\cdot}71\times y & +\ \ 5{\cdot}6\times z \\
-28{\cdot}17 = & -\ 5{\cdot}29\times x & +10{\cdot}42\times y & -\ 21{\cdot}4\times z \\
+\ 0{\cdot}36 = & +\ 2{\cdot}31\times x & -\ 1{\cdot}10\times y & +181{\cdot}3\times z
\end{array}$$

Solution.

$w=+1{\cdot}10,\quad x=-0{\cdot}463,\quad y=-2{\cdot}961,\quad z=-0{\cdot}011.$

1839 to 1847.

$$\begin{array}{llll}
+\ 2{\cdot}43=+\ 9\times w & +\ 0{\cdot}65\times x & +\ 0{\cdot}34\times y & +\ 5{\cdot}1\times z \\
-15{\cdot}34= & +64{\cdot}95\times x & +\ 0{\cdot}20\times y & -\ 2{\cdot}4\times z \\
-44{\cdot}41= & +\ 1{\cdot}37\times x & +10{\cdot}35\times y & -\ 9{\cdot}7\times z \\
+\ 0{\cdot}65= & -\ 0{\cdot}99\times x & -\ 0{\cdot}50\times y & +180{\cdot}1\times z
\end{array}$$

Solution.

$$w=+0{\cdot}45,\qquad x=-0{\cdot}223,\qquad y=-4{\cdot}270,\qquad z=-0{\cdot}010.$$

1843 to 1851.

$$\begin{array}{llll}
+\ 1{\cdot}66=+\ 9\times w & +\ 1{\cdot}37\times x & +\ 0{\cdot}33\times y & +\ 6{\cdot}2\times z \\
-18{\cdot}80= & +64{\cdot}37\times x & +\ 0{\cdot}42\times y & -\ 4{\cdot}4\times z \\
-61{\cdot}69= & +\ 0{\cdot}67\times x & +10{\cdot}53\times y & -\ 7{\cdot}0\times z \\
+\ 0{\cdot}90= & -\ 3{\cdot}31\times x & -\ 0{\cdot}32\times y & +179{\cdot}4\times z
\end{array}$$

Solution.

$$w=+0{\cdot}44,\qquad x=-0{\cdot}255,\qquad y=-5{\cdot}848,\qquad z=-0{\cdot}009.$$

1847 to 1851 (Altazimuth).

$$\begin{array}{llll}
+\ 6{\cdot}54=+\ 5\times w & +\ 0{\cdot}60\times x & -\ 0{\cdot}01\times y & +\ 4{\cdot}9\times z \\
-12{\cdot}57= & +36{\cdot}23\times x & +\ 0{\cdot}02\times y & +\ 12{\cdot}2\times z \\
-39{\cdot}03= & +\ 0{\cdot}10\times x & +\ 5{\cdot}74\times y & -\ 5{\cdot}1\times z \\
+\ 4{\cdot}62= & +\ 3{\cdot}36\times x & -\ 0{\cdot}19\times y & +103{\cdot}0\times z
\end{array}$$

Solution.

$$w=+1{\cdot}29,\qquad x=-0{\cdot}358,\qquad y=-6{\cdot}750,\qquad z=+0{\cdot}044.$$

I shall now collect, from the different solutions both of the former and of the present Memoir, the different values given for each unknown quantity, and shall infer from them the most important corrections of elements and inequalities.

I. *Values of* R, *and Correction of Moon's Epoch of Longitude and Mean Motion.*

The epochs (Damoiseau's) used in the tabular calculations include the effect of moon's acceleration, with a secular coefficient of $10''{\cdot}82$. But they do not include Hansen's Equations of Long Period produced by the attraction of *Venus*. In order to take these into account, I have formed the proper arguments in Hansen's *Tables de la Lune* for the middle of each range of years, and with these I have computed the quantities in Tables XLI. and XLII., have subtracted 7835 from the sum, and have converted the remainder into seconds of arc by the factor 0·0004704. The quantities thus formed are subtracted from the values already found for R. It will be remarked that R and all other quantities in the following table are expressed in sexagesimal seconds. In combining these results and all the others, except for V and W, the Altazimuthal results are omitted, as their range of years is not subject to the usual rules in reference to the place of Moon's Node.

Range of Years.	Value of R given by the Equations.	Hansen's Inequalities of Long Period.	R − Hansen's Inequalities.
1750 to 1759	− 3″·19	+ 26″·57	− 29″·76
1755 to 1764	− 1·38	+ 25·78	− 27·16
1760 to 1768	− 0·31	+ 24·70	− 25·01
1765 to 1773	+ 1·43	+ 23·12	− 21·69
1769 to 1778	+ 3·04	+ 21·34	− 18·30
1774 to 1782	+ 3·40	+ 19·28	− 15·88
1779 to 1787	+ 3·04	+ 16·67	− 13·63
1783 to 1791	+ 3·76	+ 14·36	− 10·60
1788 to 1796	+ 4·48	+ 11·24	− 6·76
1792 to 1801	+ 2·76	+ 8·23	− 5·47
1797 to 1805	+ 0·87	+ 5·18	− 4·31
1802 to 1810	+ 1·13	+ 1·49	− 0·36
1806 to 1815	+ 0·93	− 1·82	+ 2·75
1811 to 1819	− 0·61	− 5·14	+ 4·53
1816 to 1824	− 1·39	− 8·79	+ 7·40
1820 to 1829	− 1·11	− 11·99	+ 10·88
1825 to 1833	− 1·15	− 15·07	+ 13·92
1830 to 1838	− 1·58	− 18·30	+ 16·72
1834 to 1842	− 1·01	− 20·70	+ 19·69
1839 to 1847	+ 0·15	− 23·44	+ 23·59
1843 to 1851	+ 1·87	− 25·38	+ 27·25
1847 to 1851	+ 1·43	− 26·28	+ 27·71

If we consider the middles of the groups as equidistant (which is sufficiently accurate), and investigate on the theory of probabilities the values of the coefficients in the expression $x + y \times$ No. of group, measuring No. of group from the middle of the 11th group, or of "1797 to 1805," we find $x = -2''{\cdot}48$, $y = +2''{\cdot}771$. The substitution of these in the formula gives the following comparison :—

No. of Group.	Formula.	Excess of R−Hansen's Inequalities.	No. of Group.	Formula.	Excess of R−Hansen's Inequalities.	No. of Group.	Formula.	Excess of R−Hansen's Inequalities.
	″	″		″	″		″	″
−10	−30·19	+0·43	−3	−10·79	+0·19	+4	+8·60	−1·20
−9	−27·42	+0·26	−2	−8·02	+1·26	+5	+11·38	−0·50
−8	−24·65	−0·36	−1	−5·25	−0·22	+6	+14·15	−0·23
−7	−21·88	+0·19	0	−2·48	−1·83	+7	+16·92	−0·20
−6	−19·11	+0·81	+1	+0·29	−0·65	+8	+19·69	+0·00
−5	−16·34	+0·46	+2	+3·06	−0·31	+9	+22·46	+1·13
−4	−13·56	−0·07	+3	+5·83	−1·30	+10	+25·23	+2·02

The mean interval of groups being 4·65 years, the correction to Damoiseau's mean motion will be, for one year $+0''{\cdot}596$, and for 100 years $+59''{\cdot}6$. And Damoiseau's epoch is very nearly correct for 1806.

The difference between these numbers and those in the former Memoir arises principally from the change made by Professor Hansen in the equations of long period.

II. *Correction of the Equation depending on the Elliptic Form of the Earth.*

If we separate the corrected values of R into groups defined by the critical positions of the moon's node, we obtain the following :—

Range of Years.	**Corrected value of R for Longitude of Node.**		**Range of Years.**	**Corrected value of R for Longitude of Node.**	
	500^g to 300^g	300^g to 100^g		400^g to 200^g	200^g to 0^g
1750 to 1759		− 29″·76	1755 to 1764		− 27″·16
1760 to 1768	− 25″·01		1765 to 1773	− 21″·69	
1769 to 1778		− 18·30	1774 to 1782		− 15·88
1779 to 1787	− 13·63		1783 to 1791	− 10·60	
1788 to 1796		− 6·76	1792 to 1801		− 5·47
1797 to 1805	− 4·31		1802 to 1810	− 0·36	
1806 to 1815		+ 2·75	1811 to 1819		+ 4·53
1816 to 1824	+ 7·40		1820 to 1829	+ 10·88	
1825 to 1833		+ 13·92	1830 to 1838		+ 16·72
1834 to 1842	+ 19·69		1839 to 1847	+ 23·59	
1843 to 1851		+ 27·25			

Comparing the mean of numbers in the first column of corrected values of R with the mean of numbers in the second column,

Excess of corrected value of R (longitude of node 500^g to 300^g) above corrected value of R (longitude of node 300^g to 100^g) .. .. } = −1″·35

From which (as in the former memoir) it appears that there is a correction to the epoch equal to

−1″·06 × cos longitude of moon's node.

Again, if we compare the numbers in the third column of corrected values of R with the numbers in the fourth column,

Excess of corrected value of R (longitude of node 200^g to 0^g) above corrected value of R (longitude of node 400^g to 200^g),

By comparing 5 numbers of last column with 4 first numbers of preceding column —0″·01
By comparing 4 last numbers of last column with 5 numbers of preceding column —0·39
Mean —0·20

From which there is a correction of the epoch equal to

—0″·16 × sin longitude of moon's node.

The inequality depending on the place of the node, which has been used in computing the theoretical place of the moon, is

$$+6''{\cdot}6 \times \text{sin longitude of moon's node.}$$

Applying to this the two corrections just found, we have for the entire inequality whose argument is the mean place of node,

$$\begin{aligned} &+6''{\cdot}44 \times \text{sine longitude of moon's node} \\ &-1\ {\cdot}06 \times \text{cosine longitude of moon's node.} \end{aligned}$$

III. *Values of* S, *and Correction of the principal Coefficient of the Equation of the Centre.*

The values of S, arranged in the same form as the second table for R, are the following:—

Range of Years.	Value of S for Longitude of Node.		Range of Years.	Value of S for Longitude of Node.	
	500^s to 300^s.	300^s to 100^s.		400^s to 200^s.	200^s to 0^s.
1750 to 1759		−0·282	1755 to 1764		−0·258
1760 to 1768	−0·369		1765 to 1773	−0·484	
1769 to 1778		−0·351	1774 to 1782		−0·195
1779 to 1787	−0·301		1783 to 1791	−0·391	
1788 to 1796		−0·251	1792 to 1801		−0·239
1797 to 1805	−0·503		1802 to 1810	−0·514	
1806 to 1815		−0·431	1811 to 1819		−0·375
1816 to 1824	−0·413		1820 to 1829	−0·302	
1825 to 1833		−0·273	1830 to 1838		−0·487
1834 to 1842	−0·571		1839 to 1847	−0·353	
1843 to 1851		−0·293			

The mean of the 21 determinations of S is $-0{\cdot}364$; and the correction to the moon's longitude, in sexagesimal seconds, is therefore $-0{\cdot}364 \times$ number of centesimal grades in elliptic inequality $= \frac{-0{\cdot}364}{3240} \times$ number of sexagesimal seconds in elliptic inequality. Consequently, the coefficient of the equation of the centre ought to be diminished by 0·000112 × coefficient. The coefficient employed in the calculations is $22641''\cdot6$: hence the correction is $-2''\cdot54$; and the corrected value of the coefficient of sine of mean anomaly, when the true longitude is expressed by a function of the mean longitude, as in DAMOISEAU'S form, is

$$22639''\cdot06.$$

Again, comparing the mean of the numbers in the first column of the values of S with the mean of the numbers in the second column, we have

Excess of S (longitude of node 500^g to 300^g) above S (longitude of node 300^g to 100^g) .. .. $= -0\cdot118$

Excess of S (longitude of node 200^g to 0^g) above S (longitude of node 400^g to 200^g),

By comparing 5 numbers of last column with 4 first numbers of preceding column	$+0\cdot113$
By comparing 4 last numbers of last column with 5 numbers of preceding column	$+0\cdot085$
Mean	$+0\cdot099$

The values of these numbers in sexagesimal seconds will be found, as above, by multiplying them by $\frac{1}{3240} \times 22641''\cdot6$; and the coefficients of the corresponding inequalities depending on the moon's node will be obtained by multiplying by $\frac{\pi}{4}$. Also it will be remarked that the influence on the moon's longitude will be obtained by multiplying by sine mean anomaly. Hence the correction to the moon's longitude is

$$\{-0''\cdot65 \times \text{cosine longitude of node} + 0''\cdot54 \times \text{sine longitude of node}\} \times \text{sine mean anomaly.}$$

IV. *Values of* T, *and Correction of Epoch and Motion of Mean Anomaly.*

The following are the values of T derived from the equations, and arranged in the same form:—

Range of Years.	Value of T for Longitude of Node.		Range of Years.	Value of T for Longitude of Node.	
	500^g to 300^g.	300^g to 100^g.		400^g to 200^g.	200^g to 0^g.
1750 to 1759		+0·157	1755 to 1764		+0·961
1760 to 1768	+1·019		1765 to 1773	+0·005	
1769 to 1778		+0·730	1774 to 1782		+1·406
1779 to 1787	+0·326		1783 to 1791	−0·307	
1788 to 1796		−0·211	1792 to 1801		+0·798
1797 to 1805	−0·423		1802 to 1810	−1·877	
1806 to 1815		−0·818	1811 to 1819		+0·644
1816 to 1824	−0·828		1820 to 1829	−2·315	
1825 to 1833		−2·159	1830 to 1838		−1·011
1834 to 1842	−1·978		1839 to 1847	−3·205	
1843 to 1851		−1·326			

If we take all these numbers in one series, as not materially affected by any cause of inequality except the uniform progression of time, and express them by the formula $x + y \times$ number of groups after 1801·5, we find for the value of T,

$$-0{\cdot}496 - 0{\cdot}162 \times \text{number of groups after } 1801{\cdot}5$$

or

$$-0{\cdot}496 - 0{\cdot}0348 \times \text{number of years after } 1801{\cdot}5,$$

whence, as in the former memoir, the correction to the anomaly is

$$-4''{\cdot}96 - 0''{\cdot}348 \times \text{number of years after } 1801{\cdot}5.$$

DAMOISEAU's epoch of anomaly is nearly correct for 1787; and the motion of anomaly in 100 years must be diminished by 34″·8.

Again, taking the means of numbers in the separate columns,

Excess of T (longitude of node 500^g to 300^g) above T (longitude of node 300^g to 100^g) .. .. } = +0·229

Excess of T (longitude of node 200^g to 0^g) above T (longitude of node 400^g to 200^g),

By comparing 5 numbers of last column with 4 first numbers of preceding column +1·683

By comparing 4 numbers of last column with 5 last numbers of preceding column +2·000

Mean +1·842

For the effect in sexagesimal seconds on moon's longitude, these numbers (as in the preceding memoir) must be multiplied by $10 \times \frac{\pi}{4} \times 22641''\cdot6 \times \sin 1''$. Thus the corrections to the moon's longitude are

$$\{+0''\cdot20 \,.\, \text{cos longitude of node} + 1''\cdot59 \,.\, \text{sine longitude of node}\} \times \text{cos mean anomaly.}$$

V. *Small Terms depending on Combinations of the Mean Anomaly, and the Longitude of Node.*

The numbers at the ends of Sections III. and IV. may be combined and expressed in the following form:—

$$+0''\cdot47 \text{ sine (mean anomaly + long. node)} - 0''\cdot17 \text{ cosine (mean anomaly + long. node)}$$
$$-1\;\cdot12 \text{ sine (mean anomaly − long. node)} + 0\;\cdot37 \text{ cosine (mean anomaly − long. node).}$$

VI. *Values of* V, *and Correction of the Coefficient of Parallactic Equation.*

The following are the numbers arranged in the order of time:—

Range of Years.	Value of V.	Range of Years.	Value of V.
1750 to 1759	−0·176	1755 to 1764	−0·221
1760 to 1768	+0·013	1765 to 1773	+0·140
1769 to 1778	+0·206	1774 to 1782	+0·301
1779 to 1787	+0·064	1783 to 1791	−0·253
1788 to 1796	−0·276	1792 to 1801	−0·285
1797 to 1805	−0·427	1802 to 1810	−0·248
1806 to 1815	+0·234	1811 to 1819	+0·529
1816 to 1824	+0·635	1820 to 1829	+0·987
1825 to 1833	+0·821	1830 to 1838	+0·506
1834 to 1842	+0·389	1839 to 1847	+0·302
1843 to 1851	+0·643		
1847 to 1851	+0·902		

The mean of the 21 values given by the meridional observations is $+0{\cdot}184$. The longitude, in sexagesimal seconds, is therefore to be increased by $+ 0{\cdot}184 \times$ number of grade-minutes in the parallactic equation; and therefore the coefficient of the parallactic equation, in sexagesimal seconds, is to be increased by $0{\cdot}184 \times$ number of grade-minutes in the parallactic coefficient, or by

$$+0{\cdot}184 \times \frac{\text{number of sexagesimal seconds in the parallactic coefficient}}{32{\cdot}4}$$

$$= +0{\cdot}184 \times \frac{122''{\cdot}1}{32{\cdot}4} = +0{\cdot}184 \times 3''{\cdot}77 = +0''{\cdot}69.$$

The coefficient would thus be $122''{\cdot}1 + 0''{\cdot}69 = 122''{\cdot}79$.

The Altazimuth observations give the much larger value $+ 0{\cdot}902 \times 3''{\cdot}77 = + 3''{\cdot}40$; or coefficient $= 125''{\cdot}50$. These observations possess the rare advantage of having been extended to portions of every lunation very near to the conjunction of sun and moon, and thereby of discriminating between the entangled effects of parallactic equation and variation.

Remarking that in late years very great attention has been given to the determination of the moon's apparent semidiameter (on which the accuracy of the determination of V greatly depends), and that the use of the telescopes to which that determination applies commenced in 1811, I am induced to think that the last eight results only of the meridional observations ought to be employed. They would give for correction of coefficient $+0{\cdot}602 \times 3''{\cdot}77 = + 2''{\cdot}27$, or coefficient $= 124''{\cdot}37$.

As the altazimuth value of V is in fact exceeded by one of the meridional values of V, it will be seen that there is no actual discordance between them.

I think that the real value of increase of coefficient cannot be far from $+ 2''{\cdot}6$, or the real value of coefficient $= 124''{\cdot}7$.

VII. *Values of* W, *and Correction of the Coefficient of Variation.*

The following are the numbers, in the order of time:—

Range of Years.	Value of W.	Range of Years.	Value of W.
1750 to 1759	+0·017	1755 to 1764	+0·891
1760 to 1768	+1·496	1765 to 1773	+1·640
1769 to 1778	+0·843	1774 to 1782	+0·091
1779 to 1787	+0·008	1783 to 1791	+0·193
1788 to 1796	+1·236	1792 to 1801	+1·673
1797 to 1805	+2·247	1802 to 1810	+1·430
1806 to 1815	+0·815	1811 to 1819	+0·724
1816 to 1824	+0·608	1820 to 1829	+0·680
1825 to 1833	+1·884	1830 to 1838	+0·965
1834 to 1842	+0·185	1839 to 1847	+0·010
1843 to 1851	−1·056		
1847 to 1851	+0·563		

The mean of the 21 meridional values is + 0·790, and that of the last 8 values is + 0·500. The assumed coefficient of variation being 2370″·3, the correction = $\frac{+0{\cdot}790}{3240} \times 2370{\cdot}3 = +0''{\cdot}58$, and for the last 8 values + 0″·37. That from the altazimuth observations = + 0″·41. Adopting + 0″·40, the corrected coefficient will be + 2370″·7.

VIII. *Values of* X, *and Correction of the Coefficient of Annual Equation.*

The values, arranged in order of time, are as follows :—

Range of Years.	Value of X.	Range of Years.	Value of X.
1750 to 1759	+0·116	1755 to 1764	+0·090
1760 to 1768	+0·076	1765 to 1773	+0·113
1769 to 1778	+0·192	1774 to 1782	+0·148
1779 to 1787	+0·060	1783 to 1791	+0·080
1788 to 1796	+0·097	1792 to 1801	+0·108
1797 to 1805	+0·087	1802 to 1810	+0·056
1806 to 1815	+0·070	1811 to 1819	+0·027
1816 to 1824	−0·016	1820 to 1829	+0·006
1825 to 1833	+0·036	1830 to 1838	−0·032
1834 to 1842	+0·006	1839 to 1847	+0·104
1843 to 1851	+0·037		
1847 to 1851	−0·006		

The mean of the meridional values is +0·070. The assumed coefficient of annual equation being 668″·6, that coefficient is to be increased by $\frac{+0{\cdot}070}{32{\cdot}4} \times 668''{\cdot}6 = +1''{\cdot}44$. The eight or nine last groups, however, and the altazimuth groups, do not support this alteration. I am inclined to fix on 669″·0 as very well representing the observations.

IX. *Values of* Y, *and Correction of the Coefficient of Evection.*

The values in the order of time are as follows:—

Range of Years.	Value of Y.	Range of Years.	Value of Y.
1750 to 1759	+ 0·525	1755 to 1764	+ 0·807
1760 to 1768	+ 0·867	1765 to 1773	+ 0·757
1769 to 1778	+ 1·090	1774 to 1782	+ 0·952
1779 to 1787	+ 0·812	1783 to 1791	+ 0·662
1788 to 1796	+ 1·037	1792 to 1801	+ 1·016
1797 to 1805	+ 1·090	1802 to 1810	+ 0·843
1806 to 1815	+ 1·234	1811 to 1819	+ 1·202
1816 to 1824	+ 0·908	1820 to 1829	+ 0·609
1825 to 1833	+ 0·906	1830 to 1838	+ 0·948
1834 to 1842	+ 0·537	1839 to 1847	+ 1·200
1843 to 1851	+ 1·173		
1847 to 1851	+ 1·604		

The mean of the meridional results is +0·913. If, in respect of the altazimuth result, we increase this to +1·00; remarking that the assumed coefficient is 4585″·6, we find for the correction $\frac{+1{\cdot}00}{3240} \times 4585''{\cdot}6 = +1''{\cdot}41$; and the corrected coefficient is 4587″·01.

I now proceed with the correction of elements of Ecliptic North Polar Distance.

X. *Values of w, and Correction of the Constant Term of Parallax.*

In the former Memoir it was found that, before numbers could be exhibited which applied immediately to the correction of the tabular Constant of Parallax, it was necessary to make important alterations in the numbers derived immediately from the equations, on account of supposed systematic errors in the received elements for the reduction of the observations. In the present investigation, it is necessary to make a small alteration in several numbers on account of a systematic change in the tabular Constant of Parallax used in the reduction of the observations.

As far as the end of 1839 (see the Introduction to the *Reduction of Observations of the Moon*, 1831 *to* 1851, pages xi., xiii., and xviii., and also the Introduction to the *Reduction of Observations of the Moon*, 1750 *to* 1830, pages xxi and xxii), the Constant of Parallax employed in the reductions is 57′ 1″·8.* From the beginning of 1840 (Introduction to the *Reductions*, 1831 *to* 1851) the Constant employed is that of Mr. Adams, or 57′ 2″·3. In order to refer the reductions from 1840 to 1851 to the same Parallax-Constant which is used to 1839, we must increase the N.P.D. deduced from observation, and must increase the value of *w*, by +0″·50 × sine 51° nearly = +0″·39. For the group 1834 to 1842, we must therefore add +0″·13; for 1839 to 1847, +0″·35; afterwards +0″·39. Thus we have,

Range of Years.	Value of *w*.	Range of Years.	Value of *w*.
1750 to 1759	+ 3″·16	1755 to 1764	+ 3″·87
1760 to 1768	+ 2·58	1765 to 1773	+ 0·80
1769 to 1778	+ 1·02	1774 to 1782	+ 0·06
1779 to 1787	+ 0·02	1783 to 1791	+ 1·17
1788 to 1796	+ 1·59	1792 to 1801	+ 1·98
1797 to 1805	+ 2·30	1802 to 1810	+ 2·78
1806 to 1815	+ 2·42	1811 to 1819	+ 1·62
1816 to 1824	+ 1·25	1820 to 1829	+ 1·75
1825 to 1833	+ 1·55	1830 to 1838	+ 1·53
1834 to 1842	+ 1·23	1839 to 1847	+ 0·80
1843 to 1851	+ 0·83		
1847 to 1851	+ 1·68		

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* I regret to say that, in the former Memoir, Vol. xvii. pages 51 and 52, I have committed a most serious error in my statement of the Constant of Parallax employed in the former *Reductions*. I have given it as 57′ 3″·16; whereas it is stated explicitly in the *Reductions*, Introduction, pages xxi and xxii, and has been verified by reference to the manuscripts, that the Constant employed was 57′ 1″·8. I cannot account for this confusion, in a research which so much engaged my attention, and in which I was so much impressed with the difficulties of reconciling results.

The mean of the 22 values is + 1″·64, implying that the assumed constant term of parallax ought to be increased by about + 2″·09. The corrected constant thus becomes 57′ 3″·89.

The constant thus found is somewhat greater than Plana's, which is 57′ 3″·16, or Mr. Adams', which is 57′ 2″·3; and a little greater than that found from the Cambridge Observations (see page 51 of the former Memoir), which is 57′ 3″·46. It is worthy of remark that in each instance the constant derived from observation is greater than that derived from theory.*

If we confine ourselves to the results from 1811, which are much more certain than those preceding that year, we find from the mean of 9 values, $w = +1''{\cdot}36$; increase of constant = + 1″·74; corrected constant 57′ 3″·54. This brings the various numbers into better harmony.

XI. *Values of x; Correction of the Inclination of the Moon's Orbit; Terms of Long Period in the Inclination of the Orbit.*

It is necessary here to arrange the numbers with reference to the position of the moon's node.

Range of Years.	Value of x for Longitude of Node.	
	500^g to 300^g	300^g to 100^g
1750 to 1759		− 0·572
1760 to 1768	− 0·636	
1769 to 1778		− 0·185
1779 to 1787	− 0·560	
1788 to 1796		− 0·430
1797 to 1805	− 0·456	
1806 to 1815		− 0·256
1816 to 1824	− 0·364	
1825 to 1833		− 0·294
1834 to 1842	− 0·463	
1843 to 1851		− 0·255
1847 to 1851		− 0·358

Range of Years.	Value of x for Longitude of Node.	
	400^g to 200^g	200^g to 0^g
1755 to 1764		− 0·917
1765 to 1773	− 0·197	
1774 to 1782		− 0·537
1783 to 1791	− 0·226	
1792 to 1801		− 0·728
1802 to 1810	− 0·073	
1811 to 1819		− 0·472
1820 to 1829	− 0·229	
1830 to 1838		− 0·525
1839 to 1847	− 0·223	

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* It will be seen that the correction of the error pointed out in the last note almost entirely removes the discordance which was so strongly insisted on in the former Memoir.

The mean of the 21 meridional values is $-0\cdot410$, differing little from that given by the Altazimuth. As the assumed inclination of the moon's orbit is $18537''\cdot9$, this shows that the inclination is to be diminished by $\frac{0\cdot410 \times 18537\cdot9}{3240}$ or $2''\cdot35$, and is therefore to be reduced to,

$$18535''\cdot55.$$

And, comparing the means of the numbers in the several columns,

Excess of x (longitude of node 500^s to 300^s) above x (longitude of node 300^s to 100^s)	$-0\cdot164$
Excess of x (longitude of node 200^s to 0^s) above x (longitude of node 400^s to 200^s),	
By comparing mean of 5 numbers in third column with mean of last 4 numbers in fourth column	$-0\cdot375$
By comparing mean of first 4 numbers in third column with mean of 5 numbers in fourth column	$-0\cdot455$
Mean	$-0\cdot415$

Multiplying the numbers by $\frac{18537\cdot9}{3240} \times \frac{\pi}{4}$, we find that the inequality in inclination which they represent is,

$$-0''\cdot73 \times \text{cosine longitude of node} - 1''\cdot87 \times \text{sine longitude of node}.$$

XII. *Values of y; Correction of the Argument of Latitude and of the Motion of the Argument; Terms of Long Period in the Argument.*

The following numbers are arranged as the last.

Range of Years.	Value of y for Longitude of Node.		Range of Years.	Value of y for Longitude of Node.	
	500^s to 300^s	300^s to 100^s		400^s to 200^s	200^s to 0^s
1750 to 1759		$-1\cdot420$	1755 to 1764		$+1\cdot209$
1760 to 1768	$+3\cdot872$		1765 to 1773	$+1\cdot932$	
1769 to 1778		$-0\cdot519$	1774 to 1782		$-1\cdot194$
1779 to 1787	$+0\cdot987$		1783 to 1791	$+0\cdot356$	
1788 to 1796		$-1\cdot691$	1792 to 1801		$-0\cdot022$
1797 to 1805	$+1\cdot275$		1802 to 1810	$-1\cdot400$	
1806 to 1815		$-3\cdot266$	1811 to 1819		$-3\cdot225$
1816 to 1824	$-2\cdot339$		1820 to 1829	$-3\cdot243$	
1825 to 1833		$-4\cdot158$	1830 to 1838		$-3\cdot233$
1834 to 1842	$-2\cdot961$		1839 to 1847	$-4\cdot270$	
1843 to 1851		$-5\cdot848$			
1847 to 1851		$-6\cdot750$			

Treating these in the same manner as T, and remarking that each number is to be multiplied by 10 to give a result in sexagesimal seconds, we find for the general correction to the argument of north polar distance, (which is the same as the correction to the argument of latitude),

$$-13''\cdot92-3''\cdot183\times \text{number of groups after middle of “1797 to 1805.”}$$
$$=-13''\cdot92-0''\cdot684\times \text{number of years after } 1801\cdot5.$$

The secular motion of argument of latitude is to be diminished by $68''\cdot6$. Damoiseau's epoch is correct for 1781 nearly.

Also, comparing the means of the numbers in the several columns,

Excess of y (longitude of node 500^g to 300^g) above y (longitude of node 300^g to 100^g)	$+2\cdot984$
Excess of y (longitude of node 200^g to 0^g) above y (longitude of node 400^g to 200^g),	
By comparing mean of 5 numbers in third column with mean of last 4 numbers in fourth column	$-0\cdot594$
By comparing mean of first 4 numbers in third column with mean of 5 numbers in fourth column	$-0\cdot704$
Mean	$-0\cdot649$

Multiplying these numbers by $\frac{10\pi}{4}$, we find for the inequalities in argument of latitude,

$$+23''\cdot46\times \text{cosine longitude of node} -5''\cdot09\times \text{sine longitude of node.}$$

XIII. *Small Terms in the Moon's Latitude, produced by the Combination of the Small Terms of Long Period in the Inclination and in the Argument of Latitude.*

Put u for the moon's true longitude, y for the received true argument of latitude; then $u-y$ is the true longitude of the moon's node, which in the small terms may be considered as mean longitude of the moon's node. Hence the true argument of latitude is, as corrected,

$$y+(23\cdot46\times\cos\overline{u-y}-5\cdot09\times\sin\overline{u-y})\sin 1''$$

and the sine of argument of latitude is,

$$\sin y + 11{\cdot}73\,\{\cos \overline{u-2y} + \cos u\} \sin 1'' - 2{\cdot}54\,\{\sin \overline{u-2y} + \sin u\} \sin 1''.$$

This is to be multiplied by the inclination, or

$$18535''{\cdot}55 - 0''{\cdot}73\,.\cos \overline{u-y} - 1''{\cdot}87\,.\sin \overline{u-y}$$

The product is

$$18535''{\cdot}55 \sin y$$

$$+\,0''{\cdot}17\,.\cos \overline{u-2y} + 0''{\cdot}13\,.\sin \overline{u-2y} + 1''{\cdot}93\,.\cos u - 0''{\cdot}59\,.\sin u.$$

Omitting the two smallest terms, and combining the last term with the term $-8''{\cdot}00 \sin u$ adopted in the tabular calculations of latitude, we have for the terms of latitude depending upon the argument "Longitude from the First Point of Aries,"—

$$+1''{\cdot}93 \text{ cosine } u - 8''{\cdot}59 \text{ sine } u.$$

XIV. *Values of z, and Correction of the Coefficient of Evection in Ecliptic North Polar Distance.*

The values of z given by the equations, arranged in order of time, are as follows:—

Range of Years.	Value of z.	Range of Years.	Value of z.
1750 to 1759	+ 0·001	1755 to 1764	− 0·102
1760 to 1768	− 0·123	1765 to 1773	+ 0·004
1769 to 1778	− 0·030	1774 to 1782	− 0·017
1779 to 1787	+ 0·003	1783 to 1791	− 0·031
1788 to 1796	− 0·011	1792 to 1801	− 0·009
1797 to 1805	− 0·016	1802 to 1810	+ 0·034
1806 to 1815	+ 0·023	1811 to 1819	+ 0·005
1816 to 1824	− 0·014	1820 to 1829	+ 0·035
1825 to 1833	+ 0·085	1830 to 1838	+ 0·032
1834 to 1842	− 0·011	1839 to 1847	− 0·010
1843 to 1851	− 0·009		
1847 to 1851	+ 0·044		

The mean of the 21 meridional values is $-0{\cdot}008$; representing (as for V) a diminution of the coefficient by $\frac{0{\cdot}008 \times 527''{\cdot}5}{32{\cdot}4} = 0''{\cdot}13$. The diminution would have been smaller if the altazimuth result had been included. The tabular coefficient $527''{\cdot}5$ may be considered as sensibly correct.

Royal Observatory, Greenwich,
1859, *July* 2.

G. B. AIRY.

II. *On the Figure of the Earth.* By Captain A. R. CLARKE, R.E.

Read April 8, 1860.

THE following investigation of the figure of the earth was undertaken in consequence of the remarks of the Astronomer Royal, in the *Monthly Notices* for January 1860 (vol. xx. p. 104), on General T. F. DE SCHUBERT'S "Essai d'une Détermination de la véritable Figure de la Terre." The result arrived at in General DE SCHUBERT'S memoir is that the earth is an ellipsoid, whose elements are

Polar Semi-axis	20855605 feet,	
Maximum Compression	$\frac{1}{292\cdot109}$,	
Minimum Compression	$\frac{1}{302\cdot004}$,	
Longitude of Major Axis of Equator ..	41° 4′	221° 4′,
Longitude of Minor Axis of Equator ..	131° 4′	311° 4′,

the longitudes being measured from Greenwich eastwards.

These results are, however, dependent on so small a number of observed latitudes, that it becomes highly necessary to ascertain their probable errors before they can be received as representing the real figure of the earth. I have, therefore, repeated the calculation in a more precise manner, using a larger number of observed latitudes and bringing out the probable errors of the elements. The arcs used are the same as in General DE SCHUBERT'S memoir, with the omission of the Prussian and Pennsylvanian arcs. These arcs are both very short; the extreme latitudes of the Prussian arc are embraced both in the English and in the Russian arcs, and the Pennsylvanian arc is certainly not deserving of notice.

In the first place, whatever the real figure of the earth may be, if in the investigation we presuppose it an ellipsoid, it is quite clear that the arithmetical process must bring it out an ellipsoid of some kind or other, which ellipsoid will agree better with all the observed latitudes, as a whole, than any spheroid of revolution will. Nevertheless, it would scarcely, I conceive, be correct to say we had *proved* the earth not to be a solid of revolution. To prove this would require data which we are not in possession of at present, which must include several arcs of longitude. In the meantime it is interesting to ascertain what ellipsoid does actually best represent the existing measurements.

1.

To define meridians and parallels on the supposition of an ellipsoidal earth. Let a, b, c be the three semiaxes, a and b being the semiaxes of the equator, c the polar semiaxis, or that about which the earth revolves. Taking the origin, as usual, at the centre, the equation of the surface will be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = \text{U} = 0. \qquad (1)$$

If S be any point of the surface, the direction-cosines of the normal at S are proportional to

$$\frac{d\text{U}}{dx}, \frac{d\text{U}}{dy}, \frac{d\text{U}}{dz}, \text{ or } \frac{x}{a^2}, \frac{y}{b^2}, \frac{z}{c^2},$$

and if $\frac{\pi}{2} - \phi$ be the angle between this normal and the minor axis, so that ϕ is the latitude of S, we have

$$\sin \phi = \frac{\frac{z}{c^2}}{\left(\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}\right)^{\frac{1}{2}}}.$$

This is equivalent to

$$\frac{x^2}{a^4} + \frac{y^2}{b^4} - \frac{z^2}{c^4} \cot^2 \phi = 0, \qquad (2)$$

and if in this equation ϕ be a constant, it will represent the "parallel" in latitude ϕ. The parallel, therefore, in the case of an ellipsoidal earth is no longer a plane curve.

Let longitude be reckoned from the plane of $x\,z$ or of $a\,c$. As there are two species of latitude, geocentric and astronomical, so in an ellipsoidal earth we must distinguish two longitudes, geocentric (called u) and astronomical (called ω). Conceive a line passing through the origin, laying in the plane of $a\,b$, and directed to a point whose longitude is $\frac{\pi}{2}+\omega$, or making that angle with the semiaxis a. The direction-cosines of this line are

$$-\sin\omega,\quad \cos\omega,\quad 0.$$

We may define a "meridian" as the locus of all points whose longitude is a constant quantity, or whose zeniths lie in a great circle of the heavens, which circle has its poles in the equator. Hence those points of the surface whose normals are at right angles to the above-mentioned line lie in the meridian whose longitude is ω. This condition of perpendicularity is expressed by the equation

$$-\frac{x\sin\omega}{a^2}+\frac{y\cos\omega}{b^2}=0, \tag{3}$$

so that the meridian is still a plane curve. The relation between the geocentric and astronomical longitudes is

$$\frac{y}{x}=\tan u=\frac{b^2}{a^2}\tan\omega.$$

2.

Let us confine our attention to this meridian plane. The curve of section is, of course, an ellipse, of which the minor semiaxis is c and the major semiaxis intermediate to a and b in magnitude. Let r be the major semiaxis, or radius of the equator, then

$$\frac{1}{r^2}=\frac{\cos^2 u}{a^2}+\frac{\sin^2 u}{b^2}.$$

Let i, k be two quantities, such that

$$a^2=\frac{k^2}{1-i},\quad b^2=\frac{k^2}{1+i}.$$

Then

$$\frac{k^2}{r^2} = 1 - i \cos 2u.$$

Let

$$n = \frac{r-c}{r+c} = \frac{k - c\,(1 - i \cos 2u)^{-\frac{1}{2}}}{k + c\,(1 - i \cos 2u)^{-\frac{1}{2}}}.$$

If we expand this according to the powers of i we get

$$n = \frac{k-c}{k+c} + \frac{kc}{(k+c)^2}\, i \cos 2u + \frac{kc\,(k+3c)}{(k+c)^3} \cdot \frac{i^2}{4} \cos^2 2u + \ldots.$$

We shall neglect altogether the square of i and higher powers; also k and c are so nearly equal, we may safely put

$$n = \frac{k-c}{k+c} + \frac{i}{4} \cos 2u. \qquad (4)$$

The normal determining the zenith does not lie in the meridional plane of section, or does not pass through the minor semiaxis; consequently, the observed latitudes in a meridian arc are not exactly the quantities which should be used in the ordinary method of expressing the length of a meridian arc in terms of the extreme latitudes, and it becomes necessary to investigate the amount of the difference. Let φ be the true latitude of the point $x\,y\,z$, and φ' the latitude that would result from confining the normal to the *curve*, to the plane of section; then

$$\frac{1}{r^2} = \frac{1 - i \cos 2u}{k^2}; \quad \tan u = \frac{y}{x},$$

$$\cot^2 \varphi' = \frac{\dfrac{x^2 + y^2}{r^4}}{\dfrac{z^2}{c^4}}; \quad \cot^2 \varphi = \frac{\dfrac{x^2}{a^4} + \dfrac{y^2}{b^4}}{\dfrac{z^2}{c^4}},$$

$$\frac{\cot^2 \varphi}{\cot^2 \varphi'} = \frac{r^4}{a^4} \cos^2 u + \frac{r^4}{b^4} \sin^2 u,$$

$$\therefore \quad \frac{\tan^2 \varphi'}{\tan^2 \varphi} = \frac{(1-i)^2 \cos^2 u + (1+i)^2 \sin^2 u}{\{(1-i) \cos^2 u + (1+i) \sin^2 u\}^2}$$

$$= 1 + \left(\frac{i \sin 2u}{1 - i \cos 2u}\right)^2$$

In General DE SCHUBERT'S figure i is very small, only about $\frac{1}{8900}$, hence the difference of φ and φ' is also a very small quantity. If we call it $\delta\varphi$, we have

$$\frac{2\,\delta\varphi}{\sin\varphi\cos\varphi} = i^2 \sin^2 2u,$$

$$\therefore \quad \delta\varphi = \frac{i^2}{4} \sin 2\varphi \sin^2 2u,$$

of which the maximum value is $\frac{1}{4} i^2$, and with the preceding value of i would only amount to $0''{\cdot}00065$. We may, therefore, calculate the lengths of the meridian arcs as if the latitudes were actually observed in the plane of the ellipse.

The length of an arc measured from latitude φ_1 to latitude φ_2 on an ellipse whose minor semiaxis is c and the ratio of its axes $1 - n : 1 + n$ is

$$s = \int_{\varphi_1}^{\varphi_2} \frac{c\,(1+n)\,(1-n^2)\,d\varphi}{(1 + 2n\cos 2\varphi + n^2)^{\frac{3}{2}}}$$

$$= c\left\{\alpha\left(1 + n + \frac{5}{4}n^2 + \frac{5}{4}n^3\right) - \alpha_1\left(3n + 3n^2 + \frac{21}{8}n^3\right) + \alpha_2\left(\frac{15}{8}n^2 + \frac{15}{8}n^3\right) - \alpha_3\left(\frac{35}{24}n^3\right)\right\},$$

where

$$\left.\begin{aligned} \alpha &= \varphi_2 - \varphi_1, \\ \alpha_1 &= \sin\ (\varphi_2 - \varphi_1) \cos\ (\varphi_2 + \varphi_1), \\ \alpha_2 &= \sin 2\,(\varphi_2 - \varphi_1) \cos 2\,(\varphi_2 + \varphi_1), \\ \alpha_3 &= \sin 3\,(\varphi_2 - \varphi_1) \cos 3\,(\varphi_2 + \varphi_1). \end{aligned}\right\} \quad (5)$$

The part of s which depends on n^3 is $c\,n^3\left(\frac{5}{4}\alpha - \frac{21}{8}\alpha_1 + \frac{15}{8}\alpha_2 - \frac{35}{24}\alpha_3\right)$. Taking c as 20855000, n as $\frac{1}{590}$, the quantity $c\,n^3$ amounts to only 1·2 inches, so that this term in n^3 is very small. In the Russian arc, which is the largest we have to deal with, $\varphi_2 - \varphi_1 = 25^\circ\ 20'$; $\varphi_2 + \varphi_1 = 116^\circ\ 0'$, and the quantity within the bracket $= -1{\cdot}2317$, so that the term in question only amounts to 1·50 inches. Now, considering that the probable error of that arc is ± 40 feet, it is clear that we may omit the terms in n^3 and adopt for s the expression

$$\frac{s}{c} = \alpha\left(1 + n + \frac{5}{4}n^2\right) - \alpha_1\left(3n + 3n^2\right) + \alpha_2\left(\frac{15}{8}n^2\right). \quad (6)$$

3.

We have now from the given meridian measures to determine four quantities, namely a, b, c, and the position in longitude of the two former, or c, i, k and u, the longitude of a. Let u_1 be the longitude of one of the arcs given, then for that arc,— reckoning henceforth (geocentric) longitudes from Greenwich,—

$$n = \frac{k-c}{k+c} + \frac{i}{4} \cos 2\,(u_1 - u)$$

$$= \frac{k-c}{k+c} + \frac{i}{4} \cos 2\,u_1 \cos 2\,u + \frac{i}{4} \sin 2\,u_1 \sin 2\,u.$$

And if we put for convenience,

$$i \cos 2\,u = 4\,p\,;\; i \sin 2\,u = 4\,q, \tag{7}$$

then the foregoing equation becomes

$$n = \frac{k-c}{k+c} + p \cos 2\,u_1 + q \sin 2\,u_1 \tag{8}$$

where the unknown quantities i and u are replaced by p and q. Again assume

$$\frac{k-c}{k+c} = \frac{1}{590} + r, \tag{9}$$

then we have

$$n = \frac{1}{590} + p \cos 2\,u_1 + q \sin 2\,u_1 + r. \tag{10}$$

We shall suppose every observed latitude to be affected with error and requiring a correction x, and then determine those values of p, q, r, c, which will make the sum of the squares of the quantities x a minimum. We shall omit the influence of the correction x on the *position* of the arc in latitude, that is, on $\varphi_1 + \varphi_2$ and its multiples. Now, if in the expression for $\frac{s}{c}$ we substitute $n + dn$, $\alpha + d\alpha$, we get the following:—

$$\alpha\left(1 + n + \frac{5}{4}n^2\right) - \alpha_1\left(3\,n + 3\,n^2\right) + \alpha_2\left(\frac{15}{8}n^2\right)$$
$$+ \left\{\alpha\left(1 + \frac{5}{2}n\right) - \alpha_1\left(3 + 6\,n\right) + \alpha_2\left(\frac{15}{4}n\right)\right\} dn$$
$$+ \left\{\left(1 + n\right) - 3\,n\,\frac{d\alpha_1}{d\alpha}\right\} d\alpha,$$

where we have omitted $n^2 d\alpha$. Now, the values to be adopted in making this expression equal still to $\frac{s}{c}$, are

$$n = \frac{1}{590}; \quad d\alpha = x_2 - x_1,$$

$$dn = p \cos 2u_1 + q \sin 2u_1 + r;$$

and in computing α, α_1, α_2, the *observed* latitudes, of course, are to be used. Making these substitutions, we find

$$\left\{1 + n - 3n\cos\left(\varphi_2 - \varphi_1\right)\cos\left(\varphi_2 + \varphi_1\right)\right\}\left(x_2 - x_1\right) = \frac{s}{c} - \alpha\left(1 + n + \frac{5}{4}n^2\right) + \alpha_1\left(3n + 3n^2\right) - \alpha_2\left(\frac{15}{8}n^2\right)$$
$$-\left\{\alpha\left(1 + \frac{5}{2}n\right) - \alpha_1\left(3 + 6n\right) + \alpha_2\left(\frac{15}{4}n\right)\right\}\left\{p\cos 2u_1 + q\sin 2u_1 + r\right\}.$$

Divide this through by sin 1″ and make the following substitutions :—

$$\left.\begin{aligned} c\sin 1'' &= \frac{20855500\,.\sin 1''}{1 + \frac{T}{10000}}, \\ p &= 10\,P\sin 1'', \\ q &= 10\,Q\sin 1'', \\ r &= 10\,R\sin 1'', \\ \sigma &= 20855500\sin 1''. \end{aligned}\right\} \quad (11)$$

Also put μ for the coefficient of $x_2 - x_1$, then

$$\left.\begin{aligned} \mu\left(x_2 - x_1\right) = \frac{s}{\sigma} - \alpha''\left(1 + n + \frac{5}{4}n^2\right) + \alpha_1\left(\frac{3n + 3n^2}{\sin 1''}\right) - \alpha_2\left(\frac{15n^2}{\sin 8''}\right) \\ +10\left\{-\alpha\left(1 + \frac{5}{2}n\right) + \alpha_1\left(3 + 6n\right) - \alpha_2\left(\frac{15}{4}n\right)\right\}\left\{P\cos 2u_1 + Q\sin 2u_1 + R\right\} + \left\{\frac{s}{10000\,.\,\sigma}\right\}T. \end{aligned}\right\} \quad (12)$$

This equation establishes the relation between the six quantities x_1, x_2, P, Q, R, T, and by it we obtain expressions for, or values of, the corrections to the successive observed latitudes in an arc in terms of the correction to the lowest latitude, and the unknown elements P, Q, R, T of the earth's ellipsoidal figure. The quantity u_1, it is to be remembered, is the longitude of the arc measured from Greenwich.

4.

Although the English and French arcs are not exactly on the same meridian, they are not so far apart that they may not be used as one arc on a central meridian, of which the longitude is 0° 30′ East. To use them separately when they are fairly connected by geodetical operations, would unnecessarily diminish the accuracy of our results, and also increase slightly the labour. At the extreme northern point of the English arc are three stations, Balta, Gerth of Scaw, and Saxavord, the latter being the most northerly. I have not used the observed latitude of Saxavord, which is 60° 49′ 38″·58, because, if we transfer (by means of the meridian distance) the latitude of Gerth of Scaw to Saxavord, we get for the latter 60° 49′ 36″·59, and the transferred latitude of Balta would make Saxavord 60° 49′ 36″·46. As these three are somewhat different, but of equal weight, the mean has been adopted. The following are the data of our calculation:—

Russian Arc.

Longitude 26° 40′.

Stations.	Latitude.	Distances.	Log a_1	Log a_2
	° ′ ″	Toises.	Neg.	Neg.
Staro-Nekrassowka	45 20 2·8			
Wodolui	47 1 25·2	96415·136	7·0838447	8·7689555
Ssuprunkowzi	48 45 3·1	194973·124	7·6279317	9·0710502
Kremenetz	50 5 50·0	271724·510	7·8954010	9·2109430
Belin	52 2 42·2	382943·521	8·1763510	9·3511421
Nemesch	54 39 5·9	531753·042	8·4483194	9·4775885
Jacobstadt	56 30 4·8	637483·921	8·5990383	9·5416300
Dorpat	58 22 47·6	744764·484	8·7284751	9·5914746
Hogland	60 5 10·1	842303·102	8·8306332	9·6261828
Kilpi-maki	62 38 5·0	988016·669	8·9625738	9·6625428
Tornea	65 49 44·7	1170810·973	9·1017613	9·6856883
Stuor-oivi	68 40 58·4	1334032·877	9·2076566	9·6872028
Fuglenœs	70 40 11·3	1447786·783	9·2732664	9·6777396

Anglo-French Arc.

Longitude 0° 30′.

Stations.	Latitude.	Distances.	Log a_1	Log a_2
	° ′ ″	Feet.	Some neg.	All neg.
Formentera	38 39 56·11			
Montjouy	41 21 44·96	982672·6	7·9110440	8·9462715
Barcelona	41 22 47·90	988703·5	7·9130937	8·9491656
Carcassonne	43 12 54·30	1657290·6	8·0493274	9·1813626
Pantheon	48 50 49·37	3710833·1	7·8848801	9·5398970
Dunkirk	51 2 8·41	4509798·1	7·0480460	9·6216759
Greenwich	51 28 38·30	4671201·5	6·7427432 *n*	9·6359352
Arbury	52 13 26·59	4943840·5	7·5611008 *n*	9·6585725
Clifton	53 27 29·50	5394065·8	7·9759741 *n*	9·6922840
Kellie Law	56 14 53·60	6413222·9	8·4128865 *n*	9·7539547
Stirling	57 27 49·12	6857324·0	8·5366256 *n*	9·7753784
Saxavord	60 49 37·21	8086820·0	8·7938650 *n*	9·8199997

The distances in the French arc have been converted into feet by means of the multiplier 6·39454378, of which the logarithm is 0·8058095651 (see *Account of the Principal Triangulation of the Ordnance Survey of Great Britain and Ireland*, p. 745). And for any other departures from the original data, the reader is referred to the work just mentioned (Sect. xiv.)

Indian Arc.

Longitude 77° 40′.

Stations.	Latitude.	Distances.	Log a_1	Log a_2
	° ′ ″	Feet.		
Punnœ	8 9 31·132			
Putchapolliam	10 59 42·276	1029173·7	8·6697389	8·8896708
Dodagoontah	12 59 52·165	1756560·0	8·8958268	9·0945374
Namthabad	15 5 53·562	2518373·4	9·0453417	9·2176864
Daumergida	18 3 15·292	3591784·3	9·1880276	9·3148577
Takal Khera	21 5 51·532	4697324·1	9·2908171	9·3578701
Kalianpur	24 7 11·262	5794689·0	9·3664043	9·3563449
Kaliana	29 30 48·322	7755827·0	9·4597333	9·2345526

Cape Arc.

Longitude 18° 30′.

Stations.	Latitude.	Distances.	Log a_1	Log a_2
	° ′ ″	Feet.		Neg.
North End	29 44 17·66			
Heerenlogement Berg	31 58 9·11	811506·8	8·2660136	8·6318728
Royal Observatory	33 56 3·20	1526385·1	8·5112185	8·9470981
Zwart Kop	34 13 32·13	1632581·4	8·5358271	8·9817595
Cape Point	34 21 6·26	1678373·8	8·5458793	8·9961414

Peruvian Arc.

Longitude 281° 0′.

Stations.	Latitude.	Distances.	Log a_1	Log a_2
	° ′ ″	Toises.		
Tarqui	−3 4 32·068			
Cotchesqui	0 2 31·387	176875·5	8·7348780	9·0334345

5.

We may now proceed to form the equations of condition between the corrections to these forty observed latitudes. And, first of all, we shall write down the values of the right-hand members of equation (12): putting R_n for $P \cos 2u_n + Q \sin 2u_n + R$: the values of the reciprocals of the quantities μ are also added.

Russian.

				$\frac{1}{\mu}$
Wodolui	+3·665	− 0·3289 R_1	+0·6098 T	·99810
Ssuprunkowzi	5·210	0·7192 R_1	1·2331 T	·99794
Kremenetz	0·336	1·0611 R_1	1·7185 T	·99783
Belin	2·634	1·6137 R_1	2·4219 T	·99766
Nemesch	0·549	2·4591 R_1	3·3630 T	·99744
Jacobstadt	4·918	3·1309 R_1	4·0317 T	·99728
Dorpat	0·959	3·8727 R_1	4·7101 T	·99713
Hogland	1·810	4·5968 R_1	5·3270 T	·99700
Kilpi-maki	1·513	5·7647 R_1	6·2485 T	·99781
Tornea	6·398	7·3664 R_1	7·4046 T	·99659
Stuor-oivi	1·262	8·9172 R_1	8·4369 T	·99641
Fuglenœs	+2·729	−10·0578 R_1	+9·1563 T	·99629

Anglo-French.

				$\frac{1}{\mu}$
Montjouy	+2·134	− 0·2218 R_2	+0·9719 T	·99918
Barcelona	−1·225	0·2237 R_2	0·9778 T	·99918
Carcassonne	−3·155	0·4505 R_2	1·6391 T	·99902
Pantheon	−6·236	1·5316 R_2	3·6701 T	·99852
Dunkirk	−3·551	2·1079 R_2	4·4603 T	·99833
Greenwich	−1·570	2·2346 R_2	4·6199 T	·99830
Arbury	−1·193	2·4570 R_2	4·8896 T	·99823
Clifton	−4·911	2·8462 R_2	5·3348 T	·99813
Kellie Law	−3·578	3·8246 R_2	6·3428 T	·99789
Stirling	−3·216	4·2926 R_2	6·7820 T	·99780
Saxavord	−1·066	− 5·7149 R_2	+7·9980 T	·99753

Indian.

Putchapolliam	−0·743	+ 0·9051 R_3	+1·0179 T	1·00311
Dodagoontah	+4·600	1·5121 R_3	1·7373 T	1·00304
Namthabad	−1·241	2·1146 R_3	2·4907 T	1·00295
Daumergida	+0·502	2·8936 R_3	3·5523 T	1·00280
Takal Khera	+2·879	3·5981 R_3	4·6457 T	1·00263
Kalianpur	−3·168	4·1864 R_3	5·7311 T	1·00244
Kaliana	+1·431	+ 4·9223 R_3	+7·6707 T	1·00205

Cape.

Heerenlogement Berg	+0·294	+ 0·1671 R_4	+0·8026 T	1·00071
Royal Observatory	−0·772	0·2470 R_4	1·5096 T	1·00055
Zwart Kop	+0·813	0·2534 R_4	1·6146 T	1·00053
Cape Point	−0·344	+ 0·2557 R_4	+1·6599 T	1·00052

Peruvian.

Cotchesqui	+0·598	+ 1·0815 R_5	+1·1186 T	1·00339

Substituting in these the particular values of R_n, and multiplying through by $\frac{1}{\mu}$, we obtain the following set of corrections to the observed latitudes :—

Staro Nekrassowka	+0·000	..	..	..	..	$+x_1$
Wodolui	3·658	−0·1960 P	−0·2633 Q	− 0·3283 R	+0·6086 T	$+x_1$
Ssprunkowzi	5·199	0·4286 P	0·5757 Q	0·7177 R	1·2306 T	x_1
Kremenetz	0·336	0·6323 P	0·8493 Q	1·0588 R	1·7148 T	x_1
Belin	2·628	0·9614 P	1·2914 Q	1·6099 R	2·4162 T	x_1
Nemesch	0·548	1·4647 P	1·9675 Q	2·4528 R	3·3544 T	x_1
Jacobstadt	4·905	1·8646 P	2·5046 Q	3·1224 R	4·0208 T	x_1
Dorpat	0·956	2·3060 P	3·0975 Q	3·8616 R	4·6966 T	x_1
Hogland	1·804	2·7368 P	3·6762 Q	4·5830 R	5·3111 T	x_1
Kilpi-maki	1·508	3·4315 P	4·6093 Q	5·7463 R	6·2286 T	x_1
Tornea	6·376	4·3839 P	5·8887 Q	7·3413 R	7·3794 T	x_1
Stuor-oivi	1·257	5·3059 P	7·1271 Q	8·8853 R	8·4067 T	x_1
Fuglenœs	+2·719	−5·9839 P	−8·0378 Q	−10·0206 R	+9·1225 T	$+x_1$

Formentera	+0·000	..	..	..	..	$+x_2$
Montjouy	+2·132	−0·2216 P	−0·0039 Q	− 0·2216 R	+0·9711 T	$+x_2$
Barcelona	−1·224	0·2235 P	0·0039 Q	0·2235 R	0·9770 T	x_2
Carcassonne	−3·152	0·4500 P	0·0079 Q	0·4501 R	1·6375 T	x_2
Pantheon	−6·227	1·5291 P	0·0267 Q	1·5293 R	3·6647 T	x_2
Dunkirk	−3·545	2·1041 P	0·0367 Q	2·1044 R	4·4529 T	x_2
Greenwich	−1·568	2·2305 P	0·0389 Q	2·2308 R	4·6120 T	x_2
Arbury	−1·191	2·4523 P	0·0428 Q	2·4527 R	4·8809 T	x_2
Clifton	−4·902	2·8404 P	0·0496 Q	2·8409 R	5·3248 T	x_2
Kellie Law	−3·570	3·8160 P	0·0666 Q	3·8165 R	6·3294 T	x_2
Stirling	−3·209	4·2825 P	0·0748 Q	4·2831 R	6·7671 T	x_2
Saxavord	−1·063	−5·6999 P	−0·0995 Q	− 5·7008 R	+7·9783 T	$+x_2$

Punnœ	+0·000	..	..	..	..	$+x_3$
Putchapolliam	−0·745	−0·8251 P	+0·3789 Q	+ 0·9079 R	+1·0211 T	$+x_3$
Dodagoontah	+4·614	1·3783 P	0·6330 Q	1·5167 R	1·7426 T	x_3
Namthabad	−1·245	1·9273 P	0·8851 Q	2·1208 R	2·4981 T	x_3
Daumergida	+0·503	2·6369 P	1·2110 Q	2·9017 R	3·5623 T	x_3
Takal Khera	+2·886	3·2784 P	1·5056 Q	3·6076 R	4·6579 T	x_3
Kalianpur	−3·176	3·8137 P	1·7514 Q	4·1966 R	5·7451 T	x_3
Kaliana	+1·434	−4·4824 P	+2·0585 Q	+ 4·9324 R	+7·6865 T	$+x_3$

North End	+0·000	..	..	..	..	$+x_4$
Heerenlogement Berg	+0·295	+0·1335 P	+0·1006 Q	+ 0·1672 R	+0·8032 T	$+x_4$
Royal Observatory	−0·772	0·1974 P	0·1487 Q	0·2471 R	1·5104 T	x_4
Zwart Kop	+0·814	0·2025 P	0·1526 Q	0·2535 R	1·6155 T	x_4
Cape Point	−0·344	+0·2043 P	+0·1540 Q	+ 0·2558 R	+1·6608 T	$+x_4$

Tarqui	+0·000	..	..	..	..	$+x_5$
Cotchesqui	+0·600	−1·0061 P	−0·4065 Q	+ 1·0852 R	+1·1224 T	$+x_5$

6.

We have now to find those values of P, Q, R, T, $x_1 \ldots x_5$ which make the sum of the squares of these corrections a minimum, which will virtually be effected by supposing each = 0, and then solving by the method of least squares. Before proceeding to this, let us as a matter of curiosity determine that ellipsoid which will exactly represent the Russian, Anglo-French, Indian, and Cape arcs, dealing only with the extreme points of those arcs, and supposing the observations free from every source of error. Thus we should arrive at something like General DE SCHUBERT's figure. This will evidently be effected by equating to zero the corrections to the points Fuglenœs, Saxavord, Kaliana, and Cape Point, or making

$$\begin{aligned}
0 &= +2{\cdot}719 - 5{\cdot}9839\,P - 8{\cdot}0378\,Q - 10{\cdot}0206\,R + 9{\cdot}1225\,T \\
0 &= -1{\cdot}063 - 5{\cdot}6999\,P - 0{\cdot}0995\,Q - 5{\cdot}7008\,R + 7{\cdot}9783\,T \\
0 &= +1{\cdot}434 - 4{\cdot}4824\,P + 2{\cdot}0585\,Q + 4{\cdot}9324\,R + 7{\cdot}6865\,T \\
0 &= -0{\cdot}344 + 0{\cdot}2043\,P + 0{\cdot}1540\,Q + 0{\cdot}2558\,R + 1{\cdot}6608\,T
\end{aligned}$$

By the solution of these equations we get

$$\begin{aligned}
P &= +0{\cdot}4404 \\
Q &= +0{\cdot}7145 \\
R &= -0{\cdot}4261 \\
T &= +0{\cdot}1523
\end{aligned}$$

and from these, by the formulæ (7), (9), and (11), pages 30 and 31,

$$\begin{aligned}
p &= +{\cdot}000021351 \\
q &= +{\cdot}000034640 \\
r &= -{\cdot}000020658 \\
i &= +{\cdot}000162766
\end{aligned}$$

$$u = \tfrac{1}{2}\tan^{-1}\frac{q}{p} = 29^\circ\ 10'\ 34''$$

$$\begin{aligned}
c &= 20855182 \\
k &= 20924899 \\
a &= 20926602 \\
b &= 20923196
\end{aligned}$$

$$\frac{c}{a} = \frac{291{\cdot}054}{292{\cdot}054} \qquad \frac{c}{b} = \frac{305{\cdot}578}{306{\cdot}578}$$

These results agree tolerably with those arrived at by General DE SCHUBERT, with the exception of the position of the major-axis of the equator, which differs 12° nearly from what he obtained.

Proceeding now with the general solution by least squares, we get the following system of equations:—

$$\begin{aligned}
0&=+\ 31{\cdot}894-29{\cdot}6956\,P-39{\cdot}8884\,Q-49{\cdot}7280\,R+54{\cdot}4903\,T+13\,x_1\\
0&=-\ 27{\cdot}519-25{\cdot}8499\,P-0{\cdot}4513\,Q-25{\cdot}8537\,R+47{\cdot}5957\,T+12\,x_2\\
0&=+\ 4{\cdot}271-18{\cdot}3421\,P+8{\cdot}4235\,Q+20{\cdot}1837\,R+26{\cdot}9136\,T+8\,x_3\\
0&=-\ 0{\cdot}007+0{\cdot}7377\,P+0{\cdot}5559\,Q+0{\cdot}9236\,R+5{\cdot}5899\,T+5\,x_4\\
0&=+\ 0{\cdot}600-1{\cdot}0061\,P-0{\cdot}4065\,Q+1{\cdot}0852\,R+1{\cdot}1224\,T+2\,x_5\\
0&=-\ 15{\cdot}9575-266{\cdot}2277\,P+129{\cdot}5626\,Q+218{\cdot}5526\,R-436{\cdot}0236\,T+F_1(x_1\,x_2\ldots)\\
0&=-101{\cdot}0313+129{\cdot}5626\,P+220{\cdot}0029\,Q+289{\cdot}4383\,R-222{\cdot}9825\,T+F_2(x_1\,x_2\ldots)\\
0&=-\ 50{\cdot}1338+218{\cdot}5526\,P+289{\cdot}4383\,Q+486{\cdot}2307\,R-377{\cdot}6872\,T+F_3(x_1\,x_2\ldots)\\
0&=+\ 20{\cdot}8518-436{\cdot}0236\,P-222{\cdot}9825\,Q-377{\cdot}6872\,R+742{\cdot}6311\,T+F_4(x_1\,x_2\ldots)
\end{aligned}$$

where the values of $F_1\,F_2\ldots$ are

$$\begin{aligned}
F_1(x_1\,x_2\ldots)&=-29{\cdot}6956\,x_1-25{\cdot}8499\,x_2-18{\cdot}3421\,x_3+0{\cdot}7377\,x_4-1{\cdot}0061\,x_5\\
F_2(x_1\,x_2\ldots)&=-39{\cdot}8884\,x_1-\ 0{\cdot}4513\,x_2+\ 8{\cdot}4235\,x_3+0{\cdot}5559\,x_4-0{\cdot}4065\,x_5\\
F_3(x_1\,x_2\ldots)&=-49{\cdot}7280\,x_1-25{\cdot}8537\,x_2+20{\cdot}1837\,x_3+0{\cdot}9236\,x_4+1{\cdot}0852\,x_5\\
F_4(x_1\,x_2\ldots)&=+54{\cdot}4903\,x_1+47{\cdot}5957\,x_2+26{\cdot}9136\,x_3+5{\cdot}5899\,x_4+1{\cdot}1224\,x_5
\end{aligned}$$

By means of the first five equations we may eliminate $x_1\,x_2\ldots\ldots x_6$ from the remainder, and the result is the following in P, Q, R, T, only:—

$$\begin{aligned}
0&=+\ 7{\cdot}7121+100{\cdot}0410\,P+56{\cdot}5009\,Q+95{\cdot}9532\,R-147{\cdot}5776\,T\\
0&=-\ 8{\cdot}5789+56{\cdot}5009\,P+88{\cdot}5809\,Q+114{\cdot}7492\,R-82{\cdot}7294\,T\\
0&=+\ 1{\cdot}4793+95{\cdot}9532\,P+114{\cdot}7492\,Q+188{\cdot}6264\,R-136{\cdot}2492\,T\\
0&=-18{\cdot}3824-147{\cdot}5776\,P-82{\cdot}7294\,Q-136{\cdot}2492\,R+228{\cdot}0305\,T
\end{aligned}$$

For the purpose of obtaining the probable errors, put A, B, C, D to represent the absolute terms of this equation, and then by elimination we shall find

$$\begin{aligned}
0&=P+{\cdot}263211\,A+{\cdot}024911\,B-{\cdot}034263\,C+{\cdot}158911\,D\\
0&=Q+{\cdot}024911\,A+{\cdot}055623\,B-{\cdot}035693\,C+{\cdot}014975\,D\\
0&=R-{\cdot}034263\,A-{\cdot}035693\,B+{\cdot}033556\,C-{\cdot}015074\,D\\
0&=T+{\cdot}158911\,A+{\cdot}014975\,B-{\cdot}015074\,C+{\cdot}103656\,D
\end{aligned}\qquad(13)$$

which gives us the means of obtaining the "weight" and probable error of any such quantity as $\alpha\,P+\beta\,Q+\gamma\,R+\delta\,T$.

The actual values of P, Q, R, T are found to be

$$\begin{aligned} P &= +\,1{\cdot}1557 \\ Q &= +\,0{\cdot}6132 \\ R &= -\,0{\cdot}3687 \\ T &= +\,0{\cdot}8307 \end{aligned}$$

and by the formulæ (7), (9), (11), pages 30 and 31,

$$\begin{aligned} p &= +\,{\cdot}00005603 \\ q &= +\,{\cdot}00002973 \\ r &= -\,{\cdot}000017875 \\ i &= +\,{\cdot}0002553713 \\ u &= \frac{1}{2}\tan^{-1}\frac{q}{p} = 13^\circ\ 58'\ 30'' \\ c &= 20853768 \\ k &= 20923831 \\ a &= 20926485 \\ b &= 20921177. \end{aligned}$$

7.

The result of our investigation, then, is this: that the ellipsoid which best represents the existing meridian measurements has its major (equatoreal) axis in longitude 13° 58′·5 East from Greenwich; that the greatest and least values of the meridian compression are

$$\frac{a-c}{c} \ldots\ldots \frac{1}{286{,}779} \quad \text{in longitude } 13^\circ\, 58'{\cdot}5 \text{ E.}$$

$$\frac{b-c}{c} \ldots\ldots \frac{1}{309{,}364} \quad \text{in longitude } 103^\circ\, 58'{\cdot}5 \text{ E.}$$

and the length of the polar semiaxis 20853768 feet. The difference of the equatoreal semiaxes is 5308 feet, or, in round numbers, just one mile.

By the substitution of the values of P, Q, R, T we obtain the following set of corrections to the observed latitudes of the stations:—

Corrections to be applied to the Observed Latitudes (pages 32, 33, 34), to bring them into accordance with the Ellipsoidal Figure.

Stations.	Corrections.	Stations.	Corrections.
Staro Nekrassowka	− 2″·824	Punnœ	− 0″·394
Wodolui	+ 1·072	Putchapolliam	− 1·347
Ssuprunkowzi	+ 2·813	Dodagoontah	+ 3·903
Kremenetz	− 1·925	Namthabad	− 2·031
Belin	+ 0·501	Daumergida	− 0·307
Nemesch	− 1·485	Takal Khera	+ 2·166
Jacobstadt	+ 2·881	Kalianpur	− 3·678
Dorpat	− 1·107	Kaliana	+ 1·689
Hogland	− 0·335		
Kilpi-maki	− 0·816		
Tornea	+ 3·711		
Stuor-oivi	− 1·810		
Fuglenœs	− 0·677		
Formentera	+ 0·717	North End	− 1·098
Montjouy	+ 3·479	Heerenlogement Berg	+ 0·019
Barcelona	+ 0·126	Royal Observatory	− 0·387
Carcassonne	− 1·434	Zwart Kop	+ 1·292
Pantheon	− 3·685	Cape Point	+ 0·174
Dunkirk	− 0·808		
Greenwich	+ 1·201		
Arbury	+ 1·624	Tarqui	+ 0·140
Clifton	− 2·029	Cotchesqui	− 0·140
Kellie Law	− 0·639		
Stirling	− 0·287		
Saxavord	+ 1·735		

The sum of the squares of these corrections is 137·2168; therefore the probable error of a latitude is

$$\pm .6745\sqrt{\frac{137{\cdot}2168}{40-9}} = \pm 1''{\cdot}419$$

And the probable error of $\alpha p + \beta q + \gamma r$ (see equations (13)) is

$$= \pm 1{\cdot}419 \sin 10'' (\cdot 2632\, \alpha^2 + \cdot 0498\, \alpha\beta - \cdot 0685\, \alpha\gamma + \cdot 0556\, \beta^2 - \cdot 0714\, \beta\gamma + \cdot 0336\, \gamma^2)^{\frac{1}{2}}$$

Making $\alpha = 1$, $\beta = 0$, $\gamma = 0$, and then $\alpha = 0$, $\beta = 1$, $\gamma = 0$, we get

probable error of p $\pm \cdot 0000353$
" " q $\pm \cdot 0000162$

The expression for the minor semi-axis c involves $2085{\cdot}5$ T, hence we get

probable error of c ± 953 feet

The general expression for n in any longitude u being

$$\frac{1}{590} + p \cos 2u + q \sin 2u + r,$$

we can assign by the preceding formula the probable value of n for any meridian. If r, r', r'' be the radii of the equator in longitudes $u = 0^\circ$, $u = 45^\circ$, $u = 90^\circ$, we find the following results:—

$u = 0^\circ$ $\quad \dfrac{r}{c} = 1 + \dfrac{1}{288{\cdot}005 \pm 5{\cdot}472}$

$13^\circ\ 58'{\cdot}5$ $\quad \dfrac{a}{c} = 1 + \dfrac{1}{286{\cdot}779 \pm 4{\cdot}788}$

45° $\quad \dfrac{r'}{c} = 1 + \dfrac{1}{292{\cdot}451 \pm 1{\cdot}575}$

90° $\quad \dfrac{r''}{c} = 1 + \dfrac{1}{307{\cdot}950 \pm 7{\cdot}912}$

$103^\circ\ 58'{\cdot}5$ $\quad \dfrac{b}{c} = 1 + \dfrac{1}{309{\cdot}364 \pm 7{\cdot}990}$

As to the probable error of the position of the major axis of the equator, we may roughly estimate it from the expression for the longitude, which is

$$\frac{1}{2} \tan^{-1} \frac{q}{p} = \frac{1}{2} \tan^{-1} \left(\frac{297 \pm 162}{560 \pm 353} \right)$$

of which the extreme values are about 32° 38′ and 4° 12′, so that the probable error of the position assigned is not less than $\pm 15°$, and perhaps as much as $\pm 20°$.

8.

From the magnitudes of the probable errors exhibited above, it appears that we are not yet in possession of data sufficient to bring out the excentricity of the equator with anything like certainty. Indeed, this could hardly be expected without using arcs of longitude, and these, unfortunately, are liable to very large errors. On the supposition of an ellipsoidal figure, the parallels are no longer plane curves, and their measurements require special investigation. By means of the equations

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 \quad = 0,$$

$$\frac{x^2}{a^4} + \frac{y^2}{b^4} - \frac{z^2}{c^4} \cot^2 \varphi = 0,$$

$$\frac{x}{a^2} \sin \omega - \frac{y}{b^2} \cos \omega = 0.$$

we easily obtain by elimination the following :—

$$x = \frac{k}{N^{\frac{1}{2}}} \cdot \frac{\cos \omega}{1 - i},$$

$$y = \frac{k}{N^{\frac{1}{2}}} \cdot \frac{\sin \omega}{1 + i},$$

$$z = \frac{k}{N^{\frac{1}{2}}} \cdot \frac{c^2}{k^2} \tan \varphi,$$

where

$$N = 1 + \frac{c^2}{k^2} \tan^2 \varphi + \frac{i \cos 2\omega + i^2}{1 - i^2},$$

and by differentiation

$$dx = -\frac{k}{N^{\frac{1}{2}}} \cdot \frac{\sin\omega}{1-i}\left(\frac{1}{1+i} + \frac{c^2}{k^2}\tan^2\varphi\right) d\omega,$$

$$dy = \frac{k}{N^{\frac{1}{2}}} \cdot \frac{\cos\omega}{1+i}\left(\frac{1}{1-i} + \frac{c^2}{k^2}\tan^2\varphi\right) d\omega,$$

$$dz = \frac{k}{N^{\frac{1}{2}}} \cdot \frac{i\sin 2\omega}{1-i^2} \cdot \frac{c^2}{k^2}\tan\varphi \,.\, d\omega.$$

Expanding these, and neglecting the square and higher powers of i, we find

$$dx = \frac{-k\sin\omega}{\left(1+\frac{c^2}{k^2}\tan^2\varphi\right)^{\frac{1}{2}}}\left\{1 + i\left[\sin^2\varphi - \frac{3}{2}\cos^2\varphi\cos 2\omega\right]\right\} d\omega,$$

$$dy = \frac{k\cos\omega}{\left(1+\frac{c^2}{k^2}\tan^2\varphi\right)^{\frac{1}{2}}}\left\{1 - i\left[\sin^2\varphi + \frac{3}{2}\cos^2\varphi\cos 2\omega\right]\right\} d\omega,$$

$$dz = \frac{k\sin 2\omega}{\left(1+\frac{c^2}{k^2}\tan^2\varphi\right)^{\frac{1}{2}}} \cdot \frac{ic^2}{k^2}\tan\varphi \,.\, d\omega.$$

And therefore

$$ds^2 = \frac{k^2}{1+\frac{c^2}{k^2}\tan^2\varphi}\left\{1 - i\left[2\sin^2\varphi + 3\cos^2\varphi\right]\cos 2\omega\right\} d\omega^2,$$

$$ds = \frac{k}{\left(1+\frac{c^2}{k^2}\tan^2\varphi\right)^{\frac{1}{2}}}\left\{1 - i\,.\left(1+\frac{\cos^2\varphi}{2}\right)\cos 2\omega\right\} d\omega,$$

or finally

$$s = \frac{k}{\left(1+\frac{c^2}{k^2}\tan^2\varphi\right)^{\frac{1}{2}}}\left\{\omega_2 - \omega_1 - i\left(1+\frac{\cos^2\varphi}{2}\right)\sin(\omega_2-\omega_1)\cos(\omega_2+\omega_1)\right\},$$

where ω_1, ω_2 are the extreme astronomical, not geocentric, longitudes of the arc.

Probably, in the course of a few years we may return to the problem, armed with fresh data in the shape of arcs of longitude.

The preceding calculations were made in connexion with the geodetical operations of the Ordnance Survey; but Colonel Sir HENRY JAMES has kindly given me his permission to present them to the Royal Astronomical Society.

9.

If in the four equations (page 38) containing P, Q, R, T, we omit the two first, and make $P = 0$, $Q = 0$, in the two last, we get

$$0 = + \ 1{\cdot}4793 + 188{\cdot}6264\,R - 136{\cdot}2492\,T,$$
$$0 = - \ 18{\cdot}3824 - 136{\cdot}2492\,R + 228{\cdot}0305\,T,$$

and the values of R and T contained in these are evidently those which determine the spheroid of revolution most nearly representing the observations, that is the figure of the earth, supposing the equator a circle. The values of R and T are

$$R = +\,0{\cdot}0886, \qquad T = +\,0{\cdot}1336,$$

and the sum of the squares of the corresponding corrections to the forty observed latitudes is 147·22. On this figure, therefore, the probable error of of a latitude is

$$\pm\,{\cdot}6745\sqrt{\frac{147{\cdot}22}{40-7}} = \pm\,1''{\cdot}425.$$

And if we put A and B for the absolute terms of the above equation in R and T, we get

$$R + {\cdot}0093269\,A + {\cdot}0055729\,B = 0,$$
$$T + {\cdot}0055729\,A + {\cdot}0077152\,B = 0,$$

from whence we have directly the "weights" of any such quantity $\alpha R + \beta T$. The numerical values of R and T, with their weights, give the following elements :—

$$c = 20855221 \pm 261,$$
$$\frac{a-c}{a} = \frac{1}{294{\cdot}754 \pm 1{\cdot}155}.$$

These are almost identical with the values given at page 773 of the "Account of the Principal Triangulation ;" but the latter, involving a rather large number of stations, have a greater weight.

The sum of the squares of the corrections to the forty observed latitudes required by the ellipsoid determined at page 37, is 150·79.

III. *On the Periodical Variations of Level and Azimuth of the Transit Circle at the Royal Observatory, Greenwich.* By Mr. William Ellis, Assistant at the Royal Observatory, Greenwich.

Read November 11, 1859.

All persons accustomed to the use of instruments are aware, that when a transit or similar instrument has been placed exactly in the meridian, its position will usually fluctuate from time to time by small quantities, without in general following any very regular law. For although instances of periodical variation have been noticed (as in the case of the transit instrument at the Edinburgh Observatory, where the changes of level have been found to correspond to changes of the temperature of the rock below) the fluctuations are generally of a more irregular nature, such as in the gradual improvements of instruments, and in particular in an instrument so firmly constructed and mounted as the transit circle at Greenwich, might be expected in great part to disappear; but, instead of this, the firm construction of the Greenwich instrument appears to bring out more fully the fact, that it is subject to periodical variations of considerable extent and well-marked character.

Very soon after the erection of the instrument, its fluctuations began to attract attention, notices of which are to be found from time to time in the Annual Reports of the Astronomer Royal to the Board of Visitors. And the present object is to endeavour to show, from the observations of the eight years ending 1858, the nature and extent of these fluctuations, in the belief that such results may perhaps possess some general interest.

And, first, with respect to the error of level. The amount of this error is obtained by observing with the transit micrometer the coincidence of the central wire of the instrument with the image of the same wire as reflected from a surface of mercury. This is done every evening, and the first of the following tables contains the mean monthly results for the whole period, from 1851 to 1858. The second table contains the mean monthly temperature of the external air at the Royal Observatory, Greenwich, as extracted from the Quarterly Reports of the Registrar General.

Table I. *Mean Values of Error of Level of the Transit Circle, deduced from evening observations only* (*the morning observations of* 1851 *and* 1852 *being omitted*).

The sign + indicates that the east end of the axis is too low.

Year.	Jan.	Feb.	March.	April.	May.	June.
1851	−2″·6	−3″·5	−4″·0	−4″·4	−4″·8	−5″·7
1852	−2·2	−2·5	−2·6	−3·1	−4·0	−4·5
1853	−1·3	−1·1	−1·7	−2·6	−3·3	−3·4
1854	−1·2	−1·6	−1·7	−2·3	−1·7	−2·2
1855	+1·5	+1·6	+1·0	0·0	−0·4	−0·9
1856	+2·1	+1·6	+1·9	+0·2	+0·2	−0·9
1857	+3·0	+2·1	+1·7	+1·0	−0·1	−0·9
1858	+4·4	+4·2	+2·8	+1·9	+2·0	0·0
Mean 1851 to 1858	+0·46	+0·10	−0·33	−1·16	−1·51	−2·31

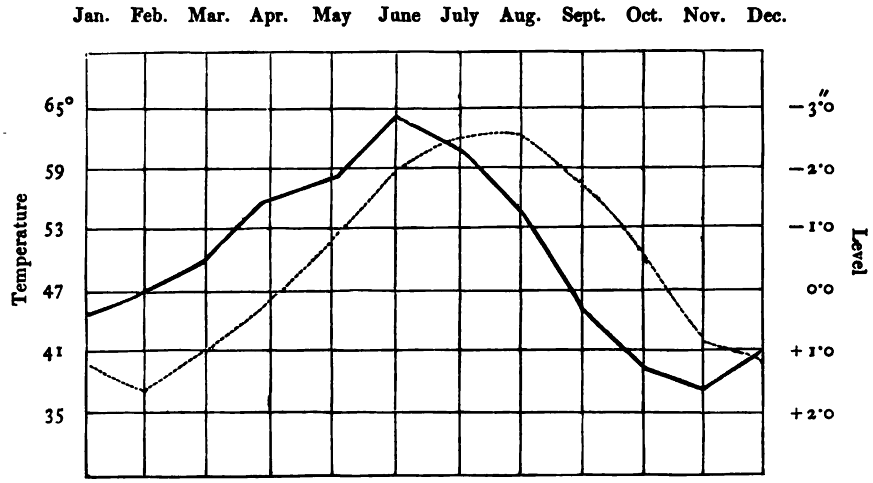
Year.	July.	August.	Sept.	October.	Nov.	Dec.	The Year.
1851	−4″·5	−4″·4	−2″·3	−1″·8	−0″·6	−1″·4	−3″·33
1852	−5·7	−2·4	−0·7	−0·2	−1·3	−1·2	−2·53
1853	−2·9	−1·7	−0·4	+0·2	+0·7	+0·8	−1·39
1854	−2·4	−1·6	−0·4	+1·7	+2·6	+2·0	−0·73
1855	−0·9	+0·1	+2·0	+2·9	+3·8	+2·8	+1·12
1856	−0·6	+0·2	+2·4	+3·1	+4·0	+3·2	+1·45
1857	+0·6	+1·8	+3·6	+4·8	+5·4	+4·7	+2·31
1858	+2·5	+3·5	+4·3	+5·6	+6·0	+5·1	+3·52
Mean 1851 to 1858	−1·74	−0·56	+1·06	+2·04	+2·57	+2·00	+0·05

TABLE II. *Mean Monthly Temperature of the External Air at the Royal Observatory, Greenwich, from the Quarterly Reports of the Registrar General.*

Year.	Jan.	Feb.	March.	April.	May.	June.
1851	43°	40°	43°	45°	51°	59°
1852	42	41	41	46	51	56
1853	42	33	39	45	52	58
1854	39	39	44	48	51	56
1855	35	29	38	46	49	57
1856	39	42	39	47	50	58
1857	37	39	42	46	54	62
1858	38	35	41	46	52	65
Mean 1851 to 1858	39·4	37·2	40·9	46·1	51·2	58·9
Mean 1853 to 1858	38·3	36·2	40·5	46·3	51·3	59·3

Year.	July.	August.	Sept.	October.	Nov.	Dec.	The Year.
1851	60°	62°	57°	53°	38°	40°	49°·3
1852	67	62	57	48	49	48	50·7
1853	60	60	55	51	42	34	47·6
1854	60	61	58	49	40	41	48·8
1855	62	62	57	51	41	36	46·9
1856	61	64	55	52	41	40	49·0
1857	64	66	60	53	46	45	51·2
1858	61	62	60	51	40	41	49·3
Mean 1851 to 1858	61·9	62·4	57·4	51·0	42·1	40·6	49·1
Mean 1853 to 1858	61·3	62·5	57·5	51·2	41·7	39·5	48·8

In order to show better to the eye the general connexion existing between the changes of level and temperature, the following diagram has been constructed from the mean of the eight years' observations, in which the thick line represents the level curve, and the dotted line the curve of temperature. The level curve has been corrected for an annual change of 1″·00, about which mention is made further on.



The rise of the level curve in the preceding diagram represents the relative rise of the east pivot of the instrument, and it appears generally to rise with rising temperature and fall with falling temperature, excepting that the points of maximum and minimum are passed earlier than in the temperature curve.

A more particular examination of the tabular numbers shows that the level error is really subject to three definite changes, firstly, a regular variation with the time, the mean value for 1851 being = −3″·33, and that for 1858 = +3″·52, the change in the intermediate years being proportional; so that from some inequality of wear or other cause, the eastern pivot sinks with great regularity at the rate of about 1″·00 in a year, very nearly. This is the change which has been allowed for in the preceding diagram. Secondly, there is a well-marked annual change depending apparently on temperature, the general effect of which is seen in the above diagram; and thirdly, changes from day to day, which always follow closely the more rapid variations of the temperature of the atmosphere. In these rapid variations, comparisons of suitable observations seem to show that an increase of 10° of mean daily external temperature causes a decrease of level, or rather a relative rise of the east pivot, amounting to about $1''\cdot00$ below 50°, and about $1''\cdot50$ above 50°.

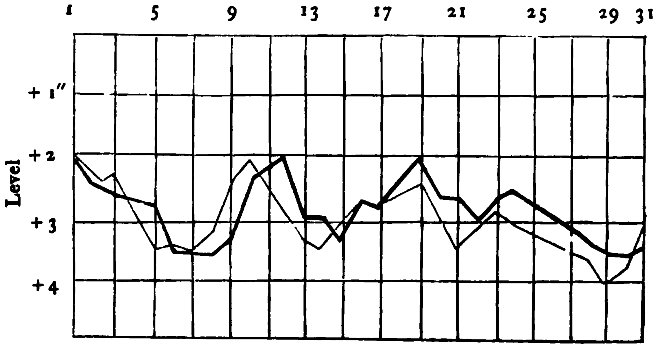
It is possible, therefore, to represent by calculation the error of level, assuming the mean values for some year as epoch, and taking into account the variation of $1''\cdot00$ yearly, and the deviation of the mean daily temperature from the average. Thus, let L = level required, and L_1 = value for the corresponding period of some year to be used as epoch (say 1855). Also let n = integral number of years elapsed, and t = excess of mean daily external temperature above the average. Then

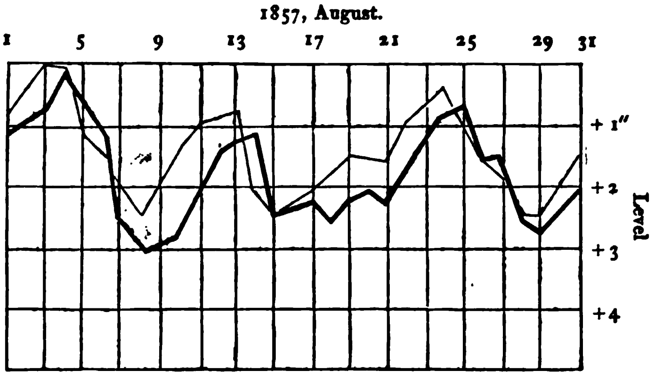
$$\text{from November to April, } L = L_1 \quad + 1''\cdot00 \;.\; n \quad - 0''\cdot10 \;.\; t$$
$$\text{,, May to October, } L = L_1 \quad + 1\cdot00 \;.\; n \quad - 0\cdot15 \;.\; t$$

By applying $+0''\cdot50$ to the means at the foot of Table I. (which are for an imaginary year, 1854–5) the proper numbers for 1855 will be obtained, each number, as an approximation, being supposed to apply to the middle of the month, and proportion being made for intermediate times.

In order to show how nearly the preceding will represent observation, the daily values of level have been computed for the two months, 1857, January and August, in both of which very great changes of temperature occurred. But instead of giving the results in a tabular shape they are given in the shape of curves, which show more clearly the kind of agreement existing between the calculated and observed values.

1857, January.





The thick line represents the observed value, the thin line the calculated value, and the rise of the curve corresponds to relative rise of the east pivot. It appears that the change of calculated level precedes by a short period the change of observed level, but the shape of the curve is in each case well represented.

On first collecting the observations of level it was proposed to compare the results with the indications of the thermometers whose bulbs are sunk in the ground, but the temperature of the external air was finally adopted, because the mean monthly results of external air temperature were, at the time, most easily obtained. The following information respecting the ground thermometers will, however, probably be interesting. The times of annual minima and maxima of temperature for the two thermometers, whose bulbs are sunk to the depths of 3 and 6 French feet respectively, do not follow by very long intervals the times of minimum and maximum of the temperature of the external air. The minimum for the thermometer whose bulb is sunk to the depth of 12 French feet, takes place about the end of March or beginning of April, and the maximum towards the latter part of the month of September. The minimum for the thermometer whose bulb is sunk to the depth of 24 French feet, takes place at the end of May or beginning of June, and the maximum near the end of November. The near coincidence of the times of minimum and maximum for the last-mentioned thermometer, with the times of greatest and least elevation of the east pivot of the transit-circle, may be indicated as a point worthy of further examination.

With respect to the azimuth of the transit circle it appears, that like the level, it undergoes periodical changes. It is known that it has been the practice at Greenwich, whenever double or more observations of *Polaris* have been obtained, to refer the resulting azimuth to the north collimator (one of the two used for determining the position of the line of collimation), mean values of its azimuth so obtained being used, during short periods, to determine, as may be required, the position of the transit circle, though this system of dependence on the stability of the collimator is now nearly given up. It is, therefore, interesting to ascertain the amount of periodic change of the north collimator as well as of the transit circle. The two following tables contain the mean monthly values of azimuth of both of these.

Table III. *Mean Values of West Azimuth of the Transit Circle, telescope pointing North (being the mean for each month of the column "Western azimuth of line of Collimation, &c." in the Table of "Azimuths of the North Collimator," in the Greenwich Observations. The means of the numbers are taken as they stand, which gives proper weight to the good determinations).*

Year.	Jan.	Feb.	March.	April.	May.	June.
	″	″	″	″	″	″
1851	−7·8	− 8·0	−7·1	−7·2	−5·3	−2·9
1852	−6·3	− 7·9	−7·7	−8·0	−7·6	−4·9
1853	−4·3	− 8·8	−9·4	−8·3	−6·3	−4·5
1854	−7·6	−10·4	−7·5	−8·1	−6·4	−5·0
1855	(−7·7)	(− 9·7)	−9·3	−6·2	−7·8	−4·3
1856	−6·2	− 6·5	−7·3	−6·4	−5·5	−2·9
1857	−6·7	− 5·6	−6·6	−5·8	−4·8	−2·3
1858	−3·7	− 5·7	−5·8	−6·1	−4·9	−0·3
Mean 1851 to 1858	−6·29	− 7·82	−7·59	−7·01	−6·08	−3·39
Mean 1853 to 1858	−6·03	− 7·78	−7·65	−6·81	−5·95	−3·22

Table III. (*continued*).

Year.	July.	August.	Sept.	October.	Nov.	Dec.	The Year.
1851	−0″·4	+1″·7	+0″·5	−1″·4	−4″·0	(−5″·7)	−3″·97
1852	−2·2	−0·9	−0·1	−2·2	−4·2	−3·0	−4·58
1853	(−2·3)	−1·3	−1·0	−1·1	−3·9	−7·2	−4·87
1854	−1·8	−2·8	−1·9	−2·8	−3·0	−5·1	−5·20
1855	−2·7	−0·6	−0·6	+0·4	−1·6	−6·3	−4·70
1856	−1·2	+1·0	−1·2	−0·9	−2·7	−3·2	−3·58
1857	+1·3	+3·5	+3·1	+2·3	+0·5	−0·9	−1·83
1858	+0·1	+1·9	+2·1	+0·9	−1·3	−4·1	−2·24
Mean 1851 to 1858	−1·15	+0·31	+0·11	−0·60	−2·53	−4·44	−3·87
Mean 1853 to 1858	−1·10	+0·28	+0·08	−0·20	−2·00	−4·47	−3·74

Table IV. *Mean Values of West Azimuth of North Collimator (being the mean for each month of the column, " Sum of excess in arc and of west azimuth of transit circle, &c." in the Table of "Azimuths of the North Collimator," in the Greenwich Observations).*

(The 1851 and 1852 results are omitted because the collimator was occasionally disturbed.)

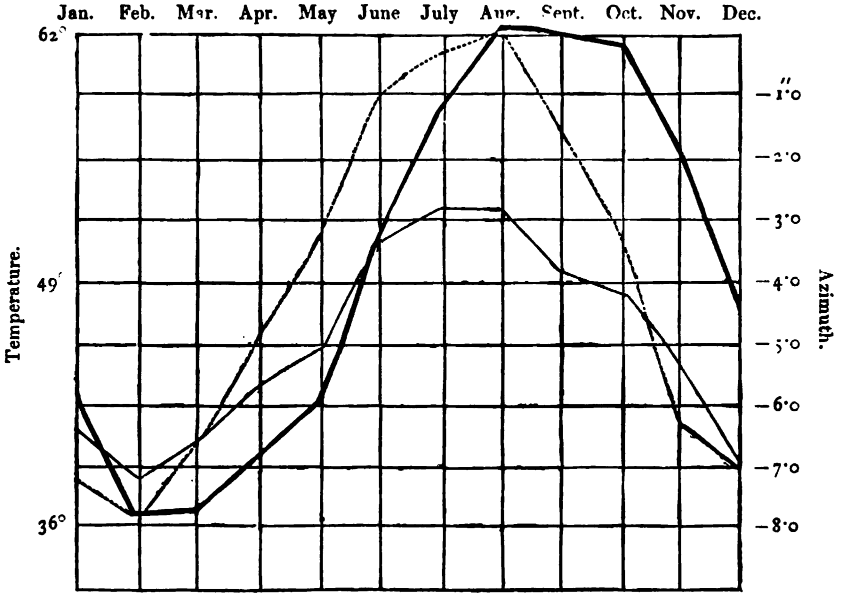
Year.	Jan.	Feb.	March.	April.	May.	June.
1853	−3″·0	−6″·2	−6″·0	−4″·7	−3″·4	−1″·5
1854	−5·2	−6·7	−4·4	−4·8	−3·4	−2·8
1855	(−7·0)	(−8·5)	−7·6	−4·5	−5·5	−2·9
1856	−6·6	−5·6	−6·2	−5·6	−4·7	−3·0
1857	−7·9	−6·9	−7·0	−6·1	−5·5	−4·6
1858	−8·0	−8·8	−8·2	−8·4	−7·8	−5·3
Mean 1853 to 1858	−6·28	−7·11	−6·57	−5·68	−5·05	−3·35

Table IV. (*continued*).

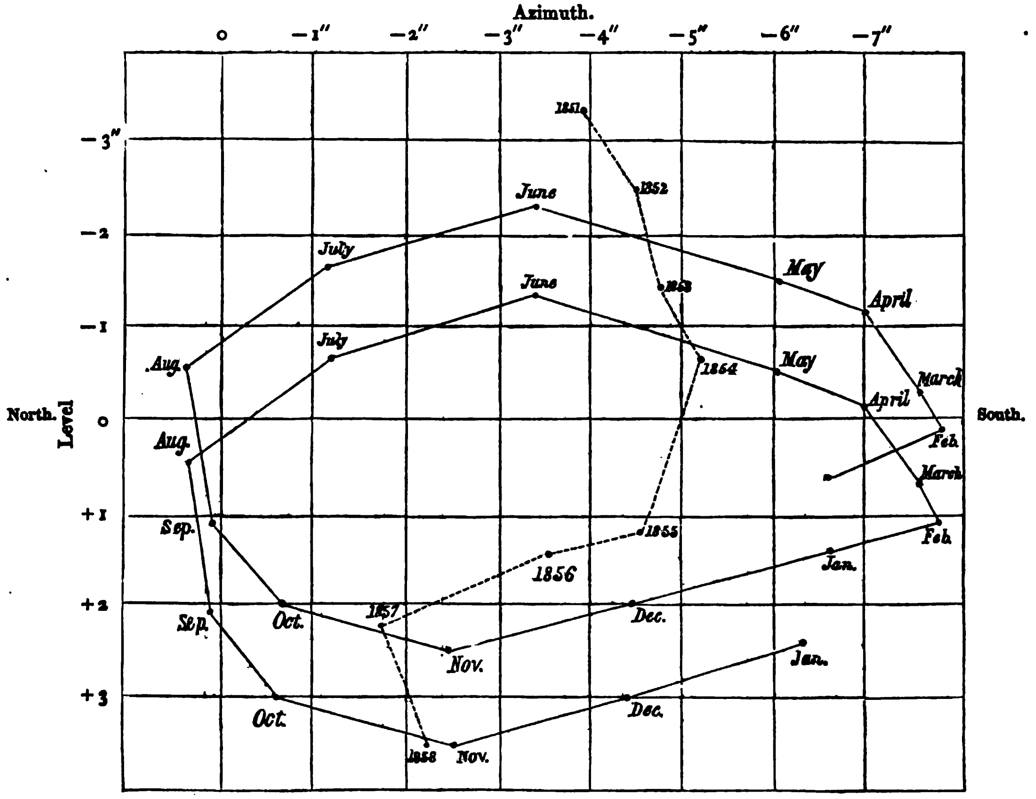
Year.	July.	August.	Sept.	October.	Nov.	Dec.	The Year.
	″	″	″	″	″	″	″
1853	(−1·5)	−1·5	−2·1	−2·0	−4·2	−5·9	−3·50
1854	−1·2	−2·9	−2·8	−3·9	−3·6	−5·1	−3·90
1855	−2·6	−2·2	−3·2	−2·8	−4·4	−7·6	−4·90
1856	−2·5	−1·7	−4·6	−4·4	−6·1	−6·3	−4·77
1857	−3·5	−3·3	−4·2	−5·0	−6·2	−7·1	−5·61
1858	−5·8	−5·8	−6·2	−7·1	−8·1	−9·3	−7·40
Mean 1853 to 1858	−2·85	−2·90	−3·85	−4·20	−5·43	−6·88	−5·01

Where a number in the two preceding tables is placed in brackets, it denotes that the azimuth is obtained from single observations of polar stars, combined, as necessary, with other stars.

The following curves show the relative changes of the azimuth of the transit-circle, and azimuth of the north collimator, and the general connexion with temperature, from the mean of the six years' observations 1853 to 1858. The thick line represents the azimuth of the transit-circle, the thin line the azimuth of the north collimator, and the dotted line the curve of temperature.



The preceding diagram shows that the great fluctuations of azimuth of transit-circle and azimuth of north collimator follow the temperature pretty closely, and that the fluctuation of the north collimator is equal to about half that of the transit-circle. In addition to these well-marked curves, it appears by examination of the observations that the azimuth of the transit-circle is subject to frequent changes of smaller amount, (somewhat similar to the rapid variations of level) accompanied by changes of nearly equal amount in the azimuth of the north collimator, but whether these changes are caused by fault of observation, or are due to some other cause, does not appear certain. They do not seem to depend, as is the case with the quick variations of level, solely on the rapid variations of the temperature of the atmosphere, although the examination of some instances of great change show that there is, to some extent, a similar connexion. These smaller variations of azimuth are the only changes, the law of which does not seem quite clear. They are particularly alluded to in the Report



by the Astronomer Royal to the Board of Visitors for 1857, and also in the reports for succeeding years, but without any certain explanation of their cause.

In order to obtain a more complete idea of the character of the changes produced by the combination of the errors of level and azimuth of the transit-circle, the diagram in the preceding page has been constructed from the mean monthly results of the eight years' observations. The spectator is supposed to be placed between the piers of the instrument and looking towards the east. The curve line shows the average motion of the east pivot (considering the west pivot stationary) continued through a period of two years, the rise of the curve indicating rise of the east pivot, and the approach of the curve to the right, corresponding to southerly motion of the east pivot. The dotted line shows the actual changes of position from year to year, the points in the line marking the mean positions for the respective years.

It will be remarked how small the movement is during the first five months of the year, (January to May) as compared with the movement in other parts of the year. The tendency of the east pivot to the south in cold, seems to explain its extreme southerly position in 1854 and 1855, the period from 1853 to 1855 being a cold period, as shown by the mean yearly temperatures in Table II. And generally it appears, on consideration of the results, that the great disturbing cause is temperature, with which in some way or other the periodical variations of all kinds are probably connected.

In the *Monthly Notices* of the Royal Astronomical Society, vol. viii. p. 134, some results, deduced by Mr. HENRY, for TROUGHTON'S transit (the instrument in use until the erection of the transit-circle) are to be found, to which it may be curious to allude. It there appears that in March (from ten years' observations) the east pivot was lower than in September by 2″·55, and more to the north in March (from five years' observations) than in September by 1″·80, but no results for other months are given. Those for March and September differ in direction from what has been found for the transit-circle. The position of the old transit was a little westward of that of the present transit-circle, it having occupied the adjoining apartment.

Greenwich, 1859, November 7.

ADDENDUM.

In consequence of the changes of level and azimuth between March and September being so different for TROUGHTON'S transit from those found for the transit-circle (see last paragraph of preceding paper), all the results for level and azimuth of TROUGHTON'S transit contained in the section of transits of the *Greenwich Observations* from 1836 to 1850 have since been collected, and the means for each separate month found, thence obtaining the mean monthly values for the whole period 1836 to 1850. The illuminated end of the axis was east during the years 1836, 1838, 1840, &c., and west during the years 1837, 1839, 1841, &c. As the periodic change in each separate year is, on the whole, nearly similar, it will be sufficient to give here the mean monthly values for the whole period only, the mean values as found for the transit-circle for the period 1851 to 1858 being added, for facility of comparison.

Month.	Troughton's Transit.		Transit-Circle.		Month.	Troughton's Transit.		Transit-Circle.	
	Level.	Azimuth.	Level.	Azimuth.		Level.	Azimuth.	Level.	Azimuth.
	″	″	″	″		″	″	″	″
January..	+1·3	+3·4	+0·5	−6·3	July	+0·4	+2·4	−1·7	−1·1
February.	+1·9	+4·4	+0·1	−7·8	August ..	−0·6	+1·5	−0·6	+0·3
March....	+2·1	+4·4	−0·3	−7·6	September	−0·7	+1·4	+1·1	+0·1
April	+2·0	+3·9	−1·2	−7·0	October ..	−0·3	+0·8	+2·0	−0·6
May	+2·2	+4·0	−1·5	−6·1	November	+0·4	+2·0	+2·6	−2·5
June.....	+1·6	+4·4	−2·3	−3·4	December	+0·7	+3·1	+2·0	−4·4

The numbers in this table show how widely different are the changes of the two instruments during the course of the year. As respects the transit-circle, it has been imagined, from the similarity of the changes of the azimuth of the instrument itself and of its collimators, that real motion of the ground exists; but if such be the true explanation of these particular variations, it is then a question how the same cause can apply to explain the variations in opposite direction shown by TROUGHTON'S transit, which occupied a position so near to that of the present transit-circle. This want of similarity would seem to indicate that the changes in each instrument are peculiar to itself. But whether produced by motion of the ground or no, there is evidently a connexion existing directly or indirectly with the changes of temperature.

Greenwich, 1860, *June* 27.

IV. *Observations of* DONATI'S *Comet, made at the Royal Observatory, Cape of Good Hope, between October* 11*th*, 1858, *and March* 4*th*, 1859. By Sir THOMAS MACLEAR, Director of the Royal Observatory, Cape of Good Hope.

Read March 11, 1860.

THE clouded state of the sky prevented the detection of this splendid Comet at the Cape Observatory before the 10th of October. On the evening of that day there was a complete view of it for a few minutes, afterwards the tail only could be seen through occasional breaks in the clouds. Rain fell on the forenoon of the 11th, after which the clouds dispersed, and as the evening twilight closed in, there is no exaggeration in the statement that the westward view from the observatory was magnificent beyond description. *Venus* in full splendour, the young moon, and the comet, presented an imposing display of celestial objects rarely witnessed, while the feelings were rendered more sensitive to the sublime and the beautiful by one of the serene evenings so delicious in the Cape climate when the atmosphere is calm and moist.

Observations for orbit being the primary consideration, and the time short for making them during the first few evenings of the appearance, the 8½-feet equatoreal could not be spared for elaborate physical examination. This, however, is of little importance, since more powerful instruments were available in Europe and America; and the decision has been verified by the published observations, up to the 20th of October, which have reached the Cape, and which are in accordance with our own so far as the latter were carried. After that date there was little to record beyond the gradual fading away of the image.

Two series of observations for orbit were made, viz., one by Mr. MANN, with the 8½-feet equatoreal, which comprehends the whole interval of visibility, commencing on the 11th of October and ending on the 4th of March following. The other with the 46-inch equatoreal by myself and

G. W. H. Maclear, conjointly, commencing on the 16th of October and ending on the 15th of November, when the comet became too faint to be observed with that instrument armed with a spider-line micrometer.

From the commencement of the observations with the 8½-feet equatoreal to Nov. 16th, the measures were taken with a position micrometer fitted with flat wires 10″ in breadth—the spider-line micrometer belonging to the instrument being then in Mr. Simms' hands for the execution of certain alterations. On November 17th it was received from England, and employed in observation up to December 29th, when the flat wire was again substituted, as the comet then had become too faint to bear the slightest illumination of the wires.

The powers generally employed were 126 with the flat-wire micrometer, and 161 with the spider-line micrometer.

The differences in R.A. were obtained by direct micrometer measurement on the parallel, using the "driving clock;" except on November 18th and 22d and on December 11th and 14th, when the transits of both objects were observed with the system of fixed lines in the spider-line micrometer.

The observations with the 46-inch equatoreal were made by means of a spider-line micrometer, having one line fixed and two moveable; and the differences in R.A. were measured in time by transits over the fixed wire—the differences in polar distance by the micrometer screws at the instants of transiting the fixed wire.

The observations are of inferior value compared with those made with the more powerful instrument, and the facilities afforded by its superior mounting; however, they are useful as vouchers for the interval they embrace, as the same stars were seldom used with both instruments.

Of the 114 stars of comparison several are of very small magnitude—a matter of no moment with respect to relative accuracy of position, as the whole list has been observed with the large transit-circle, each star at least three times.

As the headings of the several columns are sufficiently explicit, and the observations are given in the usual form, no further explanation is needed, other than that the corrections for refraction have been applied, and that the columns headed log $\frac{p}{P}$ and log $\frac{q}{P}$ contain the logarithms of the parallax factors in right ascension and polar distance respectively.

TABLE I.

Right Ascensions and North Polar Distances of DONATI'S *Comet, observed at the Royal Observatory, Cape of Good Hope, with the* 8½*-feet Equatoreal.*

Date. 1858.	Star of Comp.	Cape Mean Time.	Diff. of R.A. Comet—Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Measures.	Diff. of N.P.D. Comet—Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Oct.		h m s	m s	h m s	+		′ ″	° ′ ″	+		
11	1	7 39 10·1	+1 33·68	15 22 3·90	8·7411	4	—11 9·37	89 55 58·90	9·7444	4	W.M.
12	2	7 20 11·7	..	..	..	..	— 7 43·64	93 24 26·57	9·7353	6	
	2	7 34 0·0	—0 13·48	15 33 35·19	8·7388	6					
14	3	7 25 43·0	..	..	..	..	— 4 48·51	100 9 20·39	9·7075	6	
		7 35 57·3	+0 30·60	15 55 33·56	8·7407	6					
		7 49 35·7	..	..	..	..	— 1 34·86	100 12 34·04	9·7198	6	
		7 59 6·9	+0 40·66	15 55 43·62	8·7477	6					
15	4	8 30 34·5	..	..	..	..	—12 24·92	103 25 29·21	9·7326	4	
		8 40 23·0	+0 2·82	16 6 16·84	8·7549	3					
16	5	7 27 35·2	..	..	..	..	— 8 29·01	106 13 56·73	9·6674	6	
		7 43 51·1	—0 19·07	16 15 42·21	8·7498	8					
		7 59 7·8	..	..	..	..	— 4 45·01	106 17 40·73	9·6952	6	
		8 11 49·6	—0 7·99	16 15 53·29	8·7587	6					
18	6	7 21 4·6	..	..	..	..	— 0 45·79	111 29 9·98	9·6089	6	
		7 33 34·8	—0 45·76	16 33 48·81	8·7521	6					
		7 44 4·4	..	..	..	..	+ 1 33·86	111 31 29·63	9·6384	6	
		7 56 7·4	—0 37·59	16 33 56·98	8·7643	6					
		8 12 0·1	..	..	..	..	+ 4 22·19	111 34 17·96	9·6729	6	
		8 23 47·1	—0 27·79	16 34 6·78	8·7728	6					
21	11	7 28 39·4	..	..	..	..	— 9 39·14	117 54 28·35	9·5349	6	
	11	7 40 38·1	—0 6·36	16 57 40·64	8·7718	6					

TABLE I. *Right Ascensions and North Polar Distances of* DONATI'S *Comet (continued).*

Date. 1858.	Star of Comp.	Cape Mean Time.	Diff. of R.A. Comet – Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Measures.	Diff. of N.P.D. Comet – Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Oct.		h m s	m s	h m s	+		′ ″	° ′ ″			
21	11	7 54 50·7	..	..	..	..	− 7 36·61	117 56 30·88	9·5846	5	W. M.
	12	8 6 47·7	..	..	..	..	+ 1 18·71	117 57 25·84	9·6062	6	
	11	8 23 36·4	+0 6·54	16 57 53·54	8·7923	6					
	12	8 34 9·4	−0 44·52	16 57 56·84	8·7949	6					
	12	8 53 7·5	..	..	..	..	+ 4 52·09	118 0 59·22	9·6825	6	
	12	9 1 57·3	−0 36·01	16 58 5·35	8·7974	4					
22	15	7 20 40·4	..	..	..	..	− 0 36·48	119 40 51·81	9·4879	6	
	14	7 41 52·2	..	..	..	..	−10 27·62	119 42 20·52	9·5347	6	
	15	7 53 55·4	−0 49·32	17 4 50·56	8·7855	6					
	14	8 3 1·7	−0 19·59	17 4 53·22	8·7905	6					
	14	8 16 39·6	..	..	..	..	− 7 59·99	119 44 48·15	9·6044	4	
	15	8 24 13·3	..	..	..	..	+ 3 53·01	119 45 21·30	9·6186	4	
	14	8 34 58·3	−0 10·50	17 5 2·31	8·8019	6					
	15	8 43 8·7	−0 35·39	17 5 4·49	8·8034	6					
23	20	7 27 37·4	..	..	..	..	− 7 9·80	121 19 0·24	9·4744	6	
		8 1 5·7	−0 42·24	17 11 35·06	8·7952	6					
		8 16 9·7	..	..	..	..	− 4 1·55	121 22 8·49	9·5847	7	
		8 30 40·7	−0 34·10	17 11 43·20	8·8074	6					
		8 42 39·7	..	..	..	..	− 2 20·16	121 23 49·88	9·6369	6	
		8 56 3·5	−0 27·23	17 11 50·07	8·8118	8					
24	22	7 33 41·8	+0 29·41	17 17 49·50	8·7823	2					
		7 53 0·3	..	..	..	..	− 0 31·36	122 49 45·19	9·5114	6	
		8 4 49·5	+0 37·36	17 17 57·45	8·8030	6					
		8 14 36·8	..	..	..	..	+ 0 45·46	122 51 2·01	9·5626	2	
		8 31 29·6	+0 44·11	17 18 4·20	8·8140	6					
		8 50 39·3	..	..	..	..	+ 2 52·07	122 53 8·62	9·6385	6	
25	24	7 33 42·5	+0 48·00	17 23 50·98	8·7873	6					
		7 53 21·3	..	..	..	..	+ 1 2·71	124 11 17·30	9·4897	6	
		8 4 45·1	+0 55·47	17 23 58·45	8·8087	6					
		8 17 44·1	..	..	..	..	+ 2 21·75	124 12 36·34	9·5520	6	
	25	8 29 36·1	..	..	..	..	− 3 20·42	124 13 13·69	9·5798	6	
		8 43 5·6	+0 10·16	17 24 7·66	8·8232	6					

TABLE I. *Right Ascensions and North Polar Distances of* DONATI'S *Comet* (*continued*).

Date. 1858.	Star of Comp.	Cape Mean Time.	Diff. of R.A. Comet—Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Measures.	Diff. of N.P.D. Comet—Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Oct.		h m s	m s	h m s	+		′ ″	° ′ ″	+		
26	26	7 53 50·1	+0 19·29	17 29 38·36	8·8076	8					W. M.
		8 8 2·2	..	..	..	..	+4 39·05	125 26 40·75	9·5093	6	
		8 19 55·7	+0 25·36	17 29 44·43	8·8218	6					
		8 31 45·2	..	..	..	..	+5 49·57	125 27 51·27	9·5699	6	
27	29	7 48 34·5	+0 32·96	17 35 2·29	8·8092	5					
		8 3 17·8	+0 36·10	17 35 5·43	8·8189	6					
		8 16 38·1	..	..	..	..	−6 47·18	126 35 28·31	9·5155	6	
		8 30 5·6	+0 41·98	17 35 11·31	8·8314	6					
		8 53 39·1	..	..	..	..	−5 8·15	126 37 7·34	9·6089	6	
28	30	7 55 10·2	..	..	..	..	+8 25·97	127 37 28·51	9·4330	6	
	31	8 11 35·3	..	..	..	..	−7 51·30	127 38 6·17	9·4849	5	
	30	8 41 11·0	+0 4·71	17 40 22·60	8·8405	6					
	31	8 54 10·7	−1 9·92	17 40 25·37	8·8433	8					
	31	9 5 12·5	..	..	..	..	−5 36·18	127 40 21·29	9·6254	5	
29	32	7 37 36·6	+0 24·51	17 45 3·81	8·8107	6					
	32	7 49 17·8	..	..	..	..	−0 11·43	128 35 11·04	9·3926	6	
	32	8 4 50·1	+0 29·96	17 45 9·26	8·8303	6					
	32	8 16 47·6	..	..	..	..	+0 51·66	128 36 14·13	9·4856	6	
	32	8 27 9·1	+0 34·44	17 45 13·74	8·8413	6					
	33	8 45 49·3	..	..	..	..	−1 38·59	128 37 19·89	9·5684	6	
	33	9 1 29·1	−0 39·43	17 45 20·42	8·8499	8					
30	35	8 18 3·9	..	..	..	..	−9 37·44	129 29 39·44	9·4761	6	
		8 33 28·9	−0 46·74	17 49 55·19	8·8489	6					
		8 45 4·1	..	..	..	..	−8 39·50	129 30 37·38	9·5565	6	
		8 57 13·1	−0 42·18	17 49 59·75	8·8547	6					
31	37	7 56 44·1	..	..	..	..	−8 44·60	130 18 19·62	9·3856	6	
		8 10 20·5	−0 55·25	17 54 18·73	8·8434	6					
		8 21 51·9	..	..	..	..	−7 54·33	130 19 9·89	9·4761	6	
		8 33 50·8	−0 50·99	17 54 22·99	8·8540	6					
		8 46 45·1	..	..	..	..	−7 6·92	130 19 57·30	9·5521	6	
		8 57 0·8	−0 46·80	17 54 27·18	8·8598	6					

TABLE I. ***Right Ascensions and North Polar Distances of*** **DONATI'S** ***Comet*** (*continued*).

Date. 1858.	Star of Comp.	Cape Mean Time.	Diff. of R.A. Comet—Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Measures.	Diff. of N.P.D. Domet—Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Nov.		h m s	m s	h m s	+		′ ″	° ′ ″	+		
3	40	7 43 59·0	−0 36·25	18 6 32·31	8·8395	8					W. M.
	40	7 58 23·5	..	..	..	..	+10 34·70	132 26 16·54	9·3508	6	
	39	8 15 20·3	..	..	..	..	− 4 19·50	132 26 42·43	9·4213	6	
	39	8 29 57·1	−0 24·90	18 6 39·47	8·8666	6					
	39	8 40 13·5	..	..	..	..	− 3 39·76	132 27 22·17	9·5096	6	
4	42	7 49 12·3	..	..	..	..	+ 0 41·60	133 2 54·09	9·2949	6	
		8 0 18·6	−0 34·35	18 10 21·54	8·8557	6					
		8 10 27·3	..	..	..	..	+ 1 11·85	133 3 24·34	9·3923	6	
		8 21 59·6	−0 30·97	18 10 24·92	8·8677	8					
6	45	7 50 36·3	..	..	..	..	− 5 15·29	134 9 40·77	9·2797	6	
		8 2 14·4	−0 44·79	18 17 30·86	8·8654	6					
		8 12 33·3	..	..	..	..	− 4 47·11	134 10 8·95	9·3857	6	
		8 24 19·0	−0 41·70	18 17 33·95	8·8770	8					
7	47	8 30 35·3	−0 44·00	18 20 58·31	8·8835	10					
		8 57 59·6	..	..	..	..	− 0 5·89	134 41 15·62	9·5482	12	
		9 20 49·3	−0 36·98	18 21 5·33	8·8913	10					
9	48	9 7 28·6	−0 24·23	18 27 29·66	8·8983	6					
		9 19 23·1	..	..	..	..	+ 1 45·17	135 36 41·37	9·6080	10	
		9 32 39·2	−0 19·83	18 27 34·06	8·8969	6					
11	49	7 51 34·9	+0 11·82	18 33 26·65	8·8781	6					
		8 0 37·2	..	..	..	..	+ 5 45·52	136 24 22·04	9·3031	6	
		8 13 3·0	+0 14·46	18 33 29·29	8·8907	6					
		8 24 47·5	..	..	..	..	+ 6 7·63	136 24 44·15	9·4189	6	
		8 37 58·0	+0 17·48	18 33 32·31	8·9003	8					
12	50	7 55 17·5	+0 34·32	18 36 22·65	8·8842	6					
		8 5 44·3	..	..	..	..	+ 3 5·85	136 46 59·97	9·3277	6	
		8 22 58·8	+0 37·55	18 36 25·88	8·8986	6					
		8 49 24·3	..	..	..	..	+ 3 44·68	136 47 38·80	9·5129	6	
14	52	8 11 33·8	..	..	..	..	+ 2 5·31	137 28 39·46	9·3554	10	
		8 42 58·4	−1 21·12	18 42 5·90	8·9109	10					
		8 58 44·6	−1 19·27	18 42 7·75	8·9132	10					
		9 12 42·6	..	..	..	..	+ 2 55·97	137 29 30·12	9·5894	10	

TABLE I. *Right Ascensions and North Polar Distances of* DONATI'S *Comet* (*continued*).

Date. 1858.	Star of Comp.	Cape Mean Time.	Diff. of R.A. Comet—Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Measures.	Diff. of N.P.D. Comet—Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Nov.		h m s	m s	h m s	+		′ ″	° ′ ″	+		
15	53	7 55 16·9	..	..	..	..	+0 14·28	137 47 42·72	9·2667	10	W. M.
		8 10 5·1	+0 13·72	18 44 45·52	8·9029	10					
		8 34 7·4	..	..	..	..	+0 44·23	137 48 12·67	9·4568	10	
		8 50 3·4	+0 18·06	18 44 49·86	8·9150	9					
16	55	8 43 26·5	..	..	..	..	−2 57·50	138 6 37·31	9·4943	6	
		8 55 44·2	−0 46·80	18 47 30·02	8·9184	10					
		9 8 19·0	..	..	..	..	−2 39·71	138 6 55·10	9·5788	6	
17	56	8 19 13·6	+0 4·93	18 50 2·79	8·9128	10					
		8 38 14·3	..	..	..	..	−4 31·11	138 24 1·09	9·4759	10	
		9 12 2·0	+0 10·53	18 50 8·39	8·9209	9					
18	57	8 42 52·1	−1 20·42	18 52 38·49	8·9223	18					
		9 5 46·5	..	..	..	..	+4 30·91	138 40 59·35	9·5812	8	
19	58	8 25 38·1	+0 54·20	18 55 7·54	8·9206	10					
		8 40 16·0	..	..	..	..	+5 15·04	138 56 39·01	9·4884	6	
		8 52 11·0	+0 57·02	18 55 10·36	8·9255	6					
		9 4 19·9	..	..	..	..	+5 29·69	138 56 53·66	9·5724	6	
		9 17 42·0	+0 59·65	18 55 12·99	8·9241	3					
20	59	8 14 8·8	+0 49·78	18 57 34·48	8·9194	6					
		8 24 30·3	..	..	..	..	−2 47·27	139 11 44·25	9·4263	6	
		8 35 0·1	+0 51·79	18 57 36·49	8·9256	6					
		8 44 34·6	..	..	..	..	−2 35·85	139 11 55·67	9·5074	6	
		8 56 46·3	+0 54·21	18 57 38·91	8·9281	6					
21	60	8 31 43·6	+0 36·42	19 0 2·12	8·9273	10					
		8 42 51·3	..	..	..	..	−5 40·63	139 26 29·92	9·5041	6	
		8 53 12·0	+0 38·47	19 0 4·17	8·9301	10					
		9 4 1·1	..	..	..	..	−5 28·10	139 26 42·45	9·5769	6	
		9 17 14·2	+0 40·88	19 0 6·58	8·9287	10					
22	61	9 10 21·9	−1 26·60	19 2 29·07	8·9315	15					
		9 24 25·6	..	..	..	..	−5 43·92	139 40 50·06	9·6398	6	
		9 35 11·2	−1 23·97	19 2 31·70	8·9261	10					

TABLE I. *Right Ascensions and North Polar Distances of* DONATI'S *Comet* (*continued*).

Date. 1858.	Star of Comp.	Cape Mean Time.	Diff. of R.A. Comet—Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Measures.	Diff. of N.P.D. Comet—Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Nov.		h m s	m s	h m s	+		′ ″	° ′ ″	+		
26	62	8 33 49·9	−0 34·22	19 11 40·79	8·9387	10					W. M.
		8 45 49·5	..	..	..	..	+0 52·97	140 31 22·30	9·5332	10	
		9 0 43·9	−0 31·77	19 11 43·24	8·9395	10					
		9 14 10·8	..	..	..	..	+1 4·80	140 31 34·13	9·6239	10	
27	63	8 26 35·9	−0 42·14	19 13 54·83	8·9398	10					
		8 44 5·4	..	..	..	..	−4 14·58	140 42 51·93	9·5311	10	
		9 0 26·8	−0 39·18	19 13 57·79	8·9412	10					
29	64	8 33 30·9	−0 53·38	19 18 21·00	8·9446	10					
		8 53 14·2	..	..	..	..	+1 43·10	141 4 56·74	9·5711	10	
		9 7 55·1	−0 50·40	19 18 23·98	8·9430	10					
Dec. 2	65	8 43 12·3	+1 13·79	19 24 51·81	8·9499	10					
		8 59 35·3	..	..	..	..	+0 30·74	141 35 25·39	9·6041	7	
		9 15 58·4	+1 16·61	19 24 54·63	8·9445	10					
3	66	8 42 56·8	+0 2·62	19 26 59·25	8·9514	5					
		8 49 47·7	..	..	..	..	−0 22·13	141 44 52·89	9·5782	6	
		8 58 29·3	+0 4·02	19 27 0·65	8·9497	6					
		9 9 7·8	..	..	..	..	−0 12·77	141 45 2·25	9·6365	5	
		9 20 48·2	+0 5·93	19 27 2·56	8·9438	6					
		9 32 54·4	..	..	..	..	−0 3·51	141 45 11·51	9·6985	7	
4	67	8 36 34·7	−0 41·93	19 29 5·44	8·9530	5					
		8 57 49·0	..	..	..	..	+2 2·81	141 54 10·96	9·6076	8	
		9 8 29·5	−0 38·90	19 29 8·47	8·9486	8					
		9 20 18·5	..	..	..	..	+2 11·49	141 54 19·64	9·6708	8	
5	68	8 35 4·9	+0 32·94	19 31 11·98	8·9544	10					
		8 51 56·3	..	..	..	..	−2 39·89	142 3 10·59	9·5938	10	
		9 5 59·3	+0 35·78	19 31 14·82	8·9502	10					
		9 19 46·5	..	..	..	..	−2 31·60	142 3 18·88	9·6734	10	
6	69	8 21 16·3	+0 11·70	19 33 16·01	8·9554	6					
	70	8 27 25·8	−0 2·59	19 33 16·88	8·9558	6					
	69	8 34 28·1	..	..	..	..	+3 37·02	142 11 52·50	9·5402	6	
	70	8 40 45·1	..	..	..	..	−4 39·27	142 11 51·72	9·5621	6	

TABLE I. *Right Ascensions and North Polar Distances of* DONATI'S *Comet* (*continued*).

Date. 1858.	Star of Comp.	Cape Mean Time.	Diff. of R.A. Comet—Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Measures.	Diff. of N.P.D. Comet—Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Dec.		h m s	m s	h m s	+		′ ″	° ′ ″	+		
6	69	8 51 41·3	+0 14·20	19 33 18·51	8·9543	6					W. M.
	70	8 56 53·3	0 0·00	19 33 19·47	8·9533	1					
	69	9 5 45·8	..	..	..	..	+ 3 46·01	142 12 1·49	9·6397	6	
	70	9 13 40·3	..	..	..	..	− 4 24·64	142 12 6·35	9·6614	6	
	69	9 26 28·8	+0 17·57	19 33 21·88	8·9436	6					
	70	9 35 41·6	+0 3·27	19 33 22·74	8·9390	6					
8	71	8 28 46·6	−0 49·47	19 37 25·34	8·9586	6					
		8 45 18·3	..	..	..	..	+ 3 21·45	142 28 49·16	9·5872	10	
		9 3 26·7	−0 46·84	19 37 27·97	8·9536	10					
10	72	8 37 34·5	−0 33·20	19 41 32·25	8·9506	6					
		8 47 17·0	..	..	..	..	+ 4 32·48	142 44 57·79	9·6034	6	
		8 55 37·3	−0 31·50	19 41 33·95	8·9573	6					
		9 4 46·1	..	..	..	..	+ 4 38·80	142 45 4·11	9·6544	6	
		9 14 35·3	−0 29·84	19 41 35·61	8·9509	6					
		9 22 42·8	..	..	..	..	+ 4 43·38	142 45 8·69	9·7006	6	
		9 30 26·6	−0 28·69	19 41 36·76	8·9431	6					
11	72	8 43 15·8	+1 30·30	19 43 35·74	8·9609	20					
		9 5 39·8	..	..	..	..	+12 26·13	142 52 51·28	9·6613	10	
		9 26 49·4	+1 33·74	19 43 39·18	8·9454	18					
12	73	8 25 8·7	−0 4·44	19 45 35·85	8·9638	6					
		8 33 24·4	..	..	..	..	− 4 46·02	143 0 13·01	9·5688	6	
		8 42 56·7	−0 2·91	19 45 37·38	8·9618	8					
		8 56 16·0	..	..	..	..	− 4 36·51	143 0 22·52	9·6398	10	
14	75	8 41 41·6	−1 7·76	19 49 39·58	8·9638	15					
	74	8 56 25·1	−0 56·24	19 49 40·52	8·9599	12					
	74	9 9 0·7	..	..	..	..	− 6 42·97	143 15 12·92	9·6831	7	
	75	9 22 29·9	..	..	..	..	+ 2 33·75	143 15 18·40	9·7160	7	
	74	9 38 11·5	−0 52·45	19 49 44·31	8·9385	12					
	75	9 47 44·1	−1 2·18	19 49 45·16	8·9313	12					
19	76	8 59 25·7	−0 38·65	19 59 40·45	8·9609	6					
		9 10 1·5	..	..	..	..	+ 4 52·96	143 50 0·11	9·7068	7	
		9 20 3·7	−0 37·09	19 59 42·01	8·9500	6					

TABLE I. *Right Ascensions and North Polar Distances of* DONATI'S *Comet* (*continued*).

Date. 1858.	Star of Comp.	Cape Mean Time.	Diff. of R.A. Comet – Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Measures.	Diff. of N.P.D. Comet – Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Dec.		h m s	m s	h m s	+		′ ″	° ′ ″	+		
21	77	8 35 43·9	+0 59·50	20 3 36·71	8·9702	7					W. M.
		8 47 56·9	..	..	..	..	+1 27·71	144 3 2·62	9·6590	6	
		8 59 5·3	+1 1·46	20 3 38·67	8·9616	7					
		9 11 20·9	..	..	..	..	+1 33·90	144 3 8·81	9·7182	6	
22	78	8 42 1·8	+0 14·58	20 5 35·89	8·9688	10					
		8 58 3·5	..	..	..	..	−1 31·16	144 9 32·38	9·6902	10	
		9 13 36·7	+0 17·29	20 5 38·60	8·9538	10					
		9 28 16·5	..	..	..	..	−1 23·15	144 9 40·39	9·7596	7	
23	79	8 51 59·1	+0 43·19	20 7 35·58	8·9653	10					
		9 6 55·3	..	..	..	..	+0 59·38	144 15 55·78	9·7162	10	
		9 21 33·0	+0 45·63	20 7 38·02	8·9483	10					
		9 35 36·8	..	..	..	..	+1 7·36	144 16 3·76	9·7776	10	
24	80	8 47 15·7	+0 13·71	20 9 33·26	8·9676	10					
		9 0 44·5	..	..	..	..	−7 43·25	144 22 8·59	9·7056	10	
		9 16 35·1	+0 16·24	20 9 35·79	8·9515	10					
		9 29 26·3	..	..	..	..	−7 37·08	144 22 14·76	9·7689	10	
27	81	8 45 55·8	−1 2·34	20 15 27·09	8·9688	10					
		9 0 44·9	..	..	..	..	+1 34·94	144 40 38·86	9·7183	10	
		9 15 43·9	−1 0·06	20 15 29·37	8·9512	10					
		9 33 39·2	..	..	..	..	+1 42·65	144 40 46·57	9·7877	10	
28	82	8 49 50·2	+0 8·46	20 17 25·61	8·9671	10					
		9 6 26·2	..	..	..	..	+0 55·56	144 46 43·10	9·7354	10	
		9 23 28·3	+0 11·31	20 17 28·46	8·9447	10					
29	83	8 39 25·1	+0 11·71	20 19 22·46	8·9721	10					
		8 51 4·5	..	..	..	..	−9 27·46	144 52 36·90	9·7035	10	
		9 2 13·9	+0 14·06	20 19 24·81	8·9599	5					
30	84	8 54 31·4	..	..	..	..	−0 52·95	144 58 34·14	9·7161	5	
		9 6 2·9	−0 38·27	20 21 22·55	8·9571	6					
		9 15 26·8	..	..	..	..	−0 46·73	144 58 40·36	9·7625	5	
		9 26 7·2	−0 36·93	20 21 23·89	8·9412	6					
		9 34 38·8	..	..	..	..	−0 42·90	144 58 44·19	9·7996	5	

TABLE I. *Right Ascensions and North Polar Distances of* DONATI'S *Comet* (*continued*).

Date. 1858.	Star of Comp.	Cape Mean Time.	Diff. of R.A. Comet—Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Measures.	Diff. of N.P.D. Comet—Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Dec.		h m s	m s	h m s	+		′ ″	° ′ ″	+		
31	85	8 42 22·3	..	..	..	..	+8 26·62	145 4 29·44	9·6907	6	W. M.
		8 54 19·9	+0 16·98	20 23 19·62	8·9644	10					
		9 7 50·6	..	..	..	..	+8 30·95	145 4 33·77	9·7502	6	
		9 20 14·3	+0 18·94	20 23 21·58	8·9457	6					
1859. Jan.											
1	86	8 53 51·2	..	..	..	..	+6 47·12	145 10 20·27	9·7192	6	
		9 4 12·8	−0 7·62	20 25 17·85	8·9577	10					
		9 16 48·6	..	..	..	..	+6 56·91	145 10 30·06	9·7737	7	
4	87	8 44 36·1	..	..	..	..	−8 47·40	145 27 47·69	9·7138	6	
		8 53 16·5	−0 12·43	20 31 10·53	8·9644	6					
		9 2 50·8	..	..	..	..	−8 41·04	145 27 54·05	9·7549	7	
		9 13 19·8	−0 11·20	20 31 11·76	8·9491	6					
5	88	8 47 15·7	..	..	..	..	−2 36·34	145 33 37·61	9·7243	6	
		8 56 9·4	−0 31·79	20 33 8·26	8·9621	6					
		9 8 1·7	..	..	..	..	−2 32·25	145 33 41·70	9·7694	6	
6	89	8 49 50·3	..	..	..	..	−2 39·21	145 39 19·90	9·7344	6	
		8 57 56·6	+0 32·04	20 35 6·16	8·9604	6					
		9 5 8·8	..	..	..	..	−2 34·34	145 39 24·77	9·7672	6	
		9 16 42·0	+0 33·80	20 35 7·92	8·9446	10					
11	90	8 42 8·1	..	..	..	..	+8 21·92	146 7 59·78	9·7372	6	
		8 51 11·2	+0 25·75	20 44 54·88	8·9633	10					
		9 2 18·6	..	..	..	..	+8 25·33	146 8 3·19	9·7797	6	
		9 12 19·1	+0 27·74	20 44 56·87	8·9446	10					
		9 22 21·9	..	..	..	..	+8 30·83	146 8 8·69	9·8167	6	
12	91	9 2 38·3	..	..	..	..	−1 10·68	146 13 47·52	9·7838	14	
13	92	8 49 48·5	..	..	..	..	−0 48·88	146 19 31·12	9·7617	6	
		9 1 48·2	+0 57·35	20 48 52·03	8·9531	6					
		9 13 51·9	..	..	..	..	−0 42·34	146 19 37·66	9·8082	6	
		9 25 34·2	+1 0·02	20 48 54·70	8·9282	5					
20	93	8 37 38·0	..	..	..	..	−5 9·13	147 0 6·40	9·7624	6	
		8 44 46·4	+0 27·02	21 2 40·84	8·9639	8					

Table I. *Right Ascensions and North Polar Distances of* Donati's *Comet* (*continued*).

Date. 1859.	Star of Comp.	Cape Mean Time.	Diff. of R.A. Comet—Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Measures.	Diff. of N.P.D. Comet—Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Jan.		h m s	m s	h m s	+		′ ″	° ′ ″	+		
21	94	8 35 30·1	..	..	..	..	−2 24·23	147 5 58·00	9·7618	6	W. M.
		8 43 14·3	−0 10·80	21 4 39·40	8·9646	6					
		8 51 3·6	..	..	..	..	−2 19·27	147 6 2·96	9·7928	6	
		8 59 9·6	−0 9·65	21 4 40·55	8·9491	6					
		9 6 42·8	..	..	..	..	−2 15·97	147 6 6·26	9·8210	6	
24	95	8 43 20·0	..	..	..	..	−2 50·16	147 23 52·76	9·7883	6	
		8 50 57·3	−0 28·23	21 10 37·85	8·9550	6					
		8 57 19·8	..	..	..	..	−2 46·87	147 23 56·05	9·8141	6	
		9 8 24·0	−0 26·43	21 10 39·65	8·9353	10					
		9 23 7·0	..	..	..	..	−2 37·35	147 24 5·57	9·8558	10	
		9 34 24·7	−0 24·68	21 10 41·40	8·8990	10					
25	96	8 50 21·9	..	..	..	..	+5 51·75	147 29 55·34	9·8048	6	
		9 11 27·9	+0 0·28	21 12 39·62	8·9301	10					
28	97	8 46 44·0	..	..	..	..	+2 43·41	147 48 12·09	9·8079	6	
		8 55 8·7	+0 19·97	21 18 38·73	8·9467	10					
		9 3 45·6	..	..	..	..	+2 46·48	147 48 15·16	9·8369	8	
		9 24 37·1	+0 22·53	21 18 41·29	8·9070	10					
29	98	8 36 1·9	..	..	..	..	+7 43·42	147 54 19·44	9·7912	10	
		8 50 8·1	−0 9·82	21 20 38·54	8·9515	10					
		9 2 9·6	−0 8·58	21 20 39·78	8·9370	10					
		9 12 51·2	..	..	..	..	+7 50·89	147 54 26·91	9·8683	10	
30	99	8 37 0·5	..	..	..	..	+0 7·26	148 0 32·24	9·7964	10	
		8 49 19·0	−0 20·75	21 22 39·11	8·9515	10					
		9 3 45·5	−0 19·38	21 22 40·48	8·9337	10					
		9 15 31·8	..	..	..	..	+0 19·73	148 0 44·71	9·8604	10	
31	100	8 56 17·5	..	..	..	..	+6 44·45	148 6 56·10	9·8334	10	
		9 8 13·0	−0 19·77	21 24 41·96	8·9263	10					
Feb. 2	101	8 39 5·3	..	..	..	..	−0 59·76	148 19 37·56	9·8099	10	
		8 50 26·5	+0 8·38	21 28 43·19	8·9472	10					
		9 0 3·5	+0 9·31	21 28 44·12	8·9348	10					
		9 11 28·0	..	..	..	..	−0 52·79	148 19 44·53	9·8622	10	

TABLE I. *Right Ascensions and North Polar Distances of* DONATI'S *Comet* (*continued*).

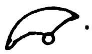
Date. 1859.	Star of Comp.	Cape Mean Time.	Diff. of R.A. Comet—Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Measures.	Diff. of N.P.D. Comet—Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Feb.		h m s	m s	h m s	+		′ ″	° ′ ″	+		
3	102	8 32 46·4	..	..	..	..	+3 31·12	148 26 0·47	9·8015	10	W. M.
		8 43 2·2	—0 7·22	21 30 44·34	8·9551	10					
		8 53 1·6	—0 6·53	21 30 45·03	8·9429	10					
		9 3 15·4	..	..	..	..	+3 39·41	148 26 8·76	9·8527	10	
4	103	8 30 57·6	..	..	..	..	+0 10·28	148 32 31·23	9·8013	10	
		8 43 27·9	—0 31·80	21 32 46·12	8·9537	10					
		8 54 56·7	—0 30·90	21 32 47·02	8·9392	10					
		9 6 8·2	..	..	..	..	+0 19·44	148 32 40·39	9·8596	10	
5	104	8 34 30·8	..	..	..	..	—2 32·93	148 39 7·58	9·8110	10	
		8 47 14·3	—0 19·03	21 34 48·85	8·9481	10					
		9 5 45·5	—0 17·14	21 34 50·74	8·9227	10					
		9 16 8·5	..	..	..	..	—2 19·15	148 39 21·36	9·8761	7	
8	105	8 41 0·2	..	..	..	..	+1 50·45	148 59 12·58	9·8310	10	
		8 53 5·7	+0 39·80	21 40 57·42	8·9369	10					
21	106	8 37 5·6	..	..	..	..	+1 22·02	150 33 44·84	9·8588	7	
		8 46 39·0	—0 34·74	22 8 6·89	8·9307	8					
		8 54 25·0	—0 33·98	22 8 7·65	8·9176	8					
		9 2 21·0	..	..	..	..	+1 32·30	150 33 55·12	9·8932	8	
22	107	8 34 16·9	..	..	..	..	—7 37·97	150 41 38·05	9·8569	7	
		8 47 27·9	+0 38·91	22 10 15·37	8·9282	10					
25	108	8 30 36·4	..	..	..	..	—0 11·22	151 5 39·68	9·8586	7	
		8 43 45·6	+0 6·13	22 16 42·69	8·9309	10					
		8 56 34·1	..	..	..	..	—0 3·02	151 5 47·88	9·8939	7	
26	109	8 17 52·8	..	..	..	..	—3 45·28	151 13 46·88	9·8411	8	
		8 28 23·0	+0 10·95	22 18 51·07	8·9545	8					
		8 39 45·4	+0 12·05	22 18 52·17	8·9365	10					
		8 52 39·3	..	..	..	..	—3 33·11	151 13 59·05	9·8910	10	
27	110	8 27 27·3	..	..	..	..	+8 28·25	152 22 9·25	9·8584	7	
		8 44 7·8	—0 8·48	22 21 3·53	8·9279	8					
28	111	8 48 30·6	..	..	..	..	—1 44·46	151 30 42·33	9·8896	6	
		8 58 4·0	—0 37·93	22 23 15·82	8·9003	9					

Table I. ***Right Ascensions and North Polar Distances of*** Donati's ***Comet*** (***continued***).

Date. 1859.	Star of Comp.	Cape Mean Time.	Diff. of R.A. Comet—Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Measures.	Diff. of N.P.D. Comet—Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Mar.		h m s	m s	h m s	+		′ ″	° ′ ″	+		
1	112	8 5 47·1	..	..	..	..	—1 34·10	151 38 57·99	9·8286	10	W. M.
		8 22 23·0	—0 16·61	22 25 24·07	8·9615	10					
		8 36 58·1	—0 15·23	22 25 25·45	8·9381	10					
		8 49 48·4	..	..	..	..	—1 23·71	151 39 8·38	9·8932	10	
2	113	8 6 30·0	..	..	..	..	+3 34·14	151 47 26·63	9·8323	8	
		8 17 7·5	+0 10·85	22 27 36·18	8·9687	8					
		8 29 44·7	+0 12·22	22 27 37·55	8·9493	10					
		8 43 49·7	..	..	..	..	+3 45·47	151 47 37·96	9·8874	10	
4	114	8 13 26·0	..	..	..	..	+7 3·60	152 5 1·33	9·8483	10	
		8 39 59·4	+1 11·28	22 32 5·27	8·9298	10					

NOTES.

Oct. 15. A thick haze covers the sky. The observations this evening are not very trustworthy, as a few glimpses only of the star were obtained when near the horizon.

Oct. 23. The comet's head when viewed with the ordinary observing power (126) appears to consist of two concentric luminous envelopes nearly surrounding the nucleus on the side next the sun. The nucleus, which is a bright disk 4″ in diameter, is situated at the apex of a sector of less intense light of somewhat this form . The sector and envelopes are separated from each other by tolerably clearly marked dark bands. The exterior envelope is fainter and traced with less distinctness than the interior envelope, and its light soon becomes blended with the general nebulosity of the head.

A few measures of the head were taken this evening, but they are not very accurate, as the "Flat-wire" micrometer is not well adapted for the purpose.

	″
Distance of nucleus from the vertex of the head of comet	156,
Breadth of head through the nucleus	165,
Nucleus to vertex of sector	15,
Breadth of sector	23.

Oct. 25. Diameter of nucleus 4″·5.

Oct. 27. Indifferent observations; the chronometer wholly inaudible, owing to the violence of the wind.

Oct. 29. The sector of light described on the 23d can now no longer be traced. The comet's head has the appearance of a nebulosity of parabolic outline, having at its focus a somewhat bright condensation of light about 10″ in diameter, enveloping the nucleus. Breadth of head at the nucleus =203″. Nucleus to vertex 151″.

Oct. 30. Diameter of nucleus $=4''\cdot4$.

Oct. 31. Diameter of nucleus $=3''\cdot9$. Breadth of head at nucleus $=153''$.

Nov. 5. Breadth of head at nucleus $=171''$. Nucleus to vertex $=100''$.

Nov. 18. The measures are not good; both objects faintly seen through cloud.

Nov. 20. Violent southerly wind, the chronometer beat inaudible.

Nov. 22. Bad observations; the comet low and faint.

Nov. 26. Very bad definition, further observation impracticable.

Nov. 27. The star very faint.

Dec. 2. The comet low and faint.

Dec. 3. Clouds; both objects very faint.

Dec. 8. Strong southerly wind and very bad definition.

Dec. 11. Calm; atmospheric circumstances favourable.

Dec. 14. Moonlight; the comet very faint.

Dec. 19. Bright moonlight; the comet very faint, the observations mere guess-work.

Dec. 23. The comet is now merely a faint nebulous body, about $90''$ in diameter, with a slight central condensation of light. No trace of a tail.

Dec. 28. Very bad definition.

Dec. 29. The second set for difference in R.A. is not very accurate, as the comet became too faint for observation.

Dec. 30. The diameter of the comet is $98''$.

1859, Jan. 4. Excessively bad definition; the observations not precise.

Jan. 5. A cirrous haze covers the sky; both objects exceedingly faint.

Jan. 11. Moonlight, the comet very faint, but the definition good and the observations satisfactory.

Jan. 13. Violent southerly wind and very bad definition. Observation difficult and not very precise.

Jan. 14 to 19. The comet wholly obliterated by moonlight.

Jan. 20. The measures continued until the moon rose, when the comet disappeared. The observations difficult, owing to the proximity of a star of 11th mag.

Jan. 21. Calm, with good definition. The observations good.

Jan. 24. Diameter of comet about $60''$.

Jan. 28. Observations difficult; the comet seen through thin stratus cloud.

Feb. 5. Haze S.W. The comet exceedingly faint. The 2d set for Δ N.P.D. is very uncertain.

Feb. 21. Extremely faint; the 2d set for Δ N.P.D. is not accurate.

Feb. 22. The worst possible definition.

Feb. 25. The proximity of the star of comparison (12th mag.) rather impairs the accuracy of the measures.

Feb. 26. Diameter of comet $=54''$.

Feb. 27. Difficult; the comet repeatedly obliterated by cloud.

Feb. 28. Compelled to wait until the comet was separated sufficiently from a star of 12th magnitude. It was then too low and faint to allow of satisfactory measurement.

TABLE II.

Right Ascensions and North Polar Distances of DONATI'S *Comet observed at the Royal Observatory, Cape of Good Hope, with the 46-inch Equatoreal.*

Date. 1858.	Star No.	Cape Mean Time.	Diff. of R.A. Comet – Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Obs.	Diff. of N.P.D. Comet – Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Oct.		h m s	m s	h m s	+		′ ″	° ′ ″	+		
16	φ Oph.	7 59 18·1	−7 14·94	16 15 47·98	8·7555	3	− 0 29·07	106 17 40·11	9·6953	3	T. M.
		8 43 36·8	−6 57·62	16 16 5·30	8·7610	2	+ 4 43·69	106 22 52·87	9·7327	2	
18	7	7 55 40·9	−6 13·51	16 33 56·66	8·7641	2	− 8 36·23	111 32 39·89	9·6530	2	
	8	7 55 40·9	−7 12·57	16 33 56·68	..	2	− 3 30·74	111 32 36·02	..	2	
	7	8 15 35·3	−6 6·56	16 34 3·61	8·7710	2	− 6 37·16	111 34 38·96	9·6772	2	
	8	8 15 35·3	−7 5·56	16 34 3·69	..	2	− 1 30·30	111 34 36·46	..	2	
	7	8 49 16·3	−5 55·31	16 34 14·86	8·7115	1	− 3 13·26	111 38 2·86	9·7160	1	
	8	8 49 16·3	−6 54·32	16 34 14·93	..	1	+ 1 54·02	111 38 0·78	..	1	
21	9	7 25 48·8	+4 30·44	16 57 36·12	8·7608	2	+10 30·11	117 54 14·53	9·5294	2	
	10	..	+2 29·46	16 57 36·37	..	2	− 8 56·74	117 54 14·56	..	2	
	9	7 45 29·6	+4 36·55	16 57 42·23	8·7749	2	+12 0·58	117 55 45·00	9·5673	2	
	10	..	+2 35·06	16 57 41·97	..	2	− 7 22·42	117 55 48·88	..	2	
	9	8 4 7·0	+4 41·61	16 57 47·29	8·7850	2	+13 30·20	117 57 14·62	9·6015	2	
	10	..	+2 40·71	16 57 47·62	..	2	− 5 52·04	117 57 19·26	..	2	
	13	..	−1 39·66	16 57 47·46	..	2	+ 2 23·71	117 57 15·91	..	2	
	9	8 23 47·2	+4 47·89	16 56 53·57	8·7924	2	+14 56·92	117 58 41·34	9·6355	2	
	10	..	+2 45·97	16 56 52·88	..	2	− 4 26·22	117 58 45·08	..	2	
22	16	7 45 54·4	−2 1·79	17 4 48·09	8·7806	2	−20 6·31	119 42 38·53	9·5431	2	
	17	..	−3 31·12	17 4 47·99	..	2	−17 45·61	119 42 36·05	..	2	
	18	..	−4 34·54	17 4 48·06	..	2	− 0 29·96	119 42 37·10	..	2	
	16	8 1 14·0	−1 57·03	17 4 52·85	8·7895	2	−19 3·81	119 43 41·03	9·5744	2	
	17	..	−3 26·48	17 4 52·63	..	2	−16 40·11	119 43 41·55	..	2	
	18	..	−4 29·78	17 4 52·82	..	2	+ 0 32·35	119 43 39·41	..	2	
	16	8 15 59·2	−1 53·09	17 4 56·79	8·7961	2	−18 1·07	119 44 43·77	9·6032	2	
	17	..	−3 22·43	17 4 56·68	..	2	−15 38·01	119 44 45·65	..	2	
	18	..	−4 26·08	17 4 56·52	..	2	+ 1 37·11	119 44 44·17	..	2	

Right Ascensions and North Polar Distances of DONATI'S *Comet* (*continued*).

Date. 1858.	Star No.	Cape Mean Time.	Diff. of R.A. Comet—Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Obs.	Diff. of N.P.D. Comet—Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
Oct.		h m s	m s	h m s	+		′ ″	° ′ ″	+		
22	16	8 30 24·9	−1 48·89	17 5 0·99	8·8008	2	−16 59·95	119 45 44·89	9·6299	2	T. M.
	17	..	−3 18·13	17 5 0·98	..	2	−14 36·66	119 45 45·00	..	2	
	18	..	−4 21·37	17 5 1·23	..	2	+ 2 38·72	119 45 45·78	..	2	
	16	8 43 58·2	−1 45·45	17 5 4·43	8·8035	2	−16 3·46	119 46 41·38	9·6536	2	
	17	..	−3 15·15	17 5 3·96	..	2	−13 38·96	119 46 42·70	..	2	
	18	..	−4 19·21	17 5 3·39	..	2	+ 3 30·05	119 46 37·11	..	2	
23	19	7 29 21·1	+1 16·43	17 11 26·05	8·7736	2	+ 6 40·65	121 19 10·94	9·4786	2	
	21	..	−1 40·56	17 11 26·91	..	2					
	19	7 48 38·9	+1 21·29	17 11 30·91	8·7879	2	+ 7 52·08	121 20 22·37	9·5246	2	
	21	..	−1 35·86	17 11 31·61	..	2	− 6 13·24	121 20 22·64	..	2	
	19	8 0 17·8	+1 24·94	17 11 34·56	8·7948	2	+ 8 34·80	121 21 5·09	9·5507	2	
	21	..	−1 32·36	17 11 35·11	..	2	− 5 27·97	121 21 7·91	..	2	
	19	8 10 8·0	+1 27·11	17 11 36·73	8·7998	2	+ 9 18·22	121 21 48·51	9·5720	2	
	21	..	−1 30·16	17 11 37·31	..	2	− 4 49·08	121 21 46·80	..	2	
	19	8 20 33·0	+1 30·21	17 11 39·83	8·8041	2	+ 9 55·03	121 22 25·32	9·5936	2	
	21	..	−1 26·44	17 11 41·03	..	2	− 4 11·84	121 22 24·04	..	2	
24	22	8 12 35·6	+0 38·50	17 17 58·59	8·8068	2	+ 0 41·04	122 50 57·59	9·5581	2	
	22	8 31 0·9	+0 43·95	17 18 4·04	8·8138	2	+ 1 43·59	122 52 0·14	9·5986	2	
	23	..	−1 42·52	17 18 4·40	..	2	− 1 5·26	122 52 2·01	..	2	
	22	8 46 11·3	+0 46·05	17 18 6·14	8·8174	1	+ 2 44·41	122 53 0·96	9·6297	1	
	23	..	−1 40·12	17 18 6·80	..	1	− 0 10·80	122 52 56·47	..	1	
	22	9 0 59·3	+0 51·17	17 18 11·26	8·8191	2	+ 3 37·69	122 53 54·24	9·6582	2	
	23	..	−1 35·01	17 18 11·91	..	2	+ 0 35·28	122 53 42·55	..	2	
25	24	7 35 7·3	+0 48·23	17 23 51·21	8·7886	6	+ 0 1·63	124 10 16·22	9·4386	6	
		7 51 11·7	+0 52·28	17 23 55·26	8·8005	6	+ 0 54·48	124 11 9·07	9·4838	6	
		8 7 18·3	+0 56·34	17 23 59·32	8·8101	6	+ 1 45·30	124 11 59·89	9·5262	6	
		8 22 53·8	+0 59·72	17 24 2·70	8·8171	6	+ 2 35·78	124 12 50·37	9·5643	6	
		8 38 34·9	+1 3·45	17 24 6·43	8·8222	6	+ 3 26·84	124 13 41·43	9·5999	6	
26	27	7 45 45·5	−1 30·97	17 29 36·62	8·8020	3	(−8 17·40)	125 25 (42·74)	9·4453	3	{T. M. {G. M.
		8 6 37·7	−1 26·03	17 29 41·56	8·8153	7	− 7 24·45	125 26 35·69	9·5055	7	G. M.
		8 36 58·8	−1 19·51	17 29 48·08	8·8278	6	− 5 53·23	125 28 6·91	9·5824	6	
		8 53 35·2	−1 15·59	17 29 52·00	8·8313	6	− 5 5·96	125 28 54·18	9·6200	6	

Right Ascensions and North Polar Distances of DONATI'S *Comet* (*continued*).

Date. 1858.	Star No.	Cape Mean Time.	Diff. of R.A. Comet—Star.	R.A. of Comet.	Log $\frac{p}{P}$	No. of Obs.	Diff. of N.P.D Comet − Star.	N.P.D. of Comet.	Log $\frac{q}{P}$	No. of Measures.	Observer.
		h m s	m s	h m s	+		′ ″	° ′ ″	+		
Oct. 27	28	7 54 25·9	+1 47·46	17 35 2·78	8·8133	3	−17 56·11	126 34 23·89	9·4502	3	G. M.
		8 19 32·4	+1 53·13	17 35 8·45	8·8272	5	−16 45·77	126 35 34·23	9·5236	5	
		8 42 28·4	+1 58·96	17 35 14·28	8·8352	6	−15 47·68	126 36 32·32	9·5907	6	
28	31	7 49 43·6	−1 23·98	17 40 11·31	8·8152	5	− 8 49·95	127 37 7·52	9·4144	5	
		8 3 53·2	−1 20·49	17 40 14·80	8·8245	5	− 8 11·45	127 37 46·02	9·4610	5	
29	32	7 47 52·0	+0 26·58	17 45 5·88	8·8189	9	− 0 16·11	128 35 6·36	9·3874	9	
		8 4 55·5	+0 29·70	17 45 9·00	8·8303	9	+ 0 24·04	128 35 46·51	9·4474	9	
		8 23 20·2	+0 33·20	17 45 12·50	8·8397	9	+ 1 6·90	128 36 29·37	9·5054	9	
30	34	8 18 37·9	−0 37·88	17 49 52·47	8·8427	9	+15 44·68	129 29 43·66	9·4777	9	
	35	8 42 53·4	−0 45·05	17 49 56·88	8·8517	5	− 8 48·95	129 30 27·93	9·5505	5	
31	36	8 14 6·0	−0 22·18	17 54 18·92	8·8454	10	−19 33·88	130 18 47·67	9·4497	10	T. M.
		8 44 22·0	−0 16·61	17 54 24·49	8·8572	10	−18 30·62	130 19 50·93	9·5454	10	
Nov. 2	38	9 56 0·5	+0 31·49	18 2 58·27	8·8643	7	+ 5 20·46	131 50 2·14	9·7081	7	G. M.
3	41	7 46 11·8	−2 1·47	18 6 32·77	8·8412	6	+ 5 37·97	132 25 56·67	9·2934	6	
		8 7 35·0	−1 58·05	18 6 36·19	8·8558	6	+ 6 9·90	132 26 28·60	9·3902	6	
4	43	7 42 25·0	−1 53·79	18 10 18·34	8·8427	3	+ 2 57·39	133 2 47·90	9·2594	3	
		8 14 47·1	−1 48·17	18 10 23·96	8·8642	9	+ 3 45·08	133 3 35·59	9·4102	9	
5	44	7 54 24·1	−0 2·56	18 13 58·67	8·8561	10	−33 18·24	133 37 25·44	9·3097	10	
		8 30 16·3	+0 2·53	18 14 3·76	8·8754	3	−32 29·76	133 38 13·92	9·4627	3	
6	46	7 54 41·4	−1 27·61	18 17 30·33	8·8605	9	+13 48·86	134 9 48·35	9·3014	9	
		8 23 3·7	−1 23·89	18 17 34·05	8·8765	9	+14 28·62	134 10 28·11	9·4293	9	
7	47	7 48 30·5	−0 49·77	18 20 52·54	8·8602	7	− 1 29·68	134 39 51·83	9·2587	7	
		8 6 6·5	−0 46·98	18 20 55·33	8·8719	7	− 1 8·97	134 40 12·54	9·3500	7	
		8 21 23·7	−0 45·55	18 20 56·76	8·8797	7	− 0 49·94	134 40 31·57	9·4175	7	
9	48	8 49 51·7	−0 26·49	18 27 27·40	8·8963	7	+ 1 13·28	135 36 9·48	9·5174	7	T. M.
		9 8 14·8	−0 22·98	18 27 30·91	8·8984	8	+ 1 27·81	135 36 24·01	9·5760	8	
12	51	8 18 31·0	−5 30·67	18 36 25·98	8·8949	4	+ 1 36·37	136 47 10·51	9·3705	4	
		8 52 30·2	−5 27·58	18 36 29·07	8·9066	5	+ 2 4·19	136 47 38·33	9·5238	5	
15	52	8 14 21·3	+1 19·11	18 44 46·12	8·9048	9	+21 25·30	137 47 59·34	9·3693	9	G. M.
	54	8 36 8·9	−1 42·04	18 44 48·18	8·9123	6	+ 2 43·05	137 48 12·84	9·4649	6	

NOTES *to Observations with 46-inch Equatoreal.*

Oct. 14. Length of tail, 10°.

Oct. 21. The comparison star (13) is too faint for this telescope.

Oct. 22. Moonlight and the glare of *Venus* prevent a correct estimate of the length of the tail: it appears to be about 4° long. The head is still bright.

Oct. 23. Diameter of the tail half way down = 8′ 24″.

Oct. 24. Frequently clouded: the observations not so precise as usual, but upon the whole pretty fair.

Oct. 30. The comet's tail is inclined northwards of the parallel 7°. The interval between the transit of the head and end of the tail = $7^m 45^s$. Breadth of head, 4′ 41″. Breadth of tail half way down = 7′ 12″.

Oct. 31. Interval between the transit of head and end of tail, $7^m 20^s$. Tail inclined northward of the parallel 6′ 35″.* The end of the tail on the north side appears like a feather-edge torn off.

Nov. 4. The star is exceedingly faint.

Nov. 5. Strong southerly wind; the chronometer often inaudible.

Nov. 6. The nucleus is much diffused, and is difficult to observe.

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* Should this be 6° 35′?—Ed.

TABLE III.

Mean Places of Stars compared with DONATI'S *Comet, deduced from observations with the Transit-circle.*

No. of Star.	Mag.	Mean R.A. Jan. 1, 1859.	Annual Prec. in R.A.	Mean N.P.D. Jan. 1, 1859.	Annual Prec. in N.P.D.
		h m s	s	° ′ ″	″
1	8	15 20 31·62	3.073	90 7 10·51	+12·85
2	11	15 33 50·08	3·137	93 32 11·36	+11·94
3	10	15 55 4·38	3·277	100 14 7·87	+10·40
4	9	16 6 15·44	3·356	103 37 51·90	+ 9·55
5	10	16 6 2·69	3·424	106 22 22·48	+ 8·79
φ Oph.	4½	16 23 4·31	3·427	106 18 5·64	+ 8·23
6	9½	16 34 35·98	3·561	111 29 50·50	+ 7·30
7	9	16 40 11·56	3·570	111 41 10·56	+ 6·84
8	6	16 41 10·64	3·569	111 36 1·18	+ 6·76
9	8	16 53 7·11	3·744	117 43 36·86	+ 5·77
10	7½	16 55 8·35	3·754	118 3 3·56	+ 5·60
11	8½	16 57 48·42	3·757	118 3 59·63	+ 5·38
12	10	16 58 42·77	3·754	117 55 59·26	+ 5·30
13	8	16 59 28·53	3·754	117 54 44·30	+ 5·23
14	10	17 5 14·22	3·817	119 52 39·52	+ 4·75
15	9	17 5 41·29	3·812	119 41 19·70	+ 4·71
16	6	17 6 51·29	3·823	120 2 36·11	+ 4·61
17	6	17 8 20·50	3·823	120 0 12·87	+ 4·48
18	7	17 9 23·99	3·815	119 42 58·30	+ 4·39
19	7	17 10 11·03	3·861	121 12 21·16	+ 4·33
20	7	17 12 18·71	3·870	121 26 0·75	+ 4·14
21	7½	17 13 8·87	3·871	121 26 26·55	+ 4·07
22	7½	17 17 21·50	3·919	122 50 6·73	+ 3·71
23	7½	17 19 48·31	3·922	122 52 57·33	+ 3·50
24	6½	17 23 4·39	3·966	124 10 4·23	+ 3·22
25	9	17 23 58·90	3·971	124 16 23·68	+ 3·14

TABLE III. *Mean Places of Stars compared with* DONATI'S *Comet* (*continued*).

No. of Star.	Mag.	Mean R.A. Jan. 1, 1859.	Annual Prec. in R.A.	Mean N.P.D. Jan. 1, 1859.	Annual Prec. in N.P.D.
		h m s	s	° ′ ″	″
26	9½	17 29 20·46	4·011	125 21 50·81	+ 2·67
27	7	17 31 8·98	4·019	125 33 49·12	+2·52
28	6	17 33 16·72	4·066	126 52 8·61	+2·33
29	8	17 34 30·73	4·061	126 42 4·10	+2·23
30	9½	17 40 19·27	4·092	127 28 50·76	+1·72
31	7	17 41 36·67	4·103	127 45 45·56	+1·61
32	7	17 44 40·69	4·135	128 35 10·29	+1·34
33	7½	17 46 1·23	4·138	128 38 46·23	+1·22
34	7	17 50 31·71	4·161	129 13 46·47	+0·83
35	8	17 50 43·30	4·178	129 39 4·25	+0·81
36	7½	17 54 42·47	4·218	130 38 8·58	+0·46
37	8	17 55 15·34	4·210	130 26 51·27	+0·41
38	8	18 2 28·13	4·263	131 44 28·27	−0·22
39	10½	18 7 5·71	4·296	132 30 48·23	−0·62
40	8½	18 7 9·90	4·285	132 15 28·20	−0·63
41	6	18 8 35·57	4·288	132 20 4·99	−0·75
42	10	18 10 57·23	4·317	133 1 58·59	−0·96
43	8	18 12 13·46	4·315	132 59 36·57	−1·07
44	6	18 14 2·58	4·367	134 10 29·46	−1·23
45	9	18 18 16·98	4·369	134 14 41·76	−1·60
46	6	18 18 59·25	4·354	133 55 45·24	−1·66
47	8	18 21 43·63	4·387	134 41 7·06	−1·90
48	7	18 27 55·20	4·425	135 34 41·50	−2·44
49	10	18 33 16·14	4·455	136 18 21·65	−2·90
50	9	18 35 49·64	4·473	136 43 39·16	−3·12
51	6	18 41 57·90	4·468	136 45 18·94	−3·65
52	6½	18 43 28·30	4·500	137 26 18·96	−3·78
53	9	18 44 33·09	4·517	137 47 13·24	−3·88
54	7½	18 46 31·49	4·512	137 45 14·53	−4·04
55	9	18 48 18·10	4·530	138 9 19·50	−4·20
56	7	18 49 59·14	4·544	138 28 16·88	−4·34
57	7	18 54 0·17	4·545	138 36 13·06	−4·68
58	9	18 54 14·62	4·558	138 51 8·65	−4·70

Table III. ***Mean Places of Stars compared with*** **Donati's** ***Comet (continued).***

No. of Star.	Mag.	Mean R.A. Jan. 1, 1859.	Annual Prec. in R.A.	Mean N.P.D. Jan. 1, 1859.	Annual Prec. in N.P.D.
		h m s	s	° ′ ″	″
59	11	18 56 45·97	4·574	139 14 16·15	— 4·92
60	8½	18 59 26·98	4·585	139 31 55·14	— 5·15
61	10	19 3 56·92	4·590	139 46 18·49	— 5·52
62	9	19 12 16·25	4·613	140 30 13·92	— 6·22
63	11	19 14 38·19	4·623	140 46 51·09	— 6·42
64	10	19 19 15·59	4·627	141 2 58·29	— 6·80
65	11	19 23 39·22	4·645	141 34 39·49	— 7·16
66	10	19 26 57·82	4·646	141 44 59·86	— 7·43
67	7	19 29 48·56	4·644	141 51 53·02	— 7·66
68	9	19 30 40·22	4·655	142 5 35·44	— 7·73
69	10½	19 33 5·48	4·650	142 8 0·49	— 7·93
70	11	19 33 20·65	4·657	142 16 15·97	— 7·95
71	10½	19 38 15·97	4·651	142 25 12·82	— 8·34
72	10½	19 42 6·58	4·653	142 40 10·58	— 8·64
73	10	19 45 41·42	4·664	143 4 44·45	— 8·92
74	9	19 50 37·87	4·663	143 21 41·43	— 9·31
75	8½	19 50 48·43	4·653	143 12 30·21	— 9·32
76	9	20 0 20·16	4·648	143 44 53·20	—10·05
77	10	20 2 38·26	4·655	144 1 21·22	—10·23
78	10	20 5 22·34	4·653	144 10 49·94	—10·43
79	9½	20 6 53·42	4·650	144 14 42·92	—10·55
80	12	20 9 20·57	4·654	144 29 38·45	—10·73
81	10½	20 16 30·40	4·631	144 38 50·87	—11·25
82	9	20 17 18·11	4·633	144 45 34·65	—11·31
83	11	20 19 11·67	4·640	145 1 51·59	—11·45
84	8	20 22 1·76	4·624	144 59 14·43	—11·65
85	10½	20 23 3·58	4·616	144 55 50·35	—11·72
86	10	20 25 26·39	4·611	145 3 20·79	—11·89
87	11	20 31 23·85	4·613	145 36 23·14	—12·31
88	11½	20 33 40·93	4·600	145 36 2·15	—12·46
89	11½	20 34 34·99	4·601	145 41 47·50	—12·53
90	9	20 44 29·93	4·563	145 59 27·04	—13·19
91	10½	20 47 1·23	4·563	146 14 47·53	—13·36

TABLE III. *Mean Places of Stars compared with* DONATI'S *Comet* (*continued*).

No. of Star.	Mag.	Mean R.A. Jan. 1, 1859.	Annual Prec. in R.A.	Mean N.P.D. Jan. 1, 1859.	Annual Prec. in N.P.D.
		h m s	s	° ′ ″	″
92	9	20 47 55·46	4·563	146 20 9·53	−13·42
93	9	21 2 14·52	4·517	147 5 6·31	−14·32
94	8	21 4 50·88	4·503	147 8 13·18	−14·48
95	10	21 11 6·72	4·478	147 26 34·46	−14·85
96	10	21 12 39·97	4·465	147 23 55·34	−14·94
97	8½	21 18 19·35	4·445	147 45 21·06	−15·27
98	10	21 20 48·94	4·428	147 46 28·59	−15·41
99	10½	21 23 0·43	4·424	148 0 17·76	−15·53
100	9	21 25 2·28	4·409	148 0 4·65	−15·64
101	10½	21 28 35·34	4·401	148 20 30·78	−15·84
102	7	21 30 52·08	4·386	148 22 23·02	−15·96
103	8	21 33 18·43	4·375	148 32 14·84	−16·09
104	11	21 35 8·38	4·369	148 41 34·64	−16·18
105	8	21 40 18·09	4·342	148 57 16·99	−16·44
106	11	22 8 41·97	4·175	150 32 20·95	−17·74
107	10	22 9 36·79	4·180	150 49 14·47	−17·77
108	12	22 16 36·87	4·126	151 5 50·15	−18·05
109	12	22 18 40·42	4·115	151 17 31·70	−18·13
110	11	22 21 12·30	4·088	151 13 40·79	−18·22
111	8	22 23 54·04	4·075	151 32 26·87	−18·32
112	11	22 25 40·97	4·063	151 40 32·46	−18·38
113	9½	22 27 25·61	4·048	151 43 53·15	−18·44
114	9	22 30 54·25	4·022	151 57 58·99	−18·56

Table IV.

Apparent Places for Dates of Observation of the Stars compared with Donati's *Comet.*

No. of Star.	Date of Comparison.	Apparent R.A. of Star.	Apparent N.P.D. of Star.	No. of Star.	Date of Comparison.	Apparent R.A. of Star.	Apparent N.P.D. of Star.
	1858.	h m s	° ′ ″		1858.	h m s	° ′ ″
1	Oct. 11	15 20 30·22	90 7 8·27	26	Oct. 26	17 29 19·07	125 22 1·70
2	12	15 33 48·67	93 32 10·21	27	26	17 31 7·59	125 34 0·14
3	14	15 55 2·96	100 14 8·90	28	27	17 33 15·32	126 52 20·00
4	15	16 6 14·02	103 37 54·13	29	27	17 34 29·33	126 42 15·49
5	16	16 16 1·28	106 22 25·74	30	28	17 40 17·89	127 29 2·54
φ Oph.	16	16 23 2·92	106 18 9·18	31	28	17 41 35·29	127 45 57·47
6	18	16 34 34·57	111 29 55·77	32	29	17 44 39·30	128 35 22·47
7	18	16 40 10·17	111 41 16·12	33	29	17 45 59·85	128 38 58·48
8	18	16 41 9·25	111 36 6·76	34	30	17 50 30·35	129 13 58·98
9	21	16 53 5·68	117 43 44·42	35	30	17 50 41·93	129 39 16·88
10	21	16 55 6·91	118 3 11·30	36	31	17 54 41·10	130 38 21·55
11	21	16 57 47·00	118 4 7·49	37	31	17 55 13·98	130 27 4·22
12	21	16 58 41·36	117 56 7·13	38	Nov. 2	18 2 26·78	131 44 41·68
13	21	16 59 27·12	117 54 52·20	39	3	18 7 4·37	132 31 1·93
14	22	17 5 12·81	119 52 48·14	40	3	18 7 8·56	132 15 41·84
15	22	17 5 39·88	119 41 28·29	41	3	18 8 34·24	132 20 18·70
16	22	17 6 49·88	120 2 44·84	42	4	18 10 55·89	133 2 12·49
17	22	17 8 19·11	120 0 21·66	43	4	18 12 12·13	132 59 50·51
18	22	17 9 22·60	119 43 7·06	44	5	18 14 1·23	134 10 43·68
19	23	17 10 9·62	121 12 30·29	45	6	18 18 15·65	134 14 56·06
20	23	17 12 17·30	121 26 10·04	46	6	18 18 57·94	133 55 59·49
21	23	17 13 7·47	121 26 35·88	47	7	18 21 42·31	134 41 21·51
22	24	17 17 20·09	122 50 16·55	48	9	18 27 53·89	135 34 56·20
23	24	17 19 46·92	122 53 7·27	49	11	18 33 14·83	136 18 36·52
24	25	17 23 2·98	124 10 14·59	50	12	18 35 48·33	136 43 54·12
25	25	17 23 57·50	124 16 34·11	51	12	18 41 56·65	136 45 34·14

Apparent Places for Dates of Observations of the Stars (continued).

No. of Star.	Date of Comparison.	Apparent R.A. of Star.	Apparent N.P.D. of Star.	No. of Star.	Date of Comparison.	Apparent R.A. of Star.	Apparent N.P.D. of Star.
	1858.	h m s	° ′ ″		1858.	h m s	° ′ ″
52	Nov. 14	18 43 27·02	137 26 34·15	83	Dec. 29	20 19 10·75	145 2 4·36
52	15	18 43 27·01	137 26 34·04	84	30	20 22 0·82	144 59 27·09
53	15	18 44 31·80	137 47 28·44	85	31	20 23 2·64	144 56 2·82
54	15	18 46 30·22	137 45 29·79	86	1859. Jan. 1	20 25 25·47	145 3 33·15
55	16	18 48 16·82	138 9 34·81	87	4	20 31 22·96	145 36 35·09
56	17	18 49 57·86	138 28 32·20	88	5	20 33 40·05	145 36 13·95
57	18	18 53 58·91	138 36 28·44	89	6	20 34 34·12	145 41 59·11
58	19	18 54 13·34	138 51 23·97	90	11	20 44 29·13	145 59 37·86
59	20	18 56 44·70	139 14 31·52	91	12	20 47 0·44	146 14 58·20
60	21	18 59 25·70	139 32 10·55	92	13	20 47 54·68	146 20 20·00
61	22	19 3 55·67	139 46 33·98	93	20	21 2 13·82	147 5 15·53
62	26	19 12 15·01	140 30 29·33	94	21	21 4 50·20	147 8 22·23
63	27	19 14 36·97	140 47 6·51	95	24	21 11 6·08	147 26 42·92
64	29	19 19 14·38	141 3 13·64	96	25	21 12 39·34	147 24 3·59
65	Dec. 2	19 23 38·02	141 34 54·65	97	28	21 18 18·76	147 45 28·68
66	3	19 26 56·63	141 45 15·02	98	29	21 20 48·36	147 46 36·02
67	4	19 29 47·37	141 52 8·15	99	30	21 22 59·86	148 0 24·98
68	5	19 30 39·04	142 5 50·48	100	31	21 25 1·73	148 0 11·65
69	6	19 33 4·31	142 8 15·48	101	Feb. 2	21 28 34·81	148 20 37·32
70	6	19 33 19·47	142 16 30·99	102	3	21 30 51·56	148 22 29·35
71	8	19 38 14·81	142 25 27·71	103	3	21 33 17·92	148 32 20·95
72	10	19 42 5·45	142 40 25·31	104	5	21 35 7·88	148 41 40·51
72	11	19 42 5·44	142 40 25·15	105	8	21 40 17·62	148 57 22·13
73	12	19 45 40·29	143 4 59·03	106	21	22 8 41·63	150 32 22·82
74	14	19 50 36·76	143 21 55·89	107	22	22 9 36·46	150 49 16·02
75	14	19 50 47·34	143 12 44·65	108	25	22 16 36·56	151 5 50·90
76	19	20 0 19·10	143 45 7·15	109	26	22 18 40·12	151 17 32·16
77	21	20 2 37·21	144 1 34·91	110	27	22 21 12·01	151 13 41·00
78	22	20 5 21·31	144 11 3·54	111	28	22 23 53·75	151 32 26·79
79	23	20 6 52·39	144 14 56·40	112	Mar. 1	22 25 40·68	151 40 32·09
80	24	20 9 19·55	144 29 51·84	113	2	22 27 25·33	151 43 52·49
81	27	20 16 29·43	144 39 3·92	114	4	22 30 53·99	151 57 57·73
82	28	20 17 17·15	144 45 47·54				

V. *A Catalogue of the Positions and Distances of* 398 *Double Stars.*

By Lord WROTTESLEY.

Read May 11, 1860.

THE positions and distances of the stars contained in the subjoined Catalogue are the result of observations made by my assistants, Messrs. SIMMS, PHILPOTT, and FREDERIC MORTON; but the far greater number of the stars have been observed since the beginning of the year 1854 by Mr. MORTON, to whose industry, ability, and skill, I can bear ample testimony. He is a good observer and an accurate computer.

The telescope employed is the equatoreal mentioned in my former communication, vol. xxiii., as forming part of the furniture of my observatory. It is 10 feet 9 inches in focal length, with a clear aperture of 7¼ inches. The flint glass is by GUINAND and the crown glass by DOLLOND. The lenses are separated by three pieces of tinfoil, and there is reason to fear that the performance of the object-glass has been somewhat impaired by irregular pressure and pinching of the two lenses. Since this was written the Astronomer Royal has seen the telescope, and suggested a remedy for the defect complained of, by removing the pieces of tinfoil, and substituting other pieces in their stead, of sufficient length to lap round and enclose the rim of the crown lens at three equidistant points, and so prevent any undue pinching of the lenses when the ring for confining the object-glass in the cell is screwed home. This method has been adopted, and there is good reason to believe with success.

The mounting is in the usual English form. The polar axis is formed of four mahogany planks, 14 feet 3 inches long and 10 inches square in the middle, and the pivots are of hard bell-metal, supported above and below on massive stone piers, resting on a solid foundation. The declination and hour-circles are each 3 feet in diameter, and their verniers read to 10″ in space and to seconds of time.

A wire-micrometer only has been employed; the screw-heads of which are divided into 100 parts, the value of each of which is 0″·1564; and the position circle of 4 inches diameter is graduated on silver, with a vernier reading to 6′ in space.

From December 8th, 1854, a slipping piece has been used.

The micrometer was at first provided with one equatoreal fixed and two vertical moveable wires; and the observations of position were then made by bisecting the stars on the single equatoreal wire; but from February 27, 1859, two very thick wires, about 12″ apart, have been substituted for the single one, and the observations of position have since that period been made in the usual manner by causing the imaginary line joining the two stars, placed between the broad wires, to be parallel to each, care being taken to test that parallelism by alternate comparisons with each wire.

The power generally employed was 450, but one of 320 was often used, and occasionally for close stars 600 or 820.

The telescope is driven by clock-work. Its performance, however, is not satisfactory, and, but for the assistance of the slipping-piece, might have been a serious impediment. Neither is the instrument itself as stable as might have been anticipated from the precautions adopted to ensure such a result.* Yet with these drawbacks it is quite capable of contending with others of its own class; and in a favourable and calm state of the atmosphere its power of showing faint companions leaves nothing to be desired.

As a general rule, ten measures, both in position and distance, were made on the same night of each object, whenever they could be obtained; but from March 1856 to October 1858 the practice was to take six measures only of each element. The ten or six observations, as the case may be, are called a set. There are a very few instances in which more than one set has been obtained of the same star on the same night. With one exception, that of *η Draconis*, all observations are rejected when less than four measures are obtained.

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* This stability has been lately much improved by introducing a solid block of brass, secured by steady pins, beneath the sliding piece which clamps the hour-circle; also two cheeks of brass, each firmly attached by three screws to the block, on each side of that sliding-piece.

The zero of the position-circle is ascertained at intervals of about three months, but its variations are very small and much within the error of observation in position. An equal number of positive and negative readings is taken for distance, and therefore no zero is required.

The individual measures are registered on printed skeleton forms, previously prepared for the use of the Observatory, bound up in volumes; and all the reductions have been made on the opposite and blank side of the form.

Anterior to the commencement of the year 1857, the probable errors and weights were computed by the usual formulæ, and on the assumption that there could be no constant cause of error affecting the mean results obtained on each separate night; but before the year 1851 I had discovered grounds for distrusting the truth of this hypothesis. "It appeared that the differences between the mean positions and distances obtained on different nights were greater than would have been anticipated by one who had merely computed the probable error of a single measure in the usual manner from the data furnished by the sums of the squares of the partial differences from the mean." For a full discussion of this question I must refer to a communication addressed by me to the Royal Society, and printed in the 10th volume of its *Proceedings*, p. 133, from which the above paragraph is quoted. It is sufficient to mention in this place, that from the beginning of 1857 the final means have been obtained and the probable errors and weights computed by considering all the observations of the star on the different nights to have been made on the same night; that is to say, all the partial observations contained in each set are grouped together as if they constituted one set only, and the sum of the squares of all their differences from the general mean of the whole is used in computing the probable error and weight of such general mean. Since 1857, therefore, the probable error and weight of the mean result of each separate night has not been computed; but inasmuch as anterior to that period, before the mode of proceeding was altered, they had been regularly obtained, they are inserted in the Catalogue, although entitled to little weight, considered as a mathematical result.

All the computations have been carefully examined either by myself or by my assistants.

The letters "M*m*" in the column headed "Power" denote that the observation to which they are attached was made with a Macro-micro or Barlow lens, by which the power is increased in the ratio of 2·1361 to 1.

The magnitudes have been carefully estimated, but not by reference to any scale. Mr. MORTON doubts whether he possesses the power of distinguishing colours with great accuracy.

The working Catalogue of Stars for observation was selected from STRUVE's great Catalogue of 1837; and contained, 1st, stars confessedly binary; 2d, others, as to which there was strong presumptive evidence of their being so; 3d, stars, of which the observations by different observers gave anomalous or discordant results. As these objects were worked out, more were added from time to time from among the brighter and closer double stars, and from those which had only been observed at one epoch. Some of these latter exhibit orbital motion.

The first of the subjoined Catalogues contains the results of the observations of each star made on separate nights; the second, the general mean of all the sets obtained in the same year. A Reference Catalogue is added, showing the names and places of the several stars; the latter are for the epoch 1 Jan. 1860.

In the first Catalogue, columns 5 and 9 contain the probable errors, and 6 and 10 the computed weights of position and distance respectively; in the column of Colours the following abbreviations are made use of:—

b. blue	brn. brown	g′. greenish	o. orange	v. very
b′. bluish	d. dark	gr. grey	p. pale	w. white
br. bright	g. green	l. light	r. red	y. yellow

In the second Catalogue, columns 7 and 12 contain the differences between this and STRUVE's Catalogue of 1837, the last epoch of which is used where there is more than one given, 5 and 10 the probable errors, and 6 and 11 the computed weights as before. The remaining columns require no explanation.

The Catalogues are followed by Notes directing attention to any particulars worthy of remark in reference to any star; but no notes have been made to such stars as *Castor*, γ *Virginis*, &c., which have received due attention from other observers, unless the fresh evidence seemed to require it.

No. I.—GENERAL CATALOGUE OF 398 DOUBLE STARS.

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
1	3	8.9–9.10	84° 23′	13′	6	6	4·719″	·044″	13	6	320	56·956	M	w ; b
2	7	9.10–10.11	213 7	17	4	8	1·295	·029	29	8	600	59·767	M	y′
3	12	6–8	151 37	8	17	8	11·995	·059	7	8	450	43·934	S	w
		6–9	150 46	12	7	10	11·741	·052	9	10	,,	·945	,,	w
		6–9	150 6	9	14	10	11·803	·014	119	10	,,	·958	,,	w
		7.8–10	149 29	10	11	10	11·682	·037	18	10	,,	44·901	P	w
		6.7–8	149 18	13	6	6	11·269	·056	8	6	,,	56·871	M	y′ ; b
		6.7–8	149 16	9	14	6	11·282	·011	189	6	,,	·917	,,	p y ; b
4	16	9–11	37 9			10	5·346			10	320	59·753	M	w ; b′
		8.9–11	37 42			10	5·396			10	320	·756	,,	w ; b
		8–11	37 20			10	5·478			10	450	·758	,,	w ; b
5	22	7–8	234 15			10	4·433			10	450	59·915	M	v p y ; v p b
		7–8	235 30			4	4·481			4	,,	·925	,,	y′
		7.8–8.9	235 20			10	4·342			10	,,	·931	,,	p y ; b′w
6	23	8–10.11	355 29	11	8	10	10·684	·018	80	10	190	54·939	M	w ; b ; a delicate object.
		8.9–10.11	356 41	17	3	6	10·866	·124	2	6	,,	56·956	,,	w ; y′ ?
7	24	8–8.9	247 17	9	13	6	5·166	·073	5	6	320	57·906	M	b′w ; p y
		8.9–9.10	247 39	8	16	6	4·947	·050	10	6	450	58·917	,,	w ; b′ w
8	40	6.7–9	312 40	9	12	4	11·352	·061	7	4	320	56·797	M	p y ; purple
		7–9.10	311 53	9	13	6	11·519	·076	4	6	,,	·805	,,	p y ; d b
9	42	8.9–10	33 33			6	5·448			6	320	57·906	M	w
		8.9–9.10	33 35			6	5·490			6	,,	·923	,,	w ; b′
10	46	6–10	194 2			6	6·244			6	320	58·767	M	br y ; purple
		6–9.10	193 44			10	6·402			10	,,	·923	,,	y ; dark b
11	48	9–9.10	332 21			6	5·412			6	320	58·676	M	w ; scarcely ½ mag. diff.
		8.9–9	332 37			6	5·565			6	,,	·723	,,	w ; b′ w

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	p.	w.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
			°	′	′			″	″						
12	59	8–9	146	19			6	2·091			6	450	58·600	M	w
		8–10	149	7			6	2·252			6	”	·723	”	w, b′
13	60	4–8	109	14	15	5	4	8·240	·024	44	4	320	54·931	M	v p y; plum colour
		4–8	110	49	11	8	10	7·991	·034	21	10	”	·966	”	p y; purp
		3.4–7.8	112	49	6	29	10	7·809	·029	30	10	”	55·928	”	br y; p b
		4–8	112	16	10	11	10	7·833	·045	12	10	”	·931	”	y; p b
		4–7.8	112	9	9	11	10	7·770	·030	28	10	”	56·025	”	y; b
		3.4–8	116	45			10	7·237			10	450	59·920	”	br y; d. red or purple
		3.4–8.9	117	20			10	6·927			10	”	·956	”	br y; r′ purp
14	61	6.7–6.7	298	14	13	6	10	4·451	·021	56	10	450	47·879	P	w
		6–6.7	296	1	13	6	10	4·556	·033	23	10	320	54·854	M	p y
		6–6.7	296	28	7	21	10	4·569	·009	278	10	”	·871	”	p y
		6–6.7	296	56	16	4	10	4·492	·039	16	10	”	·898	”	v p y
15	62	9.10–10	303	11	8	16	6	11·035	·131	1	6	320	56·917	M	w
		9.10–10	302	57	14	5	6	11·139	·062	6	6	”	·942	”	w
16	65	9.10–9.10	36	27			6	3·019			6	320	58·676	M	w
		9–9	36	41			6	2·930			6	450	·695	”	b′ w
17	73	6.7–7	340	54	44	1	4	1·311	·045	12	4	320	54·871	M	o; y
		6–6.7	340	11	13	6	10	1·353	·020	60	10	450 320	·936	”	d y
		6–6.7	339	14	17	4	10	1·085	·017	85	10	600	·939	”	y; p y
18	79	6.7–7.8	192	38	7	21	10	7·540	·043	13	10	450	47·994	P	p b
		6.7–7	192	26	5	33	10	7·744	·021	56	10	320	56·033	M	w; v p b
		7–7.8	192	33	7	19	10	7·695	·028	31	10	”	·036	”	w; b′ w
		6.7–7	192	39	7	20	10	7·687	·036	19	10	”	·038	”	w; v p b
19	97	9.10–10	99	43			10	4·548			10	320	59·756	M	w
		9.10–9.10M	98	47			10	4·633			10	”	·868	”	w
20	99	5–12 or 13	225	22	31	1	6	7·809	·096	3	6	190	56·871	M	golden o; d b
		5–12	226	6	28	1	6	7·519	·099	2	6	”	·914	”	o; purp
21	115	8–8	150	54	20	3	10	0·887	·016	98	10	600	55·931	M	w

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Dis-tance.	*p*.	*w*.	No. of Obs.	Power.	Epoch, 1800+	Observer.	Colours.
22	130	8.9–10	187° 48′	25′	2	6	7″·739	″·004	1887	6	320	57·923	M	y; b′
23	138	8.9–8.9	26 31	19	3	10	1·556	·015	116	10	450	59·758	M	y′ w
24	147	6–7	88 57	16	4	4	3·801	·132	1	4	320	56·956	M	p y; p b
		6–7	90 16	15	5	6	3·769	·061	7	6	190	·964	”	w; p b
25	154	9–9	127 7			6	5·143			6	320	57·906	M	w; A. judged the larger
		8.9–9	126 24			6	4·813			6	450	·942	”	w
26	155	7.8–8	331 1	13	6	4	4·963	·003	3378	4	320	55·972	M	w
		7.8–8	329 2	13	6	8	4·819	·053	9	8	”	56·027	”	w
		7–7.8	328 36	13	6	10	4·731	·032	24	10	”	·030	”	w
27	174	7.8–8.9	170 24			6	2·772			6	450	58·758	M	y; b
		7.8–8.9	169 46			10	2·800			10	”	·879	”	p y; p b
28	170	8.9–9.10	247 51	21	2	6	3·554	·051	9	6	320	58·758	M	w; b′
		9–10.11	248 42			10	3·319			10	450	59·849	”	r′; b′
		9–9.10	246 54			10	3·167			10	”	·906	”	w
29	178	8–8 M	196 9	9	12	10	3·208	·033	22	10	320	58·947	M	w
30	179	7.8–8	158 8	10	9	6	3·579	·011	222	6	450	56·849	M	w; b′
		7.8–8	158 51	28	1	6	3·555	·015	110	6	”	·912	”	w; p b
31	180	4–4.5	358 32	7	23	10	8·681	·045	12	10	190	56·036	M	v w
		4.5–5	359 3	6	31	10	8·740	·027	33	10	320	·038	”	w; b′ w
		4.5–4.5	359 37	3	82	10	8·748	·018	76	10	”	·079	”	v p y. No perceptible diff. in mags.
32	202	5–7.8	333 27	10	10	10	3·902	·013	152	10	450	44·027	S	w
		5.6–8	331 51	25	2	10	3·465	·013	156	10	”	·063	”	w
		5.6–8	330 15	7	24	10	3·513	·062	6	10	”	·126	”	r′
		4.5–5.6	325 52			6	3·433			6	”	58·676	M	p y; b′
		4–5	327 40			6	3·597			6	”	·723	”	p y; b
33	205	3–6	63 2			10	10·465			10	320	59·715	M	br orange; b
	$\frac{\text{A \& B+C}}{2}$	3–6.7	63 41			10	10·450			10	”	·753	”	br orange; b

No. I.— *General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
			°	′	′			″	″						
33	205	3–6.7	63	34			8	10·254			8	600	59·767	M	orange ; br b
	$\frac{\text{A \& B+C}}{2}$	3–7	62	30			10	10·195			10	450	·805	„	golden y; br b
34	218	8.9–9.10	247	22			6	4·794			6	320	58·745	M	p y; w
		9–9.10	247	21			10	4·847			10	„	·876	„	w ; p b
35	219	8.9–9.10	183	44	3	88	6	11·446	·031	25	6	320	56·914	M	w ; p b
		8.9–9.10	183	42	16	4	6	11·399	·021	57	6	„	·917	„	w ; b
36	221	8–10	145	53	15	5	10	8·565	·024	42	10	320	54·934	M	w ; d b
		8–9.10	145	13	11	8	10	8·274	·009	313	10	„	·939	„	p y; br b. A star 13th mag. in s. p. quad.
		8–9	143	43	10	9	10	8·274	·033	22	10	„	55·928	„	w ; p b
		8–9	143	24	14	5	10	8·179	·035	20	10	450 320	·931	„	w ; b
		8–9.10	144	18	18	3	10	8·316	·038	17	10	320	56·025	„	w, b. One measure of star 12th mag. gave ∠=230±
37	227	5–6	79	0	17	4	6	3·798	·030	27	6	450	53·011	*M	b
		5–6	78	46	13	6	6	3·899	·028	31	6	„	·019	„	y ; l b
		5–6.7	78	12	9	12	10	3·889	·026	35	10	„	·977	„	y ; l y
		5–6	78	20	17	4	8	3·819	·017	88	8	„	54·049	„	y ; orange
		5–7	79	36	22	2	10	3·678	·054	8	10	„	·088	C. W. and M.*	y ; orange
38	231	7–9	229	50	10	11	10	15·203	·052	9	10	320	54·936	M	p y ; br b
		7–8.9	230	22	7	20	10	15·222	·030	28	10	320 190	·966	„	p y ; b
		6.7–8.9	229	15	7	20	10	15·830	·021	57	10	140 M m	55·036	„	p y ; b
39	232	8.9–8.9	65	32			6	6·361			6	320	57·906	M	w
		8.9–8.9	65	28			6	6·249			6	450	58·008	„	w
40	238	9.10–10	357	59	10	9	6	10·789	·017	83	6	320	57·038	M	w ; b′
		10–10.11	357	31	12	7	6	10·644	·000	∞	6	„	·071	„	w ; p b

* The initials C. W. stand for "Cameron Wrottesley;" the other initials require no explanation.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Dis-tance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
			°	′	′			″	″						
41	240	8–8.9	49	5	9	12	8	4·783	·040	15	8	320	55·038	M	w
		8.9–8.9	48	24	13	6	6	5·091	·101	2	6	„	56·956	„	w
		8–8	49	29	14	5	6	4·977	·038	17	6	„	·964	„	v p b
42	245	7–8	292	20	16	4	6	10·781	·014	132	6	320	56·813	M	w ; v p b
		7–8	292	2	10	10	6	10·803	·033	22	6	„	819	„	p y ; g′
43	246	8–9.10	122	46	19	3	6	9·952	·143	1	6	320	56·964	M	p y ; v b
		8–9.10	122	31	7	19	6	9·799	·048	10	6	„	57·030	„	y′; b
44	262	5–8	263	35	9	11	10	2·090	·034	22	10	450	56·030	M	w ; b
	A & B	4.5–7.8	265	36	16	4	10	1·927	·018	75	10	„	·033	„	v p y ; b
		5–8	266	1	16	4	10	2·016	·016	100	10	320	·038	„	p y ; b′
45	A & C	4.5–9.10	107	46			6	7·265			6	320	59·821	M	y ; b
		5–10	107	52			7	7·755			6	„	·835	„	p y ; r′
		5.6–10.11	107	54			10	7·472			10	„	·854	„	y ; r′
46	268	7.8–10	127	25			10	2·753			10	600	59·756	M	b′ w ; p b
		8–10	127	50			10	2·585			10	450	·808	„	p y ; b′
47	269	7.8–9.10	346	21	16	4	10	2·090	·017	83	10	320	56·033	M	w ; p b
		8–9.10	344	34	16	4	10	1·996	·021	53	10	„	·071	„	w ; b
		8–10	344	41	13	6	10	2·021	·058	7	10	„	·077	„	w ; b
48	282	9–9	115	8	18	3	6	6·815	·071	5	6	320	57·923	M	w
49	291	7.8–8	119	16			10	3·391			10	320	58·947	M	b′ w
		7.8–8	118	13			4	3·285			4	„	59·000	„	w
		8.9–9	119	37			6	3·511			6	„	·006	„	b′ w
50	301	7.8–8	14	32	6	30	6	8·136	·034	21	6	320	56·805	M	w ; b
		8–9.10	15	30			10	8·107			10	320 190	59·049	„	p y ; g′ b
		8–10	15	36			10	8·021			10	320	·074	„	p y ; b
51	311	6.7–9	121	6	14	5	6	3·324	·079	4	6	450	56·849	M	p y ; b
	A & B	6.7–9	121	26	32	1	6	3·222	·016	94	6	320	·860	„	w ; p b
52	A & C	6–12	108	49	15	5	10	25·166	·098	3	10	190	59·868	M	p y ; b

hour-circles are each 3 feet in diameter, and their verniers read to 10″ in space and to seconds of time.

A wire-micrometer only has been employed; the screw-heads of which are divided into 100 parts, the value of each of which is 0″·1564; and the position circle of 4 inches diameter is graduated on silver, with a vernier reading to 6′ in space.

From December 8th, 1854, a slipping piece has been used.

The micrometer was at first provided with one equatoreal fixed and two vertical moveable wires; and the observations of position were then made by bisecting the stars on the single equatoreal wire; but from February 27, 1859, two very thick wires, about 12″ apart, have been substituted for the single one, and the observations of position have since that period been made in the usual manner by causing the imaginary line joining the two stars, placed between the broad wires, to be parallel to each, care being taken to test that parallelism by alternate comparisons with each wire.

The power generally employed was 450, but one of 320 was often used, and occasionally for close stars 600 or 820.

The telescope is driven by clock-work. Its performance, however, is not satisfactory, and, but for the assistance of the slipping-piece, might have been a serious impediment. Neither is the instrument itself as stable as might have been anticipated from the precautions adopted to ensure such a result.* Yet with these drawbacks it is quite capable of contending with others of its own class; and in a favourable and calm state of the atmosphere its power of showing faint companions leaves nothing to be desired.

As a general rule, ten measures, both in position and distance, were made on the same night of each object, whenever they could be obtained; but from March 1856 to October 1858 the practice was to take six measures only of each element. The ten or six observations, as the case may be, are called a set. There are a very few instances in which more than one set has been obtained of the same star on the same night. With one exception, that of *η Draconis*, all observations are rejected when less than four measures are obtained.

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* This stability has been lately much improved by introducing a solid block of brass, secured by steady pins, beneath the sliding piece which clamps the hour-circle; also two cheeks of brass, each firmly attached by three screws to the block, on each side of that sliding-piece.

The zero of the position-circle is ascertained at intervals of about three months, but its variations are very small and much within the error of observation in position. An equal number of positive and negative readings is taken for distance, and therefore no zero is required.

The individual measures are registered on printed skeleton forms, previously prepared for the use of the Observatory, bound up in volumes; and all the reductions have been made on the opposite and blank side of the form.

Anterior to the commencement of the year 1857, the probable errors and weights were computed by the usual formulæ, and on the assumption that there could be no constant cause of error affecting the mean results obtained on each separate night; but before the year 1851 I had discovered grounds for distrusting the truth of this hypothesis. "It appeared that the differences between the mean positions and distances obtained on different nights were greater than would have been anticipated by one who had merely computed the probable error of a single measure in the usual manner from the data furnished by the sums of the squares of the partial differences from the mean." For a full discussion of this question I must refer to a communication addressed by me to the Royal Society, and printed in the 10th volume of its *Proceedings*, p. 133, from which the above paragraph is quoted. It is sufficient to mention in this place, that from the beginning of 1857 the final means have been obtained and the probable errors and weights computed by considering all the observations of the star on the different nights to have been made on the same night; that is to say, all the partial observations contained in each set are grouped together as if they constituted one set only, and the sum of the squares of all their differences from the general mean of the whole is used in computing the probable error and weight of such general mean. Since 1857, therefore, the probable error and weight of the mean result of each separate night has not been computed; but inasmuch as anterior to that period, before the mode of proceeding was altered, they had been regularly obtained, they are inserted in the Catalogue, although entitled to little weight, considered as a mathematical result.

All the computations have been carefully examined either by myself or by my assistants.

The letters "M*m*" in the column headed "Power" denote that the observation to which they are attached was made with a Macro-micro or Barlow lens, by which the power is increased in the ratio of 2·1361 to 1.

The magnitudes have been carefully estimated, but not by reference to any scale. Mr. MORTON doubts whether he possesses the power of distinguishing colours with great accuracy.

The working Catalogue of Stars for observation was selected from STRUVE'S great Catalogue of 1837; and contained, 1st, stars confessedly binary; 2d, others, as to which there was strong presumptive evidence of their being so; 3d, stars, of which the observations by different observers gave anomalous or discordant results. As these objects were worked out, more were added from time to time from among the brighter and closer double stars, and from those which had only been observed at one epoch. Some of these latter exhibit orbital motion.

The first of the subjoined Catalogues contains the results of the observations of each star made on separate nights; the second, the general mean of all the sets obtained in the same year. A Reference Catalogue is added, showing the names and places of the several stars; the latter are for the epoch 1 Jan. 1860.

In the first Catalogue, columns 5 and 9 contain the probable errors, and 6 and 10 the computed weights of position and distance respectively; in the column of Colours the following abbreviations are made use of:—

b. blue
b′. bluish
br. bright
brn. brown
d. dark
g. green
g′. greenish
gr. grey
l. light
o. orange
p. pale
r. red
v. very
w. white
y. yellow

In the second Catalogue, columns 7 and 12 contain the differences between this and STRUVE'S Catalogue of 1837, the last epoch of which is used where there is more than one given, 5 and 10 the probable errors, and 6 and 11 the computed weights as before. The remaining columns require no explanation.

The Catalogues are followed by Notes directing attention to any particulars worthy of remark in reference to any star; but no notes have been made to such stars as *Castor*, γ *Virginis*, &c., which have received due attention from other observers, unless the fresh evidence seemed to require it.

No. I.—General Catalogue of 398 Double Stars.

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Dis-tance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
1	3	8.9–9.10	84°	23′	13′	6	6	4″·719	″·044	13	6	320	56·956	M	w; b
2	7	9.10–10.11	213	7	17	4	8	1·295	·029	29	8	600	59·767	M	y′
3	12	6–8	151	37	8	17	8	11·995	·059	7	8	450	43·934	S	w
		6–9	150	46	12	7	10	11·741	·052	9	10	,,	·945	,,	w
		6–9	150	6	9	14	10	11·803	·014	119	10	,,	·958	,,	w
		7.8–10	149	29	10	11	10	11·682	·037	18	10	,,	44·901	P	w
		6.7–8	149	18	13	6	6	11·269	·056	8	6	,,	56·871	M	y′; b
		6.7–8	149	16	9	14	6	11·282	·011	189	6	,,	·917	,,	p y; b
4	16	9–11	37	9			10	5·346			10	320	59·753	M	w; b′
		8.9–11	37	42			10	5·396			10	320	·756	,,	w; b
		8–11	37	20			10	5·478			10	450	·758	,,	w; b
5	22	7–8	234	15			10	4·433			10	450	59·915	M	v p y; v p b
		7–8	235	30			4	4·481			4	,,	·925	,,	y′
		7.8–8.9	235	20			10	4·342			10	,,	·931	,,	p y; b′w
6	23	8–10.11	355	29	11	8	10	10·684	·018	80	10	190	54·939	M	w; b; a delicate object.
		8.9–10.11	356	41	17	3	6	10·866	·124	2	6	,,	56·956	,,	w; y′?
7	24	8–8.9	247	17	9	13	6	5·166	·073	5	6	320	57·906	M	b′w; p y
		8.9–9.10	247	39	8	16	6	4·947	·050	10	6	450	58·917	,,	w; b′ w
8	40	6.7–9	312	40	9	12	4	11·352	·061	7	4	320	56·797	M	p y; purple
		7–9.10	311	53	9	13	6	11·519	·076	4	6	,,	·805	,,	p y; d b
9	42	8.9–10	33	33			6	5·448			6	320	57·906	M	w
		8.9–9.10	33	35			6	5·490			6	,,	·923	,,	w; b′
10	46	6–10	194	2			6	6·244			6	320	58·767	M	br y; purple
		6–9.10	193	44			10	6·402			10	,,	·923	,,	y; dark b
11	48	9–9.10	332	21			6	5·412			6	320	58·676	M	w; scarcely ½ mag. diff.
		8.9–9	332	37			6	5·565			6	,,	·723	,,	w; b′ w

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
12	59	8–9	146° 19′	′		6	2″·091	″		6	450	58·600	M	w
		8–10	149 7			6	2·252			6	,,	·723	,,	w, b′
13	60	4–8	109 14	15	5	4	8·240	·024	44	4	320	54·931	M	v p y; plum colour
		4–8	110 49	11	8	10	7·991	·034	21	10	,,	·966	,,	p y; purp
		3.4–7.8	112 49	6	29	10	7·809	·029	30	10	,,	55·928	,,	br y; p b
		4–8	112 16	10	11	10	7·833	·045	12	10	,,	·931	,,	y; p b
		4–7.8	112 9	9	11	10	7·770	·030	28	10	,,	56·025	,,	y; b
		3.4–8	116 45			10	7·237			10	450	59·920	,,	br y; d. red or purple
		3.4–8.9	117 20			10	6·927			10	,,	·956	,,	br y; r′ purp
14	61	6.7–6.7	298 14	13	6	10	4·451	·021	56	10	450	47·879	P	w
		6–6.7	296 1	13	6	10	4·556	·033	23	10	320	54·854	M	p y
		6–6.7	296 28	7	21	10	4·569	·009	278	10	,,	·871	,,	p y
		6–6.7	296 56	16	4	10	4·492	·039	16	10	,,	·898	,,	v p y
15	62	9.10–10	303 11	8	16	6	11·035	·131	1	6	320	56·917	M	w
		9.10–10	302 57	14	5	6	11·139	·062	6	6	,,	·942	,,	w
16	65	9.10–9.10	36 27			6	3·019			6	320	58·676	M	w
		9–9	36 41			6	2·930			6	450	·695	,,	b′ w
17	73	6.7–7	340 54	44	1	4	1·311	·045	12	4	320	54·871	M	o; y
		6–6.7	340 11	13	6	10	1·353	·020	60	10	450 320	·936	,,	d y
		6–6.7	339 14	17	4	10	1·085	·017	85	10	600	·939	,,	y; p y
18	79	6.7–7.8	192 38	7	21	10	7·540	·043	13	10	450	47·994	P	p b
		6.7–7	192 26	5	33	10	7·744	·021	56	10	320	56·033	M	w; v p b
		7–7.8	192 33	7	19	10	7·695	·028	31	10	,,	·036	,,	w; b′ w
		6.7–7	192 39	7	20	10	7·687	·036	19	10	,,	·038	,,	w; v p b
19	97	9.10–10	99 43			10	4·548			10	320	59·756	M	w
		9.10–9.10M	98 47			10	4·633			10	,,	·868	,,	w
20	99	5–12 or 13	225 22	31	1	6	7·809	·096	3	6	190	56·871	M	golden o; d b
		5–12	226 6	28	1	6	7·519	·099	2	6	,,	·914	,,	o; purp
21	115	8–8	150 54	20	3	10	0·887	·016	98	10	600	55·931	M	w

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	p.	w.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
22	130	8.9–10	187° 48′	25′	2	6	7″·739	″·004	1887	6	320	57·923	M	y; b′
23	138	8.9–8.9	26 31	19	3	10	1·556	·015	116	10	450	59·758	M	y′ w
24	147	6–7	88 57	16	4	4	3·801	·132	1	4	320	56·956	M	p y; p b
		6–7	90 16	15	5	6	3·769	·061	7	6	190	·964	„	w; p b
25	154	9–9	127 7			6	5·143			6	320	57·906	M	w; A. judged the larger
		8.9–9	126 24			6	4·813			6	450	·942	„	w
26	155	7.8–8	331 1	13	6	4	4·963	·003	3378	4	320	55·972	M	w
		7.8–8	329 2	13	6	8	4·819	·053	9	8	„	56·027	„	w
		7–7.8	328 36	13	6	10	4·731	·032	24	10	„	·030	„	w
27	174	7.8–8.9	170 24			6	2·772			6	450	58·758	M	y; b
		7.8–8.9	169 46			10	2·800			10	„	·879	„	p y; p b
28	170	8.9–9.10	247 51	21	2	6	3·554	·051	9	6	320	58·758	M	w; b′
		9–10.11	248 42			10	3·319			10	450	59·849	„	r′; b′
		9–9.10	246 54			10	3·167			10	„	·906	„	w
29	178	8–8 M	196 9	9	12	10	3·208	·033	22	10	320	58·947	M	w
30	179	7.8–8	158 8	10	9	6	3·579	·011	222	6	450	56·849	M	w; b′
		7.8–8	158 51	28	1	6	3·555	·015	110	6	„	·912	„	w; p b
31	180	4–4.5	358 32	7	23	10	8·681	·045	12	10	190	56·036	M	v w
		4.5–5	359 3	6	31	10	8·740	·027	33	10	320	·038	„	w; b′w
		4.5–4.5	359 37	3	82	10	8·748	·018	76	10	„	·079	„	v p y. No perceptible diff. in mags.
32	202	5–7.8	333 27	10	10	10	3·902	·013	152	10	450	44·027	S	w
		5.6–8	331 51	25	2	10	3·465	·013	156	10	„	·063	„	w
		5.6–8	330 15	7	24	10	3·513	·062	6	10	„	·126	„	r′
		4.5–5.6	325 52			6	3·433			6	„	58·676	M	p y; b′
		4–5	327 40			6	3·597			6	„	·723	„	p y; b
33	205	3–6	63 2			10	10·465			10	320	59·715	M	br orange; b
	$\frac{A\ \&\ B+C}{2}$	3–6.7	63 41			10	10·450			10	„	·753	„	br orange; b

No. I.— *General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
33	205	3–6.7	63° 34′	′		8	10″·254	″		8	600	59·767	M	orange; br b
	$\frac{\text{A \& B+C}}{2}$	3–7	62 30			10	10·195			10	450	·805	„	golden y; br b
34	218	8.9–9.10	247 22			6	4·794			6	320	58·745	M	p y; w
		9–9.10	247 21			10	4·847			10	„	·876	„	w; p b
35	219	8.9–9.10	183 44	3	88	6	11·446	·031	25	6	320	56·914	M	w; p b
		8.9–9.10	183 42	16	4	6	11·399	·021	57	6	„	·917	„	w; b
36	221	8–10	145 53	15	5	10	8·565	·024	42	10	320	54·934	M	w; d b
		8–9.10	145 13	11	8	10	8·274	·009	313	10	„	·939	„	p y; br b. A star 13th mag. in s. p. quad.
		8–9	143 43	10	9	10	8·274	·033	22	10	„	55·928	„	w; p b
		8–9	143 24	14	5	10	8·179	·035	20	10	450 320	·931	„	w; b
		8–9.10	144 18	18	3	10	8·316	·038	17	10	320	56·025	„	w, b. One measure of star 12th mag. gave ∠ = 230 ±
37	227	5–6	79 0	17	4	6	3·798	·030	27	6	450	53·011	M	b
		5–6	78 46	13	6	6	3·899	·028	31	6	„	·019	„	y; l b
		5–6.7	78 12	9	12	10	3·889	·026	35	10	„	·977	„	y; l y
		5–6	78 20	17	4	8	3·819	·017	88	8	„	54·049	„	y; orange
		5–7	79 36	22	2	10	3·678	·054	8	10	„	·088	C. W. and M.*	y; orange
38	231	7–9	229 50	10	11	10	15·203	·052	9	10	320	54·936	M	p y; br b
		7–8.9	230 22	7	20	10	15·222	·030	28	10	320 190	·966	„	p y; b
		6.7–8.9	229 15	7	20	10	15·830	·021	57	10	140 Mm	55·036	„	p y; b
39	232	8.9–8.9	65 32			6	6·361			6	320	57·906	M	w
		8.9–8.9	65 28			6	6·249			6	450	58·008	„	w
40	238	9.10–10	357 59	10	9	6	10·789	·017	83	6	320	57·038	M	w; b′
		10–10.11	357 31	12	7	6	10·644	·000	∞	6	„	·071	„	w; p b

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* The initials C. W. stand for "Cameron Wrottesley;" the other initials require no explanation.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
			°	′	′			″	″						
41	240	8–8.9	49	5	9	12	8	4·783	·040	15	8	320	55·038	M	w
		8.9–8.9	48	24	13	6	6	5·091	·101	2	6	„	56·956	„	w
		8–8	49	29	14	5	6	4·977	·038	17	6	„	·964	„	v p b
42	245	7–8	292	20	16	4	6	10·781	·014	132	6	320	56·813	M	w; v p b
		7–8	292	2	10	10	6	10·803	·033	22	6	„	819	„	p y; g′
43	246	8–9.10	122	46	19	3	6	9·952	·143	1	6	320	56·964	M	p y; v b
		8–9.10	122	31	7	19	6	9·799	·048	10	6	„	57·030	„	y′; b
44	262	5–8	263	35	9	11	10	2·090	·034	22	10	450	56·030	M	w; b
	A & B	4.5–7.8	265	36	16	4	10	1·927	·018	75	10	„	·033	„	v p y; b
		5–8	266	1	16	4	10	2·016	·016	100	10	320	·038	„	p y; b′
45	A & C	4.5–9.10	107	46			6	7·265			6	320	59·821	M	y; b
		5–10	107	52			7	7·755			6	„	·835	„	p y; r′
		5.6–10.11	107	54			10	7·472			10	„	·854	„	y; r′
46	268	7.8–10	127	25			10	2·753			10	600	59·756	M	b′ w; p b
		8–10	127	50			10	2·585			10	450	·808	„	p y; b′
47	269	7.8–9.10	346	21	16	4	10	2·090	·017	83	10	320	56·033	M	w; p b
		8–9.10	344	34	16	4	10	1·996	·021	53	10	„	·071	„	w; b
		8–10	344	41	13	6	10	2·021	·058	7	10	„	·077	„	w; b
48	282	9–9	115	8	18	3	6	6·815	·071	5	6	320	57·923	M	w
49	291	7.8–8	119	16			10	3·391			10	320	58·947	M	b′ w
		7.8–8	118	13			4	3·285			4	„	59·000	„	w
		8.9–9	119	37			6	3·511			6	„	·006	„	b′ w
50	301	7.8–8	14	32	6	30	6	8·136	·034	21	6	320	56·805	M	w; b
		8–9.10	15	30			10	8·107			10	320 190	59·049	„	p y; g′ b
		8–10	15	36			10	8·021			10	320	·074	„	p y; b
51	311	6.7–9	121	6	14	5	6	3·324	·079	4	6	450	56·849	M	p y; b
	A & B	6.7–9	121	26	32	1	6	3·222	·016	94	6	320	·860	„	w; p b
52	A & C	6–12	108	49	15	5	10	25·166	·098	3	10	190	59·868	M	p y; b

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
53	331	6–7.8	85°	13′	′		10	12″·087	″		10	320	58·821	M	w ; b
		6–7·8	85	1			4	11·884			4	„	·835	„	w ; b
		6.7–8	84	51			10	12·125			10	450	·879	„	p y ; g′
54	333	6.7–7	199	49	54	0	4	0·967	·045	12	4	600	55·931	M	w
		6–6.7	196	22	20	3	10	0·965	·027	33	10	„	56·071	„	w
		6–7	198	42	13	6	10	1·026	·012	175	10	600 450	·079	„	w
55	349	9–10	319	29			6	5·959			6	450	58·695	M	p y ; p b
		9–10	321	20			6	6·155			6	„	·723	„	w ; p b
56	362	9.10–10	141	3			10	6·999			10	320	59·758	M	w
*		9–9.10	141	31			10	7·103			10	„	·808	„	y′
57	369	7.8–8.9	29	11	9	12	10	3·443	·026	35	10	450	56·090	M	w ; l b
		7.8–8.9	29	18	22	2	6	3·538	·021	55	6	„	·914	„	w ; b′
		7.8–8.9	28	35	19	3	6	3·444	·052	9	6	„	·917	„	b′ w ; b
58	376	8.9–9	250	27	11	8	6	6·809	·050	10	6	450	57·082	M	p y
		9–9	250	39			6	6·629			6	320	·906	„	w
		8–8	250	22			6	6·553			6	„	·942	„	b′ w
59	389	7.8–9.10	63	59			10	2·773			10	450	59·854	M	w ; b
		7.8–9.10	64	22			10	2·759			10	„	·865	„	p y ; r′
60	391	8–8.9	94	8	13	6	6	3·613	·011	222	6	450	57·036	M	p y ; y
†		7.8–8	94	17	10	11	6	3·782	·023	47	6	„	·038	„	p y ; r′?
61	394	7.8–8.9	162	12	10	11	10	6·668	·025	39	10	320	55·071	M	w ; b
		7.8–8.9	161	46	12	7	4	6·883	·017	83	4	190 Mm	·071	„	w ; b
		7.8–9	162	14	12	7	10	6·830	·016	94	10	„	·077	„	p y ; b
62	401	7–7.8	269	55	6	32	6	10·983	·021	57	6	320	56·860	M	w
		7–7.8	269	32	8	14	6	11·063	·051	10	6	„	·876	„	v p y ; w

* Scarcely half a magnitude difference. A number of stars in the field.

† The colour of "B" undecided.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
63	414	10–14	185° 31′	′		6	7″·246	″		6	190	58·745	M	w
		9.10–10	185 7			10	7·422			10	320	·876	„	w
64	419	7.8–7.8M	75 0	9	13	10	2·958	·042	14	10	320	59·071	M	w
65	425	8.9–8.9	101 56			6	2·772			6	450	58·758	M	y′ w
		9–9	101 11			10	2·656			10	„	·887	„	y′ w
66	427	7.8–8	208 51	8	14	6	6·496	·035	20	6	320	57·047	M	p y; l b
		7.8–8	209 15	11	9	6	6·663	·046	12	6	„	·052	„	w
67	443	9–9.10	44 32	19	3	6	8·504	·032	24	6	320	58·008	M	w
68	448	8–10.11	16 28	11	9	6	3·316	·012	164	6	320	57·071	M	p y; br b
		8–12	16 37	30	1	6	3·256	·033	23	6	450	·077	„	y; purp
69	455	8–8.9	167 23	18	3	6	11·953	·046	12	6	320	57·923	M	y′; b′
70	470	5–6.7	347 51			6	6·835			6	320	57·942	M	y; g′ b
		5–6	347 2			6	6·671			6	450	58·151	„	y; g′
71	471	3.4–8.9	9 13	8	16	10	8·943	·029	30	10	320	56·030	M	w; b
		4–8	10 10	11	8	10	8·846	·027	34	10	„	·038	„	v p y; purp
		3.4–8.9	10 18	12	8	10	8·870	·028	30	10	„	·077	„	w; br purp
72	479	7.8–8	127 32	11	9	6	7·168	·073	5	6	320	57·052	M	w; b′
	A & B	7.8–8	127 50	9	12	6	6·995	·061	7	6	450	·068	„	y′; p b
73	494	7.8–8	187 10	10	10	10	5·207	·056	8	10	320	56·030	M	v white
		7.8–8	187 24	5	41	10	4·997	·032	24	10	„	·038	„	w
		8–8.9	187 43	8	17	10	5·057	·018	79	10	450	·071	„	w
74	495	8–11.12	219 24			10	3·896			10	320	58·879	M	p y; r′ or purp
		8–11.12	219 24			10	3·865			10	450	59·101	„	p y; d b
75	516	6–10.11	149 1	15	5	10	6·178	·068	5	10	190	56·077	M	o; b
		5.6–11	149 31	17	4	10	6·575	·069	5	10	„	·079	„	o; b
		6–11	152 26	17	3	6	6·486	·018	73	6	320	·805	„	o; d b
		5.6–10.11	148 35	19	3	6	6·624	·062	6	6	„	·849	„	d y; b

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
76	523	7.8–10.11	162° 53′	6′	25	10	10″·365	″·033	23	10	190	56·088	M	w; b
		8–11	163 47	22	2	6	10·229	·119	2	6	320	·860	,,	w; b
		8.9–11.12	162 47	23	2	3	10·062	·161	1	4	190	57·030	,,	w; b
		8–11	162 2	16	4	6	9·909	·029	29	6	320	·049	,,	p y; b
77	535	7.8–9.10	345 46			10	1·476			10	600	59·956	M	w; b′
		8–11	346 11			10	1·725			10	450	60·082	,,	w; b
78	546	8.9–10.11	185 55	6	24	10	6·727	·041	15	10	190	55·120	M	y; b
		8.9–11	186 12	14	5	10	6·740	·039	16	10	320 190	·129	,,	w; br b
		8.9–10.11	186 37	7	21	10	6·679	·039	16	10	320	·131	,,	w; b
79	550	5–7	306 57	7	20	10	10·311	·005	909	10	450	47·208	P	p y; p b
		6–7	307 47	11	9	6	10·187	·070	5	6	,,	56·917	M	p y; w
		6.7–7.8	306 36	11	8	6	10·195	·032	24	6	190 320	·964	,,	y; b′
80	552	7.8–8	113 35	10	11	6	8·857	·023	47	6	320	56·805	M	w; b′
		6.7–7	114 8	5	38	6	8·790	·064	6	6	450	·914	,,	w
81	559	7.8–8	277 31	5	38	6	3·042	·034	21	6	450	57·071	M	clear w
		7.8–8	277 4	11	8	6	3·094	·002	7752	6	,,	·077	,,	w
82	570	7–8	258 52	11	9	10	13·056	·052	9	10	450	53·082	M	b
		7–8	258 33	9	13	6	12·823	·052	9	6	,,	54·063	,,	b
		7–8	259 6	5	41	10	12·612	·023	47	10	,,	·074	,,	p b; l b
		7–8	258 53	9	12	7	12·534	·017	83	6	,,	·140	,,	p b; l b
83	572	7.8–7.8	26 24	13	6	6	3·389	·030	27	6	450	57·077	M	w
		7.8–7.8	26 34	10	10	6	3·472	·030	27	6	,,	·085	,,	p y
84	574	10–12.13	310 34	29	1	6	3·902	·015	110	6	450	57.121	M	w; b
85	590	7.8–7.8	317 29	7	21	10	8·968	·024	41	10	450	45·126	P	y; w
		7.8–7.8	316 30	14	5	10	8·966	·050	10	10	,,	·129	,,	w; y
		7.8–7.8	316 3	21	2	10	9·230	·040	15	10	,,	·131	,,	w
		7–7	316 11	11	8	10	9·037	·034	21	10	,,	46·118	,,	b′
		6.7–7	316 59			10	9·202			10	320	59·758	M	p y; y′ w
		6–6.7	317 15			10	8·828			10	,,	60·014	,,	p y; w

No. I.— *General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
			° ′	′			″	″						
86	616	5.6–9.10	351 53	10	9	6	6·296	·037	18	6	320	57·052	M	o; r′
		6.7–10	353 6	8	15	6	6·206	·080	4	6	450	·068	,,	p y; r
87	648	7.8–8	72 37	12	7	10	5·146	·099	2	10	320 190	56·038	M	w; dusky
		8–9	73 16	12	7	6	4·900	·038	17	6	320	·079	,,	b′; b
		8.9–9.10	72 38	27	1	4	4·849	·111	2	4	,,	·088	,,	w; p b
88	653	4–8	226 38	9	13	6	14·942	·124	2	6	450	52·110	M	o; b
		5.6–8	225 7	6	24	10	14·554	·025	38	10	320	55·137	,,	y; b
		5–8	225 27	13	6	10	14·619	·023	48	10	140 190	·175	,,	w or p y; b
		5.6–8	225 25	5	35	10	14·773	·023	47	10	190	·178	,,	p y; b
89	654	5–9.10	60 44			6	6·932			6	320	57·906	M	br y; purp
		5–10	61 26			4	6·981			4	,,	58·060	,,	y; v b
		5–10.11	61 22			6	6·815			6	,,	·131	,,	gold y; violet
		5–9	63 39			10	7·128			10	,,	60·110	,,	br y; v br b
		5–9	64 27			10	6·974			10	,,	·162	,,	y; b
*90	655	4.5–12.13	335 26	17	3	10	13·459?			2	190	56·077	M	w; r?
		4.5–13	337 40	28	1	6	12·763	·090	3	6	,,	57·085	,,	y; d b
91	661	5.6–8.9	359 59	12	7	10	3·250	·026	36	10	320	57·906	M	y′; b′
92	666	8.9–8.9	72 51			10	3·059			10	450	59·915	M	w
		9–9	73 7			10	3·081			10	,,	60·082	,,	w
93	668	1–9	199 49	10	10	10	9·661	·028	31	10	450	49·205	P	o; p b
94	674	7.8–11	148 25	28	1	6	10·115	·142	1	6	320	57·033	M	w; b
		7.8–11.12	147 18	22	2	6	10·378	·049	10	6	,,	·038	,,	p y; d b
95	716	7–7.8	197 34	17	4	4	4·908	·087	3	4	450	57·071	M	silv w; b′
		7–8	197 38	9	13	6	5·044	·069	5	6	,,	·077	,,	w; y′
†96	718	8.9–9	74 47			10	7·630			10	450	58·879	M	w; b′ w
		9–9	74 0			10	7·680			10	320	·887	,,	w

* A very red star of the fifth mag. precedes by about 57ˢ; angle, 274° ±.

† Scarcely half a mag. difference.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	p.	w.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
97	730	7–7	139°	14'	9'	12	6	9″·439	″·067	5	6	450	56·917	M	w; g′
		7–7	140	2	8	15	6	9·604	·078	4	6	320	·964	„	w; b′ w
98	738	4–6	44	40	5	47	10	4·591	·010	233	10	450	52·145	M	b
		4–7	46	12	21	2	10	4·597	·069	5	10	„	·167	„	y; b
99	742	8–9	250	22	11	8	10	3·507	·011	192	10	320	58·947	M	w; b′
100	752	3–9	142	1	18	3	4	11·128	·003	3333	4	450	54·074	M	y; w
	A & B	3–9	141	59	10	11	10	11·112	·063	6	10	„	·107	„	y; w
		3–8	141	52	9	13	10	11·135	·037	18	10	„	·164	„	y; w
		3.4–7	141	57	11	8	10	11·003	·028	31	10	320 450	·167	„	y; w
101	753	6–9	267	55	8	15	10	11·978	·026	37	10	450	54·175	M	y; w
		5.6–8.9	268	10	6	31	10	12·026	·040	16	10	„	·186	„	y; w
		6–8	267	46	4	59	10	12·131	·023	46	10	320	55·167	„	p y; b
		6–8	267	45	9	12	10	12·076	·040	15	10	„	·178	„	p y; p b
*		5.6–8	267	30	4	69	10	12·115	·041	14	10	„	·181	„	p y; b
		5.6–9	268	6			10	11·810			10	„	58·225	„	ruddy; b
†		5–9	268	3			10	12·217			10	„	·230	„	y′; b
102	774	2–6	159	15	14	5	10	2·759	·011	200	10	450	53·140	M	orange; y
		2.3–6	150	24	15	5	10	3·217	·020	64	10	„	54·151	„	y; d y
		2.3–5.6	152	17	15	4	10	3·100	·039	16	10	„	·153	„	y; r′
		3–7	153	0	14	6	10	2·870	·007	500	10	„	·162	„	y
103	778	9.10–11	182	22	9	13	6	3·352	·039	16	6	450	57·121	M	p y; b
104	795	7–7 Min.	200	3			10	1·553			10	600	60·167	M	p y
		7–7	200	49			10	1·459			10	„	·175	„	y
105	796	8–9	61	26			10	3·637			10	450?	59·074	M	w; b′
		8–9	60	58			10	3·641			10	450	·101	„	w; p b
		7.8–9	61	18			30	3·640			30	„	·183	„	w; b′ w
		7.8–9	60	29			10	3·576			10	600	·186	„	w; p b

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* A small star of about 14th Mag. in s. f. quadrant. One measure of position gave 114°. The distance about double of the one observed.

† One measure of "C" gave Pos. = 112° ± Dist. = 30″ ±.

No. I.—***General Catalogue of 398 Double Stars (continued).***

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
*106	825	8.9–9.10	145° 3′	16′	4	6	8″·063	″·051	9	6	320	57·033	M	w; p b
		8–9	146 15	8	15	6	8·204	·105	2	6	,,	·159	,,	w; b
107	845	5.6–7	352 25	9	11	10	7·730	·020	60	10	450	54·208	M	y
		6.7–7.8	354 8			10	8·057			10	320	59·758	,,	w; grey
		7–8	353 59			10	7·980			10	,,	·808	,,	w; dusky
108	872	8–9	217 24	9	12	10	10·794	·023	48	10	450	46·192	P	w
		8–9	217 25	8	15	10	10·859	·020	65	10	,,	·194	,,	w
		7.8–8.9	216 44	6	33	8	10·879	·032	24	8	,,	·208	,,	w
		6.7–7	218 10	10	9	10	10·744	·034	22	10	,,	54·167	M	y; orange
		6–7	218 20	6	24	10	10·739	·041	15	10	,,	·170	,,	y
		6.7–7	217 11	8	15	10	10·717	·034	21	10	,,	·197	,,	y′ w; w
109	877	7.8–8	261 54	6	29	10	5·654	·057	8	10	320	55·120	M	w
		7.8–8	262 4	9	13	10	5·540	·011	217	10	190 Mm	·120	,,	w
		8–8.9	262 11	10	10	10	5·553	·032	24	10	320	·129	,,	w
		7.8–8	262 7	9	12	10	5·527	·020	60	10	190 Mm	·131	,,	w
		8–8.9	262 41	8	17	10	5·553	·039	17	10	320	·137	,,	b′ w
		8–8.9	262 32	8	17	10	5·564	·014	130	10	190 Mm	·230	,,	b′ w; v p b
110	900	6.7–8.9	25 21	6	26	10	13·721	·059	7	10	450	45·225	P	y; w
		6.7–8.9	25 32	5	40	10	13·906	·045	12	10	,,	·249	,,	y; w
		6.7–8.9	25 17	7	19	10	13·829	·030	27	10	,,	·252	,,	y; w
		6–8	25 6	9	14	10	13·999	·052	9	10	,,	46·233	,,	y; w
		6–8	25 17	5	49	10	13·848	·054	8	10	,,	·268	,,	y; w
111	918	9–9.10	324 52			10	4·592			10	450 320	59·854	M	w; g′
		9–10.11	324 0			10	4·403			10	450	60·063	,,	w; b′
112	919 A & B	4–5	132 1	9	13	10	7·371	·023	45	10	450	52·178	M	b
		5–6	132 47	18	3	6	7·293	·058	7	6	,,	·183	,,	y; b
		6–7	131 45	9	11	10	7·623	·029	30	10	,,	·189	,,	y
		5–5.6	132 53	8	16	10	7·193	·027	33	10	,,	54·197	,,	w; b′ w

*** A star of 13th Mag. in s. f. quadrant.**

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
112	919	5.6–6.7	133° 30′	6′	29	10	7″·212	″·046	12	10	450	54·203	M	l y
	A & B	6–6.7	132 53	8	16	10	7·167	·022	52	10	„	·205	„	w
113	B & C	6.7–6.7	104 23			10	2·609			10	600	60·107	M	p y
		7–7	105 12			4	2·515			4	450	·159	„	g′ y
		6.7–6.7	104 52			6	2·678			6	600	·162	„	p y
114	928	9–10	133 35	8	17	6	3·483	·018	74	6	320	59·003	M	w ; b
115		6–8	262 56			10	16·770			10	320	59·956	M	golden o; b
		6–8.9	262 46			10	16·812			10	„	60·014	„	deep y; b′
116	948	6–7	141 9	26	2	10	1·972	·011	208	10	450	54·189	M	w ; y′
	A & B	6–6.7	142 57	25	2	10	1·841	·029	29	10	„	·222	„	v p y; p y
		6–6.7	141 58	14	5	10	1·788	·018	80	10	„	·225	„	w
		6–6.7	140 33			6	1·541			6	600	58·222	„	y
		6–6.7	139 57			6	1·567			6	450	·271	„	y
117	A & C	6–7.8	305 31	6	32	10	8·554	·021	58	10	450	54·225	M	w ; l b
		6–7.8	305 13	10	10	10	8·587	·027	33	10	„	·235	„	v p y; br b
		5.6–7	305 46	8	15	10	8·543	·009	294	10	„	·246	„	p y; l b
		6–7	305 4			6	8·473			6	600	58·222	„	y ; b
		6–7.8	305 14			6	8·662			6	450	·271	„	y ; b
118	958	6.7–6.7	259 18	11	8	6	4·935	·023	47	6	320	56·244	M	w
		6.7–6.7	258 37	20	3	6	4·888	·014	132	6	„	·246	„	w, or p y
119	979	8.9–9.10	211 2	10	11	6	7·307	·112	2	6	320	56·186	M	w ; b′
120	982	6–10.11	171 11	17	4	10	6·184	·054	8	10	450	45·214	P	o ; w
		6–10	171 33	14	5	10	6·129	·046	12	10	„	·225	„	o ; w
		6–10	171 49	12	7	10	6·179	·037	18	10	„	·257	„	y ; w
		6.7–10	170 31	12	7	10	6·148	·033	22	10	„	46·159	„	y; br b
		6.7–10	170 12	13	6	10	6·209	·039	16	10	„	·178	„	y ; b
		6.7–10	169 31	13	6	10	6·087	·043	13	10	„	·192	„	o ; b
		5–8	169 56	14	5	10	6·266	·020	65	10	„	52·194	M	y ; l b
		6–8.9	170 3	9	14	10	5·884	·030	27	10	„	54·107	„	y ; l b

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's* No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
120	982	5–9	168° 32′	6′	29	10	6″·116	″·017	86	10	450	54·162	M	y ; w
		5.6–8.9	168 18	15	5	10	5·995	·029	30	10	„	·170	„	l y ; w
121	991	8–9.10	168 27	15	4	6	3·999	·047	11	6	320	57·906	M	w ; b
122	1009	6.7–7	155 25			10	3·163			10	450	59·186	M	w
		7–7	157 1			10	3·103			10	„	·244	„	w ; p y
123	1027	9–10	355 44			10	7·032			10	320	58·876	M	w
		10–10	357 6			10	7·297			10	„	·887	„	w
124	1037	8.9–8.9	324 25			4	1·240			4	600	60·063	M	y
		8–8.9	325 15			6	1·416			6	450	·085	„	w
		7.8–8	325 12			10	1·408			10	600	·167	„	p y
125	1062	5–6.7	313 1	12	7	6	15·090	·071	5	6	320	54·164	M	p y ; w
		5.6–6.7	313 1	5	36	10	14·709	·031	25	10	450	·186	„	y
		5.6–6.7	313 21	6	26	10	14·615	·012	175	10	„	·192	„	l y; brn′ w
126	1083	7–8	44 42	10	9	10	6·528	·022	52	10	450	53·189	M	b
		7–8	45 35	21	2	4	6·566	·024	44	4	{450 320}	54·153	„	b ; w
		7–8	46 0	10	10	10	6·320	·021	53	10	450	·170	„	w
		7–8	45 52	10	9	10	6·258	·019	71	10	„	·175	„	w
127	*1102	7.8–9.10	48 45	4	74	4	7·661	·166	1	4	450	54·208	M	w
	A & B	8.9–10.11	47 36	16	4	10	7·431	·027	34	10	320	55·129	„	w ; p b
128	1110	3–4	249 33	8	14	10	5·280	·028	31	10	450	52·178	M	y
		3–3.4	244 32			6	5·165			6	820	58·219	„	y
		2.3–3	244 18			6	5·251			6	600	·298	„	y
		2.3–3.4	244 2			10	5·141			10	„	59·260	„	y ; g′ y
		2.3–3.4	243 44			10	5·171			10	„	·263	„	y ; g′ y
		2.3–3.4	243 31			10	5·526			10	320	·758	„	p y
		2.3–3.4	244 6			4	5·279			4	450	·915	„	y′ w
129	1123	11–11.12	159 54	30	1	6	3·590	·033	22	6	320	57·052	M	w ; p b
		10.11–11	160 44	10	9	6	3·633	·020	63	6	450	·121	„	w ; b′

* A quadruple star.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	p.	ϖ.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
*130		1.2–9	83°	39′	5′	46	4	327″·574	″·267	0	4	190	57·192	M	w; y
131	1126	8–8.9	140	0	16	4	10	1·528	·033	22	10	450	54·205	M	l y
		7–7.8	136	40	32	1	10	1·362	·009	294	10	,,	55·225	,,	w
		7–7.8	136	48	11	9	6	1·340	·017	88	6	,,	56·235	,,	p y, or w
		7.8–8	136	58	33	1	6	1·287	·031	26	6	,,	·241	,,	w
		7.8–8	140	7			10	1·289			10	600	59·865	,,	w; grey
		7.8–8	139	56			8	1·258			8	820	·956	,,	w
132	1127	6.7–8	340	55	12	7	6	5·490	·005	909	6	320	56·246	M	p y; b
	A & B	7–9	339	26	12	7	6	5·248	·017	88	6	,,	·301	,,	w; b
		7.8–9	339	33			7	5·337			6	,,	59·200	,,	p y; b
		8–9.10	338	37			10	5·299			10	,,	·241	,,	p y; grey
133	A & C	7.8–10.11	174	40	18	3	6	10·805	·072	5	6	190	57·222	M	p y; b
134	1140	7.8–9	272	55	10	11	6	6·045	·040	16	6	320	56·244	M	y; d b
		?	271	11			6	5·931			6	,,	57·906	,,	p y; b
		7.8–9.10	271	47			6	6·080			6	,,	58·186	,,	y; b
135	1146	7–9	17	7	11	9	10	3·339	·042	14	10	320	58·876	M	p y; b′
136	1177	7–7.8	352	19	6	24	10	3·701	·028	31	10	{190 Mm}	55·167	M	w
		7–8	352	10	10	11	10	3·832	·018	79	10	320	·178	,,	w
		6.7–7	352	25	16	4	6	3·601	·053	9	6	,,	56·257	,,	v p b
		6.7–7.8	352	40	9	13	6	3·516	·014	120	6	450	·260	,,	v p y, or w
137	1187	7.8–9	56	51			10	1·888			10	450	59·099	M	p y; w
		7–8	56	31			10	1·750			10	,,	·183	,,	p y; w
		8–9.10	55	46			6	1·896			6	600	60·274	,,	w; p b
		8.9–9.10	55	18			10	1·763			10	,,	·290	,,	w; b′ w
138	1196	5–6	144	0	15	4	4	5·271	·063	6	4	450	53·216	M	y
	A & B	5–6	141	55	21	2	4	5·240	·005	909	4	,,	·222	,,	y; b
		5–6	143	12	12	7	10	5·193	·009	278	10	,,	·238	,,	y

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* No star could be found at the distance given in the *Cycle*, although it has been looked for on many occasions.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	p.	w.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
139	1196	5.6–6.7	320°	10′	26′	1	10	1·034″	·032″	24	10	?	54·290	M	y; y′
	A & C	5.6–6	309	48	31	1	6	1·192	·070	5	6	450	56·235	„	y
		6–6.7	309	1	30	1	6	1·045	·023	47	6	600	·241	„	y
140	1223	6.7–6.7	214	15	11	8	10	4·778	·017	88	10	450	54·203	M	w
		6–6	212	48	13	6	10	5·030	·059	7	10	„	·255	„	p y
		6–6	212	55	10	10	10	4·828	·027	34	10	„	·257	„	p y; y′
141	1224	6–7	41	1	8	17	10	5·743	·026	35	10	450	52·225	M	y; b
		6–7	40	15	9	12	10	6·028	·041	14	10	„	·266	„	b
		6–7	39	52	13	6	10	6·006	·020	60	10	„	54·151	„	b
		7–8	40	14	8	15	10	6·083	·021	53	10	„	·162	„	w
		7–8	40	34	14	5	6	6·100	·032	24	6	„	·167	„	w
142	1245	7–8	24	40	3	99	10	10·395	·030	28	10	450	44·268	S	r
		7–8.9	25	2	5	46	10	10·323	·017	88	10	„	·271	„	o; r
		7–8	24	59	4	50	10	10·282	·032	24	10	„	·274	„	o; r
		7–8.9	25	12	8	15	10	10·522	·050	10	10	„	45·214	P	y; b
		7–8.9	25	12	12	7	10	10·301	·013	137	10	„	·263	„	y; w
		7–8.9	24	55	11	9	10	10·466	·034	21	10	„	·266	„	y; w
		7–8.9	24	28	7	24	10	10·387	·035	20	10	„	46·214	„	y; b
		7–9	25	46	4	53	10	10·418	·019	68	10	„	48·255	„	y; w
		6.7–8.9	26	3	14	5	10	10·202	·034	22	10	„	·285	„	y; p y
		6–7	26	23	12	8	10	10·562	·038	17	10	„	54·170	M	y
		6–7	26	2	10	11	10	10·387	·024	43	10	„	·175	„	l y; y
		5.6–7	25	27	17	4	6	10·279	·049	10	6	„	·192	„	d y; o
143	1263	7.8–8.9	16	26	7	21	10	22·382	·052	9	10	450	54·205	M	w
		7.8–8.9	16	33	7	19	10	22·502	·037	18	10	„	·222	„	w
		7.8–8	16	22	4	66	10	22·640	·037	18	10	„	·225	„	w
		8.9–8.9 Min	18	34			10	26·787			10	190	60·082	„	p y; w
		8.9–9	18	8			10	26·704			10	„	·085	„	y; p b
144	1270	8–8.9	259	25			6	4·669			6	320	57·906	M	w
		7.8–8	258	50			6	4·470			6	450	58·186	„	w

No. I.— *General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Dis-tance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
145	1273	4–9	208°	0′	21′	2	4	3″·688	″·050	10	4	450	52·268	M	y; b
		4.5–8	208	34	9	13	10	3·463	·038	17	10	„	54·274	„	y; d b
		5–8.9	208	57	14	5	10	3·493	·024	43	10	„	·277	„	p y; d b
		4.5–8.9	209	15	14	5	10	3·576	·036	19	10	„	·279	„	y′ w; b
		4–8	210	48			6	3·438			6	„	58·298	„	y; b
		3.4–8	210	28			6	3·407			6	„	·301	„	y; b
146	1280	9–10	37	24			6	6·729			6	320	59·049	M	p y
		9.10–10	35	24			6	6·647			6	450	·101	„	w
		8.9–9	35	35			10	6·464			10	320	·255	„	y; p y
147	1282	7.8–7.8	276	34	9	13	6	3·383	·023	45	6	600	56·257	M	w
		7.8–7.8 M	276	58	18	3	6	3·378	·031	26	6	450	·260	„	w
148	1288	10–10	259	8	12	8	10	7·680	·066	6	10	(450 320 190)	54·255	M	l b
		9.10–9.10	257	30	16	4	6	7·527	·055	8	6	320	·263	„	l b
		9.10–9.10	258	54	17	3	10	7·367	·038	17	10	„	·274	„	w
149	1289	9.10–10.11	1	17	25	2	4	4·254	·047	11	4	320	57·052	M	w; b′
		9–10	3	17	17	3	6	3·906	·030	27	6	„	·068	„	w
*		9–9.10	2	49	14	5	6	3·788	·018	78	6	450	·263	„	b′
150	1291	5.6–6	336	30	13	6	6	1·420	·020	63	6	600	57·285	M	y
		5.6–5.6 M	336	46	23	2	6	1·384	·027	33	6	„	·298	„	y
		6–6.7	335	19			6	1·236			6	820	60·142	„	y
		6.7–7	333	56			10	1·344			10	600	·159	„	p y
		7–7.8	335	32			10	1·297			10	„	·162	„	y
151	1295	7.8–7.8	358	30	13	6	6	4·252	·084	3	6	450	48·255	P	p y
152	1298	6.7–8.9	134	53	17	4	6	4·763	·035	20	6	320	56·241	M	p y; v b
		6.7–9	135	46	9	13	6	4·766	·020	63	6	?	·244	„	w; b
153	1306	4.5–8.9	258	33	24	2	10	3·712	·037	18	10	320	54·257	M	y; b
		5–9	256	47	19	3	10	3·558	·030	28	10	„	·260	„	g′ b; br b
		4.5–8.9	258	48	24	2	10	3·552	·036	19	10	„	·263	„	seagreen; b

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* A very neat double star follows this. One measure gave Pos. = 351°·8 ; Dist. 1″·75 ± Diff. in R.A. = $+ 1^{m}\ 22^{s}$; Diff. in Dec. = + 5′.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	p.	w.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
			° ′	′			″	″						
154	1311	7.8–8.9	201 15	14	5	10	7·153	·036	19	10	450	45·214	P	y; w
		7.8–8.9	200 26	7	20	10	7·160	·029	29	10	„	·252	„	y
		7.8–8.9	199 32	15	4	10	7·295	·035	20	10	„	·260	„	y
		8–8	199 48	8	18	10	7·316	·029	30	10	„	48·246	„	w
		8–8.9	199 55	12	7	10	7·382	·021	55	10	„	·249	„	w; p y
		7–7.8	200 9	6	24	8	7·062	·039	16	8	„	54·197	M	br y; w
		6.7–7	199 53	7	18	10	7·409	·036	19	10	„	·203	„	w
		6.7–7	200 11	9	13	10	7·259	·018	80	10	„	·205	„	p y
		7–7.8	200 24	8	15	10	7·175	·032	24	10	„	·208	„	w
155	1332	7.8–8	16 12	11	9	10	5·551	·024	41	10	450	54·208	M	w
		7.8–8	16 20	7	22	10	5·545	·013	147	10	„	·266	„	w
		6.7–7.8	16 33	7	23	10	5·563	·009	313	10	„	·271	„	w
156	1333	6.7–6.7 M	39 18			10	1·480			10	600	59·260	M	w
		6.7–7	38 35			10	1·412			10	„	·263	„	creamy w
		7–7.8	39 40			10	1·428			10	„	·285	„	p y
*157	1334	4.5–7.8	238 27			10					450	59·115	M	p y; g′
		5–8	238 10			10	2·884			10	„	·162	„	p y; b′
		4.5–7.8	238 14			6	2·945			6	600	·186	„	p y; b
158	1338	7–7	136 43	10	10	8	1·954	·018	80	8	450	53·241	M	y
		7–7	137 24	11	8	10	2·025	·027	33	10	„	54·222	„	p y
		7–7	137 40	10	11	10	1·899	·025	40	10	„	·246	„	w
		7–7	136 4	19	3	10	1·849	·015	116	10	„	·252	„	clear w
159	1340	8–10	317 51	16	4	6	5·931	·039	16	6	320	57·282	M	b′; b
		7.8–9.10	318 38	18	3	6	6·163	·029	29	6	„	·285	„	w; b
160	1346	8–9	311 38	12	7	6	5·868	·033	22	6	320	56·246	M	w; b
		8–9	310 8			10	5·868			10	„	59·099	„	w; b′
		8–9.10	310 27			10	5·695			10	450	·183	„	w; p b
161	1355	8–8.9	331 30			10	2·764			10	320	59·244	M	y
†		8.9–8.9 M	330 45			10	2·734			10	450	·301	„	p y

* The measures of distance have not been used, as after the 6th measure it was discovered that the wires fouled each other.

† The s. f. star is certainly the larger, but not half a magnitude.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
			°	′	′			″	″						
*162	1356		175	(Est.)								600	57·288	M	
163	1362	7.8–7.8	135	13	9	13	10	4·917	·014	130	10	820 450	54·285	M	w
		7.8–7.8 M	134	10			6	4·989			6	450	58·222	,,	w; b′
		7.8–7.8	134	11			6	4·900			6	600	·230	,,	w
164	1374	8–10	274	55	12	7	10	3·277	·029	30	10	450	52·282	M	y
		7–10	275	1	11	8	4	3·425	·116	2	4	,,	·287	,,	y
		7–8.9	276	52	12	7	6	3·511	·061	7	6	320	55·290	,,	p y; b
		7.8–9	277	32	5	46	10	3·599	·013	156	10	,,	·301	,,	p y; b
		7.8–8.9	277	58	7	23	10	3·489	·022	51	10	,,	·304	,,	y′; b
		7.8–8.9	277	48	10	9	10	3·547	·021	57	10	190 Mm	·307	,,	p y; b
		8–9	278	4	10	10	10	3·611	·017	89	10	,,	·312	,,	w; b
		7.8–9	278	6	10	9	10	3·495	·024	44	10	,,	·318	,,	w; p b
		8–10	279	0			10	3·444			10	450	60·118	,,	p y; b
		8–10	278	57			10	3·418			10	,,	·121	,,	p y; b
165	1376	8.9–8.9	132	57	25	2	6	5·008	·024	44	6	450	56·235	M	w
		8.9–8.9	133	48	11	8	6	5·091	·027	35	6	320	·241	,,	w
166	1394	10–11	240	54	17	3	6	3·780	·030	27	6	450	57·121	M	w
		10–11	239	55	14	5	6	3·949	·037	18	6	320	·260	,,	b′; p y
167	1421	9.10–11	334	4			10	4·409			10	450	60·162	M	w; b
		9–11	330	53			10	4·306			10	,,	·255	,,	w; b′
168	1424	2–4	105	37	15	5	10	3·180	·024	43	10	450	52·279	M	y; b
		2–4	105	32	11	9	10	3·146	·019	66	10	,,	·298	,,	y
		2.3–4	108	11	13	6	10	3·067	·012	164	10	320	55·331	,,	br gold; gr′
		2.3–3.4	109	11	12	8	10	3·067	·018	78	10	320 Mm	·337	,,	br y; g′ y
		2.3–3.4	109	10	14	5	10	3·029	·018	79	10	190 Mm	·348	,,	y; g′ y
		2.3–3.4	110	4			10	3·371			10	600	60·107	,,	golden y
		2.3–3.4	110	9			10	2·972			10	450	·142	,,	golden y; o
169	1428	8.9–9	86	8			10	3·402			10	450	59·244	M	w
		8.9–9	86	16			10	3·435			10	,,	·288	,,	w; b

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* The star appeared elongated in the direction s f. n p.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
			°	′	′			″	″						
170	1434	9.10–9.10	90	38	9	12	6	5·987	·012	164	6	320	57·263	M	w
171	1441	6.7–10	168	3	37	1	6	3·133	·112	2	6	{450 320}	58·304	M	p y; purp
172	1447	7.8–9.10	125	3	8	15	10	4·344	·017	88	10	320	55·326	M	w; b
		8–10	125	17	11	8	10	4·378	·023	48	10	,,	·331	,,	w; b
173	1448	6.7–9	256	47	13	6	4	10·961	·055	8	4	320	54·257	M	w
		7.8–10.11	257	32			4	10·284			4	,,	58·189	,,	w; b
		7.8–10.11	258	45			6	10·120			6	,,	·271	,,	w; b
		8–10.11	258	44			6	10·198			6	,,	·290	,,	w; b
*174	1457	8–9	312	58	48	0	6	0·6(Est.)				450	57·288	M	w
175	1460	9.10–10	168	18			10	3·357			10	450	60·159	M	w
		10–10 M	168	6			10	3·263			10	,,	·219	,,	w
176	1463	9.10–10	259	13	22	2	6	7·664	·070	5	6	320	57·121	M	w
		8.9–9.10	257	55	13	6	6	7·073	·058	7	6	,,	·298	,,	w; b
177	1466	6.7–8.9	240	18	11	9	10	6·697	·047	11	10	450	48·334	P	y; b
		6.7–8	240	3	10	11	10	6·657	·044	12	10	,,	·337	,,	y; p b
		6–8	241	10	10	10	10	6·725	·026	37	10	,,	52·318	M	y; b
		6.7–8	239	31	13	6	10	6·461	·036	19	10	,,	54·277	,,	l y; b
		6–8	239	29	9	13	10	6·622	·071	5	10	,,	·285	,,	y; l b
		6–7.8	239	33	7	21	10	6·760	·048	11	10	,,	·290	,,	d y; b
178	1476	8.9–9.10	356	49			10	1·847			10	450	59·162	M	w
		8–9	356	50			10	1·908			10	,,	·301	,,	w; v p b
179	1479	8.9–10	21	29	21	2	6	5·008	·023	47	6	320	57·288	M	w; b
		9.10–10.11	20	42	18	3	6	4·383	·089	3	6	,,	58·222	,,	w; b
180	1487	5–7.8	103	31	9	11	10	6·298	·019	68	10	450	48·334	P	p y; p b
		4.5–7.8	104	3	13	6	10	6·234	·013	152	10	,,	·337	,,	y; b
		4.5–7.8	102	32	5	35	10	6·397	·025	39	10	,,	·339	,,	y; b
		5.6–8	102	6	12	7	10	6·572	·041	15	10	,,	54·266	M	p y; l b
		5.6–7.8	102	16	17	4	10	6·111	·041	15	10	,,	·271	,,	p y; b

* The distance is estimated.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Dis-tance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
180	1487	5.6–7.8	103°	10′	12′	7	10	6″·339	″·032	24	10	450	54·277	M	w ; b
181	1516	7.8–8	6	31	12	7	10	2·598	·015	108	10	450	54·260	M	y
		7–8	6	36	15	4	10	2·453	·015	116	10	„	·263	„	p y
		7–8	7	30	12	7	10	2·417	·018	72	10	„	·271	„	l y
		8.9–9	54	36			10	3·244			10	„	60·121	„	y′
		8.9–5	57	55			10	3·289			10	450	·255	„	y
		9–9.10	57	29			6	3·206			6	600	·257	„	p y
182	1521	8–8.9	95	42			10	3·472			10	450	59·156	M	w
		8.9–5	95	13			10	3·682			10	„	·183	„	w
183	1523	5–6	117	34	12	8	10	2·919	·032	24	10	450	52·331	M	y
		5–6	117	45	17	4	10	2·883	·020	61	10	„	·345	„	y
		5–6	118	52	9	11	10	2·877	·036	19	10	„	·394	„	y
		4.5–5	108	24			6	2·854			6	600	58·189.	„	v p y ; p y
		4.5–5	107	56			6	2·839			6	„	·214	„	y ; orange
		5–5	105	47			10	2·804			10	„	60·082	„	y
		4.5–4.5	105	2			10	2·884			10	„	·085	„	y
184	1524	4–12 or 13	146	1			6	7·096			6	320	58·129	M	br oran. ; b′
		4–13	145	46			6	7·354			6	„	·186	„	br oran. ; b
185	1527	8–9.10	11	54	11	8	10	4·003	·026	37	10	{190 Mm}	55·337	M	w ; v p b
		8–9	11	34	9	11	10	4·067	·036	19	10	320	·348	„	w ; p b
		7.8–8.9	12	24	8	17	6	3·733	·017	83	6	450	56·356	„	w ; v p b
186	1536	4–7	83	59	16	4	10	2·714	·006	714	10	450	53·353	M	y ; b
		4–7	82	30	24	2	6	2·706	·022	51	6	„	·356	„	y ; b
187	1543	6–9	5	59	13	6	6	5·377	·029	29	6	320	57·282	M	p y ; b
188		5–5.6	211	30	8	16	10	8·800	·058	7	10	320	57·285	M	w ; b′
		5–5.6	211	47	11	8	10	8·837	·094	3	10	190	·298	„	w
189	1552	7–8.9	209	43	12	7	10	3·108	·035	20	10	450	46·397	P	p b
		6.7–9	209	14	11	8	10	3·085	·023	48	10	„	·405	„	y ; p b
		6.7–8.9	208	39	6	32	10	3·143	·019	68	10	„	48·255	„	p y ; p b
		6.7–8.9	208	50	12	7	10	3·066	·033	22	10	„	·337	„	w

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
			°	′	′			″	″						
189	1552	6.7–8.9	209	27	9	11	10	3·092	·029	30	10	450	48·364	P	w; b′
		6.7–7.8	207	40	9	14	10	3·195	·025	40	10	,,	54·361	M	p y; w
		6–7.8	208	21	15	4	10	3·164	·015	110	10	,,	·372	,,	w; b′
		6.7–7.8	209	1	9	12	10	3·568	·020	62	10	320	55·301	,,	b′; p b
		7–8	209	20	11	8	10	3·461	·011	222	10	,,	·304	,,	w; p b
		6.7–7.8	209	52	12	7	10	3·551	·019	68	10	190 Mm	·307	,,	w; p b
		7–8	209	45	9	13	10	3·455	·016	102	10	,,	·312	,,	w; l b
		7–8	210	15	10	10	10	3·379	·018	78	10	,,	·315	,,	w; p b
		7–8	209	21	13	6	10	3·369	·021	55	10	320	·318	,,	b′; p b
190	1553	8–9	170	19			10	5·165			10	320	59·205	M	p y; p b
		8.9–9	170	34			10	5·431			10	450	·301	,,	w; p lilac
191	1555	6.7–7	342	25			10	1·093			10	600	59·342	M	w
		7–7.8	343	14			10	1·192			10	,,	·364	,,	y′ w
192	1559	7–8	320	26	40	1	6	2·291	·027	34	6	320	57·288	M	p y; b
		7–8.9	320	57	40	1	6	2·088	·048	11	6	450	·345	,,	w; b
193	1561	8–10.11	264	42	12	7	10	10·503	·057	8	10	450	46·348	P	w; b′
		7.8–10	265	20	11	8	10	10·392	·047	11	10	,,	·364	,,	w
		7–9.10	265	6	11	9	10	10·549	·042	14	10	,,	·367	,,	w; p b
194	1579	8–11	32	21	9	13	10	3·646	·023	46	10	450	46·364	P	y; p b
		7–11	35	28	16	4	10	3·746	·049	10	10	,,	·367	,,	y; p b
		7–10.11	35	29	13	6	10	4·049	·058	7	10	,,	·383	,,	y; p b
195	1596	7–8	239	15	11	8	10	3·901	·022	52	10	320	55·318	M	y′; p b
		6.7–8	239	44	10	10	10	3·912	·023	47	10	,,	·331	,,	w; b
		6.7–8	239	22	15	5	10	3·954	·039	16	10	,,	·348	,,	b′ w; p b
		6.7–7.8	238	4			10	3·724			10	450	59·238	,,	w; b′
		6.7–8	238	40			6	3·785			6	,,	·241	,,	p y; b′
		6–8	238	26			40	3·721			40	600	·244	,,	p y; b′
		6.7–8	238	33			50	3·667			50	600 450	·260	,,	p y; r′
		6–8	238	49			9	3·651			8	450	·263	,,	y′ w; purp
		6–8	238	22			20	3·667			20	600 450	·288	,,	w; p purp

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
195	1596	6–8	238° 47′	′		10	3″·658	″		10	450	59·290	M	y′w ; purp
		6–8	238 36			30	3·653			30	600	·293	,,	w ; b′
		6–8	238 27			40	3·749			40	450	·296	,,	{ p y; plum col }
196	1600	7.8–8	94 15			6	7·401			6	320	58·307	M	p y; p b
		8–8.9	92 51			6	7·398			6	450	·337	,,	y′ w ; b
197	1633	7–7	63 57	10	11	10	8·712	·026	36	10	450	46·389	P	p y
		7–7	64 54	12	7	10	8·598	·062	6	10	,,	·405	,,	p y
		7–7	65 21	16	4	10	8·710	·035	21	10	,,	·408	,,	p y
198	1645	8–9	161 30	13	6	10	10·292	·028	31	10	450	48·255	P	w
		8–9	161 29	7	24	10	10·254	·014	119	10	,,	·271	,,	w
		8–9	159 44	11	9	10	10·392	·018	72	10	,,	·337	,,	w ; p b
199	1653	9–9.10	163 50	17	4	4	7·867	·026	37	4	320	57·282	M	w
		9.10–9.10	162 45	13	6	6	7·808	·012	164	6	,,	·285	,,	w ; b′
200	1670	4–4	178 21	11	9	6	2·831	·021	55	6	450	51·090	P	orange
		4–4	176 49	16	4	4	2·831	·179	1	4	,,	·167	,,	y
		4–4	176 37	14	5	10	3·021	·049	10	10	,,	·175	,,	d y
		4–4	175 25	4	57	10	2·994	·024	41	10	,,	·257	,,	g′
		3.4–3.4	171 44	5	44	10	3·408	·029	29	10	320	54·394	M	p y
		3.4–3.4	172 17	4	75	10	3·382	·044	12	10	,,	·397	,,	y′ w
		4–4	172 11	6	27	10	3·416	·016	92	10	,,	·400	,,	p y
		3.4–3.4	172 48	7	22	10	3·514	·015	116	10	,,	55·430	,,	w
		3.4–3.4	173 36	6	27	10	3·425	·026	36	10	,,	·548	,,	w
		4–4	173 29	14	5	10	3·605	·052	9	10	190	·611	,,	w
		3.4–3.4M	171 50	12	7	6	3·621	·081	4	6	450	56·424	,,	p y
		3.4–3.4	171 47	10	10	10	3·637	·012	169	10	600	·964	,,	p y
		3.4–3.4	171 32	10	10	10	3·677	·051	9	10	320	·972	,,	p y
		3.4–3.4	171 29	7	18	10	3·680	·022	51	10	,,	·975	,,	y′ w
		3.4–3.4	170 43			7	3·428			6	450	58·474	,,	w ?
		3.4–3.4	170 42			10	3·432			10	,,	·476	,,	w
		3.4–3.4	170 40			10	3·344			10	600	·479	,,	y′

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Dis- tance.	p.	w.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
200	1670	3.4–3.4	170°	36′	′		10	3″·896	″		10	450	59·115	M	y′
		3.4–3.4	171	32			10	3·865			10	„	·156	„	p y
		3.4–3.4	170	30			7	4·013			8	600	·162	„	p y
		3.4–3.4	170	15			10	4·034			10	„	·183	„	p y
201	1681	10–10	14	10	8	17	10	8·521	·031	26	10	450	46·397	P	w
202	1686	9–9	186	35	16	4	10	4·974	·038	17	10	450	46·397	P	w
		9.10–9.10	186	18			6	5·401			6	320	58·353	M	w
		9.10–10	186	17			6	5·088			6	„	·394	„	b′ w
203	1687	5–7.8	52	24	21	2	6	1·283	·051	9	6	600	56·337	M	w or p y; b′?
	A & B	5–8	48	2	36	1	6	1·364	·018	73	6	„	·356	„	y; b
		6–7.8	46	6	39	1	6	1·376	·024	41	6	450	57·268	„	y; b
		6–8	44	44	41	1	6	1·517	·028	31	6	600	·296	„	br y; b
204	A & C	5–10.11	124	1	6	29	6	28·642	·095	3	6	320	56·356	M	y; v b
		5.6–10.11	123	40	11	9	6	27·942	·072	5	6	„	57·274	„	orange; purp
		5.6–10.11	124	41	9	13	6	28·383	·073	5	6	190	·315	„	orange; d b
205	1690	8–9.10	147	56	11	9	10	6·058	·037	18	10	320	55·370	M	v p b; b
		8–11	149	17			6	5·643			6	{320 190}	58·222	„	w; b
		7.8–10.11	147	14			6	5·737			6	320	·337	„	w; b
206	1695	7.8–9	286	50			10	3·510			10	450	60·115	M	w; b′
		7.8–10	286	20			10	3·378			10	„	·159	„	w; dusky
207	1719	7.8–8	3	34	5	39	10	7·560	·046	11	10	450	53·378	M	b
		7–8	2	53	7	23	4	7·427	·061	7	4	„	·408	„	b
		7–8	2	37	15	4	4	7·442	·045	12	4	„	·413	„	b
208	1724	4.5–9.10	342	50	4	70	10	7·226	·023	47	10	320	54·386	M	w; brn′
	A & B	4–9.10	342	36	13	6	10	7·190	·045	12	10	„	·397	„	w; dusky
*209	1728	6–6.7	190	44	26	2	6	{0·65 Est.}				600	56·383	M	y
210	1755	7–9	131	44	11	9	10	4·355	·030	28	10	450	46·416	P	y; p b

* The distance estimated 0″·6 or 0″·7.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
			° ′	′			″	″						
210	1755	7.8–8.9	129 49			6	4·458			6	320	57·342	M	p y ; b
		8–9	131 50			6	4·329			6	”	·345	”	w ; b
211	1757	7.8–8.9	52 15	24	2	6	2·088	·084	3	6	320	56·419	M	y′ or w ; b′
		8–9	51 30	30	1	6	1·936	·042	14	6	450	·422	”	w
*		7–8.9	54 22	39	1	4	2·033	·021	55	4	”	57·285	”	p y ; p b
		8–9	54 5	19	3	6	1·983	·020	63	6	600	·296	”	w ; b′
212	1760	9–9	64 53	6	24	10	8·517	·028	32	10	450	46·416	P	p b
213	1763	8–8 M	43 41	14	5	10	2·833	·019	68	10	320	54·400	M	w
		8–8	44 21	25	2	6	2·789	·063	6	6	450	56·422	”	w
†		8.9–9	43 15	16	4	6	2·779	·026	37	6	”	·424	”	w
214	1770	7–8.9	118 55	17	3	10	1·841	·016	95	10	600	59·342	M	oran ; p b
215	1776	9.10–10	197 52			10	7·090			10	320	59·205	M	w ; b′
		9.10–9.10	198 29			10	7·228			10	”	·238	”	y′
216	1777	6–9	230 33			6	3·457			6	320	58·214	M	orange ; b
		6–9.10	231 0			6	3·241			6	450	·356	”	p y ; b
217	1785	7.8–9	177 9	41	1	4	3·196	·063	6	4	{450 320}	46·389	P	w
		7.8–9	175 48	15	5	10	3·260	·020	65	10	450	·408	”	y ; p y
		7–9	175 56	5	38	10	3·185	·017	88	10	”	·411	”	w
		7–9	175 52	10	10	10	3·144	·022	49	10	”	·413	”	w
		8.9–9	184 47			10	2·976			10	”	59·296	M	w
		8–8.9	186 1			10	2·806			10	”	·301	”	w ; p b
218	1789	9.10–9.10 Min	327 26			10	6·308			10	320	60·115	M	w
		9–9.10	326 26			10	6·059			10	”	·257	”	w; b′
219	1793	8–9	242 50	14	5	6	4·619	·038	17	6	320	56·427	M	w ; b
		7.8–8.9	242 51	11	8	6	4·458	·029	29	6	”	·438	”	p y ; b
220	1821	5.6–7.8	237 11	7	23	10	12·624	·040	15	10	320	54·460	M	y′ w ; b
		5–7.8	237 4	5	37	10	12·686	·034	21	10	”	·465	”	w ; p b
		5.6–8	237 52	5	42	10	12·866	·030	27	10	”	55·430	”	v p y ; b

* A small star in the s. f. quadrant.

† Scarcely half a magnitude difference.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
220	1821	5–8	238° 9′	6′	30	10	12″·697	″·019	70	10	320	55·468	M	py; brb
		5–8	238 17	5	44	10	12·677	·026	37	10	„	·487	„	p y; b
221	1826	9–10	313 4			6	4·196			6	320	57·345	M	w; b′
		9–10	313 41			6	4·325			6	„	·400	„	w; p b
222	1835	5–8	187 51	24	2	4	6·279	·092	3	4	450	44·539	W&P	y′ b; b′
		5–8	187 46	23	2	8	5·940	·082	4	8	„	·548	P	y; b′
		5–8	186 54	12	7	10	6·204	·042	14	10	„	·550	„	b′; l b
		5–8	188 50	11	8	10	6·091	·034	22	10	„	·556	„	y′; b′
		5–8	187 48	13	6	10	6·269	·030	27	10	„	·561	„	y′; l b
		6–8	183 2	5	37	10	6·375	·037	18	10	„	45·361	„	b
		6–8	183 6	14	5	10	6·465	·025	39	10	„	·378	„	b
		6.7–8.9	187 30	14	5	10	6·095	·056	8	10	„	·523	„	y; b
		6.7–8.9	187 22	5	34	10	6·330	·025	40	10	„	46·501	„	p y; p b
		6–8	187 37	7	19	10	6·252	·031	25	10	„	·504	„	p y; p b
		6.7–8.9	186 36	6	26	10	6·234	·035	20	10	„	47·501	„	p y; p b
		6–8	186 43	10	9	10	6·237	·075	4	10	„	·556	„	y; p b
		6–8	187 17	8	17	10	6·247	·059	7	10	„	48·509	„	orange; y
		6–7.8	187 4	6	26	6	6·374	·090	3	6	„	58·312	M	w; b′
223	1838	8–8	152 39	11	9	10	8·813	·051	9	10	450	46·405	P	p b
		7.8–7.8	151 56	9	14	10	8·913	·038	17	10	„	·408	„	p b
		7.8–7.8	152 49	6	24	10	8·880	·039	16	10	„	·413	„	p b
		8–8 Min.	334 5			10	8·939			10	„	60·107	M	y
		7.8–7.8	334 22			10	8·789			10	„	·255	„	w
224	1858	8.9–10.11	32 13			10	2·462			10	450	59·296	M	w; p b
		8.9–10	30 53			6	2·656			6	„	·342	„	w
		9–10	30 45			4	2·421			4	„	·348	„	y′; b′
225	1864	5–6	99 16	17	3	10	6·131	·034	21	10	450	46·416	P	y; w
		5–6	98 19	8	14	10	6·034	·042	14	10	„	·419	„	y; w
		5–6	98 47	7	23	10	6·080	·030	28	10	„	·452	„	y; w
		5–6	100 54			6	6·230			6	320	57·340	M	p y; r′

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
225	1864	4.5–5	100° 19′	′		6	6″·048	″		6	450	57·345	M	p y
226	1865	3.4–4	304 12			10	1·107			10	820	59·301	M	w
		4–4.5	305 12			10	1·258			10	600	·381	„	p y
227	1877	3–7	328 15			6	3·077			6	450	57·408	M	orange; g′
		3–6	327 23			6	2·815			6	„	·471	„	gold or; g′ b
		3–7	324 7			6	2·717			6	„	58·424	„	orange; b′g
		3–7	325 9			6	2·667			6	„	·471	„	orange; b
		3–7	325 23			10	2·761			10	600	·476	„	orange; b
228	1888	7–9	318 41	18	3	10	6·539	·033	22	10	450	45·361	P	y; w
		7–9	318 54	20	3	8	6·871	·095	3	8	„	·378	„	y; w
		6.7–8.9	318 17	20	3	10	6·855	·070	5	10	„	·465	„	r
		5.6–7.8	318 57	10	11	10	6·752	·037	18	10	„	46·457	„	y; b
		5.6–7.8	319 23	12	7	10	6·739	·020	62	10	„	·460	„	y; b
		5–8	312 58	16	4	10	6·519	·029	28	10	„	52·476	M	y
		5–7	313 12	11	9	10	6·397	·020	63	10	„	53·430	„	y; p b
		3.4–7	307 44			6	6·019			6	320	58·378	„	y; plum col
		3.4–7	307 47			6	5·839			6	450	·394	„	y; r′
229	1890	6.7–7	42 0	17	3	10	3·726	·042	14	10	450	45·408	P	y; w
		6.7–7	42 55	6	32	10	3·752	·035	20	10	„	·441	„	y; w
		6.7–7	42 13	16	4	10	3·649	·041	14	10	„	·465	„	y; w
		6.7–7	41 29	12	7	10	3·793	·014	130	10	„	46·392	„	w
		6–6	40 35	11	8	10	3·703	·033	23	10	„	·397	„	w
		6.7–6.7	40 39	10	9	10	3·540	·021	56	10	„	·402	„	w
		6–6.7	45 55	8	15	6	3·522	·027	35	6	„	58·476	M	w
		6–7	44 5			10	3·565			10	600	59·244	„	w; p b
		6–7	43 17			10	3·796			10	450	·293	„	w; b′
230	1904	7.8–7.8	346 6			6	9·744			6	320	57·408	M	w
		7.8–8	346 28			4	9·776			4	„	·455	„	p b
231	1909	5–6	238 36	9	13	10	4·622	·020	60	10	320	54·460	M	y′ w; b′ w
		5.6–6	238 18	11	8	10	4·545	·041	15	10	„	·463	„	w; v p b

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Dis-tance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
			°	′	′			″	″						
232	1910	7–7	28	21	9	12	10	3·804	·024	42	10	450	44·567	P	y′ b
		7–7	27	53	12	6	4	4·009	·005	880	4	„	·591	„	b′
		7–7	30	8	30	1	4	3·938	·000	∞	4	„	·663	„	b′
		7–7	30	5	17	4	10	4·074	·037	18	10	„	·665	„	b′
		7–7	29	25	16	4	4	3·844	·030	27	4	„	·671	„	b′
		8–8	25	40	14	5	10	3·949	·024	44	10	„	45·408	„	w
		7.8–7.8	28	28	8	16	10	4·062	·029	30	10	„	46·405	„	p b
		7–7	29	10	15	4	10	3·999	·029	29	10	„	·408	„	p b
		7–7	29	54	8	15	10	3·906	·039	16	10	„	·413	„	p b
		9–9	29	46	5	45	10	3·782	·019	68	10	„	·504	„	b
		8.9–8.9	29	51	12	6	6	3·727	·064	6	6	„	47·501	„	w
		8.9–8.9	30	1	11	9	10	3·845	·024	44	10	„	·580	„	w
		7.8–7.8	31	17			6	4·031			6	„	58·312	M	w
		8–8	31	32			6	4·142			6	„	·356	„	w
234	1928	9.10–10	275	12	17	3	6	6·120	·087	3	6	190	57·397	M	w; b
*235	1937	6.7–6.7	356	7	39	1	6	0·704	·016	94	6	450	57·476	M	y
†		6?–6.7?	2	15	16	4	10	0·55(Est.)				„	58·602	„	w or y
236	1954	3.4–5	192	46			6	3·350			6	450	57·400	M	p y
		3.4–5.6	193	22			6	3·397			6	„	·408	„	p y; b′w
237	1962	6–6	7	41	3	108	10	11·781	·017	88	10	450	46·408	P	y
		6–6	7	35	7	22	10	11·989	·041	14	10	„	·413	„	p b
		6–6	7	54	5	38	10	11·910	·060	7	10	„	·416	„	p b
238	1963	9–9.10	292	15	18	3	6	4·156	·023	47	6	450	59·446	M	w
239	1965	4–6	298	30	8	15	10	6·124	·056	8	10	450	45·408	P	y; b′
		4–6	299	8	17	3	10	5·704	·059	7	10	„	·441	„	y; b′
		4–6	300	46	23	2	10	6·163	·026	37	10	„	·446	„	y; b′
		4–6	302	40	7	18	10	6·194	·023	45	10	„	46·413	„	w

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* The star s. f. was considered the larger.

† The distance was estimated 0″·5 or 0″·6. After a careful examination at the close of the measures, the star s. p. was pronounced to be the larger.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
239	1965	4–6	301° 36′	11′	8	10	6″·219	″·031	25	10	450	46·419	P	y; p b
		4.5–6.7	299 39	7	18	10	5·997	·044	13	10	„	·452	„	p y; w
		4–5	300 53	12	7	10	6·048	·015	104	10	„	52·517	M	b′ w; p b
		4–5	301 32	8	16	10	6·162	·016	102	10	„	·528	„	l b
		4–5	300 42	12	7	10	6·187	·020	60	10	„	·553	„	b′ w
240	1985	7.8–9.10	327 2	11	8	10	5·574	·040	16	10	450	46·183	P	w; b
		7–8.9	328 5	8	15	10	5·736	·016	93	10	320	54·465	M	w; r′ brn
		8–10	330 16			6	5·854			6	„	58·378	„	p y; b′
		7–9.10	330 54			4	5·675			4	„	·394	„	w; b
		7.8–10.11	329 30			6	5·305			6	450	·479	„	p y; b
*241		4.5–6.7	337 45			10	41·024			10	320	57·345	M	p y; b′
		4.5–7.8–9	337 34			10	41·086			10	190	·441	„	y; b′
†242	2021	7–7	322 13	11	8	10	3·596	·029	30	10	320	55·468	M	w
		7–7	322 42	9	13	10	3·743	·042	14	10	„	·487	„	y′
		7.8–7.8	323 7	6	25	10	3·620	·009	333	10	190 Mm	·509	„	p y
243	2032	6–7	176 33	10	9	10	2·428	·016	93	10	450	52·572	M	y; p b
		6–7	176 32	10	9	10	2·525	·010	233	10	„	53·545	„	y
		6–7	178 18	8	14	10	2·177	·030	26	10	„	54·665	„	p y; b
		6–7	178 43	10	9	10	2·255	·022	49	10	320	·668	„	p y; b
		6–7	184 46	12	7	6	2·315	·013	147	6	450	58·613	„	w; br b
		6–7	184 54			10	2·659			10	600	59·301	„	y′ w; p b
		6–7	184 55			10	2·743			10	„	·381	„	v p y; p b
244	2046	9–9.10	222 6	36	1	6	8·298	·035	20	6	190	57·397	M	w
245		1–8?	277 40			10	3·267			10	450	58·214	M	r?; b′?
		1–8?	275 24			10	3·253			10	„	·312	„	r?
		1.2–8	273 59			8	3·341			8	320	·413	„	or; b′ or g′
		1.2–8	276 8			6	3·324			6	450	·444	„	r?
246		3–12?	140 5			2	4·716			2	190	57·471	M	y; b′?
			145 ±			1					320	·526	„	

* See Notes at the end of the Catalogue.

† The star s. f. was believed to be the larger.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
			°	′	′			″	″						
247	2052	8.9–8.9	104	13			6	2·593			6	450	58·427	M	w
		8–8.9	105	40			6	2·346			6	320	·452	„	y
		8–8	104	16			4	2·917			4	„	·457	„	p y
248	2055	4–6	16	18			6	1·303			6	450	57·523	M	y′; p b
		4–6	15	2			6	1·278			6	„	·635	„	w; b′
249	2078	6.7–8	116	13	12	7	10	3·690	·036	19	10	450	45·465	P	y
		6–7.8	115	1	18	3	10	3·866	·045	12	10	„	46·501	„	y; p y
		6–7.8	115	40	15	5	10	3·726	·029	30	10	„	·504	„	y; p y
		6–7.8	116	18	9	12	10	3·756	·010	256	10	„	·591	„	p y
		6–6.7	114	23			10	3·616			10	600	59·244	M	p b; g′
		6–7	113	57			10	3·626			10	450	·293	„	y′ w; g′ w
250	2084	3–6	74	51	20	3	10	1·478	·047	11	10	450 600	55·621	M	orange; br b
		3.4–6.7	72	59	29	1	10	1·342	·024	43	10	320 450	·660	„	y; b
		3–6	73	14	20	3	10	1·522	·029	30	10	320 Mm 190 Mm	·679	„	br y; b′
		3–6	72	10	30	1	6	1·466	·032	24	6	320 Mm	·687	„	br y; b
		3–7	59	27			6	1·655			6	450	57·446	„	y; gr?
		3–6.7	60	54			8	1·656			8	„	·457	„	y; b′
		3–7	60	34			6	1·489			6	„	·471	„	p y; b′
251	2087	9.10–10	290	32	10	11	10	5·503	·036	19	10	320	59·446	M	w
252	2104	7.8–9	18	18	12	7	10	6·115	·027	34	10	450	49·517	P	p y; w
		7–9	18	24	12	7	10	6·135	·017	85	10	„	·531	„	p y; w
		7.8–10	21	8			6	5·886			6	320	58·378	M	w; b′
		8–10	20	14			6	5·951			6	„	·413	„	w; b
253	2114	8.9–9	147	38	21	2	10	1·312	·021	58	10	600	59·301	M	w; b′ w
254	2120	7–11.12	300	20	15	5	10	2·637	·025	39	10	320	55·548	M	p y; br b
		7.8–10.11	295	49	17	4	10	2·514	·029	28	10	„	·690	„	p y; b?
255	2130	4.5–4.5	7	46	18	3	10	2·980	·026	38	10	320	54·674	M	w
		5–5	8	17	10	11	10	2·901	·013	141	10	„	·679	„	w
		5–5	9	4	10	10	10	2·897	·028	31	10	„	·684	„	w

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
256	2135	7.8–9	171°	3′	11′	9	10	6″·970	″·045	12	10	320	55·611	M	d y; b
		8–9.10	171	5	10	10	10	6·934	·060	7	10	„	·613	„	w; p b
257		5–5	212	41	6	26	10	4·478	·050	10	10	320	54·690	M	y
		5–5	213	19	12	7	10	4·409	·037	18	10	„	·693	„	y
		5–5.6	213	14			4	4·489			4	450	58·356	„	y
		5.6–6	213	21			10	4·404			10	„	·444	„	p y
		5.6–6	213	13			10	4·444			10	320	·460	„	y
258	2140	3–6	118	40	16	4	6	4·685	·023	45	6	450	46·611	P	y; w
		3–6	117	17	11	8	10	4·667	·029	28	10	„	·704	„	y; p b
		3–6	116	39	12	7	10	4·919	·047	11	10	„	52·616	M	orange; b
		3–5.6	117	54			6	4·778			6	320	57·537	„	y; br b
		3–5.6	118	43			6	4·653			6	„	·635	„	golden y; b
		3–6	117	36			6	4·282			6	„	·695	„	y; b
259	2161	5–7	309	28	18	3	6	4·071	·064	6	6	450	45·408	P	y; b′
		4–5.6	308	10	18	3	10	3·787	·045	12	10	„	·542	„	y′
		5–6.7	310	36	18	3	10	3·838	·027	34	10	„	·545	„	y′
		5–6.7	308	44	9	12	10	3·798	·017	83	10	„	·548	„	y′
		4.5–6.7	308	20	11	8	10	3·866	·008	400	10	„	46·504	„	y
		4.5–6.7	306	26	19	3	10	3·772	·009	278	10	„	·561	„	w; p y
		4.5–6.7	306	16	11	9	10	3·671	·013	152	10	„	·591	„	p y
		4.5–6	310	4	8	16	10	3·812	·017	83	10	320	55·619	M	g′; b′ g
		4.5–6	309	48	13	6	10	3·777	·027	35	10	320 450	·624	„	g′ y; g
		4–5.6	309	2	15	4	10	3·907	·012	161	10	190 Mm	·657	„	y; g′
		4–5.6	310	13	13	6	10	3·935	·009	278	10	„	·660	„	y′; g′
		4–5.6	309	22	7	23	10	3·802	·026	37	10	320	·674	„	g′; g′ b
		4–5	310	1	9	12	10	3·836	·003	3333	10	320 Mm	·693	„	y; g′ y
260	2165	8–10.11	49	56			6	7·136			6	320	57·476	M	w; b
		8–10	49	17			6	6·862			6	„	·479	„	w; ruddy
261	2168	8.9–9.10	197	5			10	2·546			10	320	59·386	M	w
		9.10–10.11	196	29			10	2·257			10	„	·709	„	w; b′

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
*262	2173	6.7–7	324°	58′	38′	1	6	0″·25 (Est.)	″			450	58·613	M	y
263	2178	7.8–9.10	129	54	13	6	10	10·385	·065	6	10	450	47·630	P	y ; p b
264	2180	7.8–7.8	85	9	33	1	4	3·120	·021	55	4	450	47·531	P	p b
		8–8	85	12	18	3	10	3·128	·014	130	10	„	·580	„	p b
		8–8	85	14	15	5	10	3·036	·020	58	10	„	·586	„	p b
		8–8.9	264	0	15	4	6	3·246	·035	20	6	320	56·490	M	v p b
		8–8.9	264	42	31	1	6	3·297	·092	3	6	„	·498	„	w
†265	2213	9–9.10	332	44			6	4·109			6	450	58·479	M	w ; b′
		8.9–9.10	331	39			6	4·351			6	320	·550	„	y′; b′
266	2218	7–8.9	353	14			10	2·310			10	450	59·244	M	p y ; b
		7.8–9	352	50			8	2·260			8	„	·457	„	w ; b′
267	2217	8.9–9	283	23			6	6·003			6	450	58·479	M	w
		9.10–10	283	33			6	6·355			6	320	·575	„	y′
268	2232	8–10.11	141	36			6	6·408			6	320	58·430	M	p y ; purp
		8–11	141	26			6	6·577			6	„	·460	„	y ; purp
‡269	2242	7.8–7.8	326	56	16	4	10	3·804	·028	31	10	320	54·646	M	w
		7.8–7.8	325	53	18	3	4	3·477	·050	10	4	„	·739	„	w
270	2245	8–8.9	294	35			10	2·510			10	450	59·548	M	w
		8.9–8.9	294	45			10	2·603			10	„	·657	„	y′ w
271	2263	8–9.10	160	3	11	9	10	7·314	·055	8	10	320 190	54·624	M	w ; b′
		8.9–10.11	161	4	9	12	10	7·361	·048	11	10	320	·638	„	w
		8.9–10.11	160	21	8	16	10	7·279	·017	88	10	„	·652	„	w ; v p b
§272	2264	5–5.6	259	36	5	37	10	6·144	·027	33	10	320	54·657	M	l g ; r
		5.6–5.6 M	259	45	8	16	10	6·145	·027	34	10	„	·663	„	p g ; r
		5.6–5.6 M	259	43	8	15	10	6·296	·025	39	10	„	·665	„	l g ; l r
		6–6	260	8			4	6·092			4	450	57·441	„	g′; r′
§		5.6–5.6	260	26			10	6·109			10	„	·457	„	g′w; orange

* When best seen, the stars were notched, but never separated. The distance was estimated 0″·2 or 0″·3.

† STRUVE's magnitudes are A = 7·5, B = 8·0; and he calls the star "B" which is here called "A."

‡ No perceptible difference in the magnitudes.

§ Scarcely half a magnitude difference.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
272	2264	5.6–5.6	260° 20′	′		10	6″·078	″		10	320	57·474	M	p g′ y; o
273	2272	5–6	110 5			6	6·253			6	320	57·635	M	y; b′
		5–7	110 25			6	6·048			6	„	·712	„	p y; r′ purp
		4.5–6.7	108 46			6	6·136			6	450	58·378	„	y; purp
		4.5–7	108 29			6	6·034			6	320	·413	„	y; r′
274	2276	7–7.8	256 59			6	6·460			6	450	58·474	M	y′; b′
		7.8–8	257 19			6	6·236			6	320	·476	„	y′; b′
275	2281	6–7.8	253 54	25	2	10	1·475	·032	25	10	320	54·668	M	v p y; w
276	2303	7.8–10	222 55	11	8	10	3·561	·031	26	10	320	55·638	M	p y; b
277	2304	8.9–10.11	69 33			6	5·271			6	320	57·476	M	w; b′
		9–11	68 58			6	4·924			6	„	·548	„	w; b′
278	2309	9.10–10	348 55	25	2	6	3·788	·026	38	6	190	54·654	M	w
		8.9–9	350 42	13	6	10	3·540	·012	169	10	320	·693	„	w; p b
279	2316	6–8	314 5	14	5	6	4·216	·028	31	6	450	53·632	M	y; orange
		6.7–8.9	314 5	13	6	6	4·020	·035	21	6	320	56·591	„	w; d b
		6–9	315 6	15	5	6	3·762	·029	29	6	450	·594	„	p y; b
280	2319	7.8–7.8	192 9	6	27	10	5·445	·007	556	10	320	54·687	M	w
		7.8–7.8	192 39	10	11	10	5·257	·031	25	10	„	·731	„	w
		7.8–7.8	191 55	9	12	10	5·379	·027	35	10	„	·734	„	w
281	2323	7–10.11	359 54	14	5	10	3·163	·024	42	10	450	45·684	P	y; b
		6.7–10.11	0 48	16	4	10	3·235	·035	20	10	„	·728	„	y; b
		6–10.11	359 50	10	11	10	3·268	·032	24	10	„	·731	„	y′; b′
		6–10	359 57	16	4	10	3·223	·024	44	10	„	46·693	„	y; b
		5.6–10	1 3	11	8	10	3·213	·022	50	10	„	·704	„	y; b
		5.6–9	359 28	17	4	6	3·402	·048	10	6	320	56·583	M	p y; purp
		5–8.9	359 47	13	6	6	3·469	·014	120	6	„	·586	„	p y; d b
282	2333	9–9.10	335 35			10	6·341			10	320	59·515	M	w; p b
		9–9.10	336 16			10	6·369			10	„	·526	„	w; b′ w
283	2339	8–10	270 17	10	11	10	2·238	·024	43	10	450	59·737	M	w; b

No. I.— *General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
284	2351	8–8.9	160° 25′	′		6	″ 5·259	″		6	320	58·476	M	y′; b
		8–8.9	159 54			6	5·193			6	450	·575	„	w
285	2362	7.8–9	179 16	12	8	10	3·690	·047	11	10	450	47·630	P	w ; p b
		7.8–9	180 51	11	8	10	3·826	·045	12	10	320	·663	„	w ; p b
		8–9.10	179 15			6	4·204			6	?	57·479	M	w ; b′
		9.10–11	180 14			6	4·121			6	450	·570	„	b′ w ; b
		8–10	180 27			6	4·211			6	320	58·457	„	p y ; purp
		8–10.11	181 17			6	3·976			6	„	·594	„	p y ; b
286	2375	7.8–8	113 7			6	2·310			6	320	58·674	M	w
		7–7.8	114 17			4	2·393			4	450	·676	„	y′
		8–8	112 3			6	2·279			6	„	·704	„	w
287	2382	6.7–8.9	18 51	18	3	4	3·099	·066	6	4	450	45·619	P	b′
		7–9	19 13	14	5	10	3·049	·020	61	10	„	·635	„	w′; b′
		7–9	22 50	11	8	10	3·234	·015	110	10	„	·643	„	w′; b′
		7–9	21 48	9	13	10	2·993	·040	15	10	„	·654	„	w′; b′
		5.6–7	18 51			6	3·100			6	„	58·427	M	y ; r′
		5–6.7	18 41			6	3·319			6	„	·430	„	p y ; y
		5–6.7	18 36			6	3·195			6	„	·444	„	p y ; r′
		5–6	19 2			6	3·186			6	320	59·583	„	p y ; b′ w
		?	18 39			6	3·136			6	450	·619	„	orange ; b′
288	2383	7–7	151 20	16	4	10	2·760	·029	29	10	450	45·597	P	y′
		7–7	150 39	14	5	10	2·854	·032	24	10	„	·643	„	w
		7–7	150 31	14	6	10	2·658	·021	54	10	„	·654	„	w
		5.6–6	147 14			6	2·377			6	600	58·378	M	p y
		5.6–6	146 36			6	2·506			6	450	·427	„	p y
		5.6–6	146 10			6	2·545			6	„	·444	„	p y ; y
289	2401	8–10.11	40 47			6	4·462			6	320	57·474	M	w ; b′
		9–11	42 16			4	4·204			4	„	·687	„	w ; b
		8.9–11	40 57			6	4·364			6	„	·706	„	w ; b

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
290	2404	7–9.10	184° 16′	14′	5	10	3″·452	″·009	294	10	450	44·684	P	y′; b
		7–9.10	184 6	9	12	8	3·701	·040	15	8	„	·695	„	orange; b
		7–9	182 38	9	11	6	3·638	·160	1	6	?	·717	„	y′; b
		7–9	183 3	12	7	10	3·454	·073	5	10	450	·734	„	y′; b
		7.8–9.10	182 29	15	5	10	3·399	·022	49	10	„	45·654	„	y; b
		7.8–9.10	181 17	16	4	10	3·557	·027	33	10	„	·684	„	y; b
		7.8–9.10	183 29	15	4	10	3·462	·045	12	10	„	·723	„	y; l b
		7.8–9	182 47	12	7	10	3·364	·028	31	10	„	46·690	„	p y; p b
		7–8.9	181 56	3	112	4	3·427	·013	147	4	„	·693	„	p y; p b
		8–9.10	182 29	8	16	10	3·443	·012	182	10	„	·704	„	y; p y
		7–9	182 59	14	5	10	3·441	·028	32	10	„	48·709	„	y
		7–9	182 4	8	16	10	3·542	·036	19	10	„	·717	„	p y
		6–8	182 55	6	24	10	3·572	·031	25	10	320	54·657	M	y; p b
		6.7–8	182 25	7	23	10	3·616	·030	27	10	„	·663	„	p y; b
		7–8	182 15	5	35	10	3·651	·028	30	10	190	·665	„	p y; b
291	2452	7.8–8.9	218 40			6	5·603			6	450	58·676	M	w; b′
		7.8–9	218 41			6	5·623			6	„	·695	„	w; p b
292	2446	7–9	153 40	15	4	10	9·732	·076	4	10	450	47·580	P	b; l b
		7–9	154 0	11	9	10	9·793	·071	5	10	„	·630	„	b; l b
		7.8–9.10	154 26	9	14	10	9·874	·032	24	10	320	54·684	M	w; b
		7.8–9.10	154 12	10	10	10	9·690	·043	13	10	„	·742	„	creamy or y′; b
293	2449	8–9	291 58	11	8	6	7·891	·037	18	6	320 190	56·490	M	w; p b
		8–9	291 47	15	5	4	7·766	·068	5	4	320	·561	„	w; ruddy
*		7.8–9	292 21	7	20	6	7·841	·012	164	6	„	·572	„	p b; r′?
294	2455	7.8–9	123 26	15	4	10	4·275	·058	7	10	320	55·654	M	w; p b
		7.8–10.11	124 51	31	1	6	3·848	·030	28	6	140 Mm	·660	„	w; b
		7–9.10	124 44	25	2	10	3·823	·031	26	10	320	·671	„	w; p b
295	2468	8.9–10	256 20	15	4	4	7·398	·053	9	4	190	57·739	M	p y; v b

* The colour of B by no means certain.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
*296	2481	8.9–8.9	51° 31′	10′	11	10	3·727″	·019″	70	10	450	48·528	P	p y
		9–9 M	230 26	10	9	6	3·981	·005	909	6	320	56·591	M	w
		9–9	230 14	26	2	6	3·945	·057	8	6	450 320	·594	”	w
297	2486	7.8–8	222 47	6	27	10	10·280	·013	137	10	450	46·706	P	w
		7–7	223 9	5	37	10	10·144	·044	13	10	”	47·572	”	p b
		7–7	222 56	8	15	10	10·264	·034	22	10	”	·580	”	b
		7–7	222 25	11	8	10	10·186	·045	12	10	”	·586	”	b
		7–7	223 0	10	10	10	10·430	·031	25	10	”	52·709	M	b
		6.7–7	222 42	5	37	10	10·260	·020	58	10	320	54·679	”	w
		6–6.7	222 22	7	23	6	9·987	·005	877	6	”	56·679	”	p y
		6.7–7	222 28	7	18	6	10·171	·018	73	6	”	·810	”	w
		7–7.8	222 18			6	10·073			6	450	59·619	”	p y; w
		7–7.8	222 35			10	10·082			10	320	·690	”	w; y
298	2504	6.7–9	286 54	9	13	10	8·912	·014	125	10	320	54·663	M	w
		6.7–8.9	287 12	7	22	10	8·868	·017	85	10	”	·665	”	w; b
		7–8.9	286 59	7	19	10	8·798	·025	41	10	”	·668	”	w
299	2523	7.8–8	150 35			6	6·105			6	450	58·474	M	w; b′
		8–8.9	150 12			6	6·056			6	”	·479	”	p y; b′
300	2524	10.11–11	100 27			6	7·082			6	190	57·570	M	w
		10.11–11	101 45			6	6·769			6	320	·709	”	w or b′
301	2534	8–8	244 50			10	6·736			10	320	59·515	M	w
		8–8	244 6			10	6·485			10	450	·657	”	w
302	43	3.4–6	55 29			10	34·635			10	190	57·457	M	gold y; b
	App. I.	3–6	55 13			10	34·519			10	320	·474	”	y; br b
		3–6	55 36			10	34·518			10	”	·476	”	br y; g′
303	2571	8–8.9	22 30	9	14	6	11·144	·105	2	6	320	56·586	M	w; p b
†304	2576	8.9–9	132 46	12	7	6	3·579	·017	88	6	320	56·594	M	w
		9–9 M	131 42	17	4	6	3·352	·006	667	6	”	·597	”	w

* Another double star in the field about 3′ south. The components 12 and 13 mag. Angle estimated 190°. Dist. 7″.

† The stars surrounded by points of light. Another very faint double star in the n. f. quadrant.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
			°	′	′			″	″						
305	2579	3–8	0	14	16	4	10					820 600	54·690	M	A y
		3–7.8	1	46	26	1	10	1·115	·023	46	10	600	·742	„	A y
		2.3–8	1	0	27	1	6	1·064			2	450	·939	„	A y
*		3–7 or 8	0	21	29	1	10					820	55·693	„	A y
†		3–7	0	49	14	5	10	1·3(Est.)				450	·723	„	A y
		3–6.7	0	37	21	2	10	1·275	·011	192	10	600	·805	„	p y ; b
306	2580	5–9.10	72	41			6	25·573			6	450	58·602	M	br y ; b
		5–9	72	34			6	25·607			6	320	·613	„	y ; br b
307	2583	6.7–7	121	52			4	1·537			4	600	58·654	M	y′
		7–7.8	120	7			6	1·447			6	600 450	·665	„	p y
308	2585	6–9.10	311	5	16	4	10	8·987	·049	10	10	320	54·638	M	b′; b
		6–9.10	312	14	12	7	10	8·651	·036	19	10	„	·652	„	p y ; br b
		5.6–9.10	312	11	11	8	10	8·688	·039	16	10	„	·654	„	w ; b
309	2603	5.6–9	359	53			10	3·016			10	600	59·737	M	y; b
		5–9.10	0	7			10	3·195			10	450	·739	„	orange ; b
		5.6–9	1	19			10	3·005			10	600	·753	„	y ; b
		5–9	0	20			10	3·036			10	450	·767	„	y ; v b
310	2605	5.6–10.11	182	32	15	5	10	3·627	·043	13	10	450	44·797	P	w ; l b
		5–10	182	45	13	6	10	3·487	·019	67	10	?	·868	„	w ; l b
		5–10.11	182	24	20	3	10	3·592	·038	17	10	450	·890	„	b′; b
		5.6–10.11	181	54	15	4	10	3·599	·027	33	10	„	45·758	„	w ; b
		5.6–10.11	183	0	29	1	6	3·590	·038	17	6	„	·780	„	y ; b
		5.6–10.11	183	29	18	3	10	3·673	·037	18	10	„	·789	„	y ; b
		5.6–8.9	182	25	12	7	10	3·440	·030	28	10	„	46·646	„	y ; p b
		5.6–8.9	181	53	13	6	10	3·457	·021	53	10	„	·668	„	y ; p b
		6–9.10	183	40	28	1	6	3·418	·042	14	6	„	48·851	„	y ; b′
		6–8.9	181	20	15	4	6	3·569	·039	16	6	„	56·671	M	w ; b′
		6.7–8.9	181	2	13	6	6	3·508	·046	12	6	320	·797	„	p y ; v b
311	2607	7.8–11	290	2	19	3	6	3·023	·018	78	6	450 600	59·715	M	w ; b

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* The vacancy = $\frac{1}{2}$ or $\frac{3}{4}$ diameter of small star.

† The dist. estimated at 1″·3, or vacancy = $\frac{1}{2}$ diameter of small star.

No. I.— *General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	p.	w.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
312	2609	7.8–9	25° 40′	′		6	2·185″	″		6	450	58·674	M	w; b′
		7.8–9	24 55			6	2·140			6	,,	·695	,,	p y; b′
*313	2610	9–9.10	298 56			6	4·218			6	320	57·750	M	w
	A & B	9–9.10	298 13			6	4·090			6	,,	·761	,,	w
314	2613	7.8–8	350 22	8	16	4	5·005	·084	3	4	320	55·657	M	w
		7.8–8	349 47	7	21	4	4·825	·012	167	4	{190 Mm}	·660	,,	w
		7.8–8	349 59	8	18	10	4·880	·028	31	10	,,	·679	,,	w
		7.8–8	349 27	12	7	6	4·645	·027	35	6	320	56·572	,,	w
		8–8.9	349 34	8	15	6	4·609	·033	23	6	,,	·583	,,	w; v p b
315	2619	9–9.10	244 40			6	4·317			6	320	57·709	M	y
		9.10–10	244 14			4	4·103			4	,,	·712	,,	w
316	2622	9–9.10	194 38	11	9	10	5·806	·005	1099	10	320	54·657	M	w; p b
	A & B	10–11	194 35			6	5·498			6	,,	58·523	,,	w; b′
		10–11	194 18			6	5·596			6	,,	·542	,,	w; b′
317	2631	9–10	341 27	10	9	10	4·475	·019	67	10	320	59·709	M	y′; b′
318	2637	6.7–7	225 44	2	210	10	72·979	·041	15	10	450	53·682	M	b
	A & B	6–7	225 52	3	112	10	73·114	·033	23	10	,,	·709	,,	d y; y
		6–7	225 34	6	24	6	72·627	·073	5	6	,,	·715	,,	y
319	A & C	9–11	324 50	5	47	10	11·468	·021	58	10	450	46·704	P	y; p b
		9–10.11	326 32	9	12	10	11·556	·035	20	10	,,	47·630	,,	y; p b
		6–8	326 41	8	15	10	11·392	·032	24	10	,,	53·676	M	y
		6–8	326 16	9	12	10	11·327	·028	32	10	,,	·679	,,	b
		6.7–9	327 18	9	13	10	11·466	·015	110	10	,,	·682	,,	b
		6.7–9	326 16	7	21	10	11·485	·032	24	10	320	54·665	,,	y; b′
		7–9	326 34	8	15	10	11·494	·025	39	10	,,	·668	,,	y; b
		7–9	326 14	8	15	10	11·362	·038	17	10	,,	·679	,,	p y; b
320	2639	8–9	302 18	7	20	6	5·518	·020	60	6	320	56·597	M	b′ w; b
		8–9	301 28	18	3	6	5·651	·059	7	6	,,	·657	,,	w; b

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* A small star in the s. p. quadrant, 12 or 13 magnitude, not noticed by STRUVE.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	p.	w.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
			°	′	′			″	″						
321	2644	8.9–8.9	210	0	12	7	8	3·330	·018	74	8	450	45·810	P	l b
		8.9–8.9	210	0	10	10	10	3·330	·021	58	10	„	·813	„	l b
		8–8	208	16	12	7	10	3·293	·045	12	10	„	·830	„	l b
		8.9–8.9	208	8	12	7	10	3·221	·025	40	10	„	·832	„	l b
		7–7	210	45	15	4	10	3·378	·021	53	10	„	52·731	M	b
		7–7	210	43	9	12	10	3·480	·018	72	10	„	53·717	„	l y; y
		7–7	212	32	15	5	6	3·457	·018	74	6	„	·728	„	y
		7–7	211	8	13	6	10	3·571	·032	24	10	„	·813	„	y
322	2655	8–8	3	20	9	13	10	6·158	·039	16	10	320	54·753	M	w
		8–8	2	55	7	20	10	6·083	·018	76	10	„	·830	„	w
		8–8	2	51	9	12	10	6·161	·012	175	10	„	·832	„	w
323	2675	4.5–9.10	122	18	17	4	6	7·198	·121	2	6	320	58·769	M	y; v b
324	2671	7–8.9	340	25			10	2·923			10	450	59·589	M	w; dusky
		7.8–9	340	49			10	3·011			10	„	·605	„	y′ w; b′
325	2695	6.7–8.9	79	12	16	4	10	1·017	·023	45	10	{450 600}	55·871	M	w; p b
*		7.8–9.10	75	6	21	2	6	0·967	·020	60	6	450	58·717	„	y′; b
†		7.8–9	77	27	21	2	10	0·75 (Est)				600	59·758	„	y′; w
326	2705	7.8–9	261	50			6	2·828			6	450	58·537	M	y; b
		8–9.10	262	34			6	2·886			6	320	·542	„	w; b
327	2708	6.7–8.9	340	31	7	22	10	16·054	·036	18	10	320	54·663	M	y; d b
		6.7–8	340	23	6	25	10	16·052	·023	46	10	„	·665	„	p y; b
		6.7–8	340	20	5	47	10	16·204	·017	88	10	„	·668	„	y; b
328	2716	5–8.9	53	16	16	4	10	3·120	·045	12	10	{450 320}	54·731	M	d y; b
		5–8.9	52	48	20	2	10	3·128	·022	52	10	320	·753	„	orange; b
		6–9	51	12	14	5	10	3·158	·033	23	10	„	·816	„	y; d b
329	2718	8–8	83	28	9	14	10	8·461	·073	5	10	450	48·873	P	p b
		8–8.9	86	10	7	23	6	8·183	·065	6	6	320	56·572	M	w
		8–8	86	28	13	6	6	8·110	·055	8	6	„	·586	„	w

* The stars were occasionally separated. † The distance estimated.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
			° ′	′			″	″						
*329	2718	8.9–8.9	85 47			10	8·257			10	320	59·619	M	p b
		8.9–8.9	86 11			10	8·074			10	450	·920	„	y′
330	2722	9–9.10	307 39			6	7·276			6	190	57·712	M	w
		8.9–9	308 18			6	7·120			6	{320 190}	·715	„	p y; b′
331	2725	7.8–8	357 23	7	22	10	4·606	·007	556	10	320	54·687	M	w; grey
		8–8.9	358 8	17	4	4	4·642	·008	400	4	„	·778	„	v w; drab
		8–8.9	358 3	8	18	10	4·566	·037	18	10	„	·791	„	w; b′
332	2726	5–10	58 58	14	5	6	6·488	·070	5	6	320	56·821	M	d or; purp
		6–10	57 57	20	3	6	6·705	·018	78	6	„	·832	„	golden; b
333	2727	5.6–7.8	272 12	7	20	8	11·727	·023	45	8	450	45·657	P	orange; y
		4.5–7	272 23	9	11	10	11·682	·023	47	10	„	·715	„	orange; y
		5–7	272 24	10	10	10	11·700	·020	60	10	„	·726	„	y
		4.5–6	272 30	7	20	6	11·375	·091	3	6	320	56·490	M	p y; p g
		4.5–6.7	272 16	4	62	6	11·186	·096	3	6	„	·531	„	oran; ṕ b
334	2736	9.10–11	217 57	27	1	6	4·867	·064	6	6	190	57·767	M	p b; b
335	2741	7–7.8	29 32	11	8	10	1·963	·031	26	10	{320 450}	55·679	M	w; v p b
		6.7–7	31 49	16	4	10	1·857	·011	220	10	450	·690	„	w; v p b
		6–7	32 9	13	6	10	1·849	·035	20	10	450?	·764	„	v p b; l b
		6–7.8	31 50	13	6	10	1·841	·024	41	10	600	·854	„	w
†336	2742	8–8.9	222 20			10	2·592			10	450	59·797	M	creamy w
		8.9–9	221 39			6	2·578			6	„	·805	„	y′ w
337	2745	6.7–10.11	192 41			6	3·066			6	450	58·665	M	p y; b
		7–10	192 28			6	3·034			6	„	·717	„	y; b
338	2747	8–8.9	259 27			6	4·520			6	320	57·709	M	w
		8.9–8.9	258 55			6	4·841			6	190	·715	„	w
339	2751	8–8.9	348 16			10	1·719			10	600	59·758	M	w
		8.9–9	349 25			10	1·750			10	450	·767	„	w

* The star n. f. is considered to be the larger.

† Scarcely half a magnitude difference.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag. *	Position.		P.	W.	No. of Obs.	Dis-tance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
			°	′	′			″	″						
340	2758	5–6	104	31	4	53	10	17·199	·050	10	10	450	52·731	M	y
		5–6	104	0	10	10	10	17·366	·025	39	10	,,	·797	,,	y
		5.6–6	105	31	5	40	10	17·408	·034	22	10	320	54·808	,,	y; p y
		5–6	105	9	3	86	10	17·549	·014	130	10	,,	·830	,,	d y; y
		5.6–6	105	20	7	23	10	17·386	·025	41	10	,,	·846	,,	orange; y
		5–6	108	18			10	17·807			10	190	59·906	,,	y
		5–5.6	108	16			10	17·952			10	320	·912	,,	y; d y
341	2760	7.8–8	224	29	4	76	10	11·013	·021	54	10	320	54·687	M	w
	A & B	7.8–8	224	23	6	24	10	10·817	·020	64	10	,,	·734	,,	w
*		7.8–8	224	6	10	11	10	10·913	·030	27	10	,,	·736	,,	w
		7.8–8	224	32	11	8	8	10·567	·033	23	8	600 450	55·860	,,	w; p b
342	2764	9–9.10	300	29			10	6·826			10	320	59·709	M	w
		9.10–10	300	48			10	6·657			10	450	·753	,,	w; b
343	2762	6.7–9.10	313	58	20	3	6	3·796	·018	73	6	320	56·531	M	w; p b
		6.7–9	313	39	18	3	6	3·718	·002	7692	6	,,	·591	,,	p b; r′
344	2776 B & C	10–11	337	4	13	6	4	6·100	·148	1	4	190	56·849	M	w; b?
345	2786	8–9	184	5			10	2·431			10	450	59·537	M	w
		8–9.10	184	40			6	2·143			6	,,	·605	,,	w; b′ w
346	2789	8.9–8.9	114	38	14	5	10	6·024	·048	11	10	450	45·635	P	b′
		8.9–8.9	116	12	16	4	4	5·899	·011	222	4	,,	·731	,,	l b
		8–8.9	116	6			10	5·646			10	,,	59·616	M	?
		8–8.9	115	24			10	6·042			10	320	·638	,,	y′
347	2797	7.8–9.10	215	2	17	4	6	3·543	·050	10	6	320	56·813	M	w; b
		7.8–9.10	213	55	16	4	6	3·508	·020	60	6	,,	·819	,,	v p y; b
348	2799	7.8–7.8 M	323	19	32	1	10	1·156	·015	112	10	600	55·854	M	w
		8–8.9	319	28			6	1·434			6	,,	59·693	,,	y′ w
		7.8-8	321	41			10	1·322			10	450	·931	,,	y

* A third star in the s. f. quadrant, 10·11 magnitude.

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Dis- tance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
349	2804	8–8.9	322°	59′	16′	4	10	3″·127	″·025	38	10	320	54·736	M	w
		7.8–8	322	51	11	9	10	2·933	·019	67	10	,,	·739	,,	v w
		8–8.9	323	7	11	8	10	3·000	·020	62	10	,,	·854	,,	w; v p b
350	2806	3–8	250	28	9	13	10	13·287	·043	13	10	320	54·827	M	v p y; b
		3–8.9	250	25	8	18	10	13·121	·018	76	10	,,	·871	,,	p y; b
		3–8.9	250	28	7	19	10	13·296	·021	55	10	,,	·901	,,	p y; br b
351	2814	8.9–10.11	163	11	9	12	4	7·617	·127	2	4	320	57·761	M	w; b
352	2816	8–10	339	5	9	12	10	19·856	·036	19	10	450	45·654	P	orange; b
	A & B	7–10	340	23	5	41	10	19·907	·092	3	10	,,	·838	,,	orange; b′
		7.8–10	339	41	6	25	10	19·852	·039	16	10	,,	·862	,,	y; b
		6–7.8	339	37	4	77	10	19·953	·021	55	10	320	54·739	M	y; w′
		6.7–8	339	25	7	18	10	19·912	·025	41	10	,,	·753	,,	y; b′ w
		6.7–8	338	56	6	25	10	19·950	·023	47	10	,,	·816	,,	y; gr
353	A & C	8–10	120	25	13	6	10	11·489	·022	51	10	450	45·654	P	orange; b
		7–10	120	27	9	14	10	11·437	·044	12	10	,,	·838	,,	orange; b′
		7.8–10	120	8	6	25	10	11·527	·055	8	10	,,	·862	,,	y; b
		6–8	121	8	6	33	10	11·728	·019	68	10	320	54·739	M	y; b
		6.7–8.9	121	4	7	24	10	11·696	·025	39	10	,,	·753	,,	y; b
		6.7–8.9	120	37	7	23	10	11·754	·050	10	10	,,	·775	,,	p y; b
354	2822	4.5–6.7	114	10	15	5	10	4·927	·056	8	10	450	47·657	P	orange; y
		4.5–6.7	115	0	12	7	10	5·066	·025	39	10	,,	·663	,,	y; p b
		4.5–6.7	113	16	10	9	10	4·835	·031	26	10	,,	·736	,,	orange; p y
		5–6.7	116	39	11	8	10	4·952	·036	19	10	320	54·830	M	v p y; b
		5–6	116	36	14	5	4	4·872	·011	222	4	,,	·835	,,	v p y; sky b
		5–6	116	29	10	10	10	4·820	·019	70	10	,,	·854	,,	v p y; b
		4.5–6	115	33	14	5	10	4·531	·037	18	10	450	55·832	,,	w; b′
		4–5.6	115	20	16	4	10	4·434	·015	116	10	,,	·838	,,	w; b′
		4.5–5.6	114	46	10	10	10	4·531	·021	58	10	600	·851	,,	w
		5.6–6.7	116	45			10	4·419			10	450	59·649	,,	y′ w; b′ w
		5–6.7	116	36			10	4·404			10	,,	·854	,,	p y; w or b′

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
			°	'	'			"	"						
355	2843	7.8–8	135	57			6	2·307			6	450	58·674	M	p y
		8–8.9	136	45			6	2·201			6	„	·712	„	y
356	2845	9.10–9.10M	174	58			6	2·077			6	450	58·745	M	y'
		9–9.10	173	9			6	1·893			6	320	·758	„	w
357	2848	8–9	55	0	8	17	10	10·397	·046	12	10	450	47·630	P	b
		7–7.8	55	45	5	39	6	10·273	·028	31	6	„	56·849	M	w
		7.8–8	55	18	8	16	6	10·234	·071	5	6	320	·871	„	p b
358		6–7.8	242	30			6	4·032			6	320	57·706	M	w; b
		6–7	242	31			10	4·098			10	„	·709	„	w; br b
		6.7–7	242	38			10	3·992			10	„	·764	„	p y
359	2861	8.9–9	220	47	13	6	6	7·262	·049	10	6	320	56·597	M	w
		8.9–9	220	25	9	13	6	7·200	·051	9	6	„	·797	„	b'
		8–8.9	220	48	17	3	6	7·112	·079	4	6	„	·813	„	w
		8–8.9	221	1	19	3	6	7·140	·025	40	6	„	·819	„	w; p b
360	2862	8–8.9	106	8	15	4	10	2·345	·024	41	10	450	59·693	M	w; y'
361	2863	5–7.8	287	53	11	8	10	5·874	·024	42	10	450	45·862	P	orange; b
		5–7.8	287	14	13	6	10	5·895	·040	15	10	„	·895	„	orange; b
		5–7.8	287	23	15	4	6	5·865	·014	132	6	„	·928	„	orange; b
		5–7	286	4			6	5·873			6	320	58·654	M	w; b
		5.6–7.8	286	44			6	5·826			6	450	·665	„	p b; b
362	2878	8–10	131	2			6	1·225			6	600	59·819	M	w; b'
		8–10	132	53			10	1·300			10	„	·868	„	w; b'
363	2881	8–8.9	105	29	12	7	10	1·749	·041	15	10	450	55·854	M	w; b'
		7.8–8	106	26	12	7	10	1·638	·030	28	10	600 450	·871	„	w
		8–8.9	106	36	9	12	10	1·494	·032	23	10	600	·873	„	w
364	2900	7–8	336	47	3	137	10	59·088	·053	9	10	320	54·734	M	y; p b
	A & B	6.7–8	336	44	8	16	6	59·153	·048	10	6	190	·895	„	y; b'
		7.8–8.9	336	45	4	69	10	59·238	·028	30	10	320	·931	„	v p y; gr'

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
365	2900	7–11	177° 54′	13′	6	10	2″·271	″·013	147	10	320	54·901	M	p y; b
	A & C	7–9.10	180 16	22	2	10	2·512	·032	24	10	„	55·931	„	w; v b
366	2902	7–8	88 30	8	17	10	6·207	·022	50	10	450	48·671	P	w; b′
		7.8–8	88 20	10	10	6	6·103	·032	24	6	320?	56·821	M	y; p b
		8–8.9	87 55	10	11	6	6·134	·061	7	6	{450 320}	·832	„	w; p b
367	2903	7.8–9	92 25	16	4	10	4·244	·049	10	10	450	44·657	P	w′
		7–9	92 25	12	7	10	4·223	·041	14	10	„	·663	„	w′
		7–9	91 46	8	17	10	4·296	·043	13	10	„	·665	„	w′
		7.8–9	94 31	22	2	10	4·209	·038	17	10	„	47·586	„	w; p b
		7.8–9	94 27	17	4	10	4·049	·015	110	10	„	·630	„	w; p b
		7.8–8.9	94 39			6	4·117			6	320	58·767	M	w; p b
		7.8–8.9	93 22			10	4·125			10	„	·810	„	p y; b′ w
		8–9	94 0			10	4·046			10	450	·821	„	y; p b
368	2910	9–9.10	344 41	20	3	6	5·479	·030	27	6	320	56·597	M	w
		9–9.10	343 47	29	1	6	5·412	·076	4	6	„	·813	„	p b
369	2909	4–4	346 55	10	10	10	3·597	·022	52	10	450	53·835	M	y
		4–4	346 13	8	15	8	3·573	·019	71	8	„	·849	„	y; orange
		4–4	346 26	7	18	10	3·626	·025	39	10	„	·860	„	y
		4–4	345 41	6	30	10	3·538	·048	11	10	600	55·797	„	w
		4.5–4.5 M	346 3	7	22	4	3·566	·042	14	4	450	·838	„	w; y′ w
		4.5–5	346 6	7	21	10	3·621	·032	25	10	„.	·851	„	w
*370	2917	9–9.10	70 50			6	4·176			6	320	58·572	M	w
		9.10–10	71 26			6	4·293			6	450	·695	„	w; b′ w
371	3134	9–10.11	77 9	13	6	6	5·917	·031	26	6	320	56·819	M	w
		9–10	75 45	18	3	6	5·967	·038	17	6	„	·821	„	w
372	2939	8–13	62 13	20	3	10	10·622	·119	2	10	320	59·756	M	{y′ w; intensely b
373	2944 A & B	7.8–8	248 52	17	3	6	3·687	·071	5	6	320	57·906	M	p y; w

* Nearly equal magnitudes.

No. I.— *General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	No. of Obs.	Distance.	*p.*	*w.*	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
374	2944	7.8–9	149° 27′	4′	61	6	51″·643	″·075	4	6	320	55·928	M	w ; b
	A & C	7.8–9	148 40			6	51·281			6	190	57·904	„	w ; b
		7.8–9	148 49			6	51·274			6	320	·906	„	y′ ; b′
375	2945	9.10–9.10	115 59	21	2	6	3·970	·011	222	6	320	56·849	M	w
		9–9	115 2	12	7	6	3·918	·015	110	6	„	·871	„	w
376	2946	8–8.9	255 41	6	29	10	5·176	·033	23	10	320	54·736	M	v p b
		7.8–8	256 1	6	26	10	5·165	·025	39	10	„	·816	„	w
		7.8–8	255 29	9	13	10	5·155	·030	28	10	„	·830	„	w
377	2947	7.8–7.8 M	70 35			6	3·258			6	450	58·717	M	w
		7–7.8	70 23			6	3·219			6	320	·723	„	p y
378	2950	6.7–8	315 34			6	1·966			6	450	58·600	M	y ; p b
		7.8–8.9	315 50			6	2·194			6	„	·695	„	p y ; b′
		6–7	315 1			10	1·949			10	600	59·638	„	p y ; p b
		5.6–7	316 34			10	2·185			10	450	·687	„	y ; b′
379	2974	8–8.9	161 10	24	2	4	2·922	·024	44	4	320	56·846	M	w
		8–8.9	160 11	20	3	6	2·604	·046	12	6	450	·871	„	w
380	2978	7.8–9	145 30	9	14	10	8·486	·023	47	10	450	47·879	P	w ; p b
		7–8	146 55	9	13	10	8·440	·019	71	10	320	54·854	M	w ; b
		7–8.9	146 7	11	9	10	8·440	·024	43	10	„	·871	„	w ; p b
		7.8–9	147 59	8	15	10	8·446	·039	16	10	„	·936	„	w ; b
		7.8–8.9	146 11	12	8	10	8·474	·013	147	10	450	55·832	„	w ; b
		7.8–9	146 37	25	2	6	8·277	·006	625	6	„	·851	„	v p b ; b
		7–8.9	145 58	12	7	10	8·362	·036	19	10	„	·871	„	w ; v b
381	2984	9–12	294 24			10	4·315			10	320	59·835	M	p y ; d b
		8–12	296 43			10	4·470			10	„	·868	„	y; d b
382	2985	7.8–9	252 24			10	14·962			10	320	59·715	M	p r ; y′ w
		7.8–8.9	252 40			6	15·128			6	„	·761	„	y′
		8–9	252 39			10	14·781			10	450	·819	„	y′ w ; r′

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800 +	Observer.	Colours.
			°	′	′			″	″						
383	2995	8–8.9	29	22			6	4·617			6	320	57·706	M	w
		8.9–9	29	32			6	4·609			6	190	·893	,,	w
384	3001	6.7–9	186	51			6	2·482			6	450	58·572	M	y; b
		6–9	186	52			6	2·726			6	320	·660	,,	y; b
385	3006	9.10–10	174	24	15	4	6	4·939	·010	263	6	320	56·849	M	w; b
		9.10–10.11	175	2	15	4	6	5·018	·010	263	6	450 320	·906	,,	w; b
386	3008	8.9–9	263	31	9	14	10	5·573	·044	13	10	320	58·923	M	y
		8–9	263	33			10	5·665			10	,,	59·915	,,	d y; r′
		8–9	264	20			10	5·496			10	450	·920	,,	r′; b′
387	3011	10–10.11	334	31			6	6·956			6	320	58·767	M	w
		10.11–11	333	32			10	7·053			10	,,	·810	,,	w; b′w
388	3019	7–9	184	59	17	3	4	10·337	·055	8	4	450	53·876	M	b; d b
		7.8–8.9	185	21	7	21	10	10·881	·044	13	10	320	54·830	,,	w; v p b
		7.8–9	186	5	8	14	10	10·711	·045	12	10	,,	·901	,,	w; p b
		7.8–8.9	186	11	6	26	8	10·656	·065	6	8	,,	·917	,,	w; b
389	3021	8.9–10.11	308	33	11	9	10	8·199	·023	47	10	320	59·906	M	w; b
390	3024	8–9	308	42	21	2	4	4·692	·015	110	4	450 320	53·887	M	w
		8.9–9.10	310	15	8	15	10	4·888	·030	26	10	320	54·739	,,	w
		9–9.10	309	24	19	3	8	4·728	·039	16	8	,,	·931	,,	w; b′w
		8.9–9.10	310	33	17	3	6	4·814	·050	10	6	,,	·934	,,	w
391	3042	7.8–7.8	89	24	6	28	10	4·375	·024	43	10	320	54·961	M	w; v p b
		7.8–7.8	89	27	10	11	6	4·325	·020	63	6	450	56·917	,,	w
		7.8–7.8	88	36	7	20	6	4·275	·020	63	6	,,	·953	,,	w
392	3049	6–10	326	41	15	4	10	2·925	·029	29	10	450	47·630	P	p y; p b
		6.7–8.9	324	47			6	2·964			6	,,	58·572	M	w; b
		6.7–8.9	324	46			6	2·956			6	,,	·712	,,	p y; purp
		6–8	323	48			10	2·909			10	450?	·821	,,	p y; b
		6–8	324	1			20	3·059			20	600	·857	,,	p y; b
		6–8.9	324	35			30	3·174			30	450	·876	,,	p y; p b

No. I.—*General Catalogue of* 398 *Double Stars* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	No. of Obs.	Distance.	*p*.	*w*.	No. of Obs.	Power.	Epoch. 1800+	Observer.	Colours.
			°	′	′			″	″						
392	3049	6.7–9	324	55			20	2·947			20	600	58·887	M	p y; b′
		6–8.9	324	15			20	2·904			20	,,	·890	,,	p y; b′
393	3050	7–7 M	196	22	7	20	10	3·658	·010	233	10	320	55·931	M	w
		7–7 M	196	43	9	12	10	3·452	·028	32	10	,,	·972	,,	w
		7–7 M	196	46	4	53	10	3·386	·042	14	10	,,	56·027	,,	v p y
394	3053	7–8.9	69	52			10	15·106			10	320	59·715	M	y; br b
		6.7–8.9	70	38			10	14·978			10	,,	·824	,,	d y; b
395	3057	8–10	296	47	15	4	6	4·095	·029	29	6	320	56·849	M	p y; l b
		7.8–10	297	29	10	9	6	3·887	·059	7	6	,,	·906	,,	p y; w
396	3060	9.10–10	113	10	24	2	5	3·774	·003	3333	4	320	56·597	M	w or p y
		8.9–9	114	19	13	6	6	3·782	·023	47	6	,,	·813	,,	w; slightly b
397	3061	7.8–8.9	148	9	40	1	4	7·711	·206	1	4	450	55·917	M	w
		7.8–8.9	146	40	11	9	10	7·444	·037	18	10	320	56·030	,,	w
398	3062	6.7–7.8	246	35	18	3	10	1·375	·022	53	10	450	55·832	M	w; b′
		6.7–7.8	247	43	19	3	10	1·239	·028	32	10	450 600	·871	,,	w
		5.6–7.8	249	34	20	2	10	1·370	·020	62	10	320 450	56·025	,,	w

REFERENCE CATALOGUE.

No.	Name of Star.	R.A.	N.P.D.	No.	Name of Star.	R.A.	N.P.D.
		h m	° ′			h m	° ′
1	Andromedæ 51	0 2	44 25	28		1 43	14 28
2		0 4	34 50	29	P. I. 191	1 45	79 53
3	35 Piscium	0 8	81 57	30	Andromedæ 241	1 45	53 24
4		0 9	36 9	31	γ Arietis	1 46	71 24
5	38 Piscium	0 10	81 54	32	α Piscium	1 55	87 55
6		0 10	90 28	33	γ Andromedæ A & $\frac{B+C}{2}$	1 55	48 21
7	Andromedæ 69	0 11	64 38	34		2 1	91 7
8	Andromedæ 112	0 28	53 58	35		2 2	57 20
9		0 28	60 45	36	H. III. 68	2 2	70 19
10	55 Piscium	0 33	69 20	37	[6] ι Trianguli	2 4	60 21
11		0 34	19 26	38	66 Ceti	2 6	93 3
12	P. O. 181	0 40	39 21	39	Trianguli 28	2 6	60 16
13	η Cassiopeæ	0 41	32 56	40		2 8	53 10
14	65 Piscium	0 42	63 3	41	Arietis 65	2 9	66 50
15		0 42	54 58	42		2 10	50 23
16		0 44	21 56	43		2 10	56 14
17	36 Andromedæ	0 47	67 8	44	Cassiopeæ B.A.C. 744 A & B	2 18	23 14
18	Andromedæ 164	0 52	46 1	45	A & C	”	”
19		1 3	39 13	46		2 19	35 7
20	φ Piscium	1 6	66 9	47	13 Trianguli P. II. 72	2 21	60 42
21		1 14	32 36	48		2 29	24 58
22		1 24	20 50	49		2 33	71 49
23	P. I. 123	1 29	83 7	50		2 37	36 41
24	[χ′] Ceti * Ceti 251	1 35	102 1	51	π Arietis A & B	2 41	73 7
25		1 37	47 1				
26		1 37	81 17				
27	P. 1. 179 1 Arietis	1 42	68 25				

* This star is called by STRUVE χ′ *Ceti*. The R.A. of χ *Ceti* in B.A.C. is $1^h 43^m$, and N.P.D. 101° 23′.

Reference Catalogue (*continued*).

No.	Name of Star.	R.A.	N.P.D.	No.	Name of Star.	R.A.	N.P.D.
		h m	° ′			h m	° ′
52	π Arietis	2 41	73 7	84		4 31	37 8
	A & C			85	55 Eridani	4 37	99 4
53	P. II. 220	2 51	38 12	86	[4] ω Aurigæ	4 50	52 19
54	ι Arietis	2 51	69 13	87		5 1	58 10
55		2 59	26 45	88	14 Aurigæ	5 6	57 29
56		3 4	30 32	89	ρ Orionis	5 6	87 19
57		3 8	50 3	90	ι Leporis	5 6	102 2
58		3 12	70 46	91	κ Leporis	5 7	103 7
59		3 19	31 8	92	H. II. 48	5 8	56 51
60		3 20	45 27	93	Rigel	5 8	98 22
61		3 20	70 2	94	P. V. 25	5 9	70 3
62		3 23	62 56	95	118 Tauri	5 21	64 58
63		3 26	70 40	96	Aurigæ 96	5 21	40 44
64	Cust. Mess. 70	3 28	20 38	97	H. III. 93	5 24	73 2
65	H. II. 52	3 31	56 20	98	λ Orionis	5 27	80 10
66	Tauri 34	3 32	61 42	99	Tauri 380	5 28	68 5
67		3 37	48 58	100	ι Orionis	5 29	96 0
68		3 39	56 50	101	26 Aurigæ	5 30	59 36
69		3 42	20 54	102	ζ Orionis	5 34	92 1
70	32 Eridani	3 47	93 22	103		5 35	59 9
71	ι Persei	3 48	50 24	104	52 Orionis	5 40	83 36
72	P. III. 213	3 53	67 12	105	P. V. 225	5 41	58 16
73	H. N. 17	3 59	67 18	106		5 52	53 31
74	Tauri 179	4 0	75 14	107	41 Aurigæ	6 1	41 16
75	39 Eridani	4 8	100 36	108	B.A.C. 1999	6 6	53 49
76		4 11	66 37	109	Orionis 277	6 7	75 23
77	Tauri 230	4 15	78 59	110	8 Monocerotis	6 16	85 20
78	H. II. 54	4 19	71 14	111	Aurigæ 229	6 22	37 27
79	1 Camelopardali	4 21	36 24	112	11 Monocerotis	6 22	96 57
80		4 21	50 20		A & B		
81	H. N. 132	4 25	72 18	113	B & C	,,	,,
82		4 29	100 3	114		6 25	51 21
83	Aurigæ 4	4 29	63 22	115	γ Canis Maj.	6 30	108 33

Reference Catalogue (*continued*).

No.	Name of Star.	R.A.	N.P.D.	No.	Name of Star.	R.A.	N.P.D.
		h m	° ′			h m	° ′
116	12 Lyncis	6 34	30 25	174	Lyncis 130	8 42	54 25
	A & B			148		8 44	61 1
117	A & C	”	”	149		8 45	45 53
118	H. II. 72	6 37	34 10	150	σ^2 Cancri	8 46	58 54
119		6 46	43 17	151	17 Hydræ	8 49	97 26
120	38 Geminor.	6 47	76 39	152	[66] σ^4 Cancri	8 53	57 12
121		6 48	64 50	153	σ^2 Ursæ Maj.	8 58	22 18
122	P. VI. 301	6 55	37 2	154	Cancri 194	8 59	66 29
123		7 0	72 52	155		9 9	65 46
124		7 4	62 31	156		9 10	54 2
125	19 Lyncis	7 11	34 28	157	38 Lyncis	9 10	52 36
126	H. III. 48	7 17	69 13	158	Lyncis 157	9 12	51 12
127		7 22	75 48	159	39 Lyncis	9 13	39 52
128	Castor	7 26	57 48		B.A.C. 3182		
129		7 32	56 18	160	21 Ursæ Maj.	9 16	35 23
130	Procyon	7 32	84 25	161		9 20	83 6
131	P. VII. 170	7 33	84 27	162	ω Leonis	9 21	80 20
132	A & B	7 34	25 36	163		9 24	16 17
133	A & C	”	”	164	Leonis Min. 130	9 32	50 24
134	Geminor. 201	7 40	71 18	165		9 36	46 8
135	ζ Navis	7 41	101 51	166		9 48	43 26
136	Cancri 17	7 57	62 2	167		10 10	61 46
137	Lyncis 85	8 0	57 21	168	γ Leonis	10 12	69 27
138	ζ Cancri	8 4	71 56	169	P. X. 58	10 17	36 40
	A & B			170		10 19	71 11
139	A & C	”	”	171	P. X. 94	10 24	96 55
140	φ^2 Cancri	8 18	62 37		B.A.C. 3603		
141	ν' Cancri	8 18	65 1	172	Leonis 178	10 26	65 54
142	P. VIII. 108	8 28	82 54	173		10 27	67 40
143		8 36	47 47	174		10 31	83 33
144	P. VIII. 160	8 38	92 6	175	Urs. Maj. 172	10 32	47 5
145	ι Hydræ	8 39	83 4	176		10 34	42 34
146	H. N. 144	8 42	18 39	177	35 Sextantis	10 36	84 31

Reference Catalogue (continued).

No.	Name of Star.	R.A.	N.P.D.	No.	Name of Star.	R.A.	N.P.D.
		h m	° ′			h m	° ′
178		10 42	93 15	208	θ Virginis	13 3	94 47
*179		10 47	6 2	209	42 Comæ Ber.	13 3	71 44
180	54 Leonis	10 48	64 30	210		13 26	52 28
181		11 6	15 45	211	P. XIII. 127	13 27	89 36
182		11 8	61 40	212		13 28	62 59
183	ξ Ursæ Maj.	11 11	57 41	213	81 Virginis	13 30	97 9
184	ν Ursæ Maj.	11 11	56 9	214	P. XIII. 156	13 32	38 34
185	Leonis 339	11 11	74 56	215		13 36	43 4
186	ι Leonis	11 17	78 42	216	84 Virginis	13 36	85 45
187	57 Ursæ Maj.	11 22	49 54	217		13 43	62 20
188	17 Crateris	11 25	118 30	218		13 48	56 30
189	90 Leonis	11 27	72 26	219	Boötis 51	13 52	63 31
190		11 29	33 5	220	κ Boötis	14 8	37 33
191	P. XI. 111	11 29	61 27	221		14 10	42 20
192	Ursæ Maj. 284	11 31	24 53	222	P. XIV. 69	14 17	80 55
193	Ursæ Maj. 290	11 31	44 8		B.A.C. 4766		
194	65 Ursæ Maj.	11 48	42 45	223		14 17	78 6
195	2 Comæ Ber.	11 57	67 46	224		14 28	53 49
196		11 58	37 16	225	π Boötis	14 34	72 59
197	Comæ Ber. 55	12 13	62 10	226	ζ Boötis	14 34	75 40
198		12 21	44 26	227	ε Boötis	14 39	62 20
199		12 26	57 11	228	ξ Boötis	14 45	70 19
200	γ Virginis	12 35	90 41	229	39 Boötis	14 45	40 42
201		12 42	85 24	230	H. N. 37	14 57	83 58
202	Virginis 359	12 46	74 11	231	44 Boötis	14 59	41 48
203	35 Comæ Ber.	12 46	68 0	232	P. XIV. 279	15 1	80 14
	A & B			234		15 9	17 1
204	A & C	”	”	235	η Cor. Bor.	15 17	59 12
205		12 49	94 3	236	δ Serpentis	15 28	78 59
206	Ursæ Maj. 417	12 50	35 9	237	Libræ 178	15 31	98 16
	B.A.C. 4348			238		15 32	59 25
207		13 0	88 42	239	ζ Cor. Bor.	15 34	52 54

* The place was obtained from the Redhill Catalogue.

Reference Catalogue (continued).

No.	Name of Star.	R.A.	N.P.D.	No.	Name of Star.	R.A.	N.P.D.
		h m	° ′			h m	° ′
240	H. II. 85	15 49	91 45	270	P. XVII. 300	17 50	71 39
	B.A.C. 5276			271	H. II. 90	17 55	63 26
241	ν Scorpii	16 4	109 5	272	95 Herculis	17 56	68 24
	A & $\frac{B+C}{2}$			273	70 Ophiuchi	17 58	87 28
				274	P. XVII. 362	17 59	78 0
242	49 Serpentis	16 7	76 6	275	73 Ophiuchi	18 3	86 2
243	σ Cor. Bor.	16 9	55 47	276	Scuti Sob. 15	18 12	98 1
244		16 19	25 17	277		18 12	49 47
245	Antares	16 21	116 7	278	H. I. 86	18 14	64 32
246	η Draconis	16 22	28 10	279	59 Serpentis	18 20	89 53
247	Herculis 71	16 23	71 15	280		18 21	70 46
248	λ Ophiuchi	16 24	87 42	281	39 Draconis	18 22	31 17
249	17 Draconis	16 33	36 48	282		18 26	57 51
250	ζ Herculis	16 36	58 8	283		18 27	72 22
251		16 37	66 6	284		18 32	48 48
252		16 44	53 49	285	P. XVIII. 151	18 33	54 5
253	P. XVI. 270	16 55	81 21	286	Tauri Pon. 75	18 38	84 40
254	Herculis 210	16 59	61 43	287	ε Lyræ	18 40	50 28
255	μ Draconis	17 2	35 21	288	5 Lyræ	18 40	50 32
256		17 6	68 35	289		18 43	69 1
257	36 Ophiuchi	17 7	116 23	290	Tauri Pon. 78	18 44	79 11
258	α Herculis	17 8	75 27	291	Draconis 233	18 58	14 24
259	ρ Herculis	17 19	52 43	292	P. XVIII. 302	18 59	83 40
260	Herculis 281	17 21	60 26	293	H. III. 109	18 59	83 4
261		17 22	54 8	294		19 1	68 3
262		17 23	90 54	295		19 3	81 33
263		17 24	54 55	296		19 6	51 28
264	P. XVII. 147	17 26	39 1	297	Cygni 6	19 8	40 26
265	Herculis 331	17 39	58 48	298		19 15	71 6
266		17 39	26 21	299	H. III. 57	19 21	69 7
267		17 40	75 8	300	H. N. 100	19 21	64 47
268		17 44	64 42	301	P. XIX. 149	19 22	53 46
269		17 47	45 4	302	β Cygni	19 25	62 20

Reference Catalogue (continued).

No.	Name of Star.	R.A.	N.P.D.	No.	Name of Star.	R.A.	N.P.D.
		h m	° ′			h m	° ′
303		19 36	12 4	335	P. XX. 429	20 54	40 5
304		19 40	56 44	336	2 Equulei	20 55	83 22
305	δ Cygni	19 41	45 13	337	12 Aquarii	20 57	96 22
306	χ Cygni	19 41	56 36	338		20 57	52 55
307	π Aquilæ	19 42	78 32	339	Cephei 83	20 58	33 55
308	ζ Sagittæ	19 43	71 12	340	61 Cygni	21 1	51 56
309	ι Draconis	19 49	20 5	341		21 1	56 28
310	χ Cygni	19 52	37 56	342		21 2	28 25
311	Cygni 116	19 53	48 7	343	P. XXI. 1	21 3	60 22
312	Cygni 118	19 53	52 17	344	H. I. 46	21 8	100 53
313		19 54	54 53	345	Equulei 27	21 13	81 8
314	Aquilæ 210	19 55	79 40	346		21 15	37 37
315		19 57	42 7	347		21 20	76 54
316	P. XIX. 392	19 58	73 25	348	Pegasi 20	21 22	79 31
317		20 1	69 18	349	Pegasi 29	21 26	69 54
318	θ Sagittæ	20 4	69 30	350	β Cephei	21 27	20 3
	A & B			351		21 33	54 17
319	A & C	,,	,,	352	P. XXI. 248	21 35	33 9
320		20 4	54 57		A & B		
321	P. XX. 26	20 5	89 33	353	A & C	,,	,,
322	H. N. 55	20 8	68 13	354	μ Cygni	21 38	61 53
323	κ Cephei	20 14	12 43	355		21 48	24 55
324	H. I. 95	20 15	35 3	356		21 48	27 36
325	Vulpec. 94	20 26	64 42	357	H. III. 74	21 51	84 45
326		20 32	57 10	358	29 Aquarii	21 55	107 38
327	H. N. 87	20 33	51 53	359		21 59	69 51
328	49 Cygni	20 35	58 11	360		22 0	90 9
329		20 36	77 48	361	ξ Cephei	22 0	26 3
330		20 37	70 48	362	Pegasi 148	22 7	82 46
331	H. II. 66	20 40	74 38	363		22 7	61 10
332	52 Cygni	20 40	59 47	364	A & B	22 17	69 52
333	γ Delphini	20 40	74 23	365	A & C	,,	,,
334		20 50	77 35	366	H. N. 64	22 17	45 23

Reference Catalogue (*continued*).

No.	Name of Star.	R.A.	N.P.D.	No.	Name of Star.	R.A.	N.P.D.
		h m	° ′			h m	° ′
367		22 18	24 2	383		23 9	92 23
368		22 21	67 11	384	ο Cephei	23 13	22 39
369	ζ Aquarii	22 22	90 44	385		23 14	55 21
370		22 25	37 13	386	P. XXIII. 69	23 16	99 14
371		22 34	60 44	387		23 19	13 42
372	P. XXII. 209	22 38	100 23	388		23 23	85 31
373	P. XXII. 219	22 41	94 57	389		23 24	74 34
	A & B			390	H. II. 94	23 25	46 58
374	A & C	”	”	391	Andromedæ 28	23 45	52 54
375		22 43	59 27	392	σ Cassiopeæ	23 52	35 2
376		22 43	50 16	393	Andromedæ 37	23 52	57 3
377		22 44	22 13	394		23 55	24 43
378	Cephei 241	22 45	29 5	395		23 57	32 16
379		22 58	57 25	396		23 58	72 43
380	P. XXII. 306	23 1	57 56	397		23 58	72 56
381		23 2	20 8	398	H. I. 39	23 59	32 22
382		23 3	42 49				

No. II.—CATALOGUE OF MEAN RESULTS.

No.	Σ's No.	Mag.	Position.	P.	W.	Δ.	No. of Obs.	Distance.	*p*.	*w*.	Δ′.	No. of Obs.	Epoch. 1800+	Observer.
3	12	6–9	150° 50′	18′	3	+ 1° 29′	28	11″·846	″·052	9	+0″·191	28	43·946	S
		6.7–8	149 17	1	2198	— 0 4	12	11·276	·004	1370	—0·379	12	56·894	M
4	16	8.9–11	37 24	8	18	— 0 50	30	5·407	·025	40	—0·097	30	59·756	„
5	22	7–8	234 54	9	13	— 2 45	24	4·419	·025	41	—0·171	24	59·924	„
8	40	7–9.10	312 17	16	4	+ 0 3	10	11·436	·056	8	—0·127	10	56·801	„
9	42	8.9–9.10	33 34	10	10	— 1 42	12	5·469	·052	9	+0·144	12	57·915	„
10	46	6–10	193 51	10	10	+ 1 7	16	6·323	·034	21	—0·044	16	58·845	„
11	48	9–9.10	332 29	9	13	+ 0 5	12	5·489	·037	18	—0·001	12	58·700	„
12	59	8–9.10	147 43	24	2	+ 2 45	12	2·172	·037	18	—0·020	12	58·662	„
13	60	4–8	110 2	32	1	+17 55	14	8·116	·084	3	—1·279	14	54·949	„
		4–7.8	112 25	8	14	+20 18	30	7·804	·012	164	—1·591	30	55·961	„
		3.4–8.9	117 3	8	17	+24 56	20	7·082	·040	15	—2·313	20	59·938	„
14	61	6–6.7	296 28	11	9	— 2 30	30	4·539	·016	94	+0·089	30	54·874	„
15	62	9.10–10	303 4	5	45	+ 0 32	12	11·087	·036	19	—0·326	12	56·930	„
16	65	9.10–9.10	36 34	10	10	+ 1 26	12	2·975	·046	12	—0·015	12	58·686	„
17	73	6–6.7	340 6	20	3	+19 38	24	1·250	·056	8	+0·313	24	54·915	„
18	79	6.7–7	192 33	3	155	+ 0 6	30	7·709	·012	164	+0·086	30	56·036	„
19	97	9.10–10	99 15	14	5	+ 0 41	20	4·591	·027	33	+0·048	20	59·812	„
20	99	5–12	225 44	15	5	— 1 47	12	7·664	·098	3	—0·316	12	56·893	„
24	147	6–7	89 37	27	1	+ 1 23	10	3·785	·011	220	—0·229	10	56·960	„
25	154	8.9–9	126 45	11	8	+ 0 5	12	4·978	·057	8	—0·192	12	57·924	„
26	155	7.8–8	329 33	30	1	— 3 16	22	4·838	·045	12	+0·238	22	56·010	„

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	Δ.			No. of Obs.	Distance.	p.	w.	Δ'.	No. of Obs.	Epoch. 1800+	Observer.
			°	'	'			°	'		"	"		"			
27	174	7.8–8.9	170	0	9	13	−	0	31	16	2·786	·019	70	+0·219	16	58·819	M
28	170	9–10	247	48	10	9	+	1	0	20	3·243	·027	33	+0·067	20	59·878	„
30	179	7.8–8	158	30	15	5	−	1	55	12	3·567	·008	400	+0·107	12	56·881	„
31	180	4.5–4.5	359	4	13	6	−	0	55	30	8·723	·014	120	+0·092	30	56·051	„
32	202	5.6–8	331	51	37	1	−	3	52	30	3·627	·093	3	−0·009	30	44·072	S
		4.5–5.6	326	46	15	5	−	8	57	12	3·515	·032	24	−0·121	12	58·700	M
33	205	3–6.7	63	11	8	16	+	0	45	38	10·341	·026	37	+0·009	38	59·760	„
34	218	9–9.10	247	21	9	13	−	2	39	16	4·821	·027	34	+0·039	16	58·811	„
35	219	8.9–9.10	183	43	1	2198	+	2	4	12	11·423	·015	110	+0·028	12	56·916	„
36	221	8–9.10	145	33	13	6	+	0	19	20	8·420	·098	3	+0·043	20	54·937	„
		8–9	143	48	11	9	−	1	26	30	8·256	·027	33	−0·121	30	55·961	„
37	227	5–6	78	53	5	45	−	1	39	12	3·849	·033	22	+0·166	12	53·015	M
		5–6.7	78	43	18	3	−	1	49	28	3·795	·042	14	+0·112	28	54·038	C. W. & M
38	231	7–8.9	229	49	13	6	+	0	6	30	15·418	·139	1	+0·068	30	54·979	M
39	232	8.9–8.9	65	30	6	27	−	0	2	12	6·305	·022	51	−0·252	12	57·957	„
40	238	10–10.11	357	45	8	16	+	1	24	12	10·717	·023	45	−0·203	12	57·055	„
41	240	8.9–8.9	48	57	22	2	+	0	55	12	5·034	·039	16	+0·321	12	56·960	„
42	245	7–8	292	11	6	27	+	0	23	12	10·792	·008	435	−0·223	12	56·816	„
43	246	8–9.10	122	39	5	39	+	0	7	12	9·876	·052	9	−0·601	12	56·997	„
44	262 A&B	5–8	265	4	30	1	−	11	37	30	2·011	·032	24	+0·149	30	56·034	„
45	A&C	5–10	107	51	7	23	+	0	33	23	7·497	·047	11	−0·129	22	59·837	„
46	268	8–10	127	37	8	17	−	1	31	20	2·669	·029	29	−0·027	20	59·782	„
47	269	8–9.10	345	12	23	2	+	4	48	30	2·036	·019	69	+0·133	30	56·060	„

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	Δ.	No. of Obs.	Distance.	*p.*	*w.*	Δ.	No. of Obs.	Epoch. 1800+	Observer.
49	291	8–8.9	119° 10′	10′	11	+0° 9′	20	3″·396	″·031	26	+0″·141	20	58·984	M
50	301	8–10	15 33	7	19	−1 3	20	8·064	·020	60	−0·163	20	59·062	„
51	311 A&B	6.7–9	121 16	7	22	+1 58	12	3·273	·034	21	−0·011	12	56·855	„
53	331	6–7.8	85 2	5	46	0 0	24	12·032	·037	18	−0·161	24	58·845	„
54	333	6–7	198 18	41	1	+9 26	24	0·986	·014	132	+0·439	24	56·027	„
55	349	9–10	320 25	15	5	+0 34	12	6·057	·045	12	−0·080	12	58·709	„
56	362	9.10–10	141 17	6	32	−1 1	20	7·051	·024	43	+0·138	20	59·783	„
57	369	7.8–8.9	28 57	15	5	+0 7	12	3·491	·032	24	+0·238	12	56·916	„
58	376	8.9–8.9	250 30	6	25	−0 43	12	6·591	·035	20	−0·193	12	57·924	„
59	389	7.8–9.10	64 10	13	6	+2 21	20	2·766	·012	172	−0·039	20	59·860	„
60	391	8–8.9	94 13	8	17	−0 37	12	3·698	·028	31	−0·092	12	57·037	„
61	394	7.8–8.9	162 4	6	27	−1 12	24	6·794	·044	13	+0·101	24	55·073	„
62	401	7–7.8	269 44	8	17	−0 17	12	11·023	·027	34	−0·097	12	56·868	„
63	414	9.10–10	185 16	6	30	−0 22	16	7·334	·047	11	+0·247	16	58·811	„
65	425	9–9	101 28	11	9	−3 8	16	2·714	·024	43	−0·159	16	58·823	„
66	427	7.8–8	209 3	7	21	+0 27	12	6·580	·036	19	−0·100	12	57·050	„
68	448	8–11.12	16 32	15	4	−2 4	12	3·286	·018	74	+0·013	12	57·074	„
70	470	5–6.7	347 26	8	17	+0 10	12	6·753	·048	11	+0·056	12	58·047	„
71	471	3.4–8.9	9 54	14	5	+0 39	30	8·886	·020	63	+0·078	30	56·048	„
72	479 A&B	7.8–8	127 41	7	20	−0 49	12	7·082	·050	10	−0·032	12	57·060	„
73	494	7.8–8	187 26	6	24	−2 26	30	5·087	·042	14	+0·007	30	56·046	„
74	495	8–11.12	219 24	11	8	+3 18	20	3·881	·019	66	+0·244	20	58·990	„

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	Δ.	No. of Obs.	Distance.	*p*.	*w*.	Δ′.	No. of Obs.	Epoch. 1800+	Observer.
75	516	6–11	149° 16′	10′	10	− 4° 26′	20	6″·377	″·134	1	+0″·117	20	56·078	M
		6–11	150 31	78	0	− 3 11	12	6·555	·046	12	+0·295	12	·827	„
76	523	8–11	162 52	21	2	− 2 11	15	10·067	·062	6	−0·223	16	56·980	„
77	535	8–10	345 58	11	8	− 7 55	20	1·601	·034	21	−0·347	20	60·019	„
78	546	8.9–10.11	186 15	8	15	− 3 39	30	6·715	·013	147	+0·062	30	55·127	„
79	550	6.7–7.8	307 12	24	2	+ 0 8	12	10·191	·003	3333	+0·058	12	56·941	„
80	552	7–7.8	113 52	11	8	− 0 33	12	8·824	·022	49	−0·140	12	56·860	„
81	559	7.8–8	277 18	7	24	− 1 22	12	3·068	·017	85	+0·033	12	57·074	„
82	570	7–8	258 51	6	24	− 0 5	23	12·656	·058	7	−0·114	22	54·092	„
83	572	7.8–7.8	26 29	8	16	− 3 49	12	3·431	·023	47	+0·255	12	57·081	„
85	590	7.8–7.8	316 41	17	3	− 1 38	30	9·055	·059	7	−0·072	30	45·129	P
		6.7–7	317 7	7	23	− 1 12	20	9·015	·052	9	−0·112	20	59·886	M
86	616	6–10	352 30	10	10	+ 0 35	12	6·251	·042	14	−0·211	12	57·060	„
87	648	8–9	72 50	9	13	− 1 26	20	4·965	·062	6	+0·285	20	56·068	„
88	653	5.6–8	225 20	4	54	− 0 9	30	14·649	·044	13	−0·004	30	55·163	„
89	654	5–10	61 9	11	8	− 2 19	16	6·909	·038	17	−0·146	16	58·032	„
		5–9	64 3	10	10	+ 0 35	20	7·051	·029	30	−0·004	20	60·136	„
92	666	9–9	72 59	9	11	+ 1 41	20	3·070	·020	63	+0·093	20	59·999	„
94	674	7.8–11.12	147 52	18	3	+ 0 32	12	10·247	·078	4	−0·300	12	57·036	„
95	716	7–8	197 36	8	16	+ 0 49	10	4·976	·052	9	+0·082	10	57·074	„
96	718	8.9–9	74 24	7	21	+ 0 14	20	7·655	·026	35	−0·128	20	58·883	„
97	730	7–7	139 38	16	4	− 2 11	12	9·522	·055	8	−0·288	12	56·941	„
98	738	4–7	45 26	31	1	+ 5 7	20	4·594	·002	5000	+0·358	20	52·156	„
100	752 A&B	3–8	141 57	1	586	− 0 13	34	11·095	·021	57	−0·225	34	54·128	„

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	Δ.	No. of Obs.	Distance.	p.	w.	Δ′.	No. of Obs.	Epoch. 1800+	Observer.
101	753	5.6–8.9	268° 3′	5′	39	+ 0° 2′	20	12″·002	″·017	88	−0″·341	20	54·181	M
		6–8	267 40	3	82	− 0 21	30	12·107	·011	222	−0·236	30	55·175	”
		5.6–9	268 4	7	19	+ 0 3	20	12·014	·056	8	−0·329	20	58·228	”
102	774	2.3–6	151 54	31	1	+ 0 34	30	3·062	·069	5	+0·512	30	54·155	”
104	795	7–7	200 26	7	21	+ 0 20	20	1·506	·016	98	−0·241	20	60·171	”
105	796	8–9	61 8	6	29	− 0 2	60	3·629	·012	167	+0·033	60	59·152	”
106	825	8.9–9.10	145 39	11	8	− 0 32	12	8·134	·057	8	−0·029	12	57·096	”
107	845	7–8	354 4	5	44	+ 0 57	20	8·019	·021	56	+0·022	20	59·783	”
108	872	8–9	217 11	9	12	− 0 11	28	10·844	·017	83	−0·183	28	46·198	P
		6.7–7	217 54	15	5	+ 0 32	30	10·733	·006	769	−0·294	30	54·178	M
109	877	8–8.9	262 15	5	41	− 1 5	60	5·565	·013	154	+0·248	60	55·145	”
110	900	6.7–8.9	25 23	3	109	− 0 29	30	13·819	·036	19	−0·046	30	45·242	P
		6–8	25 12	4	72	− 0 40	20	13·924	·051	10	+0·059	20	46·251	”
111	918	9–10	324 26	7	18	+ 2 0	20	4·498	·035	20	+0·048	20	59·959	M
112	919	5–6	132 11	13	6	+ 2 11	26	7·429	·067	5	+0·176	26	52·183	”
	A & B	5.6–6.7	133 5	8	14	+ 3 5	30	7·191	·009	333	−0·062	30	54·202	”
113	919 B & C	6.7–6.7	104 42	7	19	+ 2 58	20	2·601	·019	67	+0·138	20	60·143	”
115		6–8.9	262 51	5	45		20	16·791	·036	19		20	59·985	”
116	948	6–6.7	142 1	21	2	−11 41	30	1·867	·037	18	+0·335	30	54·212	”
	A & B	6–6.7	140 15	16	4	−13 27	12	1·554	·026	36	+0·022	12	58·247	”
117	A & C	6–7.8	305 30	6	24	+ 1 18	30	8·561	·009	333	−0·109	30	54·235	”
		6–7.8	305 9	9	13	+ 0 57	12	8·568	·042	14	−0·102	12	58·247	”
118	958	6.7–6.7	258 58	14	5	+ 2 14	12	4·912	·015	110	−0·158	12	56·245	”
120	982	6–10	171 31	7	18	− 3 22	30	6·164	·012	164	+0·428	30	45·232	P
		6.7–10	170 22	6	24	− 4 31	20	6·179	·021	55	+0·443	20	46·169	”
		5.6–8.9	168 58	22	2	− 5 55	30	5·998	·045	12	+0·262	30	54·146	M

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	Δ.		No. of Obs.	Distance.	*p*.	*w*.	Δ′.′	No. of Obs.	Epoch. 1800+	Observer.
122	1009	7–7	156°	13′	13′	6	− 2°	58′	20	3″·133	″·011	194	+0″·193	20	59·215	M
123	1027	9.10–10	356	25	11	8	+ 0	13	20	7·165	·048	11	+0·433	20	58·882	”
124	1037	8–8.9	325	3	14	5	− 2	23	20	1·355	·023	47	+0·032	20	60·105	”
125	1062	5.6–6.7	313	8	5	49	− 0	42	26	14·805	·098	3	+0·081	26	54·181	”
126	1083	7–8	45	49	5	41	+ 3	13	24	6·381	·063	6	+0·181	24	54·166	”
128	1110	3–3.4	244	25	4	62	−11	4	12	5·208	·020	65	+0·474	12	58·259	”
		2.3–3.4	243	53	7	22	−11	36	20	5·156	·014	128	+0·422	20	59·262	”
		2.3–3.4	243	37	7	23	−11	52	24	5·378	·027	34	+0·644	24	59·976	”
129	1123	11–11.12	160	19	16	4	− 2	25	12	3·612	·019	70	−0·048	12	57·087	”
131	1126	7·8–8	136	53	3	88	+ 4	52	12	1·314	·018	73	−0·150	12	56·238	”
		7.8–8	140	2	15	5	+ 8	1	18	1·274	·016	94	−0·190	18	59·911	”
132	1127	7–8.9	340	11	30	1	− 0	11	12	5·369	·082	4	+0·142	12	56·274	”
	A & B	8–9.10	339	0	7	20	− 1	22	17	5·318	·025	40	+0·091	16	59·221	”
134	1140	7.8–9.10	271	29	9	11	− 2	23	12	6·006	·031	25	−0·151	12	58·046	”
136	1177	7–8	352	15	3	107	− 2	30	20	3·767	·045	12	+0·255	20	55·173	”
		6.7–7.8	352	33	5	39	− 2	12	12	3·559	·029	31	+0·047	12	56·259	”
137	1187	7.8–8.9	56	41	8	17	−14	19	20	1·819	·030	27	+0·205	20	59·141	”
		8.9–9.10	55	28	12	7	−15	32	16	1·830	·038	17	+0·216	16	60·282	”
138	1196 A & B	5–6	143	2	25	2	− 1	6	18	5·235	·015	110	+0·002	18	53·225	”
139	A & C	6–6.7	309	25	16	4	−65	57	12	1·119	·050	10	−0·078	12	56·238	”
140	1223	6–6	213	19	19	3	+ 1	17	30	4·879	·052	9	+0·316	30	54·238	”
141	1224	6–7	40	38	16	4	+ 3	22	20	5·986	·029	29	+0·148	20	52·246	”
		6.7–7.8	40	13	8	15	+ 2	57	26	6·063	·020	63	+0·225	26	54·160	”
142	1245	7–8	24	54	5	46	− 0	33	30	10·333	·022	53	+0·004	30	44·271	S
		7–8.9	25	6	4	68	− 0	21	30	10·430	·045	12	+0·101	30	45·248	P
		7–9	25	55	6	30	+ 0	28	20	10·310	·073	5	−0·019	20	48·270	”
		6–7	25	57	11	8	+ 0	30	26	10·409	·056	8	+0·080	26	54·179	M

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	Δ.	No. of Obs.	Distance.	*p*.	*w*.	Δ′.	No. of Obs.	Epoch. 1800+	Observer.
143	1263	7.8–8.9	16° 27′	2′	213	+ 6° 51′	30	22″·508	″·050	10	+12″·183	30	54·217	M
		8.9–9	18 21	4	70	+ 8 45	20	26·746	·033	22	+16·421	20	60·084	„
144	1270	8–8.9	259 7	13	6	+ 0 1	12	4·570	·057	8	− 0·127	12	58·046	„
145	1273	4.5–8.9	208 55	8	16	+10 19	30	3·511	·023	47	+ 0·308	30	54·277	„
		4–8	210 38	12	7	+12 2	12	3·423	·036	18	+ 0·220	12	58·300	„
146	1280	9–9.10	36 2	12	8	+ 2 5	22	6·613	·043	13	− 0·819	22	59·135	„
147	1282	7.8–7.8	276 46	8	15	− 0 36	12	3·381	·002	9091	− 0·019	12	56·259	„
148	1288	9.10–9.10	258 31	21	2	− 0 23	26	7·525	·061	7	+ 0·002	26	54·264	„
149	1289	9–10	2 36	13	6	− 1 38	16	3·983	·049	10	+ 0·180	16	57·128	„
150	1291	5.6–6	336 38	13	6	+ 3 20	12	1·402	·016	97	− 0·108	12	57·292	„
		6.7–7	334 52	10	10	+ 1 34	26	1·292	·016	97	− 0·218	26	60·154	„
152	1298	6.7–9	135 20	18	3	− 2 27	12	4·765	·001	21978	+ 0·161	12	56·243	„
153	1306	4.5–8.9	258 3	26	2	− 3 57	30	3·607	·035	20	− 1·003	30	54·260	„
154	1311	7.8–8.9	200 24	20	3	− 0 6	30	7·203	·031	25	+ 0·001	30	45·242	P
		8–8.9	199 52	2	176	− 0 38	20	7·349	·022	49	+ 0·147	20	48·248	„
		7–7.8	200 .9	4	54	− 0 21	38	7·226	·049	10	+ 0·024	38	54·203	M
155	1332	7–8	16 22	4	59	+ 0 6	30	5·553	·004	1754	− 0·010	30	54·248	„
156	1333	6.7–7	39 11	10	10	− 0 13	30	1·440	·012	169	+ 0·020	30	59·269	„
157	1334	4.5–7.8	238 17	10	10	− 1 56	26	2·915	·030	27	+ 0·218	16	59·$^{154}_{174}$	„
158	1338	7–7	137 3	20	3	+15 55	30	1·924	·036	19	+ 0·162	30	54·240	„
159	1340	8–10	318 14	13	6	− 1 10	12	6·047	·041	14	− 0·013	12	57·284	„
160	1346	8–9.10	310 18	9	12	− 0 40	20	5·782	·035	20	+ 0·094	20	59·141	„
161	1355	8–8.9	331 8	10	11	+ 2 50	20	2·749	·015	109	− 0·088	20	59·273	„
163	1362	7.8–7.8	134 11	9	12	− 2 22	12	4·945	·028	31	− 0·075	12	58·226	„
164	1374	7.8–10	274 58	2	244	+ 0 18	14	3·351	·050	10	+ 0·041	14	52·285	„

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	Δ.		No. of Obs.	Distance.	*p*.	*w*.	Δ′.	No. of Obs.	Epoch. 1800+	Observer.
			°	′	′		°	′		″	″		″			
164	1374	7.8–9	277	43	7	17	+ 3	3	56	3·542	·015	114	+0·232	56	55·305	M
		8–10	278	58	9	11	+ 4	18	20	3·431	·012	161	+0·121	20	60·120	,,
165	1376	8.9–8.9	133	23	17	3	− 2	25	12	5·050	·028	31	+0·007	12	56·238	,,
166	1394	10–11	240	25	12	7	+ 3	19	12	3·865	·034	22	+0·118	12	57·191	,,
167	1421	9.10–11	332	29	19	3	+ 2	5	20	4·358	·042	14	−0·036	20	60·209	,,
168	1424	2–4	105	35	2	338	+ 2	13	20	3·163	·012	182	+0·663	20	52·289	,,
		2.3–3.4	108	51	13	6	+ 5	29	30	3·054	·010	263	+0·554	30	55·339	,,
		2.3–3.4	110	6	9	12	+ 6	44	20	3·072	·051	9	+0·672	20	60·125	,,
169	1428	8.9–9	86	12	6	24	+ 1	52	20	3·419	·016	92	−0·418	20	59·266	,,
172	1447	8–10	125	10	5	45	− 0	4	20	4·361	·011	222	+0·061	20	55·329	,,
173	1448	7.8–10.11	258	26	8	17	− 0	16	16	10·201	·052	9	−0·704	16	58·250	,,
175	1460	9.10–10	168	12	11	9	− 0	30	20	3·310	·026	35	−0·002	20	60·189	,,
176	1463	9–10	258	34	14	5	+ 0	18	12	7·369	·098	3	−0·124	12	57·210	,,
177	1466	6.7–8.9	240	11	5	39	− 0	17	20	6·677	·013	147	−0·040	20	48·336	P
		6–8	239	31	1	1649	− 0	57	30	6·614	·058	7	−0·103	30	54·284	M
178	1476	8.9–9.10	356	50	8	18	+ 3	10	20	1·878	·026	36	−0·012	20	59·232	,,
180	1487	4.5–7.8	103	22	18	3	+ 0	34	30	6·310	·032	24	+0·135	30	48·337	P
		5.6–7.8	102	31	13	6	− 0	17	30	6·341	·090	3	+0·166	30	54·271	M
181	1516	7–8	6	52	13	6	+ 64	16	30	2·489	·037	18	−5·645	30	54·265	,,
		8.9–9	56	33	16	4	+113	57	26	3·246	·012	169	−4·888	26	60·211	,,
182	1521	8.9–9	95	28	5	42	+ 0	16	20	3·577	·031	26	−0·086	20	59·170	,,
183	1523	5–6	118	4	16	4	− 53	8	30	2·893	·009	333	+0·921	30	52·357	,,
		4.5–5	108	10	9	12	− 63	2	12	2·847	·021	57	+0·875	12	58·202	,,
		5–5	105	25	6	28	− 65	47	20	2·844	·022	49	+0·872	20	60·084	,,
184	1524	4–13	145	54	15	4	− 0	40	12	7·225	·071	5	+0·129	12	58·158	,,
185	1527	8–9.10	11	44	7	22	+ 1	35	20	4·035	·021	55	+0·153	20	55·343	,,

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	Δ.		No. of Obs.	Distance.	*p*.	*w*.	Δ′.	No. of Obs.	Epoch. 1800+	Observer.
186	1536	4–7	83°	15′	30′	1	− 9°	8′	16	2″·710	″·008	370	+0″·517	16	53·355	M
188		5–5.6	211	39	7	22			20	8·819	·052	9		20	57·292	„
189	1552	7–9	209	29	10	10	+ 0	5	20	3·097	·008	400	+0·083	20	46·401	P
		6.7–8.9	208	59	10	10	− 0	25	30	3·100	·015	110	+0·086	30	48·319	„
		6–7.8	208	1	14	5	− 1	23	20	3·180	·011	222	+0·166	26	54·367	M
		7–8	209	36	7	19	+ 0	12	60	3·464	·023	47	+0·450	60	55·310	„
190	1553	8.9–9	170	27	5	40	− 1	2	20	5·298	·034	21	−0·042	20	59·253	„
191	1555	7–7.8	342	49	13	6	+ 3	27	20	1·143	·025	40	−0·103	20	59·353	„
192	1559	7–8.9	320	41	27	1	− 0	59	12	2·190	·039	16	+0·100	12	57·317	„
193	1561	7.8–10	265	3	7	18	− 0	57	30	10·481	·032	24	−0·102	30	46·360	P
194	1579	7.8–11	34	26	42	1	− 2	2	30	3·814	·082	4	+0·104	30	46·371	„
195	1596	6.7–8	239	27	6	29	− 1	10	30	3·922	·011	222	+0·192	30	55·332	M
		6–8	238	30	3	108	− 2	7	215	3·716	·006	641	−0·014	214	59·270	„
196	1600	8–8.9	93	33	10	10	+ 0	20	12	7·400	·018	73	−0·232	12	58·322	„
197	1633	7–7	64	44	17	4	− 0	23	30	8·673	·025	39	−0·072	30	46·401	P
198	1645	8–9	160	54	24	2	− 0	35	30	10·313	·028	32	−0·130	30	48·288	„
199	1653	9–9.10	163	11	12	7	+ 0	1	10	7·838	·014	119	+0·035	10	57·284	M
200	1670	4–4	176	48	24	2	−154	46	30	2·919	·034	21	+2·662	30	51·172	P
		3.4–3.4	172	4	7	21	−159	30	30	3·402	·008	435	+3·145	30	54·397	M
		3.4–3.4	173	18	10	10	−158	16	30	3·515	·035	20	+3·258	30	55·530	„
		3.4–3.4	171	36	4	71	−159	58	30	3·665	·010	263	+3·408	30	56·970	„
		3.4–3.4	170	41	3	115	−160	53	27	3·401	·010	244	+3·144	26	58·476	„
		3.4–3.4	170	44	5	41	−160	50	37	3·952	·018	78	+3·695	38	59·154	„
202	1686	9.10–10	186	17	8	16	− 1	21	12	5·245	·048	11	−0·122	12	58·374	„
203	1687	5–8	50	13	88	0	+ 24	55	12	1·324	·028	31	−0·108	12	56·347	„
	A & B	6–8	45	25	28	1	+ 20	7	12	1·447	·027	34	+0·015	12	57·282	„
204	A & C	5.6–10.11	124	11	9	12	− 0	32	12	28·163	·081	4	−0·442	12	57·295	„

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	Δ.	No. of Obs.	Distance.	*p*.	*w*.	Δ′.	No. of Obs.	Epoch. 1800 +	Observer.
205	1590	8–11	148° 16′	16′	4	− 1° 31′	12	5″·690	″·039	16	−0″·162	12	58·280	M
206	1695	7.8–9.10	286 35	8	16	− 2 31	20	3·444	·027	34	+0·184	20	60·137	”
207	1719	7–8	3 1	11	8	− 0 7	18	7·476	·028	31	+0·233	18	53·400	”
208	1724	4–9.10	342 43	5	45	− 1 15	20	7·208	·011	222	+0·135	20	54·392	”
	A & B													
210	1755	8–9	130 50	17	4	− 3 0	12	4·394	·040	15	+0·114	12	57·344	”
211	1757	8–9	51 53	15	4	+22 26	12	2·012	·051	9	+0·367	12	56·421	”
		7.8–9	54 12	18	3	+24 55	10	2·008	·015	108	+0·363	10	57·291	”
213	1763	8.9–8.9	43 48	22	2	+ 4 47	12	2·784	·003	2222	+0·099	12	56·423	”
215	1776	9.10–10	198 10	9	13	− 2 2	20	7·159	·038	17	−0·168	20	59·222	”
216	1777	6–9.10	230 46	14	5	− 4 37	12	3·349	·038	17	−0·045	12	58·285	”
217	1785	7.8–9	176 11	13	6	+11 45	34	3·196	·016	94	−0·291	34	46·405	P
		8.9–9	185 24	8	17	+20 58	20	2·891	·025	41	−0·596	20	59·299	M
218	1789	9–9.10	326 56	9	12	+ 0 58	20	6·184	·057	8	+0·101	20	60·186	”
219	1793	8–9	242 51	0	4386	+ 0 35	12	4·539	·054	8	+0·152	12	56·433	”
220	1821	5.6–7.8	237 8	2	176	− 0 35	20	12·655	·021	55	+0·058	20	54·463	”
		5–8	238 6	5	41	+ 0 23	30	12·747	·040	15	+0·150	30	55·462	”
221	1826	9–10	313 22	7	18	− 1 46	12	4·261	·025	41	−0·172	12	57·373	”
222	1835	5–8	187 50	12	7	+ 1 22	42	6·157	·043	13	+0·097	42	44·551	{W & P}
		6–8	184 33	60	0	− 1 55	30	6·312	·075	4	+0·252	30	45·421	P
		6.7–8.9	187 30	5	39	+ 1 2	20	6·291	·026	37	+0·231	20	46·503	”
		6.7–8.9	186 40	2	176	+ 0 12	20	6·236	·001	43478	+0·176	20	47·529	”
223	1838	7.8–7.8	152 28	11	8	− 1 20	30	8·869	·020	63	−0·171	30	46·409	”
		8–8	334 14	6	27	+ 0 26	20	8·864	·026	36	−0·176	20	60·181	M
224	1858	8.9–10	31 31	14	5	− 3 41	20	2·513	·038	17	+0·313	20	59·329	”
225	1864	5–6	98 47	11	8	− 0 25	30	6·082	·019	69	+0·251	30	46·429	P
		5–5.6	100 36	9	14	+ 1 24	12	6·139	·029	29	+0·308	12	57·343	M

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	Δ.		No. of Obs.	Dis-tance.	*p*.	*w*.	Δ′.	No. of Obs.	Epoch. 1800 +.	Observer.
226	1865	4–4.5	304°	42′	11′	9	− 4°	28′	20	1″·183	″·020	64	−0″·006	20	59·341	M
227	1877	3–6.7	327	49	9	14	+ 7	13	12	2·946	·055	8	+0·311	12	57·440	"
		3–7	324	58	11	9	+ 4	22	22	2·715	·021	53	+0·080	22	58·457	"
228	1888	7–9	318	37	7	19	− 9	33	28	6·755	·073	5	−0·332	28	45·401	P
		5.6–7.8	319	10	9	13	− 9	0	20	6·746	·004	1429	−0·341	20	46·459	"
		3.4–7	307	46	11	8	−20	24	12	5·929	·032	24	−1·158	12	58·386	M
229	1890	6.7–7	42	23	11	8	− 1	49	30	3·709	·021	57	−0·048	30	45·438	P
		6.7–6.7	40	54	12	7	− 3	18	30	3·679	·050	10	−0·078	30	46·397	"
		6–7	43	41	8	16	− 0	31	20	3·681	·034	21	−0·076	20	59·269	M
230	1904	7.8–8	346	15	6	25	− 0	9	10	9·760	·035	20	+0·143	10	57·432	"
231	1909	5.6–6	238	27	6	27	+ 3	35	20	4·584	·026	37	+1·284	20	54·462	"
232	1910	7–7	29	10	18	3	0	0	32	3·934	·034	21	+0·131	32	44·631	P
		7.8–7.8	29	20	13	6	+ 0	10	40	3·937	·041	14	+0·134	40	46·433	"
		8.9–8.9	29	56	3	88	+ 0	46	16	3·786	·040	16	−0·017	16	47·541	"
		8–8	31	25	11	8	+ 2	15	12	4·087	·025	40	+0·284	12	58·334	M
236	1954	3.4–5.6	193	4	12	7	− 3	52	12	3·374	·020	64	+0·807	12	57·404	"
237	1962	6–6	7	43	4	70	+ 0	35	30	11·893	·041	15	+0·080	30	46·412	P
239	1965	4–6	299	28	27	1	− 1	24	30	5·997	·099	2	−0·005	30	45·432	"
		4–6	301	18	36	1	+ 0	26	30	6·137	·048	11	+0·135	30	46·428	"
		4–5	301	2	10	10	+ 0	10	30	6·132	·029	29	+0·130	30	52·533	M
240	1985	7.8–10	330	8	11	8	+ 3	34	16	5·611	·065	6	+0·191	16	58·417	"
241		4.5–7–9	337	39	3	96			20	41·055	·023	45		20	57·393	"
242	2021	7–7	322	41	11	9	+ 5	57	30	3·653	·031	26	+0·373	30	55·488	"
243	2032	6–7	178	31	8	14	+43	47	20	2·216	·026	37	+0·781	20	54·667	"
		6–7	184	54	7	20	+50	10	20	2·701	·019	68	+1·266	20	59·341	"
245		1.2–8	275	52	16	4			34	3·296	·027	33		34	58·346	"
247	2052	8–8.9	104	47	15	5	− 4	55	16	2·619	·060	7	−0·364	16	58·445	"

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	Δ.		No. of Obs.	Distance.	*p*.	*w*.	Δ′.	No. of Obs.	Epoch. 1800+	Observer.
			°	′	′		°	′		″	″		″			
248	2055	4–6	15	40	12	7	+ 22	17	12	1·291	·022	49	+0·177	12	57·579	M
249	2078	6–7.8	115	40	15	4	− 0	48	30	3·783	·029	29	+0·040	30	46·532	P
		6–7	114	10	6	27	− 2	18	20	3·621	·021	56	−0·122	20	59·269	M
250	2084	3–6	73	19	23	2	−112	53	36	1·452	·026	36	+0·362	36	55·662	„
		3–7	60	22	18	3	−125	50	20	1·600	·027	34	+0·510	20	57·458	„
252	2104	7–9	18	21	2	244	− 1	17	20	6·125	·007	526	+0·268	20	49·524	P
		8–10	20	41	12	7	+ 1	3	12	5·919	·032	24	+0·062	12	58·396	M
254	2120	7.8–11	298	5	91	0	− 62	5	20	2·576	·042	14	−0·524	20	55·619	„
255	2130	5–5	8	22	15	4	− 14	29	30	2·926	·019	69	−0·341	30	54·679	„
256	2135	8–9.10	171	4	1	2198	+ 4	57	20	6·952	·011	222	+0·250	20	55·612	„
257		5–5	213	0	13	6			20	4·444	·024	44		20	54·692	„
		5.6–6	213	17	8	16			24	4·446	·028	32		24	58·420	„
258	2140	3–6	117	59	28	1	− 0	30	16	4·676	·006	625	+0·028	16	46·658	P
		3–5.6	118	4	12	7	− 0	25	18	4·571	·052	9	−0·077	18	57·622	M
259	2161	5–6	309	15	21	2	+ 2	2	36	3·874	·045	12	+0·274	36	45·511	P
		4.5–6.7	307	1	27	1	− 0	12	30	3·770	·038	17	+0·170	30	46·552	„
		4–5.6	309	45	8	17	+ 2	32	60	3·845	·017	83	+0·245	60	55·655	M
260	2165	8–10.11	49	37	12	7	+ 3	54	12	6·999	·053	9	+0·289	12	57·478	„
261	2168	9–10	196	47	11	9	− 2	53	20	2·402	·036	19	−0·041	20	59·548	„
264	2180	8–8	85	12	1	1015	− 0	5	24	3·095	·020	63	−0·078	24	47·566	P
		8–8.9	264	21	14	5	− 0	56	12	3·272	·018	73	+0·099	12	56·494	M
265	2213	9–9.10	332	12	11	8	− 1	4	12	4·230	·052	9	−0·220	12	58·515	„
266	2218	7.8–9	353	3	12	7	− 2	1	18	2·285	·022	51	−0·188	18	59·351	„
267	2217	9–9.10	283	28	7	23	− 1	18	12	6·179	·054	8	−0·391	12	58·527	„
268	2232	8–11	141	31	15	5	− 1	21	12	6·493	·055	8	−0·017	12	58·445	„
269	2242	7.8–7.8	326	25	21	2	− 0	35	14	3·641	·110	2	+0·178	14	54·693	„

No. II.— *Catalogue of Mean Results (continued).*

No.	Σ's No.	Mag.	Position.		P.	W.	Δ.		No. of Obs.	Distance.	p.	w.	Δ′.	No. of Obs.	Epoch. 1800+	Observer.
			°	′	′		°	′		″	″					
270	2245	8–8.9	294	40	9	12	+0	38	20	2·557	·020	63	−0·056	20	59·603	M
271	2263	8.9–10	160	29	12	7	−1	19	30	7·318	·016	101	+0·048	30	54·638	„
272	2264	5.6–5.6 M	259	41	2	293	−2	4	30	6·195	·034	21	+0·133	30	54·662	„
		5.6–5.6	260	20	4	72	−1	25	24	6·093	·015	103	+0·031	24	57·457	„
273	2272	5–6.7	110	15	9	13	−19	17	12	6·151	·051	9	+0·014	12	57·674	„
		4.5–7	108	37	7	19	−20	55	12	6·085	·039	16	−0·052	12	58·396	„
274	2276	7.8–8	257	9	9	12	−0	47	12	6·348	·038	17	−0·492	12	58·475	„
277	2304	9–11	69	16	12	7	+0	45	12	5·098	·064	6	+0·168	12	57·512	„
278	2309	9–9.10	349	49	36	1	−4	53	16	3·664	·084	3	+0·141	16	54·674	„
279	2316	6.7–9	314	36	21	2	−0	24	12	3·891	·087	3	+0·061	12	56·593	„
280	2319	7.8–7.8	192	14	9	13	+1	11	30	5·360	·037	18	−0·255	30	54·717	„
281	2323	6.7–10.11	0	11	13	6	−5	44	30	3·222	·021	57	+0·081	30	45·714	P
		6–10	0	30	22	2	−5	25	20	3·218	·003	2500	+0·077	20	46·699	„
		5.6–9	359	38	6	24	−6	17	12	3·436	·022	49	+0·295	12	56·585	M
282	2333	9–9.10	335	56	10	11	+0	38	20	6·355	·019	67	+0·073	20	59·521	„
284	2351	8–8.9	160	9	11	8	+0	23	12	5·226	·018	76	−0·009	12	58·526	„
285	2362	7.8–9	180	4	32	1	−0	11	20	3·758	·045	12	−0·202	20	47·647	P
		9–10.11	179	45	9	12	−0	30	12	4·163	·016	94	+0·203	12	57·525	M
		8–10.11	180	52	10	10	+0	37	12	4·094	·044	13	+0·134	12	58·526	„
286	2375	7.8–8	113	0	13	6	+4	53	16	2·327	·021	53	+0·091	16	58·685	„
287	2382	7–9	20	41	39	1	−5	23	34	3·094	·035	20	+0·060	34	45·638	P
		5–6.7	18	43	8	17	−7	21	18	3·205	·031	26	+0·171	18	58·434	M
		5–6	18	50	11	9	−7	14	12	3·161	·020	61	+0·127	12	59·601	„
288	2383	7–7	150	50	10	10	−4	20	30	2·757	·038	17	+0·184	30	45·631	P
		5.6–6	146	40	10	10	−8	30	18	2·476	·022	49	−0·097	18	58·416	M
289	2401	8.9–11	41	13	11	8	+3	37	16	4·343	·033	23	+0·278	16	57·622	„

No. II.— *Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	Δ.		No. of Obs.	Distance.	*p*.	*w*.	Δ′.	No. of Obs.	Epoch. 1800+	Observer.
290	2404	7–9.10	183°	31′	16′	4	+ 0°	27′	34	3·561″	·043″	13	+0·024″	34	44·708	P
		7.8–9.10	182	25	26	2	− 0	39	30	3·473	·031	26	−0·064	30	45·687	„
		7.8–9	182	24	10	10	− 0	40	24	3·411	·016	94	−0·126	24	46·696	„
		7–9	182	32	19	3	− 0	32	20	3·492	·033	22	−0·045	20	48·713	„
		6.7–8	182	32	8	15	− 0	32	30	3·613	·016	101	+0·076	30	54·662	M
291	2452	7.8–9	218	40	8	15	− 1	8	12	5·613	·008	345	−0·167	12	58·686	„
292	2446	7–9	153	50	7	22	− 0	43	20	9·763	·021	55	−0·370	20	47·605	P
		7.8–9.10	154	19	5	45	− 0	14	20	9·782	·062	6	−0·351	20	54·713	M
293	2449	8–9	292	2	7	22	− 0	18	16	7·833	·024	41	−0·181	16	56·541	„
294	2455	7.8–9.10	124	20	18	3	−20	8	26	3·982	·099	2	−0·945	26	55·662	„
296	2481	9–9	230	20	4	61	− 3	58	12	3·963	·012	169	+0·130	12	56·593	„
297	2486	7–7	222	50	9	13	− 1	51	30	10·198	·024	44	−0·325	30	47·579	P
		6.7–7	222	25	2	244	− 2	16	12	10·079	·062	6	−0·444	12	56·745	M
		7–7.8	222	29	6	27	− 2	12	16	10·078	·019	65	−0·445	16	59·655	„
298	2504	6.7–8.9	287	2	4	76	− 1	15	30	8·859	·022	49	−0·063	30	54·665	„
299	2523	8–8.9	150	24	13	6	− 1	8	12	6·081	·037	18	−0·133	12	58·477	„
300	2524	10.11–11	101	6	11	8	− 3	30	12	6·926	·057	7	−0·234	12	57·640	„
301	2534	8–8	244	28	9	11	+ 2	26	20	6·611	·045	12	−0·142	20	59·586	„
302	43 App.I.	3–6	55	26	3	88	− 0	18	30	34·557	·028	31	+0·262	30	57·469	„
304	2576	8.9–9	132	14	22	2	− 6	34	12	3·466	·076	4	−0·131	12	56·596	„
305	2579	3–8	1	0	18	3	−30	55	26	1·107			−0·693	12	54·790	„
		3–7	0	36	5	33	−31	19	30	1·275	·011	192	−0·525	10	55·740	„
306	2580	5–9.10	72	38	6	31	− 0	14	12	25·590	·031	25	−0·210	12	58·608	„
307	2583	7–7.8	120	49	21	2	+ 0	4	10	1·492	·026	37	−0·010	10	58·660	„
308	2585	6–9.10	311	50	15	4	− 1	0	30	8·775	·072	5	+0·283	30	54·648	„
309	2603	5.6–9	0	24	7	23	+ 5	52	40	3·063	·016	96	+0·273	40	59·749	„

No. II.— *Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.	P.	W.	Δ.	No. of Obs.	Distance.	*p*.	ϖ.	Δ′.	No. of Obs.	Epoch. 1800 +	Observer.
310	2605	5–10.11	182° 34′	4′	59	− 2° 1′	30	3″·569	″·028	31	+0″·247	30	44·852	P
		5.6–10.11	182 48	19	3	− 1 47	26	3·621	·018	78	+0·299	26	45·776	„
		5.6–8.9	182 9	11	9	− 2 26	20	3·448	·006	714	+0·126	20	46·657	„
		6.7–8.9	181 11	6	27	− 3 24	12	3·539	·021	55	+0·217	12	56·734	M
312	2609	7.8–9	25 18	14	5	− 3 50	12	2·163	·028	32	−0·205	12	58·685	„
313	2610 A & B	9–9.10	298 34	12	8	+ 0 10	12	4·154	·024	43	−0·111	12	57·756	„
314	2613	7.8–8	350 3	7	21	− 0 40	18	4·903	·036	19	+0·208	18	55·665	„
		8–8.9	349 31	2	176	− 1 12	12	4·627	·012	169	−0·068	12	56·578	„
315	2619	9.10–10	244 29	9	12	− 0 28	10	4·210	·042	14	−0·080	10	57·711	„
316	2622 A & B	10–11	194 27	11	9	+ 0 13	12	5·547	·032	24	−0·423	12	58·533	„
318	2637 A & B	6–7	225 43	4	81	− 0 55	26	72·907	·098	3	+2·208	26	53·702	„
319	A & C	6–8	326 45	12	7	+ 0 2	30	11·395	·029	30	−0·010	30	53·679	„
		7–9	326 21	4	54	− 0 22	30	11·447	·029	29	+0·042	30	54·671	„
320	2639	8–9	301 53	17	4	− 1 35	12	5·585	·045	12	+0·062	12	56·627	„
321	2644	8.9–8.9	209 6	21	2	+ 1 32	38	3·294	·017	83	−0·048	38	45·821	P
		7–7	211 28	22	2	+ 3 54	26	3·503	·023	45	+0·161	26	53·753	M
322	2655	8–8	3 2	6	27	+ 0 3	30	6·134	·017	83	+0·040	30	54·805	„
324	2671	7.8–9	340 37	7	18	− 0 29	20	2·967	·022	53	−0·020	20	59·597	„
326	2705	8–9.10	262 12	12	7	+ 1 24	12	2·857	·031	25	−0·173	12	58·540	„
327	2708	6.7–8	340 25	2	203	− 7 58	30	16·103	·034	21	+3·946	30	54·665	„
328	2716	5.6–8.9	52 25	25	2	+ 3 0	30	3·135	·008	435	+0·399	30	54·767	„
329	2718	8–8.9	86 19	6	27	− 0 18	12	8·147	·025	40	−0·150	12	56·579	„
		8.9–8.9	85 59	5	34	− 0 38	20	8·166	·031	26	−0·131	20	59·770	„
330	2722	9–9.10	307 59	11	8	− 0 3	12	7·198	·030	28	+0·111	12	57·714	„

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	Δ.		No. of Obs.	Distance.	*p*.	*w*.	Δ′.	No. of Obs.	Epoch. 1800 +	Observer.
331	2725	8–8.9	357°	51′	10′	11	− 0°	11′	24	4″·605	″·015	110	+0″·368	24	54·752	M
332	2726	5.6–10	58	28	21	2	+ 1	16	12	6·597	·074	5	−0·020	12	56·827	„
333	2727	5–7	272	20	3	148	− 1	26	28	11·703	·010	263	−0·201	28	45·699	P
		4.5–6.7	272	23	5	45	− 1	23	12	11·281	·064	6	−0·623	12	56·511	M
335	2741	6.7–7.8	31	20	24	2	− 4	30	40	1·878	·019	66	−0·052	40	55·747	„
336	2742	8.9–9	222	5	12	7	− 2	37	16	2·585	·019	66	+0·005	16	59·801	„
337	2745	7–10.11	192	34	6	24	+ 2	58	12	3·050	·029	29	+0·383	12	58·691	„
338	2747	8–8.9	259	11	10	9	+ 1	8	12	4·681	·059	7	+0·146	12	57·712	„
339	2751	8.9–9	348	50	9	13	+ 4	44	20	1·735	·015	103	−0·125	20	59·763	„
340	2758	5–6	104	16	10	9	+ 9	52	20	17·283	·056	8	+1·203	20	52·764	„
		5.6–6	105	20	4	55	+10	56	30	17·448	·034	21	+1·368	30	54·828	„
		5–6	108	17	3	102	+13	53	20	17·880	·033	22	+1·800	20	59·909	„
341	2760	7.8–8	224	19	5	46	+ 0	12	30	10·914	·038	17	−1·786	30	54·719	„
342	2764	9.10–10	300	38	6	29	− 1	36	20	6·742	·031	25	−0·101	20	59·731	„
343	2762	6.7–9.10	313	49	6	24	− 1	47	12	3·757	·026	37	+0·210	12	56·561	„
345	2786	8–9.10	184	18	7	19	+ 0	41	16	2·287	·037	18	−0·173	16	59·571	„
346	2789	8.9–8.9	115	25	32	1	− 0	57	14	5·962	·042	14	+0·027	14	45·683	P
		8–8.9	115	45	6	25	− 0	37	20	5·844	·052	9	−0·091	20	59·627	M
347	2797	7.8–9.10	214	29	23	2	+ 1	13	12	3·526	·011	220	+0·343	12	56·816	„
348	2799	8–8.9	320	51	21	2	−12	2	16	1·378	·020	64	+0·026	16	59·812	„
349	2804	8–8.9	322	59	3	103	+ 6	2	30	3·020	·038	17	+0·288	30	54·776	„
350	2806	3–8.9	250	27	1	2198	+ 0	25	30	13·235	·038	17	−0·339	30	54·866	„
352	2816	7.8–10	339	43	15	4	0	0	30	19·872	·011	189	−0·086	30	45·785	P
	A & B	6.7–8	339	19	8	15	− 0	24	30	19·938	·009	333	−0·020	30	54·769	M
353	A & C	7.8–10	120	20	4	61	+ 0	12	30	11·484	·018	78	−0·174	30	45·785	P
		6.7–8.9	120	56	7	23	+ 0	48	30	11·726	·011	189	+0·068	30	54·756	M

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	Δ.		No. of Obs.	Distance.	*p*.	*w*.	Δ′.	No. of Obs.	Epoch. 1800+	Observer.
354	2822	4.5–6.7	114°	9′	20′	2	−0°	24′	30	4″·943	″·045	12	−0″·614	30	47·685	P
		5–6	116	35	2	249	+2	2	24	4·881	·026	38	−0·676	24	54·840	M
		4.5–5.6	115	13	9	11	+0	40	30	4·499	·022	53	−1·058	30	55·840	”
		5.6–6.7	116	41	6	31	+2	8	20	4·412	·020	63	−1·145	20	59·752	”
355	2843	8–8.9	136	21	10	9	+3	0	12	2·254	·019	66	+0·059	12	58·693	”
356	2845	9–9.10	174	4	14	5	+5	4	12	1·985	·034	21	−0·175	12	58·752	”
357	2848	7.8–8	55	32	9	12	+0	40	12	10·254	·013	147	−0·193	12	56·860	”
358		6–7	242	33	7	21			26	4·041	·022	51		26	57·726	”
359	2861	8.9–9	220	45	5	39	+0	51	24	7·179	·022	49	+0·049	24	56·757	”
361	2863	5–7.8	287	30	8	16	−1	24	26	5·878	·006	667	+0·278	26	45·895	P
		5.6–7.8	286	24	7	19	−2	30	12	5·850	·024	41	+0·250	12	58·660	M
362	2878	8–10	132	11	14	5	+1	23	16	1·263	·017	87	−0·102	16	59·844	”
363	2881	8–8.9	106	10	14	5	−5	14	30	1·627	·050	10	−0·136	30	55·866	”
364	2900 A & B	7–8	336	45	1	2639	−5	15	26	59·160	·029	29	+2·215	26	54·853	”
366	2902	8–8.9	88	8	8	14	−1	46	12	6·119	·011	220	−0·285	12	56·827	”
367	2903	7–9	92	12	9	13	−4	16	30	4·254	·014	120	+0·004	30	44·662	P
		7.8–9	94	29	1	550	−1	59	20	4·129	·054	8	−0·121	20	47·608	”
		7.8–8.9	93	55	8	18	−2	33	26	4·096	·023	46	−0·154	26	58·799	M
368	2910	9–9.10	344	14	18	3	−3	0	12	5·446	·022	49	+0·149	12	56·705	”
369	2909	4–4	346	31	8	14	−6	12	28	3·599	·011	222	+0·210	28	53·848	”
		4.5–4.5	345	57	5	35	−6	46	24	3·575	·016	94	+0·186	24	55·829	”
370	2917	9.10–10	71	8	10	10	−1	46	12	4·235	·027	33	−0·115	12	58·634	”
371	3134	9.10–11	76	27	28	1	+0	17	12	5·942	·018	73	−0·118	12	56·820	”
374	2944 A & C	7.8–9	148	45	5	33	−7	32	12	51·278	·036	19	−3·652	12	57·905	”
375	2945	9.10–9.10	115	31	19	3	+2	57	12	3·944	·018	73	+0·064	12	56·860	”

No. II.—*Catalogue of Mean Results* (*continued*).

No.	Σ's No.	Mag.	Position.		P.	W.	Δ.		No. of Obs.	Distance.	*p*.	*w*.	Δ′.	No. of Obs.	Epoch. 1800 +	Observer.
376	2946	7.8–8	255°	44′	6′	25	+ 2°	38′	30	5″·165	″·004	1563	+0″·115	30	54·794	M
377	2947	7–7.8	70	29	9	13	— 5	33	12	3·239	·029	30	+0·262	12	58·720	„
378	2950	7–8.9	315	42	7	19	— 3	24	12	2·080	·036	19	+0·040	12	58·648	„
		6–7	315	48	13	6	— 3	18	20	2·067	·034	21	+0·027	20	59·663	„
379	2974	8–8.9	160	41	20	3	+ 0	58	10	2·763	·107	2	—0·072	10	56·859	„
380	2978	7–8.9	147	0	22	2	+ 0	50	30	8·442	·001	13158	+0·045	30	54·887	„
		7.8–8.9	146	15	8	17	+ 0	5	26	8·371	·039	16	—0·026	26	55·851	„
381	2984	8.9–12	295	34	18	3	+ 0	57	20	4·393	·031	26	—0·267	20	59·852	„
382	2985	7.8–9	252	34	4	54	+ 0	27	26	14·957	·036	19	—0·039	26	59·765	„
383	2995	8.9–9	29	27	11	8	+ 2	47	12	4·613	·032	25	+0·050	12	57·800	„
384	3001	6.7–9	186	51	9	12	+11	53	12	2·604	·041	15	+0·251	12	58·616	„
385	3006	9.10–10.11	174	43	13	6	— 8	3	12	4·979	·026	37	+0·329	12	56·878	„
386	3008	8–9	263	57	7	19	— 9	23	20	5·581	·028	31	—1·959	20	59·918	„
387	3011	10.11–11	333	54	11	9	— 0	54	16	7·005	·038	17	+0·152	16	58·789	„
388	3019	7.8–8.9	185	52	11	9	+ 0	36	28	10·749	·046	12	+0·071	28	54·883	„
390	3024	8.9–9.10	310	4	14	5	— 1	32	24	4·810	·031	25	—0·057	24	54·868	„
391	3042	7.8–7.8	89	2	17	3	— 0	18	12	4·300	·018	73	+0·068	12	56·935	„
392	3049	6–8.9	324	26	5	43	+ 0	58	112	3·014	·013	141	+0·002	112	58·833	„
393	3050	7–7 **M**	196	37	5	39	+ 5	35	30	3·499	·055	8	—0·281	30	55·977	„
394	3053	7–8.9	70	15	5	36	+ 0	17	20	15·042	·018	75	—0·111	20	59·770	„
395	3057	8–10	297	8	14	5	— 2	24	12	3·991	·070	5	+0·351	12	56·878	„
396	3060	9–9.10	113	45	23	2	+ 3	15	11	3·778	·003	3378	—0·152	10	56·705	„
397	3061	7.8–8.9	147	25	30	1	— 0	57	14	7·578	·090	3	—0·012	14	55·974	„
398	3062	6–7.8	247	57	35	1	+101	34	30	1·328	·030	27	+0·862	30	55·909	„

NOTES *to Catalogue.*

Σ 22.

The results recorded of this star, although of easy measurement, are very anomalous. If Sir W. **Herschel's** first measure in 1782, instead of being 244°·9 as recorded, be assumed to have been 236°·6 (see *Phil. Trans.* for 1824, part iii. p. 25), still there remains a difference of 6° between angles given by two different observers; the smallest being that of **Secchi**, viz. 234°·8 for the year 1855, with which the observations in this Catalogue agree very closely, and the largest that of **Mädler** in 1858, viz. 240°·5.

Σ 138.

Mädler gives the position of this star, as obtained from 4 measures in 1858, =166°·7. There must be some mistake.

Σ 262, *Cassiopeæ.*

This star does not appear to have received as much attention as it deserves; for though the fixity of the wide pair is pretty clearly established, there can be little doubt that the close pair form a true binary system, as the following measures will show:—

		°	″
Herschel I.	1782·45	290·5	1·50
Struve	1829·66	276·7	1·86
Smyth	1834·83	274·2	2·1
Bishop	1841·81	270·8	2·12
Wrottesley	1856·03	265·1	2·01
Mädler	1856·21	269·5	1·78

This result of **Mädler's** is derived from 4 measures on one night only.

Σ 1037.

The great retrograde motion assumed by **Struve**, and based on his measures of 1836, is scarcely borne out by the observations given here. He gives us no reason for throwing discredit on the result of 1827, yet if it were discarded error of observation in so close and difficult a star might account for the remaining discordances. Time alone will prove whether it be binary or no.

		°	″
Struve	1827·27	337·8	1·145
„	1831·28	330·25	1·070
„	1832·70	329·95	1·125
„	1836·26	327·43	1·323
Mädler	1855·66	323·05	1·286
„	1859·22	324·0	1·375
Wrottesley	1860·11	325·05	1·355

Mädler's first result is derived from 20 nights' observations in 1854–8, and the second from 6 nights' observations in 1859.

Σ 1187.

The observations of this star show that the angle is decreasing at the rate of about 0°·5 per year; but it is extraordinary that Mädler should have found the position in 1855 =70°, and in 1858 more than 74°, while in 1859 the angle obtained was 59°. In this latter year our observations give it 56°·7; but in order to remove all doubts, while this communication was in course of being prepared for presentation, observations on two nights were obtained which show nearly the same results, viz., 55°·5.

		°	″
Struve	1827·62	73·1	1·62
„	1832·32	67·8	1·61
Mädler	1855·32	69·7	1·738 two nights
„	1858·29	74·7	1·810 one night
„	1859·14	59·1	1·203 one night
Wrottesley	1859·14	56·7	1·819
„	1860·28	55·5	1·830

Σ 1263.

The annual displacement in distance =0″·69, ascribed by Struve to this star, is verified by these observations. The nearest approach would seem to have occurred in 1822.

Σ 1306.

Upon laying down the positions and distances of these stars to scale from the observations since 1782, it seems difficult to reconcile the differences by the assumption of orbital motion. Neither will the Proper Motion, as given by Mr. Main at p. 166 of the nineteenth vol. of the *Memoirs*, account for them.

Σ 1356, ω *Leonis.*

April 16th, 1857. With power 600 the star appeared elongated in the direction s. f., n. p. The angle estimated 175° (more probably it was 355°).

March 26th and April 22d, 1858. No elongation could be perceived on either of these nights with powers 600 and 820.

April 15th and 21st, 1859. No elongation could be perceived.

Σ 1374.

Considering that the measurement of this star in position presents no difficulties, the difference from STRUVE since 1830 can scarcely be caused by error of observation. The object appears to deserve rather more notice from astronomers than it has hitherto received.

Σ 1457.

While little or no change seems to have taken place in the distance, the angle has been increasing at the rate of 54′ per year since 1829·6, the epoch of STRUVE's result.

Σ 1516.

This star presents features analogous to Σ 1263 before noticed; but, while in the case of 1263 the nearest approach seems to have occurred about six years before any measures were taken, it appears in this instance to have happened at a very recent date. Our measures in 1854·27, and those at the Collegio Romano in 1856·29, enable us to assign the approximate epoch of nearest approach, viz., 1854·9, with an angle of about 15°, and distance about 2½″.

Σ 1536.

A longer series of observations is required to determine whether the change in position of this star is due to orbital motion. There seems some reason for believing that the proper motion of the larger component may account for it; but if this be so, the motion in R.A. ascribed to it, and which Mr. MAIN, in his paper "On the Proper Motion of the Stars contained in the Greenwich Twelve-year Catalogue," has reduced by one-half, requires to be still further modified.

Σ 1687.

The following table of results will show the great discordances in the observations of two of the components of this star. The distant star is probably unconnected.

		°	″
Struve	1828·35	22·0	1·45
„	1829·40	25·4	1·44
„	1833·37	28·4	1·38
Smyth	1834·38	30·0	1·0
Bishop	1842·39	35·74	1·518
Smyth	1843·32	42·0	1·5

Bishop	1843·35	39°·48	1″·447
Mädler	1844·34	39·17	1·322
„	1847·28	44·2	1·49
„	1854·38	40·6	1·172
Secchi	1855·29	37·55	1·29
Mädler	1855·45	44·5	1·33
„	1856·39	42·2	1·265
Wrottesley	1856·82	47·8	1·386
Mädler	1857·42	43·58	1·146
„	1858·35	43·3	1·258
„	1859·36	43·4	1·497

Now if we group these into three nearly equidistant periods, we have

1831·38	26°·45	1″·32
1844·14	40·1	1·46
1856·68	42·9	1·29

And while this gives a direct motion in position of more than 1° per year between the two first mean epochs, that motion is reduced to 0°·2 between the two later ones. Still the evidence appears sufficient to establish orbital motion.

Σ 1763.

Our observations do not confirm the retrograde motion which a comparison of Struve's measures with Sir W. Herschel's seemed to show. The following are the results:—

Herschel I.	1783·1	48°·8	1″·75 ±
Struve	1830·3	39·0	2·685
Wrottesley	1856·4	43·8	2·784

Σ 1785.

The following are the recorded positions and distances of this star:—

Struve	1830·1	164°·4	3″·487
Wrottesley	1846·4	176·2	3·196
Mädler	1855·7	183·7	3·035
Wrottesley	1859·3	185·4	2·891

A few more years will decide to which class this star belongs.

Σ 1909.

If the observations prior to 1820 are retained, the extraordinary motion of this star still remains to be accounted for, as the measures for the last 15 years show it to be increasing in distance, while the angle remains stationary, or very nearly so; but whereas between 1820 and 1840 the distance was increasing at the rate of 0″·10 per year, since that time the increase has been at the rate of 0″·04 only.

In the following table means are given derived from the results by various observers:—

		°	″
Herschel I.	1781·62	60·12	1·50 ±
Struve Herschel II. South	1820·38	228·56	1·89
Struve	1826·79	231·0	2·23
Struve Smyth	1830·16	233·95	2·778
Struve	1832·95	234·47	2·963
Smyth Struve Dawes	1835·69	235·31	3·319
Smyth Bishop	1841·31	235·75	3·917
Bishop Jacob Smyth	1846·80	237·02	4·203
Fletcher	1851·47	237·95	4·268
Mädler Wrottesley Secchi	1854·64	238·1	4·371

Σ 1985.

The observations show a continued steady increase in the angle of position, but the amount is not yet sufficient to warrant the assumption of orbital motion.

ν Scorpii.

This was one among a list of stars the observation of which was suggested by Admiral Smyth. On the first night (May 7th, 1857) on which it was observed, a note was made by my assistant, who was not then aware of the duplicity of the small star, to this effect: "I have a strong suspicion that the smaller of the two stars is itself double; the confusion of the image is so great that it cannot, however, be verified." On the second night (June 11th, 1857) there is this note: "It is seen plainly double to-night. One measure gives position 48° 40′." Singularly enough, on the 16th of the same month, Admiral Smyth wrote to ask whether "B" could be made double?

η Draconis.

There was some doubt at the time as to whether the companion of this star was really seen at the dates given in the Catalogue; but as the angle at which it was to be looked for was quite unknown to the observer prior to the observations being made, and the measures agree so well with those of Admiral SMYTH and Mr. DAWES given in the *Speculum Hartwellianum*, it was decided to insert them.

Σ 2052.

Without placing too implicit confidence on the measures here given, it is probable that the angle has decreased since 1830. There appear to be no recorded observations of this star besides those of STRUVE.

Σ 2114.

The one night's observations of this difficult star in 1859 suggest orbital motion; indeed the recorded observations exhibit a pretty regular increase of the angle, as the following table will show:—

		°	″
Struve	1830·97	135·67	1·336
Smyth	1832·41	137·0	1·5 one night
Mädler	1855·63	142·54	1·178 one night
Wrottesley	1859·30	147·63	1·312 one night

Σ 2120.

Upon projecting the places given in the following series, it would appear that this star should be included in the same category with Σ 1263 and Σ 1516. If so, the epoch of nearest approach was probably about the year 1850; and as the distance must now be steadily increasing, a very few years will determine the point. The recorded measures are,

		°	″
Struve	1829·60	11·4	3·833
„	1833·28	3·64	3·470
„	1835·29	1·81	3·213
„	1836·55	0·17	3·100
Bishop	1840·59	347·0	2·960
„	1842·53	342·37	2·748
Mädler	1854·52	301·94	2·358
Wrottesley Mädler	1855·64	298·2	2·524
Mädler	1858·28	290·8*	2·366
„	1859·39	287·4	2·646

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* In MÄDLER's Catalogue all the measures for the year 1857 are recorded as 280°+. In taking this mean it is assumed that an error of 10° has been made, and that the true position is 290°+.

Σ 2135.

The difference in angle from Struve can scarcely be due to error of observation in so easy a star. The star requires to be further observed.

Σ 2303.

This will probably turn out to be binary. The angle has increased from 213° in 1825 to 227° in 1858; the distance, however, remains probably unaltered.

Σ 2309.

Although the mean of the measures of position given differs nearly 5° from that of Struve, it is identical with the result obtained by Sir W. Herschel in 1783.

Σ 2323.

It is somewhat strange that while all the measures made here tend to confirm the decrease in angle of this star, Prof. Mädler, in 1858, should have recorded an observation of the same element differing but 1°·3 from Struve's result of 1833. It may, perhaps, be useful to give some of the principal recorded observations:—

		°	″
Herschel I.	1780·8	12·6	
Herschel II. and South	1823·6	3·9	3·60
Struve	1833·2	5·9	3·141
Smyth	1836·4	5·5	3·3
Wrottesley	1845·7	0·2	3·222
„	1846·7	0·5	3·218
„	1856·6	359·6	3·436
Mädler	1858·8	4·6	3·030 one night

Σ 2375.

A direct change in angle is shown by the measures of this star, and this is confirmed by the observations of Prof. Mädler.

The following note is extracted from the observing book, 1850, Sept. 4: "The micrometer was set to 108° (Struve's angle of 1829), but the eye refused to endorse it."

Σ 2455.

Further observations are required before we can come to any satisfactory conclusion with respect to this star, but it is probably binary.

Σ 2576.

The change of angle shown by the observations is corroborated by **Mädler**. Sir J. **South**'s measure of position in 1823 is 326°·25: this would give a retrograde motion of 25′ per year.

Σ 2603, ε *Draconis.*

The measures seem to show that this star is increasing its angle. Further observations will probably establish the fact.

Σ 2804.

The mean of the measures of position made in 1854 scarcely differs from those of Mr. **Bishop** in 1845–6, and Prof. **Mädler**'s observations up to 1858 are equally accordant. While therefore it is evident that a direct change in angle has taken place since the star was first observed by Sir J. **Herschel** and **Struve**, the amount of that change in the last 12 years has been inappreciable.

Σ 2845.

Although differing 5° from **Struve** in 1832, too much reliance must not be placed on this apparent change of angle, as the star is close and difficult.

Σ 2881.

The remark as to Σ 2845 will apply equally well to this star also.

Σ 2903.

The measures made here (though not very accordant) do not support the hypothesis of orbital motion.

Σ 2947.

A small retrograde motion may be inferred from the measures, with perhaps a slight increase in distance.

Σ 3001, *o Cephei.*

This would seem to be a true binary star. Its angle has increased 12° since **Struve**'s measures were taken in 1832. We have for comparison:—

		°	″
Struve	1832·84	174·97	2·353
Dawes	1834·02	175·12	2·302
Smyth	1834·95	173·8	2·5
Powell	1855·92	182·66	..
Wrottesley	1858·62	186·85	2·604

Σ 3006.

Notwithstanding all the discordances exhibited in the measured angles of this star, there are good grounds for believing that there is a slow retrograde change. Whether this be due to orbital motion is at present uncertain.

Σ 3008.

This star is B.A.C. 8154, and has proper motion ascribed to it in R.A. $= -0^{s}{\cdot}007$, in N.P.D. $= +0''{\cdot}09$; but as these values will not account for the decrease in position and distance which it has undergone since the first measures were made, it seems probable that it is a binary system.

Σ 3050.

The angle has undoubtedly increased, and the star deserves attention.

VI. *On the Value of the Constant of Aberration as deduced from Eight Years' Observations of γ Draconis, from* 1852 *to* 1859 *inclusive, made with the Reflex Zenith Tube at the Royal Observatory, Greenwich.* By the Rev. ROBERT MAIN, M.A., President.

Read May 11, 1860.

THE paper which I now offer to the Society may be considered as a supplement to that which is printed in the twenty-fourth volume of the *Memoirs*, on the *Values of the Constants of Nutation and Aberration*, as deduced from the observations of *γ Draconis* made with the twenty-five foot zenith-tube, which was dismounted in the year 1848. The readers of the *Memoirs* will remember that the results deduced from the observations in that paper were unsatisfactory, the constant of aberration being $20''\cdot058$, or less by about three-tenths of a second than the received value, and the deduced parallax being negative and amounting to nearly four-tenths of a second. Those results having up to the present time received no satisfactory elucidation, I have thought it desirable to discuss the observations made since with the new or reflex zenith tube, which extend with tolerable continuity from the year 1852 to the present time, especially as the instrument with which they were made is so simple in construction and so nearly perfect in theory, that the only errors which apparently can exist are those of a fluctuating character arising from the bad definition of the star, the casual disturbances of the surface of the mercury, or the bad state of the atmosphere at the time of the star's transit, all of which in their effects ought to be perfectly eliminated in the course of a long series of observations.

For a description of the reflex zenith tube I beg to refer to the first Appendix to the volume of the *Greenwich Observations* for 1854, merely giving in this place such an account of its general construction as may serve to convey to those persons who have not an opportunity of studying the detailed account given by Mr. AIRY, an adequate idea of its fitness for the observations here under discussion. It consists mainly of an object-glass fitted firmly into a cell connected with a tube placed vertical and sliding within another tube screwed down to a platform raised several feet above the floor of the small apartment containing it; of a mercury trough quite independent of the tube, and placed at the distance of nearly half the focal length of the object-glass beneath it; and of a double micrometer firmly connected with the cell of the object-glass and immediately above it. This micrometer, for the ordinary observations of the star, γ *Draconis*, has two declination wires placed at nearly double the angular distance of the star from the zenith, so that, means being provided for the rotation of the object-glass round a vertical axis, two observations may be made in reversed positions of the instrument, with a very trifling motion of the screw.

The image of the star, after being reflected from the surface of the mercury, is formed a little above the outer surface of the object-glass, in the plane of the wires of the micrometer, and is viewed by the observer conveniently by means of an apparatus consisting of a prism to reflect the rays into a horizontal direction, and of an eye-piece to convey them to the eye of the observer. A level placed parallel to the meridian, and which rotates with the rest of the apparatus, is read for the purpose of correcting for the trifling effect of shift of zero due to the want of horizontality of the object-glass, and the obstructions which the micrometer-frame in front of the object-glass offers to the pencil of light proceeding from the star are rendered perfectly symmetrical to prevent any injurious effects on the image. This explanation of the instrument may be sufficient for our present purpose.

It is, however, necessary to advert to the principal, and indeed the only source of difficulty which has been experienced in the use of the instrument, namely, that of finding a perfectly firm support for the mercury-trough, or one through which tremors arising from casual disturbances in the neighbourhood of the tube should not be transmitted to the surface of the mercury. In the year 1852, when the instrument was first used, it was placed in a small apartment to the east of the transit-circle room, but which now forms the lobby to that apartment, and there it remained till the year 1856. During this period, the mercury-trough was supported on a bed of sand heaped up to a considerable height above the level of the floor of the transit-room, but considerably below that of the observing platform to which the outer tube was screwed. This position is very near to the porter's gate of the observatory, and the effect of disturbance on the mercury was so great that the star could not be observed at all in the business hours of the day. In 1856 it was removed to its present position, but for some time the effect of disturbance through the transmission of tremor to the mercury continued as bad as before, till, in the year 1857, a caoutchouc suspension was devised for the mercury-trough, which was perfectly successful, and since that time the observations have been made at every transit of the star (except on Sundays) with no more interruptions than are due to the unfavourable state of the weather, or to accidental omissions.

For the method used in the discussion of the observations I would refer entirely to my memoir before quoted (*Mem. Roy. Ast. Soc.* vol. xxiv. pp. 147, &c.), as the tables therein given, (with the exception of a very trifling change in that for the aberration, depending on the precessional motion of the star between the middle epoch of the observations there discussed and that of the observations now before us), are perfectly applicable for this investigation.

It is of little importance that the value of the assumed mean north zenith distance of the star employed in the investigation should be absolutely correct, since its error can be determined, together with the error of the assumed constant of aberration, by means of the observations; but, on account of the shift in the position of the instrument which took place in 1856, it is necessary that the differences of north zenith distances corresponding to the old and the new positions should be accurate. By measurement on the plan of the observatory, I have found that the first position of the reflex zenith tube (near the porter's gate) was 53 feet south of that of the 25-foot zenith tube, and the new position, to which it was removed in 1856, was 75 feet south of the same. These quantities correspond respectively to $0''\cdot52$ and $0''\cdot73$ increase of north zenith distance. Hence, assuming from my former memoir that the north zenith distance of γ *Draconis*, for 1840, Jan. 1, was $117''\cdot827$; that the yearly geometrical precessional motion for 1850 is $-0''\cdot601$; and the annual proper motion, $-0''\cdot40$; we have:

for 1852, Jan. 1,	Mean N.Z.D. =	$110''\cdot56$
„ 1853	„	109·92
„ 1854	„	109·29
„ 1855	„	108·66
„ 1856	„	108·24
„ 1857	„	107·61
„ 1858	„	106·98
„ 1859	„	106·36

By means of these numbers, together with the table following Table III., in my memoir before cited (with a trifling modification), the mean of the north zenith distances corresponding to the mean of the days of observation in each group is easily formed.

The constant of aberration I have assumed to be $20''\cdot4 + x$, and the numerical part of this, or the values of $-20''\cdot4 \times \cos(1^\circ\ 23' + \odot)$, is easily computed from Table IV. of my previous memoir by adding one fiftieth part to the values therein given. I may add that to make the tables for the aberration given in that memoir strictly applicable to the middle epoch of the observations discussed at present, or to 1855 nearly, I have subtracted $3'$ from the values of $\odot$ corresponding to the means of groups throughout.

No more explanation is requisite before proceeding to the tabular arrangement, excepting that both in the present memoir and in the other which has been referred to, in the formation of the coefficients of annual parallax, the earth's orbit has been supposed to be circular.

TABLE I.

Observed Meridian Apparent North Zenith Distances of γ Draconis.

Day and Hour.		Observed Apparent N.Z.D.	Day and Hour.		Observed Apparent N.Z.D.	Day and Hour.		Observed Apparent N.Z.D.
1852.	h	″	1852.	h	″	1853.	h	″
May 31	13	105·11	Aug. 3	9	124·13	May 20	14	99·98
June 3	13	108·46	4	9	125·33	25	14	99·81
4	13	105·27	5	9	124·90	26	14	101·06
10	12	108·97	6	9	125·48	June 3	13	104·90
14	12	110·25	9	9	125·09	7	13	105·57
15	12	111·72	18	8	126·91	11	12	106·74
19	12	112·57	19	8	128·14	16	12	108·11
21	12	112·67	21	8	127·23	17	12	108·31
23	12	112·99	25	8	128·54	23	12	111·35
24	12	114·75	26	8	129·00	29	11	111·48
26	11	114·31	27	8	128·71	July 5	11	114·38
July 1	11	115·83	28	7	129·03	12	11	115·35
5	11	117·07	30	7	128·38	22	10	120·01
6	11	117·72	31	7	128·29	25	10	118·90
7	11	117·20	Sept. 1	7	128·95	Aug. 4	9	121·45
9	11	118·82	2	7	129·39	8	9	123·65
10	11	118·26	3	7	128·58	10	9	123·15
14	10	119·35	8	7	130·09	12	8	123·95
15	10	119·60	15	6	130·62	20	8	124·20
19	10	121·26	25	5	131·07	24	8	124·19
21	10	120·96	29	5	129·51	Sept. 10	7	126·97
22	10	121·78	Oct. 7	5	129·85	19	6	126·54
23	10	123·52	20	4	128·61	1854.		
27	9	123·56	1853.			Jan. 24	22	95·39
29	9	122·38	April 10	17	90·69	Mar. 14	18	88·63
30	9	123·76	26	16	92·97	April 19	16	89·57

Observed Apparent North Zenith Distances of γ Draconis (continued).

Day and Hour.	Observed Apparent N.Z.D.	Day and Hour.	Observed Apparent N.Z.D.	Day and Hour.	Observed Apparent N.Z.D.
1854. h	″	1854. h	″	1855. h	″
April 23 16	89·65	Sept. 2 7	123·03	July 19 10	113·18
May 8 15	92·31	4 7	123·03	21 10	111·77
16 14	95·97	5 7	122·75	31 9	114·22
19 14	95·44	6 7	123·91	Aug. 9 9	117·60
26 14	98·78	8 7	122·91	13 8	116·00
27 14	98·16	9 7	123·87	15 8	117·66
30 13	99·10	11 7	123·39	16 8	117·79
31 13	99·47	12 6	124·01	20 8	117·62
June 3 13	100·92	26 6	125·13	25 8	118·16
10 13	103·01	27 6	123·43	27 8	118·99
12 13	102·93	1855.		29 7	119·62
17 12	104·07	Mar. 26 18	82·48	30 7	118·49
20 12	102·61	April 17 16	85·27	31 7	117·55
28 11	109·14	19 16	84·37	Sept. 1 7	119·65
29 11	108·95	20 16	86·08	3 7	120·29
July 18 10	113·18	25 16	86·44	7 7	119·32
19 10	114·52	May 8 15	88·76	12 6	121·19
20 10	114·70	June 1 13	96·67	25 6	120·26
21 10	116·17	2 13	98·72	26 6	121·05
22 10	115·25	5 13	97·91	1856.	
24 10	118·03	9 13	99·64	May 22 14	90·77
28 10	117·03	19 12	101·72	23 14	91·81
29 9	115·47	20 12	103·01	June 2 13	94·52
Aug. 1 9	115·83	21 12	102·89	3 13	93·76
10 9	119·72	27 12	106·08	5 13	93·07
12 9	119·37	28 11	106·22	6 13	96·69
14 8	120·22	30 11	105·71	11 12	96·52
18 8	121·47	July 4 11	108·00	14 12	98·95
22 8	120·77	5 11	106·95	16 12	97·87
28 7	122·60	12 11	108·86	17 12	99·66
29 7	123·00	13 10	107·62	26 12	102·41
30 7	123·39	14 10	109·61	28 11	103·09

Observed Apparent North Zenith Distances of γ Draconis (continued).

Day and Hour.	Observed Apparent N.Z.D.	Day and Hour.	Observed Apparent N.Z.D.	Day and Hour.	Observed Apparent N.Z.D.
1856. h	″	1856. h	″	1857. h	″
July 1 11	105·57	Sept. 20 6	119·33	April 20 16	80·51
3 11	103·73	23 6	118·76	29 15	82·45
5 11	104·04	30 5	119·08	May 3 15	85·29
16 10	109·13	Oct. 1 5	118·65	6 15	84·46
17 10	108·71	20 4	116·52	7 15	86·21
18 10	109·46	22 4	115·28	8 15	85·62
22 10	109·61	23 4	116·06	11 15	86·86
23 10	109·92	25 4	114·24	14 14	86·66
24 10	110·15	27 3	114·12	25 14	90·76
25 10	109·90	Nov. 4 3	113·88	26 14	90·35
26 10	111·30	6 3	112·46	27 14	91·06
28 9	111·84	24 2	108·32	June 2 13	93·51
29 9	111·36	Dec. 11 1	102·82	4 13	94·30
30 9	112·64	15 0	102·30	8 13	94·85
31 9	112·27	19 0	98·95	10 13	95·93
Aug. 1 9	113·08	23 23	98·20	11 13	96·57
2 9	111·91	1857.		12 12	97·76
4 9	114·47	Jan. 1 23	94·67	13 12	96·66
5 9	113·21	Feb. 9 21	82·80	15 12	96·22
6 9	113·80	10 21	82·97	17 12	98·08
7 9	114·28	12 20	82·06	18 12	98·37
9 9	114·03	15 20	81·81	23 12	100·49
12 8	115·00	16 20	82·10	24 12	100·56
13 8	115·17	Mar. 12 18	77·61	25 12	99·05
14 8	115·05	15 18	78·17	26 12	101·33
23 8	115·08	16 18	79·90	27 11	100·95
28 8	117·68	17 18	78·17	30 11	102·18
Sept. 2 7	117·70	April 14 16	82·18	July 7 11	104·43
3 7	117·71	15 16	79·98	8 11	104·76
4 7	117·27	16 16	80·46	9 11	104·40
5 7	118·90	17 16	81·04	11 11	105·31
10 7	118·56	19 16	80·96	13 10	107·55

Observed Apparent North Zenith Distances of γ Draconis (continued).

Day and Hour.		Observed Apparent N.Z.D.	Day and Hour.		Observed Apparent N.Z.D.	Day and Hour.		Observed Apparent N.Z.D.
1857.	h	″	1857.	h	″	1858.	h	″
July 14	10	106·73	Oct. 1	5	117·61	Mar. 22	18	79·90
15	10	106·12	5	5	117·52	23	18	77·08
17	10	107·01	6	5	117·12	26	17	78·45
18	10	107·73	14	4	116·10	April 13	16	79·41
20	10	108·82	26	4	115·59	18	16	81·12
23	10	108·32	28	3	114·65	19	16	79·91
24	10	108·60	30	3	113·55	21	16	80·90
25	10	108·84	Nov. 11	3	111·14	May 4	15	84·09
28	9	111·29	27	1	106·33	5	15	84·12
29	9	110·52	28	1	105·97	6	15	85·94
31	9	111·35	Dec. 4	1	104·97	7	15	84·62
Aug. 5	9	112·31	10	1	102·10	8	15	85·00
12	8	113·37	11	1	102·37	12	14	87·15
14	8	114·95	31	23	95·63	13	14	86·51
17	8	113·70	1858.			18	14	87·66
22	8	115·52	Jan. 6	23	93·23	19	14	87·88
24	8	115·22	11	23	91·29	20	14	87·94
25	8	115·16	13	22	90·61	25	14	90·06
26	8	116·10	25	22	86·01	29	13	91·77
27	7	116·85	26	22	86·64	June 3	13	92·09
Sept. 3	7	116·32	27	21	86·75	5	13	94·66
4	7	115·70	31	21	86·27	9	13	94·90
5	7	116·93	Feb. 18	20	82·21	10	13	94·78
8	7	117·30	19	20	80·22	12	13	95·65
9	7	116·59	21	20	80·35	14	13	96·93
16	6	117·76	23	20	80·06	15	13	96·19
17	6	117·64	24	20	78·79	16	13	96·75
18	6	118·83	25	20	79·07	17	13	97·29
23	6	118·14	Mar. 8	19	78·73	18	13	97·14
24	6	117·96	9	19	77·71	19	13	97·32
29	5	118·06	10	19	79·33	22	12	99·11
30	5	117·05	21	18	79·25	25	12	100·76

Observed Apparent North Zenith Distances of γ Draconis (continued).

Day and Hour.			Observed Apparent N.Z.D.	Day and Hour.			Observed Apparent N.Z.D.	Day and Hour.			Observed Apparent N.Z.D.
1858.		h	″	1858.		h	″	1859.		h	″
June	28	11	102·33	Sept.	15	6	117·34	April	4	17	80·92
	29	11	101·40		16	6	117·44		6	17	80·34
July	1	11	102·32		17	6	116·95		10	17	80·44
	8	11	103·21		18	6	117·10		22	16	82·29
	12	11	105·39		20	6	115·77	May	5	15	85·96
	13	11	106·62		24	5	119·17		6	15	86·95
	14	10	105·74	Oct.	5	5	117·00		12	15	88·85
	16	10	105·99		6	5	117·09		13	14	86·96
	17	10	106·64		12	4	116·58		14	14	87·17
	19	10	107·78		30	3	113·37		23	14	91·09
	21	10	108·59	Nov.	3	3	113·90		26	14	91·42
	26	10	109·04		11	3	110·51	June	7	13	96·81
	28	10	109·68		18	2	109·20		16	12	99·63
	30	9	110·48	Dec.	1	1	106·47		17	12	99·77
	31	9	110·63		3	1	105·20		24	12	101·37
Aug.	2	9	110·59		20	0	99·03		29	11	103·52
	3	9	110·48		23	0	97·72		30	11	103·25
	5	9	110·54	1859.				July	1	11	104·75
	6	9	112·94	Jan.	23	22	87·23		4	11	104·02
	7	9	112·96	Feb.	2	22	86·19		5	11	105·37
	10	9	112·53		6	21	83·17		6	11	104·01
	11	9	113·66		17	20	80·87		7	11	103·81
	12	8	112·25		22	20	79·84		9	11	106·13
	13	8	113·10		24	20	80·78		11	11	106·57
	19	8	113·81		25	20	80·49		12	11	106·27
	26	8	115·20		27	19	78·34		13	10	107·80
	28	7	115·64	Mar.	7	19	79·49		14	10	107·11
	30	7	116·27		8	19	78·91		15	10	107·52
Sept.	1	7	116·25		9	19	79·39		16	10	107·89
	8	7	117·36		10	18	78·74		19	10	110·81
	13	6	117·68		18	18	80·05		28	9	111·43
	14	6	117·30		31	17	79·20	Aug.	2	9	111·71

Observed Apparent North Zenith Distances of γ Draconis (continued).

Day and Hour.	Observed Apparent N.Z.D.	Day and Hour.	Observed Apparent N.Z.D.	Day and Hour.	Observed Apparent N.Z.D.
1859. h	″	1859. h	″	1859. h	″
Aug. 4 9	112·90	Sept. 3 7	117·34	Nov. 10 3	113·48
5 9	112·56	6 7	117·29	17 2	109·77
16 8	114·24	18 6	119·19	19 2	110·05
19 8	114·73	21 6	118·12	21 2	109·68
20 8	115·91	24 6	118·65	23 2	109·21
22 8	115·63	Oct. 3 5	118·90	24 2	109·31
23 8	115·09	6 5	117·36	30 1	106·83
24 8	115·95	14 4	115·96	Dec. 8 1	103·30
26 8	115·95	22 4	115·14	10 1	104·09
30 7	118·29	27 4	114·96	26 0	98·52
Sept. 1 7	117·28	Nov. 3 3	114·13		

TABLE II.

Means of the Observed Apparent Zenith Distances of γ Draconis, deduced from Groups of Short Intervals.

No. of Group.	Extent of Group.	Mean Day and Hour.	Mean of Observed Zen. Dist.	Number of Obs. in Group.
		d h	″	
1	1852, May 31 to June 4 ..	June 2 21	106·28	3
2	June 10 to 21	16 7	111·24	5
3	June 23 to July 10 ..	July 2 19	116·33	9
4	July 14 to 23	19 10	121·08	6
5	July 27 to August 9	Aug. 2 21	124·33	8
6	August 18 to Sept. 3	27 8	128·43	12
7	September 8 to 29 ..	Sept. 19 12	130·32	4
8	October 7 to 20	Oct. 13 17	129·23	2
9	1853, April 10 to 26	April 18 17	91·83	2
10	May 20 to June 3 ..	May 26 20	101·44	4
11	June 7 to 29........	June 17 16	108·59	6
12	July 5 to 25........	July 14 11	117·16	4
13	August 4 to 12......	Aug. 8 21	123·05	4
14	August 20 to 24	22 8	124·20	2
15	September 10 to 19..	Sept. 14 19	126·76	2
16	1854, January 24	Jan. 24 22	95·39	1
17	March 14..........	Mar. 14 18	88·63	1
18	April 19 to 23	April 21 16	89·61	2
19	May 8 to 19........	May 14 22	94·57	3
20	May 26 to June 3 ..	30 4	99·29	5
21	June 10 to 20	June 15 6	103·15	4
22	June 28 to 29	28 23	109·05	2
23	July 18 to August 1	July 24 2	115·58	9
24	August 10 to 22	Aug. 15 13	120·31	5
25	August 28 to Sept. 12	Sept. 4 22	123·26	11
26	September 26 to 27..	26 18	124·28	2

Means of the Observed Apparent Zenith Distances, &c. (continued).

No. of Group.	Extent of Group.		Mean Day and Hour.	Mean of Observed Zen. Dist.	Number of Obs. in Group.
			d h	″	
27	1855,	March 26	Mar. 26 18	82·48	1
28		April 17 to 25	April 20 22	85·54	4
29		May 8	May 8 15	88·76	1
30		June 1 to 9	June 4 19	98·24	4
31		June 19 to 30	24 16	104·27	6
32		July 4 to 14........	July 10 1	108·21	5
33		July 19 to 31	24 2	113·06	3
34		August 9 to 20	Aug. 14 23	117·33	5
35		August 25 to Sept. 7	31 1	119·01	8
36		September 12 to 26..	Sept. 21 6	120·83	3
37	1856,	May 22 to 23	May 23 2	91·29	2
38		June 2 to 11	June 5 22	94·91	5
39		June 14 to 17	16 4	98·83	3
40		June 26 to July 5 ..	July 1 2	103·77	5
41		July 16 to 26	21 19	109·77	8
42		July 28 to August 2	30 21	112·18	6
43		August 4 to 14	Aug. 9 3	114·38	8
44		August 23 to Sept. 10	Sept. 2 4	117·56	7
45		Sept. 20 to Oct. 1 ..	26 6	118·96	4
46		Oct. 20 to 27	Oct. 23 13	115·24	5
47		November 4 to 6	Nov. 5 3	113·17	2
48		November 24	24 2	108·32	1
49		December 11 to 23 ..	Dec. 17 6	100·57	4
50	1857,	January 1..........	Jan. 1 23	94·67	1
51		February 9 to 16....	Feb. 13 6	82·35	5
52		March 12 to 17	Mar. 15 18	78·46	4
53		April 14 to 20	April 17 12	80·86	6
54		April 29 to May 14..	May 7 11	85·36	7
55		May 25 to June 8 ..	31 10	92·47	6
56		June 10 to 18	June 14 6	97·08	7
57		June 23 to 30	26 8	100·43	6
58		July 7 to 18........	July 12 21	106·00	9
59		July 20 to August 5	27 10	110·01	8

Means of the Observed Apparent Zenith Distances, &c. (continued).

No. of Group.	Extent of Group.		Mean Day and Hour.			Mean of Observed Zen. Dist.	Number of Obs. in Group.
				d	h	″	
60	1857,	August 12 to 27	Aug.	21	5	115·11	8
61		September 3 to 9....	Sept.	6	2	116·57	5
62		Sept. 16 to Oct. 1 ..		23	18	117·88	8
63		October 5 to 14	Oct.	8	13	116·91	3
64		October 26 to 30....		28	3	114·60	3
65		November 11	Nov.	11	3	111·14	1
66		Nov. 27 to Dec. 4 ..		29	17	105·76	3
67		December 10 to 11 ..	Dec.	10	13	102·24	2
68		December 31		31	23	95·63	1
69	1858,	January 6 to 13	Jan.	10	23	91·71	3
70		January 25 to 31....		28	4	86·42	4
71		February 18 to 25 ..	Feb.	22	12	80·12	6
72		March 8 to 10	Mar.	9	19	78·59	3
73		March 21 to 26		23	18	78·67	4
74		April 13 to 21	April	18	10	80·34	4
75		May 4 to 13........	May	8	11	85·35	7
76		May 18 to 29		22	19	89·06	5
77		June 3 to 19	June	13	2	95·79	11
78		June 22 to July 8 ..		29	8	101·52	6
79		July 12 to 21	July	16	10	106·68	7
80		July 26 to August 7	Aug.	1	22	110·82	9
81		August 10 to 19		13	8	113·07	5
82		August 26 to Sept. 8		31	7	116·14	5
83		September 13 to 24..	Sept.	17	9	117·34	8
84		October 5 to 12	Oct.	7	21	116·89	3
85		Oct. 30 to Nov. 11 ..	Nov.	4	11	112·59	3
86		November 18		18	2	109·20	1
87		December 1 to 3	Dec.	2	1	105·84	2
88		December 20 to 23 ..		21	12	98·38	2
89	1859,	January 23 to Feb. 6	Jan.	31	22	85·53	3
90		February 17 to 27 ..	Feb.	23	20	80·06	5
91		March 7 to 18	Mar.	11	4	79·32	5
92		March 31 to April 10	April	5	17	80·23	4

Means of the Observed Apparent Zenith Distances, &c. (continued).

No. of Group.	Extent of Group.	Mean Day and Hour.	Mean of Observed Zen. Dist.	Number of Obs. in Group.
		d h	″	
93	1859, April 22	April 22 16	82·29	1
94	May 5 to 14........	May 10 15	87·18	5
95	May 23 to June 7 ..	29 14	93·11	3
96	June 16 to July 1 ..	June 25 0	102·85	6
97	July 4 to 19........	July 11 9	106·44	12
98	July 28 to August 5	Aug. 2 9	112·15	4
99	August 16 to 26	21 18	115·36	7
100	August 30 to Sept. 6	Sept. 2 13	117·55	4
101	September 18 to 24..	21 6	118·65	3
102	October 3 to 14	Oct. 7 21	117·41	3
103	Oct. 22 to Nov. 3 ..	27 20	114·74	3
104	November 10 to 21 ..	Nov. 16 20	110·75	4
105	Nov. 23 to Dec. 10 ..	Dec. 1 1	106·55	5
106	December 26	26 0	98·52	1

TABLE III.

Values of the Sun's Longitude ☉, Moon's Longitude ☽, and Longitude of the Moon's Node ☊, for the Times corresponding to the Means of the Times of Observation in each Group.

No. of Series.	☉	☽	☊	No. of Series.	☉	☽	☊
	° ′	° ′	° ′		° ′	° ′	° ′
1	72 48	267 40	99 22	25	162 25	322 15	55 43
2	85 38	75 6	98 37	26	183 44	244 20	54 34
3	101 22	302 6	97 46	27	5 58	115 30	44 59
4	117 14	152 29	96 53	28	30 40	89 12	43 39
5	131 3	349 28	96 7	29	47 50	317 52	42 42
6	154 31	313 42	94 49	30	73 58	316 52	41 16
7	177 6	260 7	93 36	31	92 57	211 14	40 12
8	200 58	214 0	92 19	32	107 36	66 16	39 24
9	29 0	149 11	82 25	33	121 0	237 58	38 40
10	65 50	306 4	80 24	34	141 57	171 5	37 30
11	86 43	223 24	79 15	35	157 29	32 41	36 39
12	112 16	215 48	77 50	36	178 7	300 12	35 32
13	136 34	190 36	76 29	37	62 31	285 49	22 33
14	149 30	17 5	75 46	38	75 46	118 45	21 50
15	172 16	325 53	74 32	39	85 33	243 30	21 18
16	305 7	259 25	67 32	40	99 47	89 42	20 30
17	354 18	180 36	64 57	41	119 33	358 39	19 24
18	31 38	329 34	62 56	42	128 13	122 20	18 55
19	54 9	272 27	61 42	43	137 5	233 58	18 26
20	68 52	110 22	60 54	44	160 16	195 15	17 9
21	84 10	330 36	60 37	45	183 47	157 34	15 53
22	97 13	140 32	59 21	46	210 47	158 6	14 26
23	121 14	115 14	58 0	47	223 23	312 12	13 46
24	142 45	58 3	56 48	48	242 29	208 32	12 47

Longitudes of the Sun and Moon, and of the Moon's Node (continued).

No. of Series.	☉	☽	☊	No. of Series.	☉	☽	☊
49	266° 1′	159° 9′	11° 32′	78	97° 39′	315° 44′	341° 56′
50	282 2	358 26	10 42	79	113 57	192 58	341 1
51	325 0	198 46	8 28	80	129 42	37 27	340 9
52	355 34	236 29	6 51	81	140 39	200 10	339 33
53	27 50	303 53	5 7	82	157 59	66 51	338 35
54	47 14	214 58	4 3	83	174 36	293 38	337 42
55	70 16	175 44	2 48	84	194 45	210 18	336 37
56	83 29	346 35	2 4	85	222 13	213 36	335 9
57	95 1	158 16	1 25	86	235 56	22 8	334 26
58	110 47	5 44	0 33	87	250 4	217 42	333 42
59	124 39	204 8	359 46	88	269 51	110 38	332 40
60	148 29	172 44	358 28	89	312 1	294 25	330 28
61	163 50	15 22	357 37	90	335 10	242 13	329 15
62	181 4	247 46	356 41	91	350 32	73 45	328 26
63	195 39	89 59	355 54	92	15 49	50 27	327 5
64	215 7	335 37	354 52	93	32 24	280 20	326 11
65	229 10	173 49	354 7	94	49 49	158 18	325 14
66	247 57	50 7	353 8	95	68 3	38 43	324 14
67	258 57	200 9	352 34	96	93 17	25 37	322 50
68	280 46	122 22	351 26	97	108 55	253 52	321 58
69	290 58	249 15	350 54	98	129 56	185 2	320 48
70	308 28	118 53	350 0	99	148 32	67 5	319 47
71	334 4	90 4	348 39	100	159 56	236 43	319 9
72	349 24	287 17	347 50	101	178 8	112 3	318 10
73	3 16	118 26	347 6	102	193 31	333 41	317 17
74	28 31	96 22	345 44	103	214 20	243 36	316 14
75	47 58	349 40	344 41	104	234 26	141 42	315 10
76	61 45	191 55	343 55	105	248 49	327 42	314 25
77	82 10	110 16	342 48	106	274 12	299 36	313 5

TABLE IV.

Equations of Condition.

No. of Series.	Number of Obs.	Equations.
1	3	$v - 0\cdot262 \times x + 0\cdot930 \times y = -0''\cdot82$
2	5	$v - \cdot049 \quad + \cdot965 \quad -0\cdot15$
3	9	$v + \cdot213 \quad + \cdot942 \quad -0\cdot15$
4	6	$v + \cdot464 \quad + \cdot847 \quad -0\cdot07$
5	8	$v + \cdot653 \quad + \cdot712 \quad -0\cdot23$
6	12	$v + \cdot882 \quad + \cdot394 \quad -0\cdot23$
7	4	$v + \cdot966 \quad + \cdot024 \quad +0\cdot28$
8	2	$v + \cdot893 \quad - \cdot369 \quad +0\cdot89$
9	2	$v - \cdot833 \quad + \cdot490 \quad +0\cdot51$
10	4	$v - \cdot374 \quad + \cdot891 \quad +0\cdot40$
11	6	$v - \cdot030 \quad + \cdot966 \quad +0\cdot65$
12	4	$v + \cdot388 \quad + \cdot885 \quad +1\cdot11$
13	4	$v + \cdot718 \quad + \cdot646 \quad +1\cdot03$
14	2	$v + \cdot844 \quad + \cdot470 \quad -0\cdot04$
15	2	$v + \cdot959 \quad + \cdot107 \quad +0\cdot62$
16	1	$v - \cdot575 \quad - \cdot777 \quad +1\cdot01$
17	1	$v - \cdot963 \quad - \cdot072 \quad (+3\cdot52)$
18	2	$v - \cdot809 \quad + \cdot527 \quad +1\cdot37$
19	3	$v - \cdot546 \quad + \cdot797 \quad +0\cdot60$
20	5	$v - \cdot325 \quad + \cdot909 \quad +0\cdot75$
21	4	$v - \cdot074 \quad + \cdot963 \quad -0\cdot45$
22	2	$v + \cdot145 \quad + \cdot955 \quad +1\cdot16$
23	9	$v + \cdot521 \quad + \cdot813 \quad +0\cdot46$
24	5	$v + \cdot784 \quad + \cdot566 \quad +0\cdot48$
25	11	$v + \cdot928 \quad + \cdot269 \quad +1\cdot01$
26	2	$v + \cdot962 \quad - \cdot087 \quad +1\cdot53$
27	1	$v - \cdot958 \quad + \cdot125 \quad +0\cdot37$
28	4	$v - 0\cdot818 \times x + 0\cdot513 \times y = +0\cdot45$

Equations of Condition (continued).

No. of Series.	Number of Obs.	Equations.
29	1	$v - 0{\cdot}630 \times x + 0{\cdot}732 \times y = -0''{\cdot}25$
30	4	$v - {\cdot}243 \quad + {\cdot}935 \quad + 1{\cdot}12$
31	6	$v + {\cdot}075 \quad + {\cdot}963 \quad + 0{\cdot}80$
32	5	$v + {\cdot}315 \quad + {\cdot}913 \quad - 0{\cdot}06$
33	3	$v + {\cdot}518 \quad + {\cdot}815 \quad + 0{\cdot}96$
34	5	$v + {\cdot}776 \quad + {\cdot}577 \quad + 0{\cdot}64$
35	8	$v + {\cdot}901 \quad + {\cdot}347 \quad + 0{\cdot}07$
36	3	$v + {\cdot}966 \quad + {\cdot}008 \quad + 0{\cdot}79$
37	2	$v - {\cdot}424 \quad + {\cdot}868 \quad - 0{\cdot}07$
38	5	$v - {\cdot}215 \quad + {\cdot}942 \quad - 0{\cdot}79$
39	3	$v - {\cdot}051 \quad + {\cdot}965 \quad - 0{\cdot}23$
40	5	$v + {\cdot}188 \quad + {\cdot}948 \quad - 0{\cdot}09$
41	8	$v + {\cdot}497 \quad + {\cdot}826 \quad + 0{\cdot}12$
42	6	$v + {\cdot}616 \quad + {\cdot}743 \quad + 0{\cdot}19$
43	8	$v + {\cdot}723 \quad + {\cdot}640 \quad + 0{\cdot}39$
44	7	$v + {\cdot}917 \quad + {\cdot}303 \quad + 0{\cdot}23$
45	4	$v + {\cdot}962 \quad - {\cdot}088 \quad + 0{\cdot}92$
46	5	$v + {\cdot}817 \quad - {\cdot}515 \quad - 0{\cdot}03$
47	2	$v + {\cdot}686 \quad - {\cdot}681 \quad + 0{\cdot}33$
48	1	$v + {\cdot}425 \quad - {\cdot}867 \quad + 0{\cdot}58$
49	4	$v + {\cdot}043 \quad - {\cdot}965 \quad + 0{\cdot}51$
50	1	$v - {\cdot}224 \quad - {\cdot}939 \quad + 0{\cdot}16$
51	5	$v - {\cdot}805 \quad - {\cdot}534 \quad + 0{\cdot}50$
52	4	$v - {\cdot}965 \quad - {\cdot}050 \quad + 0{\cdot}20$
53	6	$v - {\cdot}842 \quad + {\cdot}473 \quad - 0{\cdot}03$
54	7	$v - {\cdot}638 \quad + {\cdot}725 \quad + 0{\cdot}05$
55	6	$v - {\cdot}303 \quad + {\cdot}917 \quad + 0{\cdot}07$
56	7	$v - {\cdot}086 \quad + {\cdot}962 \quad + 0{\cdot}17$
57	6	$v + {\cdot}108 \quad + {\cdot}960 \quad - 0{\cdot}43$
58	9	$v + {\cdot}366 \quad + {\cdot}894 \quad + 0{\cdot}08$
59	8	$v + {\cdot}569 \quad + {\cdot}781 \quad + 0{\cdot}15$
60	8	$v + 0{\cdot}836 \times x + 0{\cdot}485 \times y = + 0{\cdot}34$

Equations of Condition (continued).

No. of Series.	Number of Obs.	Equations.
61	5	$v + 0{\cdot}934 \times x + 0{\cdot}246 \times y = +0{\cdot}02''$
62	8	$v + {\cdot}965 \qquad - {\cdot}042 \qquad +0{\cdot}66$
63	3	$v + {\cdot}923 \qquad - {\cdot}284 \qquad +0{\cdot}47$
64	3	$v + {\cdot}777 \qquad - {\cdot}575 \qquad +1{\cdot}02$
65	1	$v + {\cdot}613 \qquad - {\cdot}747 \qquad +0{\cdot}67$
66	3	$v + {\cdot}339 \qquad - {\cdot}904 \qquad +0{\cdot}47$
67	2	$v + {\cdot}161 \qquad - {\cdot}952 \qquad +0{\cdot}57$
68	1	$v - {\cdot}203 \qquad - {\cdot}944 \qquad +1{\cdot}32$
69	3	$v - {\cdot}369 \qquad - {\cdot}893 \qquad +0{\cdot}84$
70	4	$v - {\cdot}619 \qquad - {\cdot}741 \qquad +0{\cdot}99$
71	6	$v - {\cdot}878 \qquad - {\cdot}401 \qquad +0{\cdot}41$
72	3	$v - {\cdot}953 \qquad - {\cdot}154 \qquad +0{\cdot}59$
73	4	$v - {\cdot}963 \qquad + {\cdot}079 \qquad +0{\cdot}91$
74	4	$v - {\cdot}837 \qquad + {\cdot}483 \qquad -0{\cdot}27$
75	7	$v - {\cdot}628 \qquad + {\cdot}734 \qquad +0{\cdot}29$
76	5	$v - {\cdot}436 \qquad + {\cdot}862 \qquad -0{\cdot}18$
77	11	$v - {\cdot}098 \qquad + {\cdot}959 \qquad -0{\cdot}51$
78	6	$v + {\cdot}152 \qquad + {\cdot}954 \qquad -0{\cdot}01$
79	7	$v + {\cdot}414 \qquad + {\cdot}872 \qquad +0{\cdot}03$
80	9	$v + {\cdot}635 \qquad + {\cdot}727 \qquad -0{\cdot}14$
81	5	$v + {\cdot}763 \qquad + {\cdot}593 \qquad -0{\cdot}24$
82	5	$v + {\cdot}904 \qquad + {\cdot}339 \qquad +0{\cdot}07$
83	8	$v + {\cdot}964 \qquad + {\cdot}067 \qquad +0{\cdot}18$
84	3	$v + {\cdot}927 \qquad - {\cdot}269 \qquad +0{\cdot}48$
85	3	$v + {\cdot}699 \qquad - {\cdot}667 \qquad +0{\cdot}36$
86	1	$v + {\cdot}521 \qquad - {\cdot}814 \qquad +0{\cdot}34$
87	2	$v + {\cdot}307 \qquad - {\cdot}916 \qquad +1{\cdot}09$
88	2	$v - {\cdot}022 \qquad - {\cdot}966 \qquad +0{\cdot}07$
89	3	$v - {\cdot}664 \qquad - {\cdot}701 \qquad +0{\cdot}75$
90	5	$v - {\cdot}886 \qquad - {\cdot}384 \qquad +0{\cdot}17$
91	5	$v - {\cdot}956 \qquad - {\cdot}135 \qquad +0{\cdot}95$
92	4	$v - 0{\cdot}922 \times x + 0{\cdot}286 \times y = +1{\cdot}10$

Equations of Condition (continued).

No. of Series.	Number of Obs.	Equations.
93	1	$v - 0{\cdot}802 \times x + 0{\cdot}538 \times y = +0''{\cdot}38$
94	5	$v - {\cdot}604 \quad + {\cdot}754 \quad +1{\cdot}00$
95	3	$v - {\cdot}338 \quad + {\cdot}904 \quad +1{\cdot}09$
96	6	$v + {\cdot}080 \quad + {\cdot}963 \quad +1{\cdot}34$
97	12	$v + {\cdot}336 \quad + {\cdot}906 \quad +0{\cdot}44$
98	4	$v + {\cdot}638 \quad + {\cdot}725 \quad +0{\cdot}41$
99	7	$v + {\cdot}836 \quad + {\cdot}484 \quad -0{\cdot}29$
100	4	$v + {\cdot}916 \quad + {\cdot}308 \quad +0{\cdot}44$
101	3	$v + {\cdot}966 \quad + {\cdot}008 \quad +0{\cdot}53$
102	3	$v + {\cdot}933 \quad - {\cdot}248 \quad -0{\cdot}03$
103	3	$v + {\cdot}785 \quad - {\cdot}564 \quad -0{\cdot}16$
104	4	$v + {\cdot}542 \quad - {\cdot}799 \quad +0{\cdot}42$
105	5	$v + {\cdot}326 \quad - {\cdot}909 \quad +0{\cdot}36$
106	1	$v - 0{\cdot}094 \times x - 0{\cdot}962 \times y = +0{\cdot}60$

With regard to the equations above, I may remark that I have rejected No. 17, which depends on a single observation, evidently affected by an error amounting to 2″ or 3″.

Proceeding to solve the remaining 105 equations by the method of least squares, and using precisely the same method and the same notation as in my previous Memoir, I find

$\Sigma(n) = +486$

$\Sigma(na) = +113{\cdot}510$

$\Sigma(nb) = +201{\cdot}696$

$\Sigma(na^2) = +209{\cdot}944$

$\Sigma(nb^2) = +243{\cdot}474$

$\Sigma(nd) = -144{\cdot}24$

$\Sigma(nab) = +41{\cdot}550$

$\Sigma(nad) = -23{\cdot}035$

$\Sigma(nbd) = -21{\cdot}554$

and the final equations are

$$\begin{aligned}
486\,v + 113{\cdot}510\,x + 201{\cdot}696\,y - 144{\cdot}24 &= 0\\
113{\cdot}510\,v + 209{\cdot}944\,x + 41{\cdot}550\,y - 23{\cdot}035 &= 0\\
201{\cdot}696\,v + 41{\cdot}550\,x + 243{\cdot}474\,y - 21{\cdot}554 &= 0
\end{aligned}$$

from which we derive

$$v = +\,0''{\cdot}412$$
$$x = -\,0{\cdot}065$$
$$y = -\,0{\cdot}242$$

Substituting these values in the equations, and calling Δ the residual error of each, we have for the values of Δ,—

No. of Series.	Δ.	No. of Series.	Δ.	No. of Series.	Δ.	No. of Series.	Δ.
1	−1″·03	29	−0″·52	56	−0″·02	83	−0″·15
2	−0·33	30	+0·92	57	−0·60	84	+0·06
3	−0·32	31	+0·63	58	−0·09	85	−0·16
4	−0·25	32	−0·23	59	−0·03	86	−0·24
5	−0·43	33	+0·78	60	+0·10	87	+0·48
6	−0·49	34	+0·42	61	−0·27	88	−0·57
7	−0·07	35	−0·20	62	+0·30	89	+0·12
8	+0·45	36	+0·44	63	+0·05	90	−0·39
9	+0·17	37	−0·30	64	+0·52	91	+0·45
10	+0·17	38	−0·98	65	+0·12	92	+0·70
11	+0·47	39	−0·41	66	−0·14	93	+0·05
12	+0·94	40	−0·26	67	−0·06	94	+0·73
13	+0·83	41	−0·06	68	+0·67	95	+0·88
14	−0·28	42	0·00	69	+0·19	96	+1·17
15	+0·30	43	+0·18	70	+0·36	97	+0·27
16	−0·37	44	−0·05	71	−0·16	98	+0·21
18	+1·04	45	+0·55	72	+0·08	99	−0·53
19	+0·34	46	−0·51	73	+0·46	100	+0·16
20	+0·54	47	−0·20	74	−0·61	101	+0·18
21	−0·64	48	−0·01	75	+0·02	102	−0·44
22	+0·99	49	−0·13	76	−0·41	103	−0·66
23	+0·28	50	−0·49	77	−0·70	104	−0·15
24	+0·26	51	−0·09	78	−0·18	105	−0·25
25	+0·72	52	−0·28	79	−0·14	106	−0·05
26	+1·16	53	−0·38	80	−0·33		
27	−0·07	54	−0·23	81	−0·46		
28	+0·11	55	−0·14	82	−0·20		

Hence, n being the number of observations in a group, we shall find $\Sigma\,(n\,\Delta^2) = 94{\cdot}468$.

And hence we shall find, by the usual mode of calculation,

$$\begin{aligned}\text{Probable error of } v &= 0''{\cdot}014\\ \text{,,} \qquad x &= 0{\cdot}023\\ \text{,,} \qquad y &= 0{\cdot}024\end{aligned}$$

In looking at the results above, the first point to be observed is the negative value of the parallax, $-\,0''{\cdot}24$, which is ten times as great as the probable error, and is therefore an indication of something having affected the observations injuriously, though not to so large an amount as in the case of the 25-foot zenith tube. The correction to the assumed value of the constant of aberration, $20''{\cdot}4$, is however small, and the value deduced from the observations is therefore in moderate agreement with the most recent determinations, and in this respect the result of the investigation is satisfactory. To submit this, however, to a further test, I have solved the equations again under the supposition that the parallax of the star is insensible; and I find the following values of v and x:—

$$\begin{aligned}v &= +\,0''{\cdot}310\\ x &= -\,0{\cdot}058\end{aligned}$$

Here the value of x is so closely in agreement with its former value $-\,0''{\cdot}065$, that we may fairly assume that the value $-\,0''{\cdot}06$ is very near the truth, and that, consequently, the value of the constant of aberration is very nearly $20''{\cdot}34$, and that considerable credit is due to this determination.

Substituting, however, these latter values of v and x in the equations, we find that the value of $\Sigma\,(n\,\Delta^2)$ is larger than before, namely, its value is $105{\cdot}57$, and, therefore, that the former values of v, x, and y, on the whole, satisfy the equations better than the latter values of v and y. Assuming those values, we have finally,—

$$\begin{aligned}\text{Mean N.Z.D. of } \gamma\ \textit{Draconis}\text{, 1859, Jan. 1} &= 106''{\cdot}77 \pm 0''{\cdot}014\\ \text{Constant of Aberration} \quad .. &= 20{\cdot}335 \pm 0{\cdot}023\end{aligned}$$

Royal Observatory, Greenwich,
1860, *May* 7.

ROBERT MAIN.

VII. *Tables of the Developments of Functions in the Theory of Elliptic Motion.* By A. CAYLEY, Esq.

Read January 12, 1859.

THE notation made use of is as follows; viz., r, f, a, e, g denote

r, the radius vector;
f, the true anomaly;
a, the mean distance;
e, the excentricity;
g, the mean anomaly;

so that

$$\frac{r}{a}, = \text{elqr}\,(e, g), \text{ and}$$

$$f, = \text{elta}\,(e, g),$$

are known functions of e, g. Moreover, x denotes the periodic part of the quotient radius $\frac{r}{a}$, and y the equation of the centre, or periodic part of the true anomaly f; so that

$$\frac{r}{a} = 1 + x,$$

$$f = g + y,$$

and (x, y) are therefore also known functions of e, g.

Formulæ for the development in multiple cosines or sines, up to the terms in e^7, of the functions

$$(x^0, x^1, \ldots x^7) \begin{matrix}\cos\\ \sin\end{matrix}\, jy,$$

where j is an indeterminate symbol, are given by Leverrier in the *Annales de l'Observatoire de Paris*, tom. i. (1855), pp. 346–348. It occurred to me that it would be desirable to form tables such as are contained in the present memoir, of various functions of the forms

$$x^m \begin{matrix}\cos\\ \sin\end{matrix} jf, \qquad \left(\frac{r}{a}\right)^{\pm m} \begin{matrix}\cos\\ \sin\end{matrix} jf,$$

obtainable from Leverrier's formulæ by assigning particular integer values to the symbol j. The calculations were performed under my superintendence by Messrs. Creedy and Davis; and I have to acknowledge my obligation for a grant to defray the expenses, made to me out of the Donation Fund at the disposal of the Council of the Royal Society.

A cosine series is in general represented in the form

$$\Sigma\ [\cos]^i \cos ig,$$

where i extends from $-\infty$ to $+\infty$, and the coefficients $[\cos]^i$ satisfy the condition

$$[\cos]^{-i} = [\cos]^i;$$

and in like manner a sine series is represented in the form

$$\Sigma\ [\sin]^i \sin ig,$$

where i extends from $-\infty$ to $+\infty$, and the coefficients $[\sin]^i$ satisfy the condition

$$\begin{aligned}[\sin]^{-i} &= -\ [\sin]^i, \text{ and in particular}\\ [\sin]^0 &= 0.\end{aligned}$$

Thus $[\cos]^i$ is a general symbol denoting the coefficient of $\cos ig$, in a cosine series considered as involving as well negative as positive multiples of the argument g, or, what is the same thing, $[\cos]^0$ is the constant term and $[\cos]^i$ is the half coefficient of $\cos ig$, when the series is considered as containing only positive multiples of the argument. And in like manner $[\sin]^i$ is a general symbol for the coefficient of $\sin ig$, in a sine series considered as involving as well negative as positive multiples of the argument g, or, what is the same thing, $[\sin]^i$ is the half coefficient of sin ig, when the series is considered as containing only positive multiples of the argument.

In the case of a pair of corresponding functions $x^m \cos jf$ and $x^m \sin jf$, or $\left(\frac{r}{a}\right)^{\pm m} \cos jf$ and $\left(\frac{r}{a}\right)^{\pm m} \sin jf$, one of them expanded in the form

$$\Sigma\,[\cos]^i \cos ig,$$

and the other in the form

$$\Sigma\,[\sin]^i \sin ig,$$

the sums and differences of the corresponding coefficients $[\cos]^i$, $[\sin]^i$, are represented by the notation

$$[\cos \pm \sin]^i,$$

and it is clear that we have

$$[\cos + \sin]^{-i} = [\cos - \sin]^i,\ [\cos \pm \sin]^0 = [\cos]^0.$$

These coefficients $[\cos \pm \sin]^i$ are for many purposes equally useful with the coefficients $[\cos]^i$, $[\sin]^i$, and they are here tabulated accordingly.

The functions tabulated are as follows :—

$$\left(x^1,\ x^2\ .\ .\ .\ .\ x^7\right) \qquad , [\cos],$$

$$\left(x^0,\ x^1,\ x^2\ .\ .\ .\ x^7\right) \begin{matrix}\cos\\ \sin\end{matrix}\, jf,\ \ j = 1 \text{ to } j = 7, [\cos], [\sin], [\cos \pm \sin],$$

$$\left(\left(\frac{r}{a}\right)^{+4}, \ldots \left(\frac{r}{a}\right)^{+1}, \log\frac{r}{a}, \left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \qquad , [\cos],$$

$$\left(\left(\frac{r}{a}\right)^{+4}, \ldots \left(\frac{r}{a}\right)^{+1},\ \ \left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \begin{matrix}\cos\\ \sin\end{matrix}\, jf, j = 1 \text{ to } j = 5, [\cos], [\sin], [\cos \pm \sin],$$

all the developments being carried up to e^7, the limit of the formulæ from which they are deduced.

The calculations are based upon Leverrier's formulæ for

$$(x^0,\ x^1,\ .\ .\ .\ x^7) \begin{matrix}\cos\\ \sin\end{matrix}\, jy,$$

which are made use of under the following forms, which I call the Datum Tables $(x^0 \,..\, x^7) \begin{matrix}\cos\\ \sin\end{matrix}\, jy$.

Cosine Table.	$[\ ]^0$				$[\ ]^1$			
$\frac{1}{2}$ of	$\frac{e^0}{2}$	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e}{4}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$
$x^0 \cos jy$	$+1$	$-8j^2$	$+96j^4$ $-54j^2$	$-1280j^6$ $+3920j^4$ $-3440j^2$	0	$-30j^2$	$+800j^4$ $-120j^2$	$-16800j^6$ $+30590j^4$ $-29470j^2$
$x \cos jy$		$+4$	$-48j^2$	$-360j^2$	-1	$+12j^2$ $+9$	$-160j^4$ $-370j^2$ -50	$+2240j^6$ $-980j^4$ $+9695j^2$ $+245$
$x^2 \cos jy$		$+4$	$-96j^2$	$+1920j^4$ $-1320j^2$		-12	$-240j^2$ $+80$	$+16800j^4$ $-6300j^2$ -420
$x^3 \cos jy$			$+144$	$-2880j^2$		-18	$+480j^2$ $+360$	$-10080j^4$ $-22050j^2$ -3150
$x^4 \cos jy$			$+144$	$-5760j^2$			-960	$-5040j^2$ $+10080$
$x^5 \cos jy$				$+14400$			-1200	$+50400j^2$ $+37800$
$x^6 \cos jy$				$+14400$				-151200
$x^7 \cos jy$				0				-176400

Datum Table $(x^0 \ldots x^7) \cos jy \ldots$

Cosine Table.	$[\ ]^2$			$[\ ]^3$		
$\frac{1}{2}$ of	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$
$x^0 \cos jy$	$+4j^2$	$-64j^4$ $-256j^2$	$+960j^6$ $+11820j^4$ $+4710j^2$	$+30j^2$	$-1200j^4$ $-3240j^2$	$+30240j^6$ $+243810j^4$ $+173502j^2$
$x \cos jy$	-2	$+192j^2$ $+64$	$-4800j^4$ $-8190j^2$ -1440	$-12j^2$ -9	$+240j^4$ $+2745j^2$ $+675$	$-4032j^6$ $-122220j^4$ $-225099j^2$ -35721
$x^2 \cos jy$	$+2$	$0j^2$ -96	$-480j^4$ $+1590j^2$ $+2400$	$+12$	$-840j^2$ -1080	$+10080j^4$ $+102060j^2$ $+61236$
$x^3 \cos jy$		0	$+7920j^2$ $+1440$	-6	$-240j^2$ $+540$	$+10080j^4$ $+85050j^2$ -17010
$x^4 \cos jy$		$+96$	$-1440j^2$ -8640		$+480$	$-95760j^2$ -90720
$x^5 \cos jy$			$+3600$		-600	$-10080j^2$ $+113400$
$x^6 \cos jy$			$+10800$			$+30240$
$x^7 \cos jy$			0			-105840

. . . **Datum Table (x^0 . . x^7) $\cos jy$** . . .

Cosine Table. $\frac{1}{2}$ of	$[\ ]^4$		$[\ ]^5$		$[\ ]^6$	$[\ ]^7$
	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$	$\frac{e^6}{92160}$	$\frac{e^7}{1290240}$
$x^0 \cos jy$	$+16j^4$ $+283j^2$	$-384j^6$ $-22440j^4$ $-51528j^2$	$+400j^4$ $+3360j^2$	$-16800j^6$ $-467530j^4$ $-972790j^2$	$+64j^6$ $+8660j^4$ $+48538j^2$	$+3360j^6$ $+193130j^4$ $+828758j^2$
$x \cos jy$	$-168j^2$ -64	$+7680j^4$ $+46260j^2$ $+9216$	$-80j^4$ $-2375j^2$ -625	$+2240j^6$ $+205660j^4$ $+905135j^2$ $+153125$	$-2880j^4$ $-37890j^2$ -7776	$-448j^6$ $-82460j^4$ $-689731j^2$ -117649
$x^2 \cos jy$	$+48j^2$ $+96$	$-960j^4$ $-22380j^2$ -15360	$+1080j^2$ $+1000$	$-50400j^4$ $-540540j^2$ -262500	$+480j^4$ $+21450j^2$ $+12960$	$+23520j^4$ $+444780j^2$ $+201684$
$x^3 \cos jy$	-72	$+1440j^2$ $+11520$	$-240j^2$ -900	$+3360j^4$ $+147630j^2$ $+236250$	$-7920j^2$ -12960	$-3360j^4$ $-210630j^2$ -216090
$x^4 \cos jy$	$+24$	$+2880j^2$ $+0$	$+480$	$+35280j^2$ -84000	$+1440j^2$ $+8640$	$+65520j^2$ $+164640$
$x^5 \cos jy$		-7200	-120	$-30240j^2$ -63000	-3600	$-10080j^2$ -88200
$x^6 \cos jy$		$+4320$		$+90720$	$+720$	$+30240$
$x^7 \cos jy$		0		-35280	0	-5040

. . . **Datum Table** $(x^0 \,.\,.\, x^7) \cos jy$.

Sine Table. ½ of	[ ]°				[ ]^1			
	$\frac{e^0}{2}$	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e}{4}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$
$x^0 \sin jy$					$+2j$	$-24j^3$ $-6j$	$+320j^5$ $+260j^3$ $+100j$	$-4480j^7$ $-1400j^5$ $-17290j^3$ $+7490j$
$x \sin jy$						$+21j$	$-400j^3$ $-180j$	$+5040j^5$ $+35j^3$ $+6125j$
$x^2 \sin jy$						$+12j$	$-320j^3$ $+160j$	$+6720j^5$ $-420j^3$ $+3780j$
$x^3 \sin jy$							$+840j$	$-25200j^3$ $-10080j$
$x^4 \sin jy$							$+480j$	$-20160j^3$ $+25200j$
$x^5 \sin jy$								$+88200j$
$x^6 \sin jy$								$+50400j$
$x^7 \sin jy$								0

Datum Table $(x^0 \,.\,.\, x^7) \sin jy$ . . .

Sine Table. $\frac{1}{2}$ of	$[\ ]^2$			$[\ ]^3$		
	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$
$x^0 \sin jy$	$+5j$	$-240j^3$ $-88j$	$+6000j^5$ $+8895j^3$ $+2040j$	$+8j^3$ $+26j$	$-160j^5$ $-3070j^3$ $-1290j$	$+2688j^7$ $+125160j^5$ $+257166j^3$ $+59850j$
$x \sin jy$	$-4j$	$+64j^3$ $+184j$	$-960j^5$ $-8340j^3$ $-5700j$	$-27j$	$+1400j^3$ $+2300j$	$-36624j^5$ $-207165j^3$ $-145971j$
$x^2 \sin jy$		$-72j$	$-1200j^3$ $+4260j$	$+12j$	$-160j^3$ $-1600j$	$+1344j^5$ $+39900j^3$ $+130788j$
$x^3 \sin jy$		$-96j$	$+2880j^3$ $+6120j$		$-180j$	$+55440j^3$ $+32760j$
$x^4 \sin jy$			$-8280j$		$+720j$	$-20160j^3$ $-156240j$
$x^5 \sin jy$			$-7200j$			$+37800j$
$x^6 \sin jy$			0			$+90720j$
$x^7 \sin jy$			0			0

. . . Datum Table (x^0 . . x^7) $\sin jy$ . . .

Sine Table.	$[\ ]^4$		$[\ ]^5$		$[\ ]^6$	$[\ ]^7$
$\frac{1}{2}$ of	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$	$\frac{e^6}{92160}$	$\frac{e^7}{1290240}$
$x^0 \sin jy$	$+120j^3$ $+206j$	$-4800j^5$ $-49200j^3$ $-21648j$	$+32j^5$ $+1790j^3$ $+2194j$	$-896j^7$ $-124600j^5$ $-939722j^3$ $-416990j$	$+1200j^5$ $+29835j^3$ $+29352j$	$+128j^7$ $+35560j^5$ $+563486j^3$ $+472730j$
$x \sin jy$	$-32j^3$ $-236j$	$+768j^5$ $+27840j^3$ $+35736j$	$-760j^3$ $-2640j$	$+34608j^5$ $+599935j^3$ $+657601j$	$-192j^5$ $-15780j^3$ $-36324j$	$-9744j^5$ $-339745j^3$ $-595231j$
$x^2 \sin jy$	$+156j$	$-7680j^3$ $-30120j$	$+160j^3$ $+2080j$	$-4032j^5$ $-237300j^3$ $-610764j$	$+5520j^3$ $+31620j$	$+1344j^5$ $+152460j^3$ $+552468j$
$x^3 \sin jy$	$-48j$	$0j^3$ $+8640j$	$-1020j$	$+35280j^3$ $+303240j$	$-960j^3$ $-19320j$	$-45360j^3$ $-384720j$
$x^4 \sin jy$		$+7200j$	$+240j$	$+6720j^3$ $+11760j$	$+7560j$	$+6720j^3$ $+193200j$
$x^5 \sin jy$		$-5760j$		$-113400j$	$-1440j$	$-63000j$
$x^6 \sin jy$		0		$+50400j$	0	$+10080j$
$x^7 \sin jy$		0		0	0	0

. . . **Datum Table (x^0 . . x^7) sin jy.**

It may be remarked that with a view to the convenience of calculation, the Datum Tables give the half values of the several functions; thus, *e.g.*, we have

$$\frac{1}{2}x^0\cos jy=\left\{+1\frac{e^0}{2}-8j^2\frac{e^2}{16}+(96j^4-54j^2)\frac{e^4}{768}+\&c.\right\}+\left\{0\frac{e}{4}-30j^2\frac{e^3}{96}+\&c.\right\}2\cos g$$

$$+\left\{4j^2\frac{e^2}{16}+\&c.\right\}2\cos 2g+\&c.$$

and so for the other functions in the two tables.

The calculation, for a given value of j, of the functions $(x^0, x^1 \ldots x^7)\,{}^{\cos}_{\sin}\,jf$ is performed as follows; the numerical value of j is in the first instance substituted throughout the two Datum tables, so as to obtain the numerical values of the separate terms $\pm Aj^m$; when this is done, the terms in each compartment are added together with the proper signs, and the results written down in a single table as follows, viz. if $j=1$, and if we write down only the portion of the table which corresponds to $x^0\,{}^{\cos}_{\sin}\,y$, (the portions corresponding to $x\,{}^{\cos}_{\sin}\,y$, $x^2\,{}^{\cos}_{\sin}\,y$ &c. . . $x^7\,{}^{\cos}_{\sin}\,y$, being of a precisely similar form, and coming into the table in order one under the other) this is

	[ ]°				[ ]¹			
$\frac{1}{2}x^0\,{}^{\cos}_{\sin}\,y$	$\frac{e^0}{2}$	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e}{4}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$
[cos]	+1	−8	+42	−800	0	−30	+ 680	−15680
[sin]	0	0	0	0	2	−30	+ 680	−15680
[cos+sin]	+1	−8	+42	−800	+2	−60	+1360	−31360
[cos−sin]	Do.	,,	,,	,,	−2	0	0	0

	[ ]²			[ ]³		
$\frac{1}{2}x^0\,{}^{\cos}_{\sin}\,y$	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$
[cos]	+4	−320	+17490	+30	−4440	+447552
[sin]	+5	−328	+16935	+34	−4520	+444864
[cos+sin]	+9	−648	+34425	+64	−8960	+892416
[cos−sin]	−1	+ 8	+ 555	− 4	+ 80	+ 2688

$\frac{1}{2}x^0 \frac{\cos}{\sin} y$	[ ]^4		[ ]^5		[ ]^6	[ ]^7
	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$	$\frac{e^6}{92160}$	$\frac{e^7}{1290240}$
[cos]	+299	− 74352	+3760	−1457120	+ 57262	+1025248
[sin]	+326	− 75648	+4016	−1482208	+ 60387	+1071904
[cos+sin]	+625	−150000	+7776	−2939328	+117649	+2097152
[cos−sin]	− 27	+ 1296	− 256	+ 25088	− 3125	− 46656

where the lines [cos + sin], [cos − sin] are obtained by the addition and subtraction of the lines [cos], [sin].

The lines [cos + sin], [cos − sin], are then rearranged by reading them backwards and forwards from the terms which are in the [cos − sin] line, under [ ]^j; viz., in the present case, where $j = 1$, from the terms −2, 0, 0, 0 under [ ]^1, as follows:—

$x^0 \frac{\cos}{\sin} f$	[ ]^0				[ ]^1			
	$\frac{e}{4}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$	$\frac{e^0}{2}$	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$
$\frac{1}{2}$ [cos+sin]	−2	0	0	0	+1	−8	+42	− 800
$\frac{1}{2}$ [cos−sin]	Do.	„	„	„	0	−1	+ 8	+ 555
[cos]	−4	0	0	0	+1	−9	+50	− 245
[sin]	0	0	0	0	+1	−7	+34	−1355

$x^0 \frac{\cos}{\sin} f$	[ ]^2				[ ]^3			[ ]^4		
	$\frac{e}{4}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$
$\frac{1}{2}$ [cos+sin]	+2	−60	+1360	−31360	+9	−648	+34425	+64	−8960	+892416
$\frac{1}{2}$ [cos−sin]	0	− 4	+ 80	+ 2688	0	− 27	+ 1296	0	− 256	+ 25088
[cos]	+2	−64	+1440	−28672	+9	−675	+35721	+64	−9216	+917504
[sin]	+2	−56	+1280	−24048	+9	−621	+33129	+64	−8704	+867328

$x^0 \frac{\cos}{\sin} f$	$[\ ]^5$		$[\ ]^6$		$[\ ]^7$	$[\ ]^8$
	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$	$\frac{e^6}{92160}$	$\frac{e^7}{1290240}$
$\frac{1}{2}$ [cos+sin]	+625	−150000	+7776	−2939328	+117649	+2097152
$\frac{1}{2}$ [cos−sin]	0	− 3125	0	− 46656	0	0
[cos]	+625	−153125	+7776	−2985984	+117649	+2097152
[sin]	+625	−146875	+7776	−2892672	+117649	+2097152

where the lines [cos], [sin] are obtained by the addition and subtraction of the lines $\frac{1}{2}$ [cos + sin], $\frac{1}{2}$ [cos − sin]; and it then only remains to reduce the fractions, at the same time multiplying by 2 the lines $\frac{1}{2}$ [cos + sin], $\frac{1}{2}$ [cos − sin], to obtain instead the lines [cos + sin], [cos − sin]. We have thus,

$x^0 \frac{\cos}{\sin} f$	$[\ ]^0$				$[\ ]^1$			
	e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
[cos+sin]	−1	0	0	0	+1	−1	$+\frac{7}{64}$	$-\frac{5}{288}$
[cos−sin]	Do.	"	"	"	0	$-\frac{1}{8}$	$+\frac{1}{48}$	$+\frac{37}{3072}$
[cos]	−1	0	0	0	$+\frac{1}{2}$	$-\frac{9}{16}$	$+\frac{25}{384}$	$-\frac{49}{18432}$
[sin]	0	0	0	0	$+\frac{1}{2}$	$-\frac{7}{16}$	$+\frac{17}{384}$	$-\frac{271}{18432}$

$x^0 \frac{\cos}{\sin} f$	$[\ ]^2$				$[\ ]^3$			$[\ ]^4$		
	e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7
[cos+sin]	+1	$-\frac{5}{4}$	$+\frac{17}{48}$	$-\frac{7}{144}$	$+\frac{9}{8}$	$-\frac{27}{16}$	$+\frac{765}{1024}$	$+\frac{4}{3}$	$-\frac{7}{3}$	$+\frac{83}{60}$
[cos−sin]	0	$-\frac{1}{12}$	$+\frac{1}{48}$	$+\frac{1}{240}$	0	$-\frac{9}{128}$	$+\frac{9}{320}$	0	$-\frac{1}{15}$	$+\frac{7}{180}$
[cos]	$+\frac{1}{2}$	$-\frac{2}{3}$	$+\frac{3}{16}$	$-\frac{1}{45}$	$+\frac{9}{16}$	$-\frac{225}{256}$	$+\frac{3969}{10240}$	$+\frac{2}{3}$	$-\frac{6}{5}$	$+\frac{32}{45}$
[sin]	$+\frac{1}{2}$	$-\frac{7}{12}$	$+\frac{1}{6}$	$-\frac{19}{720}$	$+\frac{9}{16}$	$-\frac{207}{256}$	$+\frac{3681}{10240}$	$+\frac{2}{3}$	$-\frac{17}{15}$	$+\frac{121}{180}$

$x^0 \overset{\cos}{\underset{\sin}{}} f$	$[\ ]^5$		$[\ ]^6$		$[\ ]^7$	$[\ ]^8$
	e^4	e^6	e^5	e^7	e^6	e^7
[cos + sin]	$+\frac{625}{384}$	$-\frac{625}{192}$	$+\frac{81}{40}$	$-\frac{5103}{1120}$	$+\frac{117649}{46080}$	$+\frac{1024}{315}$
[cos − sin]	0	$-\frac{625}{9216}$	0	$-\frac{81}{1120}$	0	0
[cos]	$+\frac{625}{768}$	$-\frac{30625}{18432}$	$+\frac{81}{80}$	$-\frac{81}{35}$	$+\frac{117649}{92160}$	$+\frac{512}{315}$
[sin]	$+\frac{625}{768}$	$-\frac{29375}{18432}$	$+\frac{81}{80}$	$-\frac{2511}{1120}$	$+\frac{117649}{92160}$	$+\frac{512}{315}$

which is, in fact, part of the table $(x^0 \ldots x^7) \overset{\cos}{\underset{\sin}{}} f$. It is to be remarked, that when in any column the second term is zero, the third and fourth terms are each equal to one-half of the first term, and that I have in the tables represented them by the letters H, H.

The foregoing method depends, it is easy to see, on the following proposition; viz., writing

$$\overset{m}{x} \cos jf = \Sigma\, [\cos]^{i}_{f} \cos ig, \qquad \overset{m}{x} \sin jf = \Sigma\, [\sin]^{i}_{f} \sin ig,$$

$$\overset{m}{x} \cos jy = \Sigma\, [\cos]^{i}_{y} \cos ig, \qquad \overset{m}{x} \sin jy = \Sigma\, [\sin]^{i}_{y} \sin ig,$$

where, as before, $f = g + y$, then we have

$$[\cos + \sin]^{i}_{f} = [\cos + \sin]^{i-j}_{y},$$

$$[\cos - \sin]^{i}_{f} = [\cos - \sin]^{i+j}_{y},$$

the latter of which follows immediately from the former. To prove this, it is to be remarked that in general

$$\begin{aligned}
\cos \alpha\, \Sigma\, [\cos]^{i} \cos ig &= \Sigma\, [\cos]^{i} \cos (ig + \alpha), \\
\sin \alpha\, \Sigma\, [\sin]^{i} \sin ig &= -\Sigma\, [\sin]^{i} \cos (ig + \alpha), \\
\sin \alpha\, \Sigma\, [\cos]^{i} \cos ig &= \Sigma\, [\cos]^{i} \sin (ig + \alpha), \\
\cos \alpha\, \Sigma\, [\sin]^{i} \sin ig &= \Sigma\, [\sin]^{i} \sin (ig + \alpha);
\end{aligned}$$

whence, putting $\alpha = jg$,

$$\cos jf = \cos(jy + jg) = \Sigma\,[\cos+\sin]_{y}^{i} \cos(i+j)\,g$$

$$= \Sigma\,\tfrac{1}{2}\left\{[\cos+\sin]_{y}^{i-j} + [\cos+\sin]_{y}^{-i-j}\right\} \cos ig,$$

$$\sin jf = \sin(jy + jg) = \Sigma\,[\cos+\sin]_{y}^{i} \sin(i+j)\,g$$

$$= \Sigma\,\tfrac{1}{2}\left\{[\cos+\sin]_{y}^{i-j} - [\cos+\sin]_{y}^{-i-j}\right\} \sin ig;$$

and therefore

$$[\cos]_{f}^{i} = \tfrac{1}{2}\left\{[\cos+\sin]_{y}^{i-j} + [\cos+\sin]_{y}^{-i-j}\right\},$$

$$[\sin]_{f}^{i} = \tfrac{1}{2}\left\{[\cos+\sin]_{y}^{i-j} - [\cos+\sin]_{y}^{-i-j}\right\};$$

and, adding and subtracting, we have the theorem. The coefficients contained in the Datum Tables $(x^{\circ} \ldots x^{7})\ \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, jy$ were taken to be $\tfrac{1}{2}[\cos]_{y}^{i}$, $\tfrac{1}{2}[\sin]_{y}^{i}$, in order that the calculations might in the first instance give $\tfrac{1}{2}[\cos+\sin]_{f}^{i}$, $\tfrac{1}{2}[\cos+\sin]_{f}^{i}$, and thence by an addition and subtraction, without a division by 2, give $[\cos]_{f}^{i}$, $[\sin]_{f}^{i}$.

The resulting values admit of a very simple verification; in fact, writing down the two equations

$$x^{m} \cos jf = \Sigma\,[\cos]^{i} \cos ig, \qquad x^{m} \sin jf = \Sigma\,[\text{in}]^{i} \sin ig,$$

or, what is the same thing, the summation now extending only from $i = 1$ to $i = \infty$,

$$\tfrac{1}{2} x^{m} \cos jf = \tfrac{1}{2}[\cos]^{\circ} + \Sigma'\,[\cos]^{i} \cos ig, \qquad \tfrac{1}{2} x^{m} \sin jf = \Sigma'\,[\sin]^{i} \sin ig,$$

if in the first equation, and in the second equation differentiated with respect to g, we put $g = 0$; then, observing that $g = 0$ gives $f = 0$, $x = -e$, $\frac{df}{dy} = \frac{(1+e)^2}{(1-e^2)^{\frac{3}{2}}}$, we have

$$\tfrac{1}{2}\,[\cos]^{\circ} + \Sigma'\,[\cos]^{i} = \tfrac{1}{2}\,j\,(-e)^{m},$$

$$\Sigma'\,i\,[\sin]^{i} = \tfrac{1}{2}\,j\,(-e)^{m}\left(1+2\,e+\frac{5}{2}\,e^2+3\,e^3+\frac{27}{8}\,e^4+\frac{15}{4}\,e^5+\frac{65}{16}\,e^6+\frac{35}{8}\,e^7+\,..\right).$$

The values of $(x^{\circ}\,.\,.\,.\,x^7)\,{}^{\cos}_{\sin}\,jf$ being known, those of $\left(\frac{r}{a}\right)^{\pm m}\,{}^{\cos}_{\sin}\,jf$ can, of course, be determined by merely substituting for $\frac{r}{a}$ its value $(1+x)$, expanding and reducing; and a convenient mode of arranging the calculations presents itself so obviously, that no explanation is required. The verification is similar to that for $x^{m}\,{}^{\cos}_{\sin}\,jf$. The only difference is, that for $g = 0$ we have the value of r, viz. $1-e$, in the place of the value $-e$ of x. A similar verification is applicable to the Datum Tables.

To make the arrangement of the Tables clear, I have given at the end of them some examples to show how they are to be read off.

Table $(x^0, x^1, x^2, x^3)\,{}^{\cos}_{\sin} f \ldots$

${}^{\cos}_{\sin} f$		$[\ ]^0$				$[\ ]^1$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
x^0	[cos + sin]	-1	0	0	0	$+1$	-1	$+\frac{7}{64}$	$-\frac{5}{288}$
	[cos − sin]	„	„	„	„	0	$-\frac{1}{8}$	$+\frac{1}{48}$	$+\frac{37}{3072}$
	[cos]	-1	0	0	0	H	$-\frac{9}{16}$	$+\frac{25}{384}$	$-\frac{49}{18432}$
	[sin]	0	0	0	0	H	$-\frac{7}{16}$	$+\frac{17}{384}$	$-\frac{271}{18432}$
x^1	[cos + sin]	$-\frac{1}{2}$	0	0	0		$+\frac{1}{2}$	$-\frac{1}{8}$	$-\frac{1}{128}$
	[cos − sin]	„	„	„	„		$+\frac{1}{4}$	$+\frac{1}{48}$	$+\frac{19}{1536}$
	[cos]	$-\frac{1}{2}$	0	0	0		$+\frac{3}{8}$	$-\frac{5}{96}$	$+\frac{7}{3072}$
	[sin]	0	0	0	0		$+\frac{1}{8}$	$-\frac{7}{96}$	$-\frac{31}{3072}$
x^2	[cos + sin]		$-\frac{1}{2}$	0	0		$+\frac{1}{2}$	$-\frac{1}{4}$	$+\frac{5}{384}$
	[cos − sin]		„	„	„		$+\frac{1}{4}$	$-\frac{1}{16}$	$+\frac{5}{512}$
	[cos]		$-\frac{1}{2}$	0	0		$+\frac{3}{8}$	$-\frac{5}{32}$	$+\frac{35}{3072}$
	[sin]		0	0	0		$+\frac{1}{8}$	$-\frac{3}{32}$	$+\frac{5}{3072}$
x^3	[cos + sin]		$-\frac{3}{8}$	0	0			$+\frac{3}{8}$	$-\frac{1}{16}$
	[cos − sin]		„	„	„			$+\frac{1}{4}$	$+\frac{1}{128}$
	[cos]		$-\frac{3}{8}$	0	0			$+\frac{5}{16}$	$-\frac{7}{256}$
	[sin]		0	0	0			$+\frac{1}{16}$	$-\frac{9}{256}$

N.B.—In all the Tables H, H, in the third and fourth lines of a set of four lines, denote one half of the number above them in the first line. The terms [cos−sin]°, denoted by „ „ … are the same as the terms [cos + sin]° immediately above them.

. . . *Table* $(x^0, x^1, x^2, x^3)\ {}^{\cos}_{\sin} f$ . . .

	${}^{\cos}_{\sin} f$	$[\ ]^2$				$[\ ]^3$			$[\ ]^4$		
		e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7
x^0	[cos + sin]	$+1$	$-\frac{5}{4}$	$+\frac{17}{48}$	$-\frac{7}{144}$	$+\frac{9}{8}$	$-\frac{27}{16}$	$+\frac{765}{1024}$	$+\frac{4}{3}$	$-\frac{7}{3}$	$+\frac{83}{60}$
	[cos − sin]	0	$-\frac{1}{12}$	$+\frac{1}{48}$	$+\frac{1}{240}$	0	$-\frac{9}{128}$	$+\frac{9}{320}$	0	$-\frac{1}{15}$	$+\frac{7}{180}$
	[cos]	H	$-\frac{2}{3}$	$+\frac{3}{16}$	$-\frac{1}{45}$	H	$-\frac{225}{256}$	$+\frac{3969}{10240}$	H	$-\frac{6}{5}$	$+\frac{32}{45}$
	[sin]	H	$-\frac{7}{12}$	$+\frac{1}{6}$	$-\frac{19}{720}$	H	$-\frac{207}{256}$	$+\frac{3681}{10240}$	H	$-\frac{17}{15}$	$+\frac{121}{180}$
x^1	[cos + sin]	$-\frac{1}{2}$	$+\frac{7}{8}$	$-\frac{29}{96}$	$+\frac{5}{144}$	$-\frac{3}{4}$	$+\frac{21}{16}$	$-\frac{327}{512}$	-1	$+\frac{23}{12}$	$-\frac{289}{240}$
	[cos − sin]	0	$+\frac{1}{8}$	$-\frac{1}{96}$	$+\frac{1}{240}$	0	$+\frac{3}{32}$	$-\frac{33}{1280}$	0	$+\frac{1}{12}$	$-\frac{29}{720}$
	[cos]	H	$+\frac{1}{2}$	$-\frac{5}{32}$	$+\frac{7}{360}$	H	$+\frac{45}{64}$	$-\frac{1701}{5120}$	H	$+1$	$-\frac{28}{45}$
	[sin]	H	$+\frac{3}{8}$	$-\frac{7}{48}$	$+\frac{11}{720}$	H	$+\frac{39}{64}$	$-\frac{1569}{5120}$	H	$+\frac{11}{12}$	$-\frac{419}{720}$
x^2	[cos + sin]		0	$-\frac{1}{12}$	$+\frac{1}{32}$	$+\frac{1}{4}$	$-\frac{7}{16}$	$+\frac{73}{512}$	$+\frac{1}{2}$	$-\frac{23}{24}$	$+\frac{257}{480}$
	[cos − sin]		0	$-\frac{1}{24}$	$+\frac{1}{480}$	0	$-\frac{1}{32}$	$-\frac{5}{256}$	0	$-\frac{1}{24}$	$-\frac{1}{480}$
	[cos]		0	$-\frac{1}{16}$	$+\frac{1}{60}$	H	$-\frac{15}{64}$	$+\frac{63}{1024}$	H	$-\frac{1}{2}$	$+\frac{4}{15}$
	[sin]		0	$-\frac{1}{48}$	$+\frac{7}{480}$	H	$-\frac{13}{64}$	$+\frac{83}{1024}$	H	$-\frac{11}{24}$	$+\frac{43}{160}$
x^3	[cos + sin]		$-\frac{3}{8}$	$+\frac{7}{16}$	$-\frac{7}{64}$		$-\frac{1}{4}$	$+\frac{51}{128}$	$-\frac{1}{8}$	$+\frac{1}{32}$	$+\frac{33}{128}$
	[cos − sin]		$-\frac{1}{8}$	$+\frac{1}{8}$	$-\frac{1}{64}$		$-\frac{1}{16}$	$+\frac{3}{32}$	0	$-\frac{1}{32}$	$+\frac{29}{384}$
	[cos]		$-\frac{1}{4}$	$+\frac{9}{32}$	$-\frac{1}{16}$		$-\frac{5}{32}$	$+\frac{63}{256}$	H	0	$+\frac{1}{6}$
	[sin]		$-\frac{1}{8}$	$+\frac{5}{32}$	$-\frac{3}{64}$		$-\frac{3}{32}$	$+\frac{39}{256}$	H	$+\frac{1}{32}$	$+\frac{35}{384}$

. . . *Table* $(x^0, x^1, x^2, x^3)\,{\cos \atop \sin} f$.

${\cos \atop \sin} f$		$[\ ]^5$		$[\ ]^6$		$[\ ]^7$	$[\ ]^8$
		e^4	e^6	e^5	e^7	e^6	e^7
x^0	[cos + sin]	$+\frac{625}{384}$	$-\frac{625}{192}$	$+\frac{81}{40}$	$-\frac{5103}{1120}$	$+\frac{117649}{46080}$	$+\frac{1024}{315}$
	[cos − sin]	0	$-\frac{625}{9216}$	0	$-\frac{81}{1120}$	0	0
	[cos]	H	$-\frac{30625}{18432}$	H	$-\frac{81}{35}$	H	H
	[sin]	H	$-\frac{29375}{18432}$	H	$-\frac{2511}{1120}$	H	H
x^1	[cos + sin]	$-\frac{125}{96}$	$+\frac{2125}{768}$	$-\frac{27}{16}$	$+\frac{1269}{320}$	$-\frac{16807}{7680}$	$-\frac{128}{45}$
	[cos − sin]	0	$+\frac{125}{1536}$	0	$+\frac{27}{320}$	0	0
	[cos]	H	$+\frac{4375}{3072}$	H	$+\frac{81}{40}$	H	H
	[sin]	H	$+\frac{1375}{1024}$	H	$+\frac{621}{320}$	H	H
x^2	[cos + sin]	$+\frac{25}{32}$	$-\frac{425}{256}$	$+\frac{9}{8}$	$-\frac{423}{160}$	$+\frac{2401}{1536}$	$+\frac{32}{15}$
	[cos − sin]	0	$-\frac{25}{512}$	0	$-\frac{9}{160}$	0	0
	[cos]	H	$-\frac{875}{1024}$	H	$-\frac{27}{20}$	H	H
	[sin]	H	$-\frac{825}{1024}$	H	$-\frac{207}{160}$	H	H
x^3	[cos + sin]	$-\frac{5}{16}$	$+\frac{15}{32}$	$-\frac{9}{16}$	$+\frac{9}{8}$	$-\frac{343}{384}$	$-\frac{4}{3}$
	[cos − sin]	0	$-\frac{5}{384}$	0	0	0	0
	[cos]	H	$+\frac{175}{768}$	H	H	H	H
	[sin]	H	$+\frac{185}{768}$	H	H	H	H

Table $(x^4, x^5, x^6, x^7)\,{}^{\cos}_{\sin} f\,\ldots$

	${}^{\cos}_{\sin} f$	$[\ ]^0$				$[\ ]^1$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
x^4	[cos + sin]			$-\frac{3}{8}$	0			$+\frac{3}{8}$	$-\frac{1}{8}$
	[cos − sin]			”	”			$+\frac{1}{4}$	$-\frac{5}{128}$
	[cos]			$-\frac{3}{8}$	0			$+\frac{5}{16}$	$-\frac{21}{256}$
	[sin]			0	0			$+\frac{1}{16}$	$-\frac{11}{256}$
x^5	[cos + sin]			$-\frac{5}{16}$	0				$+\frac{5}{16}$
	[cos − sin]			”	”				$+\frac{15}{64}$
	[cos]			$-\frac{5}{16}$	0				$+\frac{35}{128}$
	[sin]			0	0				$+\frac{5}{128}$
x^6	[cos + sin]				$-\frac{5}{16}$				$+\frac{5}{16}$
	[cos − sin]				”				$+\frac{15}{64}$
	[cos]				$-\frac{5}{16}$				$+\frac{35}{128}$
	[sin]				0				$+\frac{5}{128}$
x^7	[cos + sin]				$-\frac{35}{128}$				
	[cos − sin]				”				
	[cos]				$-\frac{35}{128}$				
	[sin]				0				

. . . *Table* $(x^4, x^5, x^6, x^7) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} f$ . . .

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} f$	$[\;]^2$				$[\;]^3$			$[\;]^4$		
		e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7
x^4	[cos + sin]			$-\frac{1}{8}$	$+\frac{1}{64}$		$+\frac{1}{4}$	$-\frac{51}{128}$		$+\frac{5}{16}$	$-\frac{9}{16}$
	[cos − sin]			$-\frac{1}{16}$	$-\frac{1}{64}$		$+\frac{1}{16}$	$-\frac{3}{32}$		$+\frac{1}{16}$	$-\frac{5}{48}$
	[cos]			$-\frac{3}{32}$	0		$+\frac{5}{32}$	$-\frac{63}{256}$		$+\frac{3}{16}$	$-\frac{1}{3}$
	[sin]			$-\frac{1}{32}$	$+\frac{1}{64}$		$+\frac{3}{32}$	$-\frac{39}{256}$		$+\frac{1}{8}$	$-\frac{11}{48}$
x^5	[cos + sin]			$-\frac{5}{16}$	$+\frac{35}{128}$			$-\frac{5}{64}$		$-\frac{5}{32}$	$+\frac{7}{32}$
	[cos − sin]			$-\frac{5}{32}$	$+\frac{13}{128}$			$-\frac{1}{32}$		$-\frac{1}{32}$	$+\frac{1}{32}$
	[cos]			$-\frac{15}{64}$	$+\frac{3}{16}$			$-\frac{7}{128}$		$-\frac{3}{32}$	$+\frac{1}{8}$
	[sin]			$-\frac{5}{64}$	$+\frac{11}{128}$			$-\frac{3}{128}$		$-\frac{1}{16}$	$+\frac{3}{32}$
x^6	[cos + sin]				$-\frac{5}{32}$			$+\frac{15}{64}$			$+\frac{3}{16}$
	[cos − sin]				$-\frac{3}{32}$			$+\frac{3}{32}$			$+\frac{1}{16}$
	[cos]				$-\frac{1}{8}$			$+\frac{21}{128}$			$+\frac{1}{8}$
	[sin]				$-\frac{1}{32}$			$+\frac{9}{128}$			$+\frac{1}{16}$
x^7	[cos + sin]				$-\frac{35}{128}$						$-\frac{21}{128}$
	[cos − sin]				$-\frac{21}{128}$						$-\frac{7}{128}$
	[cos]				$-\frac{7}{32}$						$-\frac{7}{64}$
	[sin]				$-\frac{7}{128}$						$-\frac{7}{128}$

. . . *Table* $(x^4, x^5, x^6, x^7) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} f$.

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} f$	$[\ ]^5$		$[\ ]^6$		$[\ ]^7$	$[\ ]^8$
		e^4	e^6	e^5	e^7	e^6	e^7
x^4	$[\cos + \sin]$	$+\frac{1}{16}$	$+\frac{7}{32}$	$+\frac{3}{16}$	$-\frac{3}{64}$	$+\frac{49}{128}$	$+\frac{2}{3}$
	$[\cos - \sin]$	0	$+\frac{7}{128}$	0	$+\frac{3}{64}$	0	0
	$[\cos]$	H	$+\frac{35}{256}$	H	0	H	H
	$[\sin]$	H	$+\frac{21}{256}$	H	$-\frac{3}{64}$	H	H
x^5	$[\cos + \sin]$		$-\frac{9}{32}$	$-\frac{1}{32}$	$-\frac{41}{128}$	$-\frac{7}{64}$	$-\frac{1}{4}$
	$[\cos - \sin]$		$-\frac{3}{64}$	0	$-\frac{7}{128}$	0	0
	$[\cos]$		$-\frac{21}{128}$	H	$-\frac{3}{16}$	H	H
	$[\sin]$		$-\frac{15}{128}$	H	$-\frac{17}{128}$	H	H
x^6	$[\cos + \sin]$		$+\frac{3}{32}$		$+\frac{7}{32}$	$+\frac{1}{64}$	$+\frac{1}{16}$
	$[\cos - \sin]$		$+\frac{1}{64}$		$+\frac{1}{32}$	0	0
	$[\cos]$		$+\frac{7}{128}$		$+\frac{1}{8}$	H	H
	$[\sin]$		$+\frac{5}{128}$		$+\frac{3}{32}$	H	H
x^7	$[\cos + \sin]$				$-\frac{7}{128}$		$-\frac{1}{128}$
	$[\cos - \sin]$				$-\frac{1}{128}$		0
	$[\cos]$				$-\frac{1}{32}$		H
	$[\sin]$				$-\frac{3}{128}$		H

Table $(x^0, x^1, x^2, x^3) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 2f \ldots$

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 2f$		$[\ ]^0$			$[\ ]^1$				$[\ ]^2$			
		e^2	e^4	e^6	e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
x^0	[cos + sin]	$+\frac{3}{4}$	$+\frac{1}{8}$	$+\frac{3}{64}$	-2	$+\frac{7}{4}$	$-\frac{5}{96}$	$+\frac{271}{4608}$	$+1$	-4	$+\frac{55}{16}$	$-\frac{103}{144}$
	[cos − sin]	"	"	"	0	$+\frac{1}{12}$	$+\frac{5}{192}$	$+\frac{49}{7680}$	0	0	$+\frac{1}{24}$	$+\frac{1}{60}$
	[cos]	$+\frac{3}{4}$	$+\frac{1}{8}$	$+\frac{3}{64}$	H	$+\frac{11}{12}$	$-\frac{5}{384}$	$+\frac{751}{23040}$	H	H	$+\frac{167}{96}$	$-\frac{503}{1440}$
	[sin]	0	0	0	H	$+\frac{5}{6}$	$-\frac{5}{128}$	$+\frac{151}{5760}$	H	H	$+\frac{163}{96}$	$-\frac{527}{1440}$
x	[cos + sin]	$+\frac{3}{4}$	$-\frac{1}{8}$	$-\frac{3}{64}$	$-\frac{1}{2}$	$+\frac{5}{16}$	$-\frac{53}{384}$	$-\frac{203}{18432}$		$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{32}$
	[cos − sin]	"	"	"	0	$-\frac{1}{16}$	$-\frac{61}{768}$	$-\frac{271}{6144}$		0	$-\frac{1}{48}$	$-\frac{17}{480}$
	[cos]	$+\frac{3}{4}$	$-\frac{1}{8}$	$-\frac{3}{64}$	H	$+\frac{1}{8}$	$-\frac{167}{1536}$	$-\frac{127}{4608}$		H	$-\frac{25}{96}$	$-\frac{1}{30}$
	[sin]	0	0	0	H	$+\frac{3}{16}$	$-\frac{15}{512}$	$+\frac{305}{18432}$		H	$-\frac{23}{96}$	$+\frac{1}{480}$
x^2	[cos + sin]	$+\frac{1}{4}$	$+\frac{1}{8}$	$+\frac{3}{64}$		$-\frac{3}{4}$	$+\frac{17}{48}$	$+\frac{19}{512}$		$+\frac{1}{2}$	-1	$+\frac{53}{96}$
	[cos − sin]	"	"	"		$-\frac{1}{4}$	$+\frac{1}{96}$	$+\frac{83}{7680}$		0	$-\frac{1}{16}$	$+\frac{1}{32}$
	[cos]	$+\frac{1}{4}$	$+\frac{1}{8}$	$+\frac{3}{64}$		$-\frac{1}{2}$	$+\frac{35}{192}$	$+\frac{23}{960}$		H	$-\frac{17}{32}$	$+\frac{7}{24}$
	[sin]	0	0	0		$-\frac{1}{4}$	$+\frac{11}{64}$	$+\frac{101}{7680}$		H	$-\frac{15}{32}$	$+\frac{25}{96}$
x^3	[cos + sin]		$+\frac{1}{2}$	$-\frac{3}{64}$		$-\frac{3}{8}$	$+\frac{5}{32}$	$-\frac{49}{1024}$			$+\frac{3}{8}$	$-\frac{1}{4}$
	[cos − sin]		"	"		$-\frac{1}{8}$	$-\frac{1}{64}$	$-\frac{39}{1024}$			$+\frac{1}{16}$	0
	[cos]		$+\frac{1}{2}$	$-\frac{3}{64}$		$-\frac{1}{4}$	$+\frac{9}{128}$	$-\frac{11}{256}$			$+\frac{7}{32}$	$-\frac{1}{8}$
	[sin]		0	0		$-\frac{1}{8}$	$+\frac{11}{128}$	$-\frac{5}{1024}$			$+\frac{5}{32}$	$-\frac{1}{8}$

. . . *Table* $(x^0, x^1, x^2, x^3) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 2f$ . . .

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 2f$	$[\ ]^3$				$[\ ]^4$			$[\ ]^5$		
		e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7
x^0	[cos + sin]	$+2$	$-\frac{27}{4}$	$+\frac{207}{32}$	$-\frac{1147}{512}$	$+\frac{13}{4}$	$-\frac{259}{24}$	$+\frac{559}{48}$	$+\frac{59}{12}$	$-\frac{3221}{192}$	$+\frac{31087}{1536}$
	[cos − sin]	0	0	$+\frac{9}{320}$	$+\frac{27}{2560}$	0	0	$+\frac{1}{45}$	0	0	$+\frac{625}{32256}$
	[cos]	H	H	$+\frac{2079}{640}$	$-\frac{1427}{1280}$	H	H	$+\frac{8401}{1440}$	H	H	$+\frac{163363}{16128}$
	[sin]	H	H	$+\frac{2061}{640}$	$-\frac{25929}{23040}$	H	H	$+\frac{8369}{1440}$	H	H	$+\frac{326101}{32256}$
x	[cos + sin]	$-\frac{1}{2}$	$+\frac{33}{16}$	$-\frac{255}{128}$	$+\frac{1081}{2048}$	$-\frac{5}{4}$	$+\frac{107}{24}$	$-\frac{229}{48}$	$-\frac{37}{16}$	$+\frac{6259}{768}$	$-\frac{59723}{6144}$
	[cos − sin]	0	0	$-\frac{3}{256}$	$-\frac{231}{10240}$	0	0	$-\frac{1}{120}$	0	0	$-\frac{125}{18432}$
	[cos]	H	H	$-\frac{513}{512}$	$+\frac{2587}{10240}$	H	H	$-\frac{1147}{480}$	H	H	$-\frac{89647}{18432}$
	[sin]	H	H	$-\frac{507}{512}$	$+\frac{1409}{5120}$	H	H	$-\frac{381}{160}$	H	H	$-\frac{44761}{9216}$
x^2	[cos + sin]		$+\frac{1}{4}$	$-\frac{13}{16}$	$+\frac{367}{512}$	$+\frac{1}{4}$	$-\frac{5}{8}$	0	$+\frac{3}{4}$	$-\frac{223}{96}$	$+\frac{2987}{1536}$
	[cos − sin]		0	$-\frac{1}{32}$	$+\frac{71}{2560}$	0	0	$-\frac{1}{48}$	0	0	$-\frac{25}{1536}$
	[cos]		H	$-\frac{27}{64}$	$+\frac{953}{2560}$	H	H	$-\frac{1}{96}$	H	H	$+\frac{1481}{1536}$
	[sin]		H	$-\frac{25}{64}$	$+\frac{441}{1280}$	H	H	$+\frac{1}{96}$	H	H	$+\frac{251}{256}$
x^3	[cos + sin]		$-\frac{3}{8}$	$+\frac{33}{32}$	$-\frac{753}{1024}$		$-\frac{1}{2}$	$+\frac{95}{64}$	$-\frac{1}{8}$	$-\frac{13}{64}$	$+\frac{1577}{1024}$
	[cos − sin]		0	$+\frac{3}{64}$	$-\frac{13}{1024}$		0	$+\frac{7}{192}$	0	0	$+\frac{95}{3072}$
	[cos]		H	$+\frac{69}{128}$	$-\frac{383}{1024}$		H	$+\frac{73}{96}$	H	H	$+\frac{2413}{3072}$
	[sin]		H	$+\frac{63}{128}$	$-\frac{185}{512}$		H	$+\frac{139}{192}$	H	H	$+\frac{1159}{1536}$

. . . *Table* $(x^0, x^1, x^2, x^3) \begin{matrix}\cos\\ \sin\end{matrix} 2f$.

	$\begin{matrix}\cos\\ \sin\end{matrix} 2f$	$[\ ]^6$		$[\ ]^7$		$[\ ]^8$	$[\ ]^9$
		e^4	e^6	e^5	e^7	e^6	e^7
x^0	[cos + sin]	$+\frac{115}{16}$	$-\frac{2049}{80}$	$+\frac{9893}{960}$	$-\frac{889303}{23040}$	$+\frac{42037}{2880}$	$+\frac{367439}{17920}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x	[cos + sin]	$-\frac{61}{16}$	$+\frac{69}{5}$	$-\frac{4553}{768}$	$+\frac{2061389}{92160}$	$-\frac{8551}{960}$	$-\frac{26809}{2048}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^2	[cos + sin]	$+\frac{25}{16}$	$-\frac{21}{4}$	$+\frac{269}{96}$	$-\frac{77143}{7680}$	$+\frac{297}{64}$	$+\frac{18749}{2560}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^3	[cos + sin]	$-\frac{7}{16}$	$+\frac{3}{4}$	$-\frac{65}{64}$	$+\frac{8425}{3072}$	$-\frac{379}{192}$	$-\frac{3563}{1024}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H

Table $(x^4, x^5, x^6, x^7) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}{}' 2f$ . . .

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 2f$		$[\ ]^0$			$[\ ]^1$				$[\ ]^2$			
		e^2	e^4	e^6	e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
x^4	[cos + sin]		$+\frac{1}{4}$	$+\frac{3}{64}$			$-\frac{1}{2}$	$+\frac{5}{32}$			$+\frac{3}{8}$	$-\frac{1}{2}$
	[cos − sin]		”	”			$-\frac{1}{4}$	0			$+\frac{1}{16}$	$-\frac{1}{16}$
	[cos]		$+\frac{1}{4}$	$+\frac{3}{64}$			$-\frac{3}{8}$	$+\frac{5}{64}$			$+\frac{7}{32}$	$-\frac{9}{32}$
	[sin]		0	0			$-\frac{1}{8}$	$+\frac{5}{64}$			$+\frac{5}{32}$	$-\frac{7}{32}$
x^5	[cos + sin]			$+\frac{25}{64}$			$-\frac{5}{16}$	$+\frac{25}{256}$				$+\frac{5}{16}$
	[cos − sin]			”			$-\frac{5}{32}$	$-\frac{1}{256}$				$+\frac{3}{32}$
	[cos]			$+\frac{25}{64}$			$-\frac{15}{64}$	$+\frac{3}{64}$				$+\frac{13}{64}$
	[sin]			0			$-\frac{5}{64}$	$+\frac{13}{256}$				$+\frac{7}{64}$
x^6	[cos + sin]			$+\frac{15}{64}$				$-\frac{25}{64}$				$+\frac{5}{16}$
	[cos − sin]			”				$-\frac{15}{64}$				$+\frac{3}{32}$
	[cos]			$+\frac{15}{64}$				$-\frac{5}{16}$				$+\frac{13}{64}$
	[sin]			0				$-\frac{5}{64}$				$+\frac{7}{64}$
x^7	[cos + sin]							$-\frac{35}{128}$				
	[cos − sin]							$-\frac{21}{128}$				
	[cos]							$-\frac{7}{32}$				
	[sin]							$-\frac{7}{128}$				

. . . *Table* $(x^4, x^5, x^6, x^7)\,{\cos \atop \sin}\,2f$. . .

${\cos \atop \sin}\,2f$		$[\ ]^3$				$[\ ]^4$			$[\ ]^5$		
		e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7
x^4	[cos + sin]			0	$-\frac{3}{16}$		$+\frac{1}{4}$	$-\frac{43}{64}$		$+\frac{1}{2}$	$-\frac{47}{32}$
	[cos − sin]			0	$-\frac{1}{32}$		0	$-\frac{1}{64}$		0	$-\frac{1}{48}$
	[cos]			0	$-\frac{7}{64}$		H	$-\frac{11}{32}$		H	$-\frac{143}{192}$
	[sin]			0	$-\frac{5}{64}$		H	$-\frac{21}{64}$		H	$-\frac{139}{192}$
x^5	[cos + sin]			$-\frac{5}{16}$	$+\frac{165}{256}$			$-\frac{15}{64}$		$-\frac{5}{32}$	$+\frac{59}{256}$
	[cos − sin]			$-\frac{1}{32}$	$+\frac{17}{256}$			$-\frac{1}{64}$		0	$-\frac{1}{256}$
	[cos]			$-\frac{11}{64}$	$+\frac{91}{256}$			$-\frac{1}{8}$		H	$+\frac{29}{256}$
	[sin]			$-\frac{9}{64}$	$+\frac{37}{128}$			$-\frac{7}{64}$		H	$+\frac{15}{128}$
x^6	[cos + sin]				$-\frac{5}{64}$			$+\frac{15}{64}$			$+\frac{21}{64}$
	[cos − sin]				$-\frac{1}{64}$			$+\frac{1}{64}$			$+\frac{1}{64}$
	[cos]				$-\frac{3}{64}$			$+\frac{1}{8}$			$+\frac{11}{64}$
	[sin]				$-\frac{1}{32}$			$+\frac{7}{64}$			$+\frac{5}{32}$
x^7	[cos + sin]				$-\frac{35}{128}$						$-\frac{21}{128}$
	[cos − sin]				$-\frac{7}{128}$						$-\frac{1}{128}$
	[cos]				$-\frac{21}{128}$						$-\frac{11}{128}$
	[sin]				$-\frac{7}{64}$						$-\frac{5}{64}$

. . . *Table* $(x^4, x^5, x^6, x^7) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 2f$.

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 2f$	[ ]^6		[ ]^7		[ ]^8	[ ]^9
		e^4	e^6	e^5	e^7	e^6	e^7
x^4	[cos + sin]	$+\frac{1}{16}$	$+\frac{9}{16}$	$+\frac{1}{4}$	$+\frac{5}{24}$	$+\frac{41}{64}$	$+\frac{43}{32}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^5	[cos + sin]		$-\frac{13}{32}$	$-\frac{1}{32}$	$-\frac{163}{256}$	$-\frac{9}{64}$	$-\frac{101}{256}$
	[cos − sin]		0	0	0	0	0
	[cos]		H	H	H	H	H
	[sin]		H	H	H	H	H
x^6	[cos + sin]		$+\frac{3}{32}$		$+\frac{19}{64}$	$+\frac{1}{64}$	$+\frac{5}{64}$
	[cos − sin]		0		0	0	0
	[cos]		H		H	H	H
	[sin]		H		H	H	H
x^7	[cos + sin]				$-\frac{7}{128}$		$-\frac{1}{128}$
	[cos − sin]				0		0
	[cos]				H		H
	[sin]				H		H

Table $(x^0, x^1, x^2, x^3)\ \begin{matrix}\cos\\ \sin\end{matrix}\ 3f\ldots$

$\begin{matrix}\cos\\ \sin\end{matrix}\ 3f$		$[\ ]^0$			$[\ ]^1$			$[\ ]^2$			
		e^3	e^5	e^7	e^2	e^4	e^6	e	e^3	e^5	e^7
x^0	[cos + sin]	$-\frac{1}{2}$	$-\frac{3}{16}$	$-\frac{3}{32}$	$+\frac{21}{8}$	$-\frac{31}{16}$	$-\frac{103}{1024}$	-3	$+\frac{33}{4}$	$-\frac{89}{16}$	$+\frac{163}{192}$
	[cos − sin]	"	"	"	o	$-\frac{5}{128}$	$-\frac{11}{320}$	o	o	$-\frac{1}{80}$	$-\frac{17}{960}$
	[cos]	$-\frac{1}{2}$	$-\frac{3}{16}$	$-\frac{3}{32}$	H	$-\frac{253}{256}$	$-\frac{691}{10240}$	H	H	$-\frac{223}{80}$	$+\frac{133}{320}$
	[sin]	o	o	o	H	$-\frac{243}{256}$	$-\frac{339}{10240}$	H	H	$-\frac{111}{40}$	$+\frac{13}{30}$
x^1	[cos + sin]	$-\frac{3}{4}$	$+\frac{1}{32}$	$+\frac{3}{64}$	$+\frac{5}{4}$	$-\frac{61}{48}$	$+\frac{129}{512}$	$-\frac{1}{2}$	$+\frac{9}{8}$	$-\frac{125}{96}$	$+\frac{709}{1152}$
	[cos − sin]	"	"	"	o	$-\frac{1}{96}$	$+\frac{59}{1280}$	o	o	$-\frac{1}{96}$	$+\frac{89}{5760}$
	[cos]	$-\frac{3}{4}$	$+\frac{1}{32}$	$+\frac{3}{64}$	H	$-\frac{41}{64}$	$+\frac{763}{5120}$	H	H	$-\frac{21}{32}$	$+\frac{1817}{5760}$
	[sin]	o	o	o	H	$-\frac{121}{192}$	$+\frac{527}{5120}$	H	H	$-\frac{31}{48}$	$+\frac{3}{10}$
x^2	[cos + sin]	$-\frac{1}{2}$	$+\frac{1}{8}$	o	$+\frac{1}{4}$	$+\frac{5}{16}$	$-\frac{85}{1536}$		-1	$+\frac{19}{12}$	$-\frac{49}{96}$
	[cos − sin]	"	"	"	o	$+\frac{5}{32}$	$+\frac{53}{768}$		o	$+\frac{1}{24}$	$+\frac{7}{480}$
	[cos]	$-\frac{1}{2}$	$+\frac{1}{8}$	o	H	$+\frac{15}{64}$	$+\frac{7}{1024}$		H	$+\frac{13}{16}$	$-\frac{119}{480}$
	[sin]	o	o	o	H	$+\frac{5}{64}$	$-\frac{191}{3072}$		H	$+\frac{37}{48}$	$-\frac{21}{80}$
x^3	[cos + sin]	$-\frac{1}{8}$	$-\frac{9}{32}$	$-\frac{3}{64}$		$+\frac{3}{4}$	$-\frac{65}{128}$		$-\frac{3}{8}$	$+\frac{9}{16}$	$-\frac{61}{128}$
	[cos − sin]	"	"	"		$+\frac{3}{16}$	$-\frac{1}{32}$		o	o	$-\frac{5}{128}$
	[cos]	$-\frac{1}{8}$	$-\frac{9}{32}$	$-\frac{3}{64}$		$+\frac{15}{32}$	$-\frac{69}{256}$		H	H	$-\frac{33}{128}$
	[sin]	o	o	o		$+\frac{9}{32}$	$-\frac{61}{256}$		H	H	$-\frac{7}{32}$

. . . *Table* $(x^0, x^1, x^2, x^3)\ {\cos \atop \sin}\ 3f$ . . .

	${\cos \atop \sin}\ 3f$	$[\ ]^3$				$[\ ]^4$				$[\ ]^5$		
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7	e^2	e^4	e^6
x^0	[cos+sin]	$+1$	-9	$+\frac{1215}{64}$	$-\frac{449}{32}$	$+3$	$-\frac{39}{2}$	$+\frac{155}{4}$	$-\frac{767}{24}$	$+\frac{51}{8}$	$-\frac{593}{16}$	$+\frac{75643}{1024}$
	[cos−sin]	o	o	o	$-\frac{27}{5120}$	o	o	o	$-\frac{1}{420}$	o	o	o
	[cos]	H	H	H	$-\frac{71867}{10240}$	H	H	H	$-\frac{8949}{560}$	H	H	H
	[sin]	H	H	H	$-\frac{71813}{10240}$	H	H	H	$-\frac{26843}{1680}$	H	H	H
x^1	[cos+sin]		$+\frac{1}{2}$	$-\frac{9}{8}$	$-\frac{9}{128}$	$-\frac{1}{2}$	$+\frac{15}{4}$	$-\frac{173}{24}$	$+\frac{161}{36}$	$-\frac{7}{4}$	$+\frac{509}{48}$	$-\frac{10439}{512}$
	[cos−sin]		o	o	$-\frac{21}{2560}$	o	o	o	$-\frac{1}{144}$	o	o	o
	[cos]		H	H	$-\frac{201}{5120}$	H	H	H	$+\frac{643}{288}$	H	H	H
	[sin]		H	H	$-\frac{159}{5120}$	H	H	H	$+\frac{215}{96}$	H	H	H
x^2	[cos+sin]		$+\frac{1}{2}$	$-\frac{9}{4}$	$+\frac{399}{128}$		$+\frac{1}{2}$	$-\frac{8}{3}$	$+\frac{437}{96}$	$+\frac{1}{4}$	$-\frac{13}{16}$	$-\frac{1393}{1536}$
	[cos−sin]		o	o	$+\frac{11}{512}$		o	o	$+\frac{7}{480}$	o	o	o
	[cos]		H	H	$+\frac{1607}{1024}$		H	H	$+\frac{137}{60}$	H	H	H
	[sin]		H	H	$+\frac{1585}{1024}$		H	H	$+\frac{363}{160}$	H	H	H
x^3	[cos+sin]			$+\frac{3}{8}$	$-\frac{9}{16}$		$-\frac{3}{8}$	$+\frac{15}{8}$	$-\frac{343}{128}$		$-\frac{3}{4}$	$+\frac{469}{128}$
	[cos−sin]			o	$-\frac{1}{128}$		o	o	$-\frac{1}{128}$		o	o
	[cos]			H	$-\frac{73}{256}$		H	H	$-\frac{43}{32}$		H	H
	[sin]			H	$-\frac{71}{256}$		H	H	$-\frac{171}{128}$		H	H

. . . *Table* $(x^0, x^1, x^2, x^3)\ {\cos \atop \sin}\ 3f.$

	${\cos \atop \sin}\ 3f$	$[\ ]^6$			$[\ ]^7$		$[\ ]^8$		$[\ ]^9$	$[\ ]^{10}$
		e^3	e^5	e^7	e^4	e^6	e^6	e^7	e^6	e^7
x^0	[cos+sin]	$+\frac{47}{4}$	$-\frac{525}{8}$	$+\frac{43041}{320}$	$+\frac{2567}{128}$	$-\frac{35563}{320}$	$+\frac{2611}{80}$	$-\frac{87599}{480}$	$+\frac{263351}{5120}$	$+\frac{106469}{1344}$
	[cos−sin]	0	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H	H
x^1	[cos+sin]	$-\frac{33}{8}$	$+\frac{373}{16}$	$-\frac{5919}{128}$	$-\frac{787}{96}$	$+\frac{58143}{1280}$	$-\frac{1423}{96}$	$+\frac{237373}{2880}$	$-\frac{64653}{2560}$	$-\frac{47603}{1152}$
	[cos−sin]	0	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H	H
x^2	[cos+sin]	$+1$	$-\frac{37}{8}$	$+\frac{891}{160}$	$+\frac{83}{32}$	$-\frac{9871}{768}$	$+\frac{133}{24}$	$-\frac{857}{30}$	$+\frac{5431}{512}$	$+\frac{1817}{96}$
	[cos−sin]	0	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H	H
x^3	[cos+sin]	$-\frac{1}{8}$	$-\frac{9}{16}$	$+\frac{627}{128}$	$-\frac{9}{16}$	$+\frac{35}{32}$	$-\frac{51}{32}$	$+\frac{367}{64}$	$-\frac{467}{128}$	$-\frac{945}{128}$
	[cos−sin]	0	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H	H

Table $(x^4, x^5, x^6, x^7) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 3f \ldots$

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 3f$		[ ]°			[ ]¹			[ ]²			
		e^3	e^5	e^7	e^2	e^4	e^6	e	e^3	e^5	e^7
x^4	[cos + sin]		$-\frac{7}{16}$	$+\frac{3}{32}$		$+\frac{1}{4}$	$+\frac{9}{128}$			$-\frac{5}{8}$	$+\frac{43}{64}$
	[cos − sin]		”	”		$+\frac{1}{16}$	$+\frac{3}{32}$			$-\frac{1}{16}$	$+\frac{5}{192}$
	[cos]		$-\frac{7}{16}$	$+\frac{3}{32}$		$+\frac{5}{32}$	$+\frac{21}{256}$			$-\frac{11}{32}$	$+\frac{67}{192}$
	[sin]		o	o		$+\frac{3}{32}$	$-\frac{3}{256}$			$-\frac{9}{32}$	$+\frac{31}{96}$
x^5	[cos + sin]		$-\frac{5}{32}$	$-\frac{9}{64}$			$+\frac{35}{64}$			$-\frac{5}{16}$	$+\frac{45}{128}$
	[cos − sin]		”	”			$+\frac{7}{32}$			$-\frac{1}{32}$	$+\frac{1}{128}$
	[cos]		$-\frac{5}{32}$	$-\frac{9}{64}$			$+\frac{49}{128}$			$-\frac{11}{64}$	$+\frac{23}{128}$
	[sin]		o	o			$+\frac{21}{128}$			$-\frac{9}{64}$	$+\frac{11}{64}$
x^6	[cos + sin]			$-\frac{3}{8}$			$+\frac{15}{64}$				$-\frac{15}{32}$
	[cos − sin]			”			$+\frac{3}{32}$				$-\frac{3}{32}$
	[cos]			$-\frac{3}{8}$			$+\frac{21}{128}$				$-\frac{9}{32}$
	[sin]			o			$+\frac{9}{128}$				$-\frac{3}{16}$
x^7	[cos + sin]			$-\frac{21}{128}$							$-\frac{35}{128}$
	[cos − sin]			”							$-\frac{7}{128}$
	[cos]			$-\frac{21}{128}$							$-\frac{21}{128}$
	[sin]			o							$-\frac{7}{64}$

. . . *Table* $(x^4, x^5, x^6, x^7)\,{}^{\cos}_{\sin}\,3f$ . . .

${}^{\cos}_{\sin}\,3f$		$[\ ]^3$				$[\ ]^4$				$[\ ]^5$		
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7	e^2	e^4	e^6
x^4	[cos + sin]			$+\frac{3}{8}$	$-\frac{9}{8}$			$+\frac{1}{8}$	$-\frac{25}{32}$		$+\frac{1}{4}$	$-\frac{129}{128}$
	[cos − sin]			0	$-\frac{3}{128}$			0	$-\frac{1}{96}$		0	0
	[cos]			H	$-\frac{147}{256}$			H	$-\frac{19}{48}$		H	H
	[sin]			H	$-\frac{141}{256}$			H	$-\frac{37}{96}$		H	H
x^5	[cos + sin]				$+\frac{5}{16}$			$-\frac{5}{16}$	$+\frac{75}{64}$			$-\frac{25}{64}$
	[cos − sin]				$+\frac{1}{64}$			0	$+\frac{1}{64}$			0
	[cos]				$+\frac{21}{128}$			H	$+\frac{19}{32}$			H
	[sin]				$+\frac{19}{128}$			H	$+\frac{37}{64}$			H
x^6	[cos + sin]				$+\frac{5}{16}$				0			$+\frac{15}{64}$
	[cos − sin]				$+\frac{1}{64}$				0			0
	[cos]				$+\frac{21}{128}$				0			H
	[sin]				$+\frac{19}{128}$				0			H
x^7	[cos + sin]								$-\frac{35}{128}$			
	[cos − sin]								$-\frac{1}{128}$			
	[cos]								$-\frac{9}{64}$			
	[sin]								$-\frac{17}{128}$			

. . . *Table* $(x^4, x^5, x^6, x^7) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 3f.$

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 3f$		$[\ ]^6$			$[\ ]^7$		$[\ ]^8$		$[\ ]^9$	$[\ ]^{10}$
		e^3	e^5	e^7	e^4	e^6	e^5	e^7	e^6	e^7
x^4	[cos + sin]		$+\frac{11}{16}$	$-\frac{195}{64}$	$+\frac{1}{16}$	$+\frac{33}{32}$	$+\frac{5}{16}$	$+\frac{67}{96}$	$+\frac{123}{128}$	$+\frac{451}{192}$
	[cos − sin]		0	0	0	0	0	0	0	0
	[cos]		H	H	H	H	H	H	H	H
	[sin]		H	H	H	H	H	H	H	H
x^5	[cos + sin]		$-\frac{5}{32}$	$+\frac{27}{128}$		$-\frac{17}{32}$	$-\frac{1}{32}$	$-\frac{67}{64}$	$-\frac{11}{64}$	$-\frac{73}{128}$
	[cos − sin]		0	0		0	0	0	0	0
	[cos]		H	H		H	H	H	H	H
	[sin]		H	H		H	H	H	H	H
x^6	[cos + sin]			$+\frac{15}{32}$		$+\frac{3}{32}$		$+\frac{3}{8}$	$+\frac{1}{64}$	$+\frac{3}{32}$
	[cos − sin]			0		0		0	0	0
	[cos]			H		H		H	H	H
	[sin]			H		H		H	H	H
x^7	[cos + sin]			$-\frac{21}{128}$				$-\frac{7}{128}$		$-\frac{1}{128}$
	[cos − sin]			0				0		0
	[cos]			H				H		H
	[sin]			H				H		H

Table $(x^0, x^1, x^2, x^3)\ {\cos \atop \sin} 4f \ldots$

${\cos \atop \sin} 4f$		$[\ ]^0$		$[\ ]^1$				$[\ ]^2$		
		e^4	e^6	e	e^3	e^5	e^7	e^2	e^4	e^6
x^0	[cos + sin]	$+\frac{5}{16}$	$+\frac{3}{16}$		$-\frac{17}{6}$	$+\frac{161}{96}$	$+\frac{907}{3840}$	$+\frac{11}{2}$	$-\frac{149}{12}$	$+\frac{325}{48}$
	[cos − sin]	,,	,,		0	$+\frac{7}{480}$	$+\frac{277}{11520}$	0	0	$+\frac{1}{720}$
	[cos]	$+\frac{5}{16}$	$+\frac{3}{16}$		H	$+\frac{203}{240}$	$+\frac{1499}{11520}$	H	H	$+\frac{1219}{360}$
	[sin]	0	0		H	$+\frac{133}{160}$	$+\frac{611}{5760}$	H	H	$+\frac{2437}{720}$
x^1	[cos + sin]	$+\frac{5}{8}$	$+\frac{3}{32}$		$-\frac{31}{16}$	$+\frac{1447}{768}$	$-\frac{4537}{30720}$	$+\frac{7}{4}$	$-\frac{53}{12}$	$+\frac{743}{192}$
	[cos − sin]	,,	,,		0	$+\frac{19}{768}$	$-\frac{793}{92160}$	0	0	$+\frac{11}{960}$
	[cos]	$+\frac{5}{8}$	$+\frac{3}{32}$		H	$+\frac{733}{768}$	$-\frac{3601}{46080}$	H	H	$+\frac{621}{320}$
	[sin]	0	0		H	$+\frac{119}{128}$	$-\frac{6409}{92160}$	H	H	$+\frac{463}{240}$
x^2	[cos + sin]	$+\frac{5}{8}$	$-\frac{5}{32}$		$-\frac{3}{4}$	$+\frac{53}{96}$	$-\frac{2123}{7680}$	$+\frac{1}{4}$	$+\frac{1}{2}$	$-\frac{49}{64}$
	[cos − sin]	,,	,,		0	$-\frac{7}{96}$	$-\frac{649}{7680}$	0	0	$-\frac{1}{64}$
	[cos]	$+\frac{5}{8}$	$-\frac{5}{32}$		H	$+\frac{23}{96}$	$-\frac{231}{1280}$	H	H	$-\frac{25}{64}$
	[sin]	0	0		H	$+\frac{5}{16}$	$-\frac{737}{7680}$	H	H	$-\frac{3}{8}$
x^3	[cos + sin]	$+\frac{5}{16}$	0		$-\frac{1}{8}$	$-\frac{43}{64}$	$+\frac{389}{1024}$		$+1$	$-\frac{7}{4}$
	[cos − sin]	,,	,,		0	$-\frac{11}{64}$	$-\frac{59}{3072}$		0	$-\frac{1}{48}$
	[cos]	$+\frac{5}{16}$	0		H	$-\frac{27}{64}$	$+\frac{277}{1536}$		H	$-\frac{85}{96}$
	[sin]	0	0		H	$-\frac{1}{4}$	$+\frac{613}{3072}$		H	$-\frac{83}{96}$

. . . *Table* $(x^0, x^1, x^2, x^3)\,{\cos \atop \sin}\,4f$ . . .

	${\cos \atop \sin}\,4f$	$[\ ]^3$				$[\ [^4$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
x^0	[cos + sin]	-4	$+\frac{45}{2}$	$-\frac{591}{16}$	$+\frac{5737}{256}$	$+1$	-16	$+\frac{247}{4}$	$-\frac{3355}{36}$
	[cos − sin]	0	0	0	$-\frac{9}{8960}$	0	0	0	0
	[cos]	H	H	H	$+\frac{100393}{8960}$	H	H	H	H
	[sin]	H	H	H	$+\frac{50201}{4480}$	H	H	H	H
x^1	[cos + sin]	$-\frac{1}{2}$	$+\frac{39}{16}$	$-\frac{687}{128}$	$+\frac{12355}{2048}$		$+\frac{1}{2}$	-2	$-\frac{1}{8}$
	[cos − sin]	0	0	0	$+\frac{69}{10240}$		0	0	0
	[cos]	H	H	H	$+\frac{15461}{5120}$		H	H	H
	[sin]	H	H	H	$+\frac{30853}{10240}$		H	H	H
x^2	[cos + sin]		$-\frac{5}{4}$	$+\frac{67}{16}$	$-\frac{2119}{512}$		$+\frac{1}{2}$	-4	$+\frac{245}{24}$
	[cos − sin]		0	0	$-\frac{17}{2560}$		0	0	0
	[cos]		H	H	$-\frac{2653}{1280}$		H	H	H
	[sin]		H	H	$-\frac{5289}{2560}$		H	H	H
x^3	[cos + sin]		$-\frac{3}{8}$	$+\frac{39}{32}$	$-\frac{2037}{1024}$			$+\frac{3}{8}$	-1
	[cos − sin]		0	0	$-\frac{7}{1024}$			0	0
	[cos]		H	H	$-\frac{511}{512}$			H	H
	[sin]		H	H	$-\frac{1015}{1024}$			H	H

. . . *Table* $(x^0, x^1, x^2, x^3)\ {\cos \atop \sin} 4f$ . . .

	${\cos \atop \sin} 4f$	$[\ ]^5$				$[\ ]^6$			$[\ ]^7$		
		e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7
x^0	[cos + sin]	$+4$	$-\frac{85}{2}$	$+\frac{6845}{48}$	$-\frac{490585}{2304}$	$+\frac{21}{2}$	$-\frac{377}{4}$	$+\frac{597}{2}$	$+\frac{137}{6}$	$-\frac{18113}{96}$	$+\frac{2249741}{3840}$
	[cos − sin]	0	0	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H	H	H
x^1	[cos + sin]	$-\frac{1}{2}$	$+\frac{95}{16}$	$-\frac{7325}{384}$	$+\frac{407635}{18432}$	$-\frac{9}{4}$	$+\frac{83}{4}$	$-\frac{4029}{64}$	$-\frac{103}{16}$	$+\frac{40967}{768}$	$-\frac{4894577}{30720}$
	[cos − sin]	0	0	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H	H	H
x^2	[cos + sin]		$+\frac{3}{4}$	$-\frac{295}{48}$	$+\frac{8785}{512}$	$+\frac{1}{4}$	-1	$-\frac{215}{64}$	$+\frac{5}{4}$	$-\frac{779}{96}$	$+\frac{103901}{7680}$
	[cos − sin]		0	0	0	0	0	0	0	0	0
	[cos]		H	H	H	H	H	H	H	H	H
	[sin]		H	H	H	H	H	H	H	H	H
x^3	[cos + sin]		$-\frac{3}{8}$	$+\frac{95}{32}$	$-\frac{7285}{1024}$		-1	$+\frac{117}{16}$	$-\frac{1}{8}$	$-\frac{67}{64}$	$+\frac{12069}{1024}$
	[cos − sin]		0	0	0		0	0	0	0	0
	[cos]		H	H	H		H	H	H	H	H
	[sin]		H	H	H		H	H	H	H	H

. . . *Table* $(x^0, x^1, x^2, x^3) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 4f$.

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 4f$	$[\ ]^8$		$[\ ]^9$		$[\ ]^{10}$	$[\ ]^{11}$
		e^4	e^6	e^5	e^7	e^6	e^7
x^0	[cos + sin]	$+\frac{2141}{48}$	$-\frac{84857}{240}$	$+\frac{13011}{160}$	$-\frac{809813}{1280}$	$+\frac{5087}{36}$	$+\frac{19116073}{80640}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^1	[cos + sin]	$-\frac{359}{24}$	$+\frac{56527}{480}$	$-\frac{7887}{256}$	$+\frac{2427367}{10240}$	$-\frac{11263}{192}$	$-\frac{9742939}{92160}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^2	[cos + sin]	$+\frac{31}{8}$	$-\frac{855}{32}$	$+\frac{307}{32}$	$-\frac{172907}{2560}$	$+\frac{3995}{192}$	$+\frac{317653}{7680}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^3	[cos + sin]	$-\frac{11}{16}$	$+\frac{3}{2}$	$-\frac{147}{64}$	$+\frac{10999}{1024}$	$-\frac{145}{24}$	$-\frac{42325}{3072}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	0	H	H	H

Table $(x^4, x^5, x^6, x^7) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 4f \ldots$

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 4f$	$[\ ]^0$		$[\ ]^1$			$[\ ]^2$			$[\ ]^3$			
		e^4	e^6	e^3	e^5	e^7	e^2	e^4	e^6	e^1	e^3	e^5	e^7
x^4	[cos + sin]	$+\frac{1}{16}$	$+\frac{3}{8}$		$-\frac{5}{8}$	$+\frac{29}{64}$		$+\frac{1}{4}$	$+\frac{1}{32}$			$-\frac{3}{4}$	$+\frac{111}{64}$
	[cos − sin]	"	"		$-\frac{1}{8}$	$+\frac{1}{192}$		0	$+\frac{1}{32}$			0	$+\frac{1}{64}$
	[cos]	$+\frac{1}{16}$	$+\frac{3}{8}$		$-\frac{3}{8}$	$+\frac{11}{48}$		H	$+\frac{1}{32}$			H	$+\frac{7}{8}$
	[sin]	0	0		$-\frac{1}{4}$	$+\frac{43}{192}$		H	0			H	$+\frac{55}{64}$
x^5	[cos + sin]		$+\frac{11}{32}$		$-\frac{5}{32}$	$-\frac{79}{256}$			$+\frac{45}{64}$			$-\frac{5}{16}$	$+\frac{195}{256}$
	[cos − sin]		"		$-\frac{1}{32}$	$-\frac{37}{256}$			$+\frac{3}{64}$			0	$+\frac{1}{256}$
	[cos]		$+\frac{11}{32}$		$-\frac{3}{32}$	$-\frac{29}{128}$			$+\frac{3}{8}$			H	$+\frac{49}{128}$
	[sin]		0		$-\frac{1}{16}$	$-\frac{21}{256}$			$+\frac{21}{64}$			H	$+\frac{97}{256}$
x^6	[cos + sin]		$+\frac{3}{32}$			$-\frac{33}{64}$			$+\frac{15}{64}$				$-\frac{35}{64}$
	[cos − sin]		"			$-\frac{11}{64}$			$+\frac{1}{64}$				$-\frac{1}{64}$
	[cos]		$+\frac{3}{32}$			$-\frac{11}{32}$			$+\frac{1}{8}$				$-\frac{9}{32}$
	[sin]		0			$-\frac{11}{64}$			$+\frac{7}{64}$				$-\frac{17}{64}$
x^7	[cos + sin]					$-\frac{21}{128}$							$-\frac{35}{128}$
	[cos − sin]					$-\frac{7}{128}$							$-\frac{1}{128}$
	[cos]					$-\frac{7}{64}$							$-\frac{9}{64}$
	[sin]					$-\frac{7}{128}$							$-\frac{17}{128}$

. . . *Table* $(x^4, x^5, x^6, x^7)\,\genfrac{}{}{0pt}{}{\cos}{\sin}\,4f$ . . .

$\genfrac{}{}{0pt}{}{\cos}{\sin}\,4f$		$[\ ]^4$				$[\ ]^5$				$[\ ]^6$		
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7	e^2	e^4	e^6
x^4	[cos + sin]			$+\frac{3}{8}$	-2			$+\frac{1}{4}$	$-\frac{125}{64}$		$+\frac{1}{4}$	$-\frac{45}{32}$
	[cos − sin]			0	0			0	0		0	0
	[cos]			H	H			H	H		H	H
	[sin]			H	H			H	H		H	H
x^5	[cos + sin]				$+\frac{5}{16}$			$-\frac{5}{16}$	$+\frac{475}{256}$			$-\frac{35}{64}$
	[cos − sin]				0			0	0			0
	[cos]				H			H	H			H
	[sin]				H			H	H			H
x^6	[cos + sin]				$+\frac{5}{16}$				$+\frac{5}{64}$			$+\frac{15}{64}$
	[cos − sin]				0				0			0
	[cos]				H				H			H
	[sin]				H				H			H
x^7	[cos + sin]								$-\frac{35}{128}$			
	[cos − sin]								0			
	[cos]								H			
	[sin]								H			

. . . *Table* $(x^4, x^5, x^6, x^7)\genfrac{}{}{0pt}{}{\cos}{\sin}4f$.

$\genfrac{}{}{0pt}{}{\cos}{\sin}4f$		$[\ ]^7$			$[\ ]^8$		$[\ ]^9$		$[\ ]^{10}$	$[\ ]^{11}$
		e^3	e^5	e^7	e^4	e^6	e^5	e^7	e^6	e^7
x^4	[cos + sin]		$+\frac{7}{8}$	$-\frac{351}{64}$	$+\frac{1}{16}$	$+\frac{13}{8}$	$+\frac{3}{8}$	$+\frac{95}{64}$	$+\frac{43}{32}$	$+\frac{719}{192}$
	[cos − sin]		o	o	o	o	o	o	o	o
	[cos]		H	H	H	H	H	H	H	H
	[sin]		H	H	H	H	H	H	H	H
x^5	[cos + sin]		$-\frac{5}{32}$	$+\frac{41}{256}$		$-\frac{21}{32}$	$-\frac{1}{32}$	$-\frac{397}{256}$	$-\frac{13}{64}$	$-\frac{199}{256}$
	[cos − sin]		o	o		o	o	o	o	o
	[cos]		H	H		H	H	H	H	H
	[sin]		H	H		H	H	H	H	H
x^6	[cos + sin]			$+\frac{39}{64}$		$+\frac{3}{32}$		$+\frac{29}{64}$	$+\frac{1}{64}$	$+\frac{7}{64}$
	[cos − sin]			o		o		o	o	o
	[cos]			H		H		H	H	H
	[sin]			H		H		H	H	H
x^7	[cos + sin]			$-\frac{21}{128}$				$-\frac{7}{128}$		$-\frac{1}{128}$
	[cos − sin]			o				o		o
	[cos]			H				H		H
	[sin]			H				H		H

Table (x°, x^1, x^2, x^3) $\genfrac{}{}{0pt}{}{\cos}{\sin}\, 5f$ . . .

$\genfrac{}{}{0pt}{}{\cos}{\sin}\, 5f$		[ ]°		[ ]¹		[ ]²			[ ]³		
		e^5	e^7	e^4	e^6	e^3	e^5	e^7	e^2	e^4	e^6
x°	[cos + sin]	$-\frac{3}{16}$	$-\frac{5}{32}$	$+\frac{1045}{384}$	$-\frac{229}{192}$	$-\frac{95}{12}$	$+\frac{185}{12}$	$-\frac{1295}{192}$	$+\frac{75}{8}$	$-\frac{665}{16}$	$+\frac{58515}{1024}$
	[cos − sin].	„	„	o	$-\frac{37}{9216}$	o	o	$+\frac{5}{4032}$	o	o	o
	[cos]	$-\frac{3}{16}$	$-\frac{5}{32}$	H	$-\frac{11029}{18432}$	H	H	$-\frac{13595}{4032}$	H	H	H
	[sin]	o	o	H	$-\frac{10955}{18432}$	H	H	$-\frac{425}{126}$	H	H	H
x^1	[cos + sin]	$-\frac{15}{32}$	$-\frac{11}{64}$	$+\frac{229}{96}$	$-\frac{7747}{3840}$	$-\frac{29}{8}$	$+\frac{205}{24}$	$-\frac{11831}{1920}$	$+\frac{9}{4}$	$-\frac{169}{16}$	$+\frac{9609}{512}$
	[cos − sin]	„	„	o	$-\frac{151}{7680}$	o	o	$-\frac{37}{5760}$	o	o	o
	[cos]	$-\frac{15}{32}$	$-\frac{11}{64}$	H	$-\frac{1043}{1024}$	H	H	$-\frac{3553}{1152}$	H	H	H
	[sin]	o	o	H	$-\frac{15343}{15360}$	H	H	$-\frac{277}{90}$	H	H	H
x^2	[cos + sin]	$-\frac{5}{8}$	$+\frac{1}{16}$	$+\frac{43}{32}$	$-\frac{357}{256}$	-1	$+\frac{37}{24}$	$-\frac{691}{480}$	$+\frac{1}{4}$	$+\frac{11}{16}$	$-\frac{1435}{512}$
	[cos − sin]	„	„	− o	$+\frac{37}{1536}$	o	o	$+\frac{1}{480}$	o	o	o
	[cos]	$-\frac{5}{8}$	$+\frac{1}{16}$	H	$-\frac{2105}{3072}$	H	H	$-\frac{23}{32}$	H	H	H
	[sin]	o	o	H	$-\frac{2179}{3072}$	H	H	$-\frac{173}{240}$	H	H	H
x^3	[cos + sin]	$-\frac{15}{32}$	$+\frac{5}{32}$	$+\frac{7}{16}$	$+\frac{3}{32}$	$-\frac{1}{8}$	$-\frac{19}{16}$	$+\frac{261}{128}$		$+\frac{5}{4}$	$-\frac{531}{128}$
	[cos − sin]	„	„	o	$+\frac{47}{384}$	o	o	$+\frac{7}{384}$		o	o
	[cos]	$-\frac{15}{32}$	$+\frac{5}{32}$	H	$+\frac{83}{768}$	H	H	$+\frac{395}{384}$		H	H
	[sin]	o	o	H	$-\frac{11}{768}$	H	H	$+\frac{97}{96}$		H	H

. . . *Table* $(x^0, x^1, x^2, x^3) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 5f$ . . .

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 5f$		$[\;]^4$				$[\;]^5$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
x^0	[cos + sin]	-5	$+\frac{95}{2}$	$-\frac{1675}{12}$	$+\frac{25085}{144}$	$+1$	-25	$+\frac{9775}{64}$	$-\frac{110225}{288}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
x^1	[cos + sin]	$-\frac{1}{2}$	$+\frac{17}{4}$	$-\frac{365}{24}$	$+\frac{8413}{288}$		$+\frac{1}{2}$	$-\frac{25}{8}$	$-\frac{25}{128}$
	[cos − sin]	0	0	0	0		0	0	0
	[cos]	H	H	H	H		H	H	H
	[sin]	H	H	H	H		H	H	H
x^2	[cos + sin]		$-\frac{3}{2}$	$+\frac{26}{3}$	$-\frac{527}{32}$		$+\frac{1}{2}$	$-\frac{25}{4}$	$+\frac{9725}{384}$
	[cos − sin]		0	0	0		0	0	0
	[cos]		H	H	H		H	H	H
	[sin]		H	H	H		H	H	H
x^3	[cos + sin]		$-\frac{3}{8}$	$+\frac{17}{8}$	$-\frac{725}{128}$			$+\frac{3}{8}$	$-\frac{25}{16}$
	[cos − sin]		0	0	0			0	0
	[cos]		H	H	H			H	H
	[sin]		H	H	H			H	H

. . . *Table* $(x^\circ, x^1, x^2, x^3)\ {\cos \atop \sin}\ 5f$ . . . .

	${\cos \atop \sin}\ 5f$	$[\ ]^6$				$[\ ]^7$			$[\ ]^8$		
		e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7
x°	[cos + sin]	$+5$	$-\frac{315}{4}$	$+\frac{6375}{16}$	$-\frac{59585}{64}$	$+\frac{125}{8}$	$-\frac{9605}{48}$	$+\frac{2825155}{3072}$	$+\frac{235}{6}$	$-\frac{21515}{48}$	$+\frac{187915}{96}$
	[cos − sin]	0	0	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H	H	H
x^1	[cos + sin]	$-\frac{1}{2}$	$+\frac{69}{8}$	$-\frac{1335}{32}$	$+\frac{10003}{128}$	$-\frac{11}{4}$	$+\frac{1723}{48}$	$-\frac{242573}{1536}$	$-\frac{37}{4}$	$+\frac{10145}{96}$	$-\frac{425783}{960}$
	[cos − sin]	0	0	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H	H	H
x^2	[cos + sin]		$+1$	$-\frac{47}{4}$	$+\frac{1553}{32}$	$+\frac{1}{4}$	$-\frac{19}{16}$	$-\frac{4295}{512}$	$+\frac{3}{2}$	$-\frac{313}{24}$	$+\frac{6977}{240}$
	[cos − sin]		0	0	0	0	0	0	0	0	0
	[cos]		H	H	H	H	H	H	H	H	H
	[sin]		H	H	H	H	H	H	H	H	H
x^3	[cos + sin]		$-\frac{3}{8}$	$+\frac{69}{16}$	$-\frac{1995}{128}$		$-\frac{5}{4}$	$+\frac{1639}{128}$	$-\frac{1}{8}$	$-\frac{53}{32}$	$+\frac{769}{32}$
	[cos − sin]		0	0	0		0	0	0	0	0
	[cos]		H	H	H		H	H	H	H	H
	[sin]		H	H	H		H	H	H	H	H

. . . . *Table* $(x^0, x^1, x^2, x^3)\ \begin{matrix}\cos\\ \sin\end{matrix}\ 5f.$

	$\begin{matrix}\cos\\ \sin\end{matrix}\ 5f$	$[\ ]^9$		$[\ ]^{10}$		$[\ ]^{11}$	$[\ ]^{12}$
		e^4	e^6	e^5	e^7	e^6	e^7
x^0	[cos + sin]	$+\frac{11035}{128}$	$-\frac{59127}{64}$	$+\frac{8359}{48}$	$-\frac{1033885}{576}$	$+\frac{3050417}{9216}$	$+\frac{67335}{112}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^1	[cos + sin]	$-\frac{787}{32}$	$+\frac{334011}{1280}$	$-\frac{5455}{96}$	$+\frac{665623}{1152}$	$-\frac{918191}{7680}$	$-\frac{37647}{160}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^2	[cos + sin]	$+\frac{173}{32}$	$-\frac{12697}{256}$	$+\frac{365}{24}$	$-\frac{13481}{96}$	$+\frac{18859}{512}$	$+\frac{12907}{160}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^3	[cos + sin]	$-\frac{13}{16}$	$+\frac{63}{32}$	$-\frac{25}{8}$	$+\frac{7115}{384}$	$-\frac{3563}{384}$	$-\frac{3011}{128}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H

Table $(x^4, x^5, x^6, x^7) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 5f \ldots$

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 5f$		$[\ ]^0$		$[\ ]^1$		$[\ ]^2$			$[\ ]^3$		
		e^5	e^7	e^4	e^6	e^3	e^5	e^7	e^2	e^4	e^6
x^4	[cos + sin]	$-\frac{3}{16}$	$-\frac{5}{32}$	$+\frac{1}{16}$	$+\frac{25}{32}$		$-\frac{13}{16}$	$+\frac{81}{64}$		$+\frac{1}{4}$	$-\frac{9}{128}$
	[cos − sin]	”	”	0	$+\frac{19}{128}$		0	$-\frac{1}{192}$		0	0
	[cos]	$-\frac{3}{16}$	$-\frac{5}{32}$	H	$+\frac{119}{256}$		H	$+\frac{121}{192}$		H	H
	[sin]	0	0	H	$+\frac{81}{256}$		H	$+\frac{61}{96}$		H	H
x^5	[cos + sin]	$-\frac{1}{32}$	$-\frac{25}{64}$		$+\frac{15}{32}$		$-\frac{5}{32}$	$-\frac{65}{128}$			$+\frac{55}{64}$
	[cos − sin]	”	”		$+\frac{5}{64}$		0	$-\frac{5}{128}$			0
	[cos]	$-\frac{1}{32}$	$-\frac{25}{64}$		$+\frac{35}{128}$		H	$-\frac{35}{128}$			H
	[sin]	0	0		$+\frac{25}{128}$		H	$-\frac{15}{64}$			H
x^6	[cos + sin]		$-\frac{1}{4}$		$+\frac{3}{32}$			$-\frac{21}{32}$			$+\frac{15}{64}$
	[cos − sin]		”		$+\frac{1}{64}$			$-\frac{1}{32}$			0
	[cos]		$-\frac{1}{4}$		$+\frac{7}{128}$			$-\frac{11}{32}$			H
	[sin]		0		$+\frac{5}{128}$			$-\frac{5}{16}$			H
x^7	[cos + sin]		$-\frac{7}{128}$					$-\frac{21}{128}$			
	[cos − sin]		”					$-\frac{1}{128}$			
	[cos]		$-\frac{7}{128}$					$-\frac{11}{128}$			
	[sin]		0					$-\frac{5}{64}$			

. . . *Table* $(x^4, x^5, x^6, x^7) \substack{\cos \\ \sin} 5f$ . . .

$\substack{\cos \\ \sin} 5f$		[ ]^4				[ ]^5				[ ]^6				[ ]^7		
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6	e	e^3	e^5	e^7	e^2	e^4	e^6
x^4	[cos + sin]			$-\frac{7}{8}$	$+\frac{113}{32}$			$+\frac{3}{8}$	$-\frac{25}{8}$			$+\frac{3}{8}$	$-\frac{249}{64}$		$+\frac{1}{4}$	$-\frac{239}{128}$
	[cos − sin]			o	o			o	o			o	o		o	o
	[cos]			H	H			H	H			H	H		H	H
	[sin]			H	H			H	H			H	H		H	H
x^5	[cos + sin]			$-\frac{5}{16}$	$+\frac{85}{64}$				$+\frac{5}{16}$			$-\frac{5}{16}$	$+\frac{345}{128}$			$-\frac{45}{64}$
	[cos − sin]			o	o				o			o	o			o
	[cos]			H	H				H			H	H			H
	[sin]			H	H				H			H	H			H
x^6	[cos + sin]				$-\frac{5}{8}$				$+\frac{5}{16}$				$+\frac{5}{32}$			$+\frac{15}{64}$
	[cos − sin]				o				o				o			o
	[cos]				H				H				H			H
	[sin]				H				H				H			H
x^7	[cos + sin]				$-\frac{35}{128}$								$-\frac{35}{128}$			
	[cos − sin]				o								o			
	[cos]				H								H			
	[sin]				H								H			

. . . *Table* $(x^4, x^5, x^6, x^7) \begin{matrix}\cos\\ \sin\end{matrix} 5f.$

	$\begin{matrix}\cos\\ \sin\end{matrix} 5f$	$[\ ]^8$			$[\ ]^9$		$[\ ]^{10}$		$[\ ]^{11}$	$[\ ]^{12}$
		e^3	e^5	e^7	e^4	e^6	e^5	e^7	e^6	e^7
x^4	[cos + sin]		$+\frac{17}{16}$	$-\frac{287}{32}$	$+\frac{1}{16}$	$+\frac{75}{32}$	$+\frac{7}{16}$	$+\frac{505}{192}$	$+\frac{229}{128}$	$+\frac{179}{32}$
	[cos − sin]		0	0	0	0	0	0	0	0
	[cos]		H	H	H	H	H	H	H	H
	[sin]		H	H	H	H	H	H	H	H
x^5	[cos + sin]		$-\frac{5}{32}$	$+\frac{5}{64}$		$-\frac{25}{32}$	$-\frac{1}{32}$	$-\frac{275}{128}$	$-\frac{15}{64}$	$-\frac{65}{64}$
	[cos − sin]		0	0		0	0	0	0	0
	[cos]		H	H		H	H	H	H	H
	[sin]		H	H		H	H	H	H	H
x^6	[cos + sin]			$+\frac{3}{4}$		$+\frac{3}{32}$		$+\frac{17}{32}$	$+\frac{1}{64}$	$+\frac{1}{8}$
	[cos − sin]			0		0		0	0	0
	[cos]			H		H		H	H	H
	[sin]			H		H		H	H	H
x^7	[cos + sin]			$-\frac{21}{128}$				$-\frac{7}{128}$		$-\frac{1}{128}$
	[cos − sin]			0				0		0
	[cos]			H				H		H
	[sin]			H				H		H

Table $(x^0, x^1, x^2, x^3)\,{}^{\cos}_{\sin}\,6f\ldots$

	${}^{\cos}_{\sin}\,6f$	$[\;]^0$	$[\;]^1$		$[\;]^2$		$[\;]^3$		
		e^6	e^5	e^7	e^4	e^6	e^3	e^5	e^7
x^0	[cos + sin]	$+\frac{7}{64}$	$-\frac{773}{320}$	$+\frac{5167}{7680}$	$+\frac{157}{16}$	$-\frac{1339}{80}$	$-\frac{67}{4}$	$+\frac{4053}{64}$	$-\frac{192531}{2560}$
	[cos − sin]	„	0	$+\frac{17}{53760}$	0	0	0	0	0
	[cos]	$\frac{7}{64}$	H	$+\frac{6031}{17920}$	H	H	H	H	H
	[sin]	0	H	$+\frac{4519}{13440}$	H	H	H	H	H
x^1	[cos + sin]	$+\frac{21}{64}$	$-\frac{1961}{768}$	$+\frac{162773}{92160}$	$+\frac{277}{48}$	$-\frac{993}{80}$	$-\frac{93}{16}$	$+\frac{6289}{256}$	$-\frac{384789}{10240}$
	[cos − sin]	„	0	$+\frac{1091}{92160}$	0	0	0	0	0
	[cos]	$\frac{21}{64}$	H	$+\frac{20483}{23040}$	H	H	H	H	H
	[sin]	0	H	$+\frac{26947}{30720}$	H	H	H	H	H
x^2	[cos + sin]	$+\frac{35}{64}$	$-\frac{179}{96}$	$+\frac{14689}{7680}$	$+\frac{37}{16}$	$-\frac{235}{48}$	$-\frac{5}{4}$	$+\frac{107}{32}$	$-\frac{12123}{2560}$
	[cos − sin]	„	0	$-\frac{17}{7680}$	0	0	0	0	0
	[cos]	$+\frac{35}{64}$	H	$+\frac{917}{960}$	H	H	H	H	H
	[sin]	0	H	$+\frac{2451}{2560}$	H	H	H	H	H
x^3	[cos + sin]	$+\frac{35}{64}$	$-\frac{57}{64}$	$+\frac{739}{1024}$	$+\frac{9}{16}$	$+\frac{1}{4}$	$-\frac{1}{8}$	$-\frac{117}{64}$	$+\frac{6249}{1024}$
	[cos − sin]	„	0	$-\frac{75}{1024}$	0	0	0	0	0
	[cos]	$+\frac{35}{64}$	H	$+\frac{83}{256}$	H	H	H	H	H
	[sin]	0	H	$+\frac{407}{1024}$	H	H	H	H	H

. . . Table $(x^0, x^1, x^2, x^3) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 6f$ . . .

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 6f$		$[\ ]^4$			$[\ ]^5$			
		e^2	e^4	e^6	e	e^3	e^5	e^7
x^0	[cos + sin]	$+\frac{57}{4}$	$-\frac{829}{8}$	$+\frac{8117}{32}$	-6	$+\frac{345}{4}$	$-\frac{12605}{32}$	$+\frac{1246315}{1536}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
x^1	[cos + sin]	$+\frac{11}{4}$	$-\frac{497}{24}$	$+\frac{1933}{32}$	$-\frac{1}{2}$	$+\frac{105}{16}$	$-\frac{13325}{384}$	$+\frac{1838665}{18432}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
x^2	[cos + sin]	$+\frac{1}{4}$	$+\frac{7}{8}$	$-\frac{685}{96}$		$-\frac{7}{4}$	$+\frac{745}{48}$	$-\frac{72955}{1536}$
	[cos − sin]	0	0	0		0	0	0
	[cos]	H	H	H		H	H	H
	[sin]	H	H	H		H	H	H
x^3	[cos + sin]		$+\frac{3}{2}$	$-\frac{517}{64}$		$-\frac{3}{8}$	$+\frac{105}{32}$	$-\frac{13265}{1024}$
	[cos − sin]		0	0		0	0	0
	[cos]		H	H		H	H	H
	[sin]		H	H		H	H	H

. . . *Table* $(x^0, x^1, x^2, x^3)\ {\cos \atop \sin}\ 6f$ . . .

	${\cos \atop \sin}\ 6f$	$[\ ]^6$				$[\ ]^7$			
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7
x^0	[cos + sin]	$+1$	-36	$+\frac{5103}{16}$	$-\frac{19015}{16}$	$+6$	$-\frac{525}{4}$	$+\frac{29813}{32}$	$-\frac{4795063}{1536}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
x^1	[cos + sin]	0	$+\frac{1}{2}$	$-\frac{9}{2}$	$-\frac{9}{32}$	$-\frac{1}{2}$	$+\frac{189}{16}$	$-\frac{30821}{384}$	$+\frac{4080685}{18432}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	0	H	H	H	H	H	H	H
	[sin]	0	H	H	H	H	H	H	H
x^2	[cos + sin]		$+\frac{1}{2}$	-9	$+\frac{1695}{32}$		$+\frac{5}{4}$	$-\frac{959}{48}$	$+\frac{175553}{1536}$
	[cos − sin]		0	0	0		0	0	0
	[cos]		H	H	H		H	H	H
	[sin]		H	H	H		H	H	H
x^3	[cos + sin]			$+\frac{3}{8}$	$-\frac{9}{4}$		$-\frac{3}{8}$	$+\frac{189}{32}$	$-\frac{30737}{1024}$
	[cos − sin]			0	0		0	0	0
	[cos]			H	H		H	H	H
	[sin]			H	H		H	H	H

... *Table* $(x^0, x^1, x^2, x^3)\ {\cos \atop \sin}\ 6f$ ...

	${\cos \atop \sin}\ 6f$	$[\ ]^8$			$[\ ]^9$		
		e^2	e^4	e^6	e^3	e^5	e^7
x^0	[cos + sin]	$+\frac{87}{4}$	$-\frac{3011}{8}$	$+\frac{151205}{64}$	$+\frac{247}{4}$	$-\frac{59781}{64}$	$+\frac{13947303}{2560}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^1	[cos + sin]	$-\frac{13}{4}$	$+\frac{1369}{24}$	$-\frac{21969}{64}$	$-\frac{201}{16}$	$+\frac{48449}{256}$	$-\frac{10874049}{10240}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^2	[cos + sin]	$+\frac{1}{4}$	$-\frac{11}{8}$	$-\frac{3317}{192}$	$+\frac{7}{4}$	$-\frac{629}{32}$	$+\frac{145449}{2560}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^3	[cos + sin]		$-\frac{3}{2}$	$+\frac{1313}{64}$	$-\frac{1}{8}$	$-\frac{153}{64}$	$+\frac{44889}{1024}$
	[cos − sin]		0	0	0	0	0
	[cos]		H	H	H	H	H
	[sin]		H	H	H	H	H

. . . *Table* $(x^0, x^1, x^2, x^3)\ \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\ 6f$.

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\ 6f$	$[\ ]^{10}$		$[\ ]^{11}$		$[\ ]^{12}$	$[\ ]^{13}$
		e^4	e^6	e^5	e^7	e^6	e^7
x^0	[cos + sin]	$+\frac{605}{4}$	$-\frac{16829}{8}$	$+\frac{107333}{320}$	$-\frac{33928027}{7680}$	$+\frac{110793}{160}$	$+\frac{72815971}{53760}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^1	[cos + sin]	$-\frac{1805}{48}$	$+\frac{16547}{32}$	$-\frac{73961}{768}$	$+\frac{115203737}{92160}$	$-\frac{35499}{160}$	$-\frac{43634809}{92160}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^2	[cos + sin]	$+\frac{115}{16}$	$-\frac{8135}{96}$	$+\frac{2173}{96}$	$-\frac{2039459}{7680}$	$+\frac{1937}{32}$	$+\frac{1111819}{7680}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^3	[cos + sin]	$-\frac{15}{16}$	$+\frac{5}{2}$	$-\frac{261}{64}$	$+\frac{30707}{1024}$	$-\frac{863}{64}$	$-\frac{38507}{1024}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H

Table $(x^4, x^5, x^6, x^7)\,{}^{\cos}_{\sin}\,6f\ldots$

$^{\cos}_{\sin}\,6f$		$[\ ]^0$	$[\ ]^1$		$[\ ]^2$		$[\ ]^3$			$[\ ]^4$		
		e^6	e^5	e^7	e^4	e^6	e^3	e^5	e^7	e^2	e^4	e^6
x^4	[cos + sin]	$+\frac{21}{64}$	$-\frac{1}{4}$	$-\frac{25}{48}$	$+\frac{1}{16}$	$+\frac{21}{16}$		-1	$+\frac{87}{32}$		$+\frac{1}{4}$	$-\frac{15}{64}$
	[cos − sin]	”	0	$-\frac{13}{96}$	0	0		0	0		0	0
	[cos]	$+\frac{21}{64}$	H	$-\frac{21}{64}$	H	H		H	H		H	H
	[sin]	0	H	$-\frac{37}{192}$	H	H		H	H		H	H
x^5	[cos + sin]	$+\frac{7}{64}$	$-\frac{1}{32}$	$-\frac{187}{256}$		$+\frac{19}{32}$		$-\frac{5}{32}$	$-\frac{189}{256}$			$+\frac{65}{64}$
	[cos − sin]	”	0	$-\frac{29}{256}$		0		0	0			0
	[cos]	$+\frac{7}{64}$	H	$-\frac{27}{64}$		H		H	H			H
	[sin].	0	H	$-\frac{79}{256}$		H		H	H			H
x^6	[cos + sin]	$+\frac{1}{64}$		$-\frac{21}{64}$		$+\frac{3}{32}$			$-\frac{51}{64}$			$+\frac{15}{64}$
	[cos − sin]	”		$-\frac{3}{64}$		0			0			0
	[cos]	$+\frac{1}{64}$		$-\frac{3}{16}$		H			H			H
	[sin]	0		$-\frac{9}{64}$		H			H			H
x^7	[cos + sin]			$-\frac{7}{128}$					$-\frac{21}{128}$			
	[cos − sin]			$-\frac{1}{128}$					0			
	[cos]			$-\frac{1}{32}$					H			
	[sin]			$-\frac{3}{128}$					H			

. . . *Table* $(x^4, x^5, x^6, x^7) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 6f$ . . .

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 6f$	$[\ ]^5$			$[\ ]^6$		$[\ ]^7$			$[\ ]^8$	
		e^3	e^5	e^7	e^4	e^6	e^3	e^5	e^7	e^4	e^6
x^4	[cos + sin]		-1	$+\frac{25}{4}$	$+\frac{3}{8}$	$-\frac{9}{2}$		$+\frac{1}{2}$	$-\frac{217}{32}$	$+\frac{1}{4}$	$-\frac{153}{64}$
	[cos − sin]		0	0	0	0		0	0	0	0
	[cos]		H	H	H	H		H	H	H	H
	[sin]		H	H	H	H		H	H	H	H
x^5	[cos + sin]		$-\frac{5}{16}$	$+\frac{525}{256}$		$+\frac{5}{16}$		$-\frac{5}{16}$	$+\frac{945}{256}$		$-\frac{55}{64}$
	[cos − sin]		0	0		0		0	0		0
	[cos]		H	H		H		H	H		H
	[sin]		H	H		H		H	H		H
x^6	[cos + sin]			$-\frac{45}{64}$		$+\frac{5}{16}$			$+\frac{15}{64}$		$+\frac{15}{64}$
	[cos − sin]			0		0			0		0
	[cos]			H		H			H		H
	[sin]			H		H			H		H
x^7	[cos + sin]			$-\frac{35}{128}$					$-\frac{35}{128}$		
	[cos − sin]			0					0		
	[cos]			H					H		
	[sin]			H					H		

. . . *Table* $(x^4, x^5, x^6, x^7) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 6f$.

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 6f$		$[\ ]^9$			$[\ ]^{10}$		$[\ ]^{11}$		$[\ ]^{12}$	$[\ ]^{13}$
		e^3	e^5	e^7	e^4	e^6	e^5	e^7	e^6	e^7
x^4	[cos + sin]		$+\frac{5}{4}$	$-\frac{219}{16}$	$+\frac{1}{16}$	$+\frac{51}{16}$	$+\frac{1}{2}$	$+\frac{403}{96}$	$+\frac{147}{64}$	$+\frac{191}{24}$
	[cos − sin]		0	0	0	0	0	0	0	0
	[cos]		H	H	H	H	H	H	H	H
	[sin]		H	H	H	H	H	H	H	H
x^5	[cos + sin]		$-\frac{5}{32}$	$-\frac{9}{256}$		$-\frac{29}{32}$	$-\frac{1}{32}$	$-\frac{727}{256}$	$-\frac{17}{64}$	$-\frac{329}{256}$
	[cos − sin]		0	0		0	0	0	0	0
	[cos]		H	H		H	H	H	H	H
	[sin]		H	H		H	H	H	H	H
x^6	[cos + sin]			$+\frac{57}{64}$		$+\frac{3}{32}$		$+\frac{39}{64}$	$+\frac{1}{64}$	$+\frac{9}{64}$
	[cos − sin]			0		0		0	0	0
	[cos]			H		H		H	H	H
	[sin]			H		H		H	H	H
x^7	[cos + sin]			$-\frac{21}{128}$				$-\frac{7}{128}$		$-\frac{1}{128}$
	[cos − sin]			0				0		0
	[cos]			H				H		H
	[sin]			H				H		H

Table $(x^0, x^1, x^2, x^3) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 7f \ldots$

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 7f$		$[\ ]^0$	$[\ ]^1$	$[\ ]^2$		$[\ ]^3$	
		e^7	e^6	e^5	e^7	e^4	e^6
x^0	[cos + sin]	$-\frac{1}{16}$	$+\frac{93289}{46080}$	$-\frac{329}{30}$	$+\frac{11809}{720}$	$+\frac{3227}{128}$	$-\frac{26943}{320}$
	[cos − sin]	„	0	0	0	0	0
	[cos]	$-\frac{1}{16}$	H	H	H	H	H
	[sin]	0	H	H	H	H	H
x^1	[cos + sin]	$-\frac{7}{32}$	$+\frac{19081}{7680}$	$-\frac{187}{24}$	$+\frac{10927}{720}$	$+\frac{361}{32}$	$-\frac{55317}{1280}$
	[cos − sin]	„	0	0	0	0	0
	[cos]	$-\frac{7}{32}$	H	H	H	H	H
	[sin]	0	H	H	H	H	H
x^2	[cos + sin]	$-\frac{7}{16}$	$+\frac{1131}{512}$	$-\frac{97}{24}$	$+\frac{4223}{480}$	$+\frac{113}{32}$	$-\frac{3177}{256}$
	[cos − sin]	„	0	0	0	0	0
	[cos]	$-\frac{7}{16}$	H	H	H	H	H
	[sin]	0	H	H	H	H	H
x^3	[cos + sin]	$-\frac{35}{64}$	$+\frac{529}{384}$	$-\frac{23}{16}$	$+\frac{391}{192}$	$+\frac{11}{16}$	$+\frac{15}{32}$
	[cos − sin]	„	0	0	0	0	0
	[cos]	$-\frac{35}{64}$	H	H	H	H	H
	[sin]	0	H	H	H	H	H

. . . *Table* $(x^0, x^1, x^2, x^3) \, {}^{\cos}_{\sin} \, 7f$ . . .

$^{\cos}_{\sin} \, 7f$		$[\;]^4$			$[\;]^5$		
		e^3	e^5	e^7	e^2	e^4	e^6
x^0	[cos + sin]	$-\frac{91}{3}$	$+\frac{4445}{24}$	$-\frac{47299}{120}$	$+\frac{161}{8}$	$-\frac{10409}{48}$	$+\frac{2509759}{3072}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^1	[cos + sin]	$-\frac{17}{2}$	$+\frac{2689}{48}$	$-\frac{3395}{24}$	$+\frac{13}{4}$	$-\frac{1721}{48}$	$+\frac{236923}{1536}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^2	[cos + sin]	$-\frac{3}{2}$	$+\frac{149}{24}$	$-\frac{5897}{480}$	$+\frac{1}{4}$	$+\frac{17}{16}$	$-\frac{7671}{512}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
x^3	[cos + sin]	$-\frac{1}{8}$	$-\frac{83}{32}$	$+\frac{1807}{128}$		$+\frac{7}{4}$	$-\frac{1781}{128}$
	[cos − sin]	0	0	0		0	0
	[cos]	H	H	H		H	H
	[sin]	H	H	H		H	H

. . . *Table* $(x^0, x^1, x^2, x^3)\ {}^{\cos}_{\sin}\,7f$ . . .

	$^{\cos}_{\sin}\,7f$	$[\ ]^6$				$[\ ]^7$			
		e^1	e^3	e^5	e^7	e^0	e^2	e^4	e^6
x^0	[cos + sin]	-7	$+\frac{567}{4}$	$-\frac{14805}{16}$	$+\frac{89999}{32}$	$+1$	-49	$+\frac{37975}{64}$	$-\frac{883421}{288}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
x^1	[cos + sin]	$-\frac{1}{2}$	$+\frac{75}{8}$	$-\frac{2199}{32}$	$+\frac{17549}{64}$		$+\frac{1}{2}$	$-\frac{49}{8}$	$-\frac{49}{128}$
	[cos − sin]	0	0	0	0		0	0	0
	[cos]	H	H	H	H		H	H	H
	[sin]	H	H	H	H		H	H	H
x^2	[cos + sin]		-2	$+\frac{101}{4}$	$-\frac{3611}{32}$		$+\frac{1}{2}$	$-\frac{49}{4}$	$+\frac{37877}{384}$
	[cos − sin]		0	0	0		0	0	0
	[cos]		H	H	H		H	H	H
	[sin]		H	H	H		H	H	H
x^3	[cos + sin]		$-\frac{3}{8}$	$+\frac{75}{16}$	$-\frac{411}{16}$			$+\frac{3}{8}$	$-\frac{49}{16}$
	[cos − sin]		0	0	0			0	0
	[cos]		H	H	H			H	H
	[sin]		H	H	H			H	H

. . . *Table* $(x^0, x^1, x^2, x^3) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 7f$ . . .

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 7f$		$[\ ]^8$				$[\ ]^9$		
		e	e^3	e^5	e^7	e^2	e^4	e^6
x^0	[cos + sin]	$+7$	-203	$+\frac{5768}{3}$	$-\frac{1255219}{144}$	$+\frac{231}{8}$	$-\frac{10381}{16}$	$+\frac{5454687}{1024}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
x^1	[cos + sin]	$-\frac{1}{2}$	$+\frac{31}{2}$	$-\frac{845}{6}$	$+\frac{154651}{288}$	$-\frac{15}{4}$	$+\frac{1363}{16}$	$-\frac{344031}{512}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
x^2	[cos + sin]		$+\frac{3}{2}$	$-\frac{94}{3}$	$+\frac{3791}{16}$	$+\frac{1}{4}$	$-\frac{25}{16}$	$-\frac{16155}{512}$
	[cos − sin]		0	0	0	0	0	0
	[cos]		H	H	H	H	H	H
	[sin]		H	H	H	H	H	H
x^3	[cos + sin]		$-\frac{3}{8}$	$+\frac{31}{4}$	$-\frac{3373}{64}$		$-\frac{7}{4}$	$+\frac{3945}{128}$
	[cos − sin]		0	0	0		0	0
	[cos]		H	H	H		H	H
	[sin]		H	H	H		H	H

. . . *Table* $(x^0, x^1, x^2, x^3)\ {\cos \atop \sin} 7f$ . . .

	${\cos \atop \sin} 7f$	$[\ ]^{10}$			$[\ ]^{11}$	
		e^3	e^5	e^7	e^4	e^6
x^0	[cos + sin]	$+\frac{1099}{12}$	$-\frac{84889}{48}$	$+\frac{1273433}{96}$	$+\frac{94885}{384}$	$-\frac{4151693}{960}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
x^1	[cos + sin]	$-\frac{131}{8}$	$+\frac{30193}{96}$	$-\frac{436457}{192}$	$-\frac{5231}{96}$	$+\frac{3618557}{3840}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
x^2	[cos + sin]	$+2$	$-\frac{677}{24}$	$+\frac{9889}{96}$	$+\frac{295}{32}$	$-\frac{34789}{256}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
x^3	[cos + sin]	$-\frac{1}{8}$	$-\frac{13}{4}$	$+\frac{2361}{32}$	$-\frac{17}{16}$	$+\frac{99}{32}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H

. . . *Table* $(x^{\circ}, x^{1}, x^{2}, x^{3})\, {}^{\cos}_{\sin}\, 7f$.

	${}^{\cos}_{\sin}\, 7f$	$[\ ]^{12}$		$[\ ]^{13}$	$[\ ]^{14}$
		e^5	e^7	e^6	e^7
x°	[cos + sin]	$+\frac{23877}{40}$	$-\frac{390873}{40}$	$+\frac{61307827}{46080}$	$+\frac{1004057}{360}$
	[cos − sin]	o	o	o	o
	[cos]	H	H	H	H
	[sin]	H	H	H	H
x^{1}	[cos + sin]	$-\frac{2451}{16}$	$+\frac{49413}{20}$	$-\frac{2945503}{7680}$	$-\frac{254051}{288}$
	[cos − sin]	o	o	o	o
	[cos]	H	H	H	H
	[sin]	H	H	H	H
x^{2}	[cos + sin]	$+\frac{257}{8}$	$-\frac{74701}{160}$	$+\frac{144373}{1536}$	$+\frac{116977}{480}$
	[cos − sin]	o	o	o	o
	[cos]	H	H	H	H
	[sin]	H	H	H	H
x^{3}	[cos + sin]	$-\frac{165}{32}$	$+\frac{5905}{128}$	$-\frac{7213}{384}$	$-\frac{10969}{192}$
	[cos − sin]	o	o	o	o
	[cos]	H	H	H	H
	[sin]	H	H	H	H

Table $(x^4, x^5, x^6, x^7)\,{\cos \atop \sin}\,7f\ldots$

	${\cos \atop \sin}\,7f$	$[\ ]^0$	$[\ ]^1$	$[\ ]^2$		$[\ ]^3$		$[\ ]^4$			$[\ ]^5$	
		e^7	e^6	e^5	e^7	e^4	e^6	e^3	e^5	e^7	e^4	e^6
x^4	[cos + sin]	$-\frac{7}{16}$	$+\frac{73}{128}$	$-\frac{5}{16}$	$-\frac{221}{192}$	$+\frac{1}{16}$	$+\frac{63}{32}$		$-\frac{19}{16}$	$+5$	$+\frac{1}{4}$	$-\frac{59}{128}$
	[cos − sin]	„	0	0	0	0	0		0	0	0	0
	[cos]	$-\frac{7}{16}$	H	H	H	H	H		H	H	H	H
	[sin]	0	H	H	H	H	H		H	H	H	H
x^5	[cos + sin]	$-\frac{7}{32}$	$+\frac{9}{64}$	$-\frac{1}{32}$	$-\frac{149}{128}$		$+\frac{23}{32}$		$-\frac{5}{32}$	-1		$+\frac{75}{64}$
	[cos − sin]	„	0	0	0		0		0	0		0
	[cos]	$-\frac{7}{32}$	H	H	H		H		H	H		H
	[sin]	0	H	H	H		H		H	H		H
x^6	[cos + sin]	$-\frac{1}{16}$	$+\frac{1}{64}$		$-\frac{13}{32}$		$+\frac{3}{32}$			$-\frac{15}{16}$		$+\frac{15}{64}$
	[cos − sin]	„	0		0		0			0		0
	[cos]	$-\frac{1}{16}$	H		H		H			H		H
	[sin]	0	H		H		H			H		H
x^7	[cos + sin]	$-\frac{1}{128}$			$-\frac{7}{128}$					$-\frac{21}{128}$		
	[cos − sin]	„			0					0		
	[cos]	$-\frac{1}{128}$			H					H		
	[sin]	0			H					H		

. . . *Table* $(x^4, x^5, x^6, x^7) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 7f$ . . .

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 7f$	$[\ ]^6$		$[\ ]^7$		$[\ ]^8$				$[\ ]^9$		
		e^5	e^7	e^4	e^6	e	e^3	e^5	e^7	e^2	e^4	e^6
x^4	[cos + sin]	$-\frac{9}{8}$	$+\frac{645}{64}$	$+\frac{3}{8}$	$-\frac{49}{8}$			$+\frac{5}{8}$	$-\frac{173}{16}$		$+\frac{1}{4}$	$-\frac{381}{128}$
	[cos − sin]	o	o	o	o			o	o		o	o
	[cos]	H	H	H	H			H	H		H	H
	[sin]	H	H	H	H			H	H		H	H
x^5	[cos + sin]	$-\frac{5}{16}$	$+\frac{375}{128}$		$+\frac{5}{16}$			$-\frac{5}{16}$	$+\frac{155}{32}$			$-\frac{65}{64}$
	[cos − sin]	o	o		o			o	o			o
	[cos]	H	H		H			H	H			H
	[sin]	H	H		H			H	H			H
x^6	[cos + sin]		$-\frac{25}{32}$		$+\frac{5}{16}$				$+\frac{5}{16}$			$+\frac{15}{64}$
	[cos − sin]		o		o				o			o
	[cos]		H		H				H			H
	[sin]		H		H				H			H
x^7	[cos + sin]		$-\frac{35}{128}$						$-\frac{35}{128}$			
	[cos − sin]		o						o			
	[cos]		H						H			
	[sin]		H						H			

. . . *Table* $(x^4, x^5, x^6, x^7)\,{}^{\cos}_{\sin}\,7f.$

	${}^{\cos}_{\sin}\,7f$	$[\;]^{10}$		$[\;]^{11}$		$[\;]^{12}$		$[\;]^{13}$	$[\;]^{14}$
		e^5	e^7	e^4	e^6	e^5	e^7	e^6	e^7
x^4	[cos + sin]	$+\frac{23}{16}$	$-\frac{3807}{192}$	$+\frac{1}{16}$	$+\frac{133}{32}$	$+\frac{9}{16}$	$+\frac{25}{4}$	$+\frac{367}{128}$	$+\frac{2093}{192}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
x^5	[cos + sin]	$-\frac{5}{32}$	$-\frac{23}{128}$		$-\frac{33}{32}$	$-\frac{1}{32}$	$-\frac{29}{8}$	$-\frac{19}{64}$	$-\frac{203}{128}$
	[cos − sin]	0	0		0	0	0	0	0
	[cos]	H	H		H	H	H	H	H
	[sin]	H	H		H	H	H	H	H
x^6	[cos + sin]		$+\frac{33}{32}$		$+\frac{3}{32}$		$+\frac{11}{16}$	$+\frac{1}{64}$	$+\frac{5}{32}$
	[cos − sin]		0		0		0	0	0
	[cos]		H		H		H	H	H
	[sin]		H		H		H	H	H
x^7	[cos + sin]		$-\frac{21}{128}$				$-\frac{7}{128}$		$-\frac{1}{128}$
	[cos − sin]		0				0		0
	[cos]		H				H		H
	[sin]		H				H		H

Table $\left(\left(\frac{r}{a}\right)^{-5}\ldots\left(\frac{r}{a}\right)^{-1},\ \log\frac{r}{a},\ \left(\frac{r}{a}\right)^{1}\ldots\left(\frac{r}{a}\right)^{4}\right)\ldots$

		[ ]°				[ ]¹			
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^{-5}$	[cos]	$+1$	$+5$	$+\frac{105}{8}$	$+\frac{105}{4}$	$+\frac{5}{2}$	$+\frac{135}{16}$	$+\frac{7285}{384}$	$+\frac{643015}{18432}$
$\left(\frac{r}{a}\right)^{-4}$		$+1$	$+3$	$+\frac{45}{8}$	$+\frac{35}{4}$	$+2$	$+\frac{17}{4}$	$+\frac{685}{96}$	$+\frac{48293}{4608}$
$\left(\frac{r}{a}\right)^{-3}$		$+1$	$+\frac{3}{2}$	$+\frac{15}{8}$	$+\frac{35}{16}$	$+\frac{3}{2}$	$+\frac{27}{16}$	$+\frac{261}{128}$	$+\frac{14309}{6144}$
$\left(\frac{r}{a}\right)^{-2}$		$+1$	$+\frac{1}{2}$	$+\frac{3}{8}$	$+\frac{5}{16}$	$+1$	$+\frac{3}{8}$	$+\frac{65}{192}$	$+\frac{2675}{9216}$
$\left(\frac{r}{a}\right)^{-1}$		$+1$	0	0	0	$+\frac{1}{2}$	$-\frac{1}{16}$	$+\frac{1}{384}$	$-\frac{1}{18432}$
$\log\frac{r}{a}$		0	$+\frac{1}{4}$	$+\frac{1}{32}$	$+\frac{1}{96}$	$-\frac{1}{2}$	$+\frac{3}{16}$	$+\frac{1}{128}$	$+\frac{127}{18432}$
$\left(\frac{r}{a}\right)^{1}$		$+1$	$+\frac{1}{2}$	0	0	$-\frac{1}{2}$	$+\frac{3}{16}$	$-\frac{5}{384}$	$+\frac{7}{18432}$
$\left(\frac{r}{a}\right)^{2}$		$+1$	$+\frac{3}{2}$	0	0	-1	$+\frac{1}{8}$	$-\frac{1}{192}$	$+\frac{1}{9216}$
$\left(\frac{r}{a}\right)^{3}$		$+1$	$+3$	$+\frac{3}{8}$	0	$-\frac{3}{2}$	$-\frac{9}{16}$	$+\frac{15}{128}$	$-\frac{35}{6144}$
$\left(\frac{r}{a}\right)^{4}$		$+1$	$+5$	$+\frac{15}{8}$	0	-2	$-\frac{9}{4}$	$+\frac{19}{96}$	$-\frac{29}{4608}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-5} \ldots \left(\frac{r}{a}\right)^{-1}, \log\frac{r}{a}, \left(\frac{r}{a}\right)^{1} \ldots \left(\frac{r}{a}\right)^{4}\right)$ . . .

		$[\ ]^2$			$[\ ]^3$			$[\ ]^4$	
		e^2	e^4	e^6	e^3	e^5	e^7	e^4	e^6
$\left(\frac{r}{a}\right)^{-5}$	[cos]	$+5$	$+\frac{155}{12}$	$+\frac{835}{32}$	$+\frac{145}{16}$	$+\frac{4715}{256}$	$+\frac{70893}{2048}$	$+\frac{745}{48}$	$+\frac{197}{8}$
$\left(\frac{r}{a}\right)^{-4}$		$+\frac{7}{2}$	$+\frac{67}{12}$	$+\frac{421}{48}$	$+\frac{23}{4}$	$+\frac{435}{64}$	$+\frac{27027}{2560}$	$+\frac{437}{48}$	$+\frac{899}{120}$
$\left(\frac{r}{a}\right)^{-3}$		$+\frac{9}{4}$	$+\frac{7}{4}$	$+\frac{141}{64}$	$+\frac{53}{16}$	$+\frac{393}{256}$	$+\frac{24753}{10240}$	$+\frac{77}{16}$	$+\frac{129}{160}$
$\left(\frac{r}{a}\right)^{-2}$		$+\frac{5}{4}$	$+\frac{1}{6}$	$+\frac{21}{64}$	$+\frac{13}{8}$	$-\frac{25}{128}$	$+\frac{393}{1024}$	$+\frac{103}{48}$	$-\frac{129}{160}$
$\left(\frac{r}{a}\right)^{-1}$		$+\frac{1}{2}$	$-\frac{1}{6}$	$+\frac{1}{48}$	$+\frac{9}{16}$	$-\frac{81}{256}$	$+\frac{729}{10240}$	$+\frac{2}{3}$	$-\frac{8}{15}$
$\log\frac{r}{a}$		$-\frac{3}{8}$	$+\frac{11}{48}$	$-\frac{3}{128}$	$-\frac{17}{48}$	$+\frac{77}{256}$	$-\frac{743}{10240}$	$-\frac{71}{192}$	$+\frac{129}{320}$
$\left(\frac{r}{a}\right)^{1}$		$-\frac{1}{4}$	$+\frac{1}{6}$	$-\frac{1}{32}$	$-\frac{3}{16}$	$+\frac{45}{256}$	$-\frac{567}{10240}$	$-\frac{1}{6}$	$+\frac{1}{5}$
$\left(\frac{r}{a}\right)^{2}$		$-\frac{1}{4}$	$+\frac{1}{12}$	$-\frac{1}{96}$	$-\frac{1}{8}$	$+\frac{9}{128}$	$-\frac{81}{5120}$	$-\frac{1}{12}$	$+\frac{1}{15}$
$\left(\frac{r}{a}\right)^{3}$		0	$-\frac{1}{4}$	$+\frac{3}{32}$	$+\frac{1}{16}$	$-\frac{45}{256}$	$+\frac{189}{2048}$	$+\frac{1}{16}$	$-\frac{3}{20}$
$\left(\frac{r}{a}\right)^{4}$		$+\frac{1}{2}$	$-\frac{7}{12}$	$+\frac{1}{8}$	$+\frac{1}{4}$	$-\frac{19}{64}$	$+\frac{261}{2560}$	$+\frac{7}{48}$	$-\frac{1}{5}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-5} \ldots \left(\frac{r}{a}\right)^{-1}, \log\frac{r}{a}, \left(\frac{r}{a}\right)^{1} \ldots \left(\frac{r}{a}\right)^{4}\right)$.

		[ ]^5		[ ]^6	[ ]^7
		e^5	e^7	e^6	e^7
$\left(\frac{r}{a}\right)^{-5}$	[cos]	$+\frac{19669}{768}$	$+\frac{565135}{18432}$	$+\frac{1317}{32}$	$+\frac{1195093}{18432}$
$\left(\frac{r}{a}\right)^{-4}$		$+\frac{2701}{192}$	$+\frac{31997}{4608}$	$+\frac{1709}{80}$	$+\frac{737707}{23040}$
$\left(\frac{r}{a}\right)^{-3}$		$+\frac{1773}{256}$	$-\frac{4987}{6144}$	$+\frac{3167}{320}$	$+\frac{432091}{30720}$
$\left(\frac{r}{a}\right)^{-2}$		$+\frac{1097}{384}$	$-\frac{16621}{9216}$	$+\frac{1223}{320}$	$+\frac{47273}{9216}$
$\left(\frac{r}{a}\right)^{-1}$		$+\frac{625}{768}$	$-\frac{15625}{18432}$	$+\frac{81}{80}$	$+\frac{117649}{92160}$
$\log\frac{r}{a}$		$-\frac{523}{1280}$	$+\frac{10039}{18432}$	$-\frac{899}{1920}$	$-\frac{355081}{645120}$
$\left(\frac{r}{a}\right)^{1}$		$-\frac{125}{768}$	$+\frac{4375}{18432}$	$-\frac{27}{160}$	$-\frac{16807}{92160}$
$\left(\frac{r}{a}\right)^{2}$		$-\frac{25}{384}$	$+\frac{625}{9216}$	$-\frac{9}{160}$	$-\frac{2401}{46080}$
$\left(\frac{r}{a}\right)^{3}$		$+\frac{15}{256}$	$-\frac{875}{6144}$	$+\frac{9}{160}$	$+\frac{343}{6144}$
$\left(\frac{r}{a}\right)^{4}$		$+\frac{19}{192}$	$-\frac{725}{4608}$	$+\frac{3}{40}$	$+\frac{1421}{23040}$

Table $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} f \ldots$

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} f$	$[\ ]^\circ$				$[\ ]^1$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$-\frac{3}{2}$	0	0	0	$+1$	$-\frac{1}{2}$	$-\frac{1}{64}$	$-\frac{29}{1152}$
	[cos − sin]	"	"	"	"	0	$+\frac{1}{8}$	$+\frac{1}{24}$	$+\frac{25}{1024}$
	[cos]	$-\frac{3}{2}$	0	0	0	H	$-\frac{3}{16}$	$+\frac{5}{384}$	$-\frac{7}{18432}$
	[sin]	0	0	0	0	H	$-\frac{5}{16}$	$-\frac{11}{384}$	$-\frac{457}{18432}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	-2	$-\frac{1}{2}$	0	0	$+1$	$+\frac{1}{2}$	$-\frac{25}{64}$	$-\frac{23}{1152}$
	[cos − sin]	"	"	"	"	0	$+\frac{5}{8}$	0	$+\frac{143}{3072}$
	[cos]	-2	$-\frac{1}{2}$	0	0	H	$+\frac{9}{16}$	$-\frac{25}{128}$	$+\frac{245}{18432}$
	[sin]	0	0	0	0	H	$-\frac{1}{16}$	$-\frac{25}{128}$	$-\frac{613}{18432}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$-\frac{5}{2}$	$-\frac{15}{8}$	0	0	$+1$	$+2$	$-\frac{41}{64}$	$-\frac{37}{576}$
	[cos − sin]	"	"	"	"	0	$+\frac{11}{8}$	$+\frac{7}{48}$	$+\frac{265}{3072}$
	[cos]	$-\frac{5}{2}$	$-\frac{15}{8}$	0	0	H	$+\frac{27}{16}$	$-\frac{95}{384}$	$+\frac{203}{18432}$
	[sin]	0	0	0	0	H	$+\frac{5}{16}$	$-\frac{151}{384}$	$-\frac{1387}{18432}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	-3	$-\frac{9}{2}$	$-\frac{3}{8}$	0	$+1$	$+4$	$-\frac{1}{64}$	$-\frac{199}{576}$
	[cos − sin]	"	"	"	"	0	$+\frac{19}{8}$	$+\frac{47}{48}$	$+\frac{115}{1024}$
	[cos]	-3	$-\frac{9}{2}$	$-\frac{3}{8}$	0	H	$+\frac{51}{16}$	$+\frac{185}{384}$	$-\frac{2149}{18432}$
	[sin]	0	0	0	0	H	$+\frac{13}{16}$	$-\frac{191}{384}$	$-\frac{4219}{18432}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) {\cos \atop \sin} f$ . . .

	${\cos \atop \sin} f$	$[\ ]^2$				$[\ ]^3$			$[\ ]^4$		
		e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{1}{2}$	$-\frac{3}{8}$	$+\frac{5}{96}$	$-\frac{1}{72}$	$+\frac{3}{8}$	$-\frac{3}{8}$	$+\frac{111}{1024}$	$+\frac{1}{3}$	$-\frac{5}{12}$	$+\frac{43}{240}$
	[cos − sin]	0	$+\frac{1}{24}$	$+\frac{1}{96}$	$+\frac{1}{120}$	0	$+\frac{3}{128}$	$+\frac{3}{1280}$	0	$+\frac{1}{60}$	$-\frac{1}{720}$
	[cos]	H	$-\frac{1}{6}$	$+\frac{1}{32}$	$-\frac{1}{360}$	H	$-\frac{45}{256}$	$+\frac{567}{10240}$	H	$-\frac{1}{5}$	$+\frac{4}{45}$
	[sin]	H	$-\frac{5}{24}$	$+\frac{1}{48}$	$-\frac{1}{90}$	H	$-\frac{51}{256}$	$+\frac{543}{10240}$	H	$-\frac{13}{60}$	$+\frac{13}{144}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	0	$+\frac{1}{2}$	$-\frac{1}{3}$	$+\frac{5}{96}$	$-\frac{1}{8}$	$+\frac{1}{2}$	$-\frac{397}{1024}$	$-\frac{1}{6}$	$+\frac{13}{24}$	$-\frac{47}{96}$
	[cos − sin]	0	$+\frac{1}{6}$	$-\frac{1}{24}$	$+\frac{7}{480}$	0	$+\frac{11}{128}$	$-\frac{11}{256}$	0	$+\frac{7}{120}$	$-\frac{7}{160}$
	[cos]	0	$+\frac{1}{3}$	$-\frac{3}{16}$	$+\frac{1}{30}$	H	$+\frac{75}{256}$	$-\frac{441}{2048}$	H	$+\frac{3}{10}$	$-\frac{4}{15}$
	[sin]	0	$+\frac{1}{6}$	$-\frac{7}{48}$	$+\frac{3}{160}$	H	$+\frac{53}{256}$	$-\frac{353}{2048}$	H	$+\frac{29}{120}$	$-\frac{107}{480}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$-\frac{1}{2}$	$+1$	$-\frac{35}{96}$	$+\frac{23}{576}$	$-\frac{3}{8}$	$+\frac{11}{16}$	$-\frac{351}{1024}$	$-\frac{7}{24}$	$+\frac{55}{96}$	$-\frac{701}{1920}$
	[cos − sin]	0	$+\frac{1}{6}$	$-\frac{1}{96}$	$+\frac{7}{960}$	0	$+\frac{7}{128}$	$-\frac{9}{640}$	0	$+\frac{13}{480}$	$-\frac{73}{5760}$
	[cos]	H	$+\frac{7}{12}$	$-\frac{3}{16}$	$+\frac{17}{720}$	H	$+\frac{95}{256}$	$-\frac{1827}{10240}$	H	$+\frac{3}{10}$	$-\frac{17}{90}$
	[sin]	H	$+\frac{5}{12}$	$-\frac{17}{96}$	$+\frac{47}{2880}$	H	$+\frac{81}{256}$	$-\frac{1683}{10240}$	H	$+\frac{131}{480}$	$-\frac{203}{1152}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	-1	$+\frac{3}{4}$	$+\frac{13}{48}$	$-\frac{83}{576}$	$-\frac{3}{8}$	$+\frac{3}{16}$	$+\frac{249}{1024}$	$-\frac{1}{6}$	$+\frac{1}{48}$	$+\frac{119}{480}$
	[cos − sin]	0	$-\frac{1}{12}$	$+\frac{1}{6}$	$-\frac{43}{960}$	0	$-\frac{9}{128}$	$+\frac{57}{640}$	0	$-\frac{11}{240}$	$+\frac{91}{1440}$
	[cos]	H	$+\frac{1}{3}$	$+\frac{7}{32}$	$-\frac{17}{180}$	H	$+\frac{15}{256}$	$+\frac{1701}{10240}$	H	$-\frac{1}{80}$	$+\frac{7}{45}$
	[sin]	H	$+\frac{5}{12}$	$+\frac{5}{96}$	$-\frac{143}{2880}$	H	$+\frac{33}{256}$	$+\frac{789}{10240}$	H	$+\frac{1}{30}$	$+\frac{133}{1440}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) \begin{matrix}\cos\\ \sin\end{matrix} f.$

	$\begin{matrix}\cos\\ \sin\end{matrix} f$	[ ]^5		[ ]^6		[ ]^7	[ ]^8
		e^4	e^6	e^5	e^7	e^6	e^7
$\left(\frac{r}{a}\right)^1$	[cos+sin]	$+\frac{125}{384}$	$-\frac{125}{256}$	$+\frac{27}{80}$	$-\frac{189}{320}$	$+\frac{16807}{46080}$	$+\frac{128}{315}$
	[cos−sin]	0	$+\frac{125}{9216}$	0	$+\frac{27}{2240}$	0	0
	[cos]	H	$-\frac{4375}{18432}$	H	$-\frac{81}{280}$	H	H
	[sin]	H	$-\frac{4625}{18432}$	H	$-\frac{135}{448}$	H	H
$\left(\frac{r}{a}\right)^2$	[cos+sin]	$-\frac{25}{128}$	$+\frac{475}{768}$	$-\frac{9}{40}$	$+\frac{117}{160}$	$-\frac{2401}{9216}$	$-\frac{32}{105}$
	[cos−sin]	0	$+\frac{425}{9216}$	0	$+\frac{9}{224}$	0	0
	[cos]	H	$+\frac{6125}{18432}$	H	$+\frac{27}{70}$	H	H
	[sin]	H	$+\frac{5275}{18432}$	H	$+\frac{387}{1120}$	H	H
$\left(\frac{r}{a}\right)^3$	[cos+sin]	$-\frac{95}{384}$	$+\frac{205}{384}$	$-\frac{9}{40}$	$+\frac{171}{320}$	$-\frac{9947}{46080}$	$-\frac{68}{315}$
	[cos−sin]	0	$+\frac{155}{9216}$	0	$+\frac{27}{2240}$	0	0
	[cos]	H	$+\frac{5075}{18432}$	H	$+\frac{153}{560}$	H	H
	[sin]	H	$+\frac{4765}{18432}$	H	$+\frac{117}{448}$	H	H
$\left(\frac{r}{a}\right)^4$	[cos+sin]	$-\frac{31}{384}$	$-\frac{7}{128}$	$-\frac{3}{80}$	$-\frac{33}{320}$	$-\frac{539}{46080}$	$+\frac{2}{315}$
	[cos−sin]	0	$-\frac{301}{9216}$	0	$-\frac{57}{2240}$	0	0
	[cos]	H	$-\frac{805}{18432}$	H	$-\frac{9}{140}$	H	H
	[sin]	H	$-\frac{203}{18432}$	H	$-\frac{87}{2240}$	H	H

Table $\left(\left(\frac{r}{a}\right)^{-1}\dots\left(\frac{r}{a}\right)^{-5}\right)\begin{matrix}\cos\\ \sin\end{matrix} f\dots$

	$\begin{matrix}\cos\\ \sin\end{matrix} f$	$[\;]^0$				$[\;]^1$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$-\frac{1}{2}$	$-\frac{1}{8}$	$-\frac{1}{16}$	$-\frac{5}{128}$	$+1$	-1	$-\frac{1}{64}$	$-\frac{17}{288}$
	[cos − sin]	"	"	"	"	0	$-\frac{1}{8}$	$-\frac{1}{16}$	$-\frac{115}{3072}$
	[cos]	$-\frac{1}{2}$	$-\frac{1}{8}$	$-\frac{1}{16}$	$-\frac{5}{128}$	H	$-\frac{9}{16}$	$-\frac{5}{128}$	$-\frac{889}{18432}$
	[sin]	0	0	0	0	H	$-\frac{7}{16}$	$+\frac{3}{128}$	$-\frac{199}{18432}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	0	0	0	0	$+1$	$-\frac{1}{2}$	$-\frac{1}{64}$	$-\frac{29}{1152}$
	[cos − sin]	"	"	"	"	0	$+\frac{1}{8}$	$+\frac{1}{24}$	$+\frac{25}{1024}$
	[cos]	0	0	0	0	H	$-\frac{3}{16}$	$+\frac{5}{384}$	$-\frac{7}{18432}$
	[sin]	0	0	0	0	H	$-\frac{5}{16}$	$-\frac{11}{384}$	$-\frac{457}{18432}$
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{1}{2}$	$+\frac{3}{4}$	$+\frac{15}{16}$	$+\frac{35}{32}$	$+1$	$+\frac{1}{2}$	$+\frac{55}{64}$	$+\frac{1177}{1152}$
	[cos − sin]	"	"	"	"	0	$+\frac{5}{8}$	$+\frac{5}{6}$	$+\frac{3103}{3072}$
	[cos]	$+\frac{1}{2}$	$+\frac{3}{4}$	$+\frac{15}{16}$	$+\frac{35}{32}$	H	$+\frac{9}{16}$	$+\frac{325}{384}$	$+\frac{18725}{18432}$
	[sin]	0	0	0	0	H	$-\frac{1}{16}$	$+\frac{5}{384}$	$+\frac{107}{18432}$
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+1$	$+\frac{5}{2}$	$+\frac{35}{8}$	$+\frac{105}{16}$	$+1$	$+2$	$+\frac{239}{64}$	$+\frac{3323}{576}$
	[cos − sin]	"	"	"	"	0	$+\frac{11}{8}$	$+\frac{49}{16}$	$+\frac{15665}{3072}$
	[cos]	$+1$	$+\frac{5}{2}$	$+\frac{35}{8}$	$+\frac{105}{16}$	H	$+\frac{27}{16}$	$+\frac{435}{128}$	$+\frac{100163}{18432}$
	[sin]	0	0	0	0	H	$+\frac{5}{16}$	$+\frac{43}{128}$	$+\frac{6173}{18432}$
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{3}{2}$	$+\frac{45}{8}$	$+\frac{105}{8}$	$+\frac{1575}{64}$	$+1$	$+4$	$+\frac{647}{64}$	$+\frac{11465}{576}$
	[cos − sin]	"	"	"	"	0	$+\frac{19}{8}$	$+\frac{371}{48}$	$+\frac{17179}{1024}$
	[cos]	$+\frac{3}{2}$	$+\frac{45}{8}$	$+\frac{105}{8}$	$+\frac{1575}{64}$	H	$+\frac{51}{16}$	$+\frac{3425}{384}$	$+\frac{338051}{18432}$
	[sin]	0	0	0	0	H	$+\frac{13}{16}$	$+\frac{457}{384}$	$+\frac{28829}{18432}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) {}^{\cos}_{\sin} f$ . . .

	${}^{\cos}_{\sin} f$	$[\ ]^3$				$[\ ]^4$		
		e	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{3}{2}$	$-\frac{7}{4}$	$+\frac{31}{96}$	$-\frac{1}{12}$	$+\frac{17}{8}$	$-\frac{47}{16}$	$+\frac{1069}{1024}$
	[cos − sin]	o	$-\frac{1}{12}$	$-\frac{1}{24}$	$-\frac{7}{240}$	o	$-\frac{9}{128}$	$-\frac{9}{320}$
	[cos]	H	$-\frac{11}{12}$	$+\frac{9}{64}$	$-\frac{9}{160}$	H	$-\frac{385}{256}$	$+\frac{5201}{10240}$
	[sin]	H	$-\frac{5}{6}$	$+\frac{35}{192}$	$-\frac{13}{480}$	H	$-\frac{367}{256}$	$+\frac{5489}{10240}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+2$	$-\frac{3}{2}$	$+\frac{5}{24}$	$-\frac{1}{18}$	$+\frac{27}{8}$	$-\frac{27}{8}$	$+\frac{999}{1024}$
	[cos − sin]	o	$+\frac{1}{6}$	$+\frac{1}{24}$	$+\frac{1}{30}$	o	$+\frac{27}{128}$	$+\frac{27}{1280}$
	[cos]	H	$-\frac{2}{3}$	$+\frac{1}{8}$	$-\frac{1}{90}$	H	$-\frac{405}{256}$	$+\frac{5103}{10240}$
	[sin]	H	$-\frac{5}{6}$	$+\frac{1}{12}$	$-\frac{2}{45}$	H	$-\frac{459}{256}$	$+\frac{4887}{10240}$
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{5}{2}$	$-\frac{1}{8}$	$+\frac{103}{96}$	$+\frac{1255}{1152}$	$+\frac{39}{8}$	-2	$+\frac{1803}{1024}$
	[cos − sin]	o	$+\frac{19}{24}$	$+\frac{43}{48}$	$+\frac{2063}{1920}$	o	$+\frac{131}{128}$	$+\frac{237}{256}$
	[cos]	H	$+\frac{1}{3}$	$+\frac{63}{64}$	$+\frac{779}{720}$	H	$-\frac{125}{256}$	$+\frac{2751}{2048}$
	[sin]	H	$-\frac{11}{24}$	$+\frac{17}{192}$	$+\frac{43}{5760}$	H	$-\frac{387}{256}$	$+\frac{855}{2048}$
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+3$	$+\frac{11}{4}$	$+\frac{245}{48}$	$+\frac{463}{64}$	$+\frac{53}{8}$	$+\frac{39}{16}$	$+\frac{7041}{1024}$
	[cos − sin]	o	$+\frac{23}{12}$	$+\frac{89}{24}$	$+\frac{5663}{960}$	o	$+\frac{343}{128}$	$+\frac{2819}{640}$
	[cos]	H	$+\frac{7}{3}$	$+\frac{141}{32}$	$+\frac{197}{30}$	H	$+\frac{655}{256}$	$+\frac{57757}{10240}$
	[sin]	H	$+\frac{5}{12}$	$+\frac{67}{96}$	$+\frac{641}{960}$	H	$-\frac{31}{256}$	$+\frac{12653}{10240}$
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{7}{2}$	$+\frac{15}{2}$	$+\frac{1529}{96}$	$+\frac{16207}{576}$	$+\frac{69}{8}$	$+\frac{183}{16}$	$+\frac{23865}{1024}$
	[cos − sin]	o	$+\frac{11}{3}$	$+\frac{997}{96}$	$+\frac{20279}{960}$	o	$+\frac{711}{128}$	$+\frac{8733}{640}$
	[cos]	H	$+\frac{67}{12}$	$+\frac{421}{32}$	$+\frac{8867}{360}$	H	$+\frac{2175}{256}$	$+\frac{189189}{10240}$
	[sin]	H	$+\frac{23}{12}$	$+\frac{133}{48}$	$+\frac{10099}{2880}$	H	$+\frac{753}{256}$	$+\frac{49461}{10240}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} f$ . . .

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} f$		$[\ ]^4$			$[\ ]^5$	
		e^3	e^5	e^7	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{71}{24}$	$-\frac{229}{48}$	$+\frac{1169}{480}$	$+\frac{523}{128}$	$-\frac{1451}{192}$
	[cos − sin]	0	$-\frac{1}{15}$	$-\frac{1}{60}$	0	$-\frac{625}{9216}$
	[cos]	H	$-\frac{387}{160}$	$+\frac{387}{320}$	H	$-\frac{70273}{18432}$
	[sin]	H	$-\frac{1129}{480}$	$+\frac{1177}{960}$	H	$-\frac{69023}{18432}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+\frac{16}{3}$	$-\frac{20}{3}$	$+\frac{43}{15}$	$+\frac{3125}{384}$	$-\frac{3125}{256}$
	[cos − sin]	0	$+\frac{4}{15}$	$-\frac{1}{45}$	0	$+\frac{3125}{9216}$
	[cos]	H	$-\frac{16}{5}$	$+\frac{64}{45}$	H	$-\frac{109375}{18432}$
	[sin]	H	$-\frac{52}{15}$	$+\frac{13}{9}$	H	$-\frac{115625}{18432}$
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{103}{12}$	$-\frac{593}{96}$	$+\frac{721}{192}$	$+\frac{5485}{384}$	$-\frac{11053}{768}$
	[cos − sin]	0	$+\frac{643}{480}$	$+\frac{2569}{2880}$	0	$+\frac{16289}{9216}$
	[cos]	H	$-\frac{387}{160}$	$+\frac{1673}{720}$	H	$-\frac{116347}{18432}$
	[sin]	H	$-\frac{451}{120}$	$+\frac{4123}{2880}$	H	$-\frac{148925}{18432}$
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+\frac{77}{6}$	$-\frac{25}{48}$	$+\frac{4751}{480}$	$+\frac{2955}{128}$	$-\frac{3463}{384}$
	[cos − sin]	0	$+\frac{899}{240}$	$+\frac{2441}{480}$	0	$+\frac{48203}{9216}$
	[cos]	H	$+\frac{129}{80}$	$+\frac{899}{120}$	H	$-\frac{34909}{18432}$
	[sin]	H	$-\frac{32}{15}$	$+\frac{77}{32}$	H	$-\frac{131315}{18432}$
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{437}{24}$	$+\frac{1361}{96}$	$+\frac{63341}{1920}$	$+\frac{13505}{384}$	$+\frac{1541}{128}$
	[cos − sin]	0	$+\frac{3983}{480}$	$+\frac{100849}{5760}$	0	$+\frac{113027}{9216}$
	[cos]	H	$+\frac{899}{80}$	$+\frac{36359}{1440}$	H	$+\frac{223979}{18432}$
	[sin]	H	$+\frac{1411}{480}$	$+\frac{44587}{5760}$	H	$-\frac{2075}{18432}$

$\ldots$ *Table* $\left(\left(\frac{r}{a}\right)^{-1}\ldots\left(\frac{r}{a}\right)^{-5}\right)\begin{matrix}\cos\\ \sin\end{matrix} f.$

$\begin{matrix}\cos\\ \sin\end{matrix} f$		$[\ ]^6$		$[\ ]^7$	$[\ ]^8$
		e^5	e^7	e^6	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{899}{160}$	$-\frac{1879}{160}$	$+\frac{355081}{46080}$	$+\frac{47259}{4480}$
	[cos − sin]	o	$-\frac{81}{1120}$	o	o
	[cos]	H	$-\frac{6617}{1120}$	H	H
	[sin]	H	$-\frac{817}{140}$	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+\frac{243}{20}$	$-\frac{1701}{80}$	$+\frac{823543}{46080}$	$+\frac{8192}{315}$
	[cos − sin]	o	$+\frac{243}{560}$	o	o
	[cos]	H	$-\frac{729}{70}$	H	H
	[sin]	H	$-\frac{1215}{112}$	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{3669}{160}$	$-\frac{18847}{640}$	$+\frac{330911}{9216}$	$+\frac{556403}{10080}$
	[cos − sin]	o	$+\frac{2101}{896}$	o	o
	[cos]	H	$-\frac{7589}{560}$	H	H
	[sin]	H	$-\frac{71217}{4480}$	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+\frac{3167}{80}$	$-\frac{8999}{320}$	$+\frac{3024637}{46080}$	$+\frac{178331}{1680}$
	[cos − sin]	o	$+\frac{16337}{2240}$	o	o
	[cos]	H	$-\frac{729}{70}$	H	H
	[sin]	H	$-\frac{7933}{448}$	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{5127}{80}$	$-\frac{753}{320}$	$+\frac{5163949}{46080}$	$+\frac{3830591}{20160}$
	[cos − sin]	o	$+\frac{40263}{2240}$	o	o
	[cos]	H	$+\frac{2187}{280}$	H	H
	[sin]	H	$-\frac{22767}{2240}$	H	H

Table $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right)\genfrac{}{}{0pt}{}{\cos}{\sin} 2f \ldots$

	$\genfrac{}{}{0pt}{}{\cos}{\sin} 2f$	$[\ ]^0$			$[\ ]^1$			
		e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{3}{2}$	0	0	$-\frac{5}{2}$	$+\frac{33}{16}$	$-\frac{73}{384}$	$+\frac{881}{18432}$
	[cos − sin]	”	”	”	0	$+\frac{1}{48}$	$-\frac{41}{768}$	$-\frac{1159}{30720}$
	[cos]	$+\frac{3}{2}$	0	0	H	$+\frac{25}{24}$	$-\frac{187}{1536}$	$+\frac{29}{5760}$
	[sin]	0	0	0	H	$+\frac{49}{48}$	$-\frac{35}{512}$	$+\frac{3941}{92160}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{5}{2}$	0	0	-3	$+\frac{13}{8}$	$+\frac{5}{192}$	$+\frac{227}{3072}$
	[cos − sin]	”	”	”	0	$-\frac{7}{24}$	$-\frac{47}{384}$	$-\frac{1091}{15360}$
	[cos]	$+\frac{5}{2}$	0	0	H	$+\frac{2}{3}$	$-\frac{37}{768}$	$+\frac{11}{7680}$
	[sin]	0	0	0	H	$+\frac{23}{24}$	$+\frac{19}{256}$	$+\frac{371}{5120}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{15}{4}$	$+\frac{5}{8}$	0	$-\frac{7}{2}$	$+\frac{1}{16}$	$+\frac{289}{384}$	$+\frac{1645}{18432}$
	[cos − sin]	”	”	”	”	$-\frac{47}{48}$	$-\frac{151}{768}$	$-\frac{4043}{30720}$
	[cos]	$+\frac{15}{4}$	$+\frac{5}{8}$	0	H	$-\frac{11}{24}$	$+\frac{427}{1536}$	$-\frac{61}{2880}$
	[sin]	0	0	0	H	$+\frac{25}{48}$	$+\frac{243}{512}$	$+\frac{10177}{92160}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{21}{4}$	$+\frac{21}{8}$	0	-4	-3	$+\frac{79}{48}$	$+\frac{233}{1152}$
	[cos − sin]	”	”	”	”	$-\frac{13}{6}$	$-\frac{13}{24}$	$-\frac{989}{3840}$
	[cos]	$+\frac{21}{4}$	$+\frac{21}{8}$	0	H	$-\frac{31}{12}$	$+\frac{53}{96}$	$-\frac{637}{23040}$
	[sin]	0	0	0	H	$-\frac{5}{12}$	$+\frac{35}{32}$	$+\frac{5297}{23040}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4\right) \begin{matrix}\cos\\ \sin\end{matrix}\, 2f$ . . .

$\begin{matrix}\cos\\ \sin\end{matrix}\, 2f$		$[\ ]^2$				$[\ ]^3$			
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+1$	$-\frac{7}{2}$	$+\frac{47}{16}$	$-\frac{215}{288}$	$+\frac{3}{2}$	$-\frac{75}{16}$	$+\frac{573}{128}$	$-\frac{3507}{2048}$
	[cos − sin]	0	0	$+\frac{1}{48}$	$-\frac{3}{160}$	0	0	$+\frac{21}{1280}$	$-\frac{123}{10240}$
	[cos]	H	H	$+\frac{71}{48}$	$-\frac{551}{1440}$	H	H	$+\frac{5751}{2560}$	$-\frac{8829}{10240}$
	[sin]	H	H	$+\frac{35}{24}$	$-\frac{131}{360}$	H	H	$+\frac{5709}{2560}$	$-\frac{4353}{5120}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+1$	$-\frac{5}{2}$	$+\frac{23}{16}$	$-\frac{65}{288}$	$+1$	$-\frac{19}{8}$	$+\frac{107}{64}$	$-\frac{479}{1024}$
	[cos − sin]	0	0	$-\frac{1}{16}$	$-\frac{11}{480}$	0	0	$-\frac{17}{640}$	$-\frac{7}{1024}$
	[cos]	H	H	$+\frac{11}{16}$	$-\frac{179}{1440}$	H	H	$+\frac{1053}{1280}$	$-\frac{243}{1024}$
	[sin]	H	H	$+\frac{3}{4}$	$-\frac{73}{720}$	H	H	$+\frac{1087}{1280}$	$-\frac{59}{256}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+1$	-1	$-\frac{11}{16}$	$+\frac{43}{72}$	$+\frac{1}{2}$	$-\frac{3}{16}$	$-\frac{117}{128}$	$+\frac{1553}{2048}$
	[cos − sin]	0	0	$-\frac{7}{48}$	$+\frac{1}{240}$	0	0	$-\frac{69}{1280}$	$+\frac{137}{10240}$
	[cos]	H	H	$-\frac{5}{12}$	$+\frac{433}{1440}$	H	H	$-\frac{1239}{2560}$	$+\frac{3951}{10240}$
	[sin]	H	H	$-\frac{13}{48}$	$+\frac{427}{1440}$	H	H	$-\frac{1101}{2560}$	$+\frac{1907}{5120}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+1$	$+1$	$-\frac{43}{16}$	$+\frac{35}{36}$		$+\frac{3}{2}$	$-\frac{9}{4}$	$+\frac{267}{256}$
	[cos − sin]	0	0	$-\frac{5}{48}$	0		0	$-\frac{3}{160}$	$+\frac{3}{640}$
	[cos]	H	H	$-\frac{67}{48}$	$+\frac{35}{72}$		H	$-\frac{363}{320}$	$+\frac{1341}{2560}$
	[sin]	H	H	$-\frac{31}{24}$	$+\frac{35}{72}$		H	$-\frac{357}{320}$	$+\frac{1329}{2560}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) \begin{matrix}\cos\\ \sin\end{matrix}\, 2f$ . . .

$\begin{matrix}\cos\\ \sin\end{matrix}\, 2f$		$[\ ]^4$			$[\ ]^5$		
		e^2	e^4	e^6	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+2$	$-\frac{19}{3}$	$+\frac{55}{8}$	$+\frac{125}{48}$	$-\frac{6625}{768}$	$+\frac{64625}{6144}$
	[cos − sin]	0	0	$+\frac{1}{72}$	0	0	$+\frac{1625}{129024}$
	[cos]	H	H	$+\frac{31}{9}$	H	H	$+\frac{679375}{129024}$
	[sin]	H	H	$+\frac{247}{72}$	H	H	$+\frac{338875}{64512}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+1$	$-\frac{5}{2}$	$+\frac{101}{48}$	$+\frac{25}{24}$	$-\frac{1075}{384}$	$+\frac{8425}{3072}$
	[cos − sin]	0	0	$-\frac{11}{720}$	0	0	$-\frac{75}{7168}$
	[cos]	H	H	$+\frac{47}{45}$	H	H	$+\frac{29375}{21504}$
	[sin]	H	H	$+\frac{763}{720}$	H	H	$+\frac{925}{672}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{1}{4}$	$+\frac{5}{24}$	$-\frac{227}{192}$	$+\frac{5}{48}$	$+\frac{385}{768}$	$-\frac{9515}{6144}$
	[cos − sin]	0	0	$-\frac{83}{2880}$	0	0	$-\frac{2435}{129024}$
	[cos]	H	H	$-\frac{109}{180}$	H	H	$-\frac{101125}{129024}$
	[sin]	H	H	$-\frac{1661}{2880}$	H	H	$-\frac{49345}{64512}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$-\frac{1}{4}$	$+\frac{37}{24}$	$-\frac{139}{64}$	$-\frac{1}{3}$	$+\frac{151}{96}$	$-\frac{877}{384}$
	[cos − sin]	0	0	$-\frac{17}{2880}$	0	0	$-\frac{41}{16128}$
	[cos]	H	H	$-\frac{49}{45}$	H	H	$-\frac{36875}{32256}$
	[sin]	H	H	$-\frac{3119}{2880}$	H	H	$-\frac{36793}{32256}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 2f$.

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 2f$		$[\ ]^6$		$[\ ]^7$		$[\ ]^8$	$[\ ]^9$
		e^4	e^6	e^5	e^7	e^6	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{27}{8}$	$-\frac{189}{16}$	$+\frac{16807}{3840}$	$-\frac{1495823}{92160}$	$+\frac{256}{45}$	$+\frac{531441}{71680}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{9}{8}$	$-\frac{261}{80}$	$+\frac{2401}{1920}$	$-\frac{12005}{3072}$	$+\frac{64}{45}$	$+\frac{59049}{35840}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	0	$+\frac{63}{80}$	$-\frac{343}{3840}$	$+\frac{102557}{92160}$	$-\frac{8}{45}$	$-\frac{19683}{71680}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	0	H	H	H	H	H
	[sin]	0	H	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$-\frac{3}{8}$	$+\frac{33}{20}$	$-\frac{49}{120}$	$+\frac{20531}{11520}$	$-\frac{4}{9}$	$-\frac{2187}{4480}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H

Table $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \begin{matrix}\cos\\ \sin\end{matrix}\, 2f \ldots$

$\begin{matrix}\cos\\ \sin\end{matrix}\, 2f$		$[\;]^0$			$[\;]^1$			
		e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{1}{4}$	$+\frac{1}{8}$	$+\frac{5}{64}$	$-\frac{3}{2}$	$+\frac{17}{16}$	$+\frac{37}{384}$	$+\frac{197}{2048}$
	[cos − sin]	"	"	"	0	$+\frac{1}{48}$	$+\frac{29}{768}$	$+\frac{1013}{30720}$
	[cos]	$+\frac{1}{4}$	$+\frac{1}{8}$	$+\frac{5}{64}$	H	$+\frac{13}{24}$	$+\frac{103}{1536}$	$+\frac{31}{480}$
	[sin]	0	0	0	H	$+\frac{25}{48}$	$+\frac{15}{512}$	$+\frac{971}{30720}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	0	0	0	-1	$+\frac{3}{8}$	$+\frac{7}{192}$	$+\frac{295}{9216}$
	[cos − sin]	"	"	"	0	$-\frac{1}{24}$	$-\frac{13}{384}$	$-\frac{389}{15360}$
	[cos]	0	0	0	H	$+\frac{1}{6}$	$+\frac{1}{768}$	$+\frac{77}{23040}$
	[sin]	0	0	0	H	$+\frac{5}{24}$	$+\frac{9}{256}$	$+\frac{1321}{46080}$
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	0	0	0	$-\frac{1}{2}$	$+\frac{1}{16}$	$-\frac{5}{384}$	$-\frac{143}{18432}$
	[cos − sin]	"	"	"	0	$+\frac{1}{48}$	$+\frac{11}{768}$	$+\frac{313}{30720}$
	[cos]	0	0	0	H	$+\frac{1}{24}$	$+\frac{1}{1536}$	$+\frac{7}{5760}$
	[sin]	0	0	0	H	$+\frac{1}{48}$	$-\frac{7}{512}$	$-\frac{827}{92160}$
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+\frac{1}{4}$	$+\frac{5}{8}$	$+\frac{35}{32}$	0	$+\frac{1}{2}$	$+\frac{11}{12}$	$+\frac{1099}{768}$
	[cos − sin]	"	"	"	0	$+\frac{1}{3}$	$+\frac{73}{96}$	$+\frac{2441}{1920}$
	[cos]	$+\frac{1}{4}$	$+\frac{5}{8}$	$+\frac{35}{32}$	0	$+\frac{5}{12}$	$+\frac{161}{192}$	$+\frac{3459}{2560}$
	[sin]	0	0	0	0	$+\frac{1}{12}$	$+\frac{5}{64}$	$+\frac{613}{7680}$
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{3}{4}$	$+\frac{21}{8}$	$+\frac{189}{32}$	$+\frac{1}{2}$	$+\frac{33}{16}$	$+\frac{1865}{384}$	$+\frac{169229}{18432}$
	[cos − sin]	"	"	"	0	$+\frac{49}{48}$	$+\frac{2545}{768}$	$+\frac{219221}{30720}$
	[cos]	$+\frac{3}{4}$	$+\frac{21}{8}$	$+\frac{189}{32}$	H	$+\frac{37}{24}$	$+\frac{6275}{1536}$	$+\frac{23497}{2880}$
	[sin]	0	0	0	H	$+\frac{25}{48}$	$+\frac{395}{512}$	$+\frac{94241}{92160}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \begin{matrix}\cos\\ \sin\end{matrix}\ 2f$ . . .

$\begin{matrix}\cos\\ \sin\end{matrix}\ 2f$		$[\ ]^2$				$[\ ]^3$			
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+1$	-4	$+\frac{47}{16}$	$-\frac{55}{144}$	$+\frac{5}{2}$	$-\frac{131}{16}$	$+\frac{887}{128}$	$-\frac{3999}{2048}$
	[cos − sin]	o	o	o	$+\frac{1}{48}$	o	o	$-\frac{9}{1280}$	$+\frac{153}{10240}$
	[cos]	H	H	H	$-\frac{13}{72}$	H	H	$+\frac{8861}{2560}$	$-\frac{9921}{10240}$
	[sin]	H	H	H	$-\frac{29}{144}$	H	H	$+\frac{8879}{2560}$	$-\frac{5037}{5120}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+1$	$-\frac{7}{2}$	$+\frac{29}{16}$	$-\frac{53}{288}$	$+3$	$-\frac{69}{8}$	$+\frac{369}{64}$	$-\frac{1401}{1024}$
	[cos − sin]	o	o	$-\frac{1}{24}$	$-\frac{3}{80}$	o	o	$-\frac{27}{640}$	$-\frac{189}{5120}$
	[cos]	H	H	$+\frac{85}{96}$	$-\frac{319}{2880}$	H	H	$+\frac{3663}{1280}$	$-\frac{3597}{5120}$
	[sin]	H	H	$+\frac{89}{96}$	$-\frac{211}{2880}$	H	H	$+\frac{3717}{1280}$	$-\frac{213}{320}$
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+1$	$-\frac{5}{2}$	$+\frac{13}{16}$	$-\frac{35}{288}$	$+\frac{7}{2}$	$-\frac{123}{16}$	$+\frac{489}{128}$	$-\frac{1763}{2048}$
	[cos − sin]	o	o	$+\frac{1}{24}$	$+\frac{7}{240}$	o	o	$+\frac{81}{1280}$	$+\frac{81}{2048}$
	[cos]	H	H	$+\frac{41}{96}$	$-\frac{133}{2880}$	H	H	$+\frac{4971}{2560}$	$-\frac{841}{2048}$
	[sin]	H	H	$+\frac{37}{96}$	$-\frac{217}{2880}$	H	H	$+\frac{4809}{2560}$	$-\frac{461}{1024}$
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+1$	-1	$+\frac{17}{16}$	$+\frac{85}{72}$	$+4$	-5	$+\frac{51}{16}$	$+\frac{143}{128}$
	[cos − sin]	o	o	$+\frac{7}{16}$	$+\frac{109}{120}$	o	o	$+\frac{23}{40}$	$+\frac{1369}{1280}$
	[cos]	H	H	$+\frac{3}{4}$	$+\frac{47}{45}$	H	H	$+\frac{301}{160}$	$+\frac{2799}{2560}$
	[sin]	H	H	$+\frac{5}{16}$	$+\frac{49}{360}$	H	H	$+\frac{209}{160}$	$+\frac{61}{2560}$
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+1$	$+1$	$+\frac{65}{16}$	$+\frac{139}{18}$	$+\frac{9}{2}$	$-\frac{3}{16}$	$+\frac{963}{128}$	$+\frac{22737}{2048}$
	[cos − sin]	o	o	$+\frac{67}{48}$	$+\frac{333}{80}$	o	o	$+\frac{2451}{1280}$	$+\frac{53193}{10240}$
	[cos]	H	H	$+\frac{131}{48}$	$+\frac{8557}{1440}$	H	H	$+\frac{12081}{2560}$	$+\frac{83439}{10240}$
	[sin]	H	H	$+\frac{4}{3}$	$+\frac{2563}{1440}$	H	H	$+\frac{7179}{2560}$	$+\frac{15123}{5120}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, 2f$ . . .

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, 2f$		$[\ ]^4$			$[\ ]^5$		
		e^2	e^4	e^6	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos+sin]	$+\frac{19}{4}$	$-\frac{121}{8}$	$+\frac{707}{48}$	$+\frac{389}{48}$	$-\frac{20267}{768}$	$+\frac{179141}{6144}$
	[cos−sin]	0	0	$-\frac{1}{90}$	0	0	$-\frac{625}{43008}$
	[cos]	H	H	$+\frac{10597}{1440}$	H	H	$+\frac{626681}{43008}$
	[sin]	H	H	$+\frac{10613}{1440}$	H	H	$+\frac{104551}{7168}$
$\left(\frac{r}{a}\right)^{-2}$	[cos+sin]	$+\frac{13}{2}$	$-\frac{55}{3}$	$+\frac{239}{16}$	$+\frac{295}{24}$	$-\frac{13745}{384}$	$+\frac{105175}{3072}$
	[cos−sin]	0	0	$-\frac{2}{45}$	0	0	$-\frac{3125}{64512}$
	[cos]	H	H	$+\frac{10723}{1440}$	H	H	$+\frac{1102775}{64512}$
	[sin]	H	H	$+\frac{10787}{1440}$	H	H	$+\frac{276475}{16128}$
$\left(\frac{r}{a}\right)^{-3}$	[cos+sin]	$+\frac{17}{2}$	$-\frac{115}{6}$	$+\frac{601}{48}$	$+\frac{845}{48}$	$-\frac{32525}{768}$	$+\frac{208225}{6144}$
	[cos−sin]	0	0	$+\frac{4}{45}$	0	0	$+\frac{15625}{129024}$
	[cos]	H	H	$+\frac{9079}{1440}$	H	H	$+\frac{2194175}{129024}$
	[sin]	H	H	$+\frac{8951}{1440}$	H	H	$+\frac{1089275}{64512}$
$\left(\frac{r}{a}\right)^{-4}$	[cos+sin]	$+\frac{43}{4}$	$-\frac{129}{8}$	$+\frac{1985}{192}$	$+\frac{145}{6}$	$-\frac{1015}{24}$	$+\frac{23581}{768}$
	[cos−sin]	0	0	$+\frac{437}{576}$	0	0	$+\frac{2701}{2688}$
	[cos]	H	H	$+\frac{799}{144}$	H	H	$+\frac{56823}{3584}$
	[sin]	H	H	$+\frac{2759}{576}$	H	H	$+\frac{159665}{10752}$
$\left(\frac{r}{a}\right)^{-5}$	[cos+sin]	$+\frac{53}{4}$	$-\frac{179}{24}$	$+\frac{977}{64}$	$+\frac{1541}{48}$	$-\frac{23479}{768}$	$+\frac{216917}{6144}$
	[cos−sin]	0	0	$+\frac{7579}{2880}$	0	0	$+\frac{467069}{129024}$
	[cos]	H	H	$+\frac{6443}{720}$	H	H	$+\frac{2511163}{129024}$
	[sin]	H	H	$+\frac{18193}{2880}$	H	H	$+\frac{1022047}{64512}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, 2f.$

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, 2f$		$[\ ]^6$		$[\ ]^7$		$[\ ]^8$	$[\ ]^9$
		e^4	e^6	e^5	e^7	e^6	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{209}{16}$	$-\frac{887}{20}$	$+\frac{78077}{3840}$	$-\frac{2228929}{30720}$	$+\frac{17807}{576}$	$+\frac{3313213}{71680}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+\frac{345}{16}$	$-\frac{10569}{160}$	$+\frac{69251}{1920}$	$-\frac{5394109}{46080}$	$+\frac{42037}{720}$	$+\frac{3306951}{35840}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{533}{16}$	$-\frac{13827}{160}$	$+\frac{228347}{3840}$	$-\frac{3071075}{18432}$	$+\frac{73369}{720}$	$+\frac{12144273}{71680}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+49$	-98	$+\frac{44569}{480}$	$-\frac{133707}{640}$	$+\frac{241517}{1440}$	$+\frac{2619177}{8960}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{555}{8}$	$-\frac{7149}{80}$	$+\frac{533617}{3840}$	$-\frac{20449667}{92160}$	$+\frac{379691}{1440}$	$+\frac{34431741}{71680}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H

Table $\left(\left(\frac{r}{a}\right) \cdots \left(\frac{r}{a}\right)^4\right) {}^{\cos}_{\sin} 3f \ldots$

	${}^{\cos}_{\sin} 3f$	$[\ ]^0$			$[\ ]^1$		
		e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$-\frac{5}{4}$	$-\frac{5}{32}$	$-\frac{3}{64}$	$+\frac{31}{8}$	$-\frac{77}{24}$	$+\frac{155}{1024}$
	[cos − sin]	„	„	„	0	$-\frac{19}{384}$	$+\frac{3}{256}$
	[cos]	$-\frac{5}{4}$	$-\frac{5}{32}$	$-\frac{3}{64}$	H	$-\frac{417}{256}$	$+\frac{167}{2048}$
	[sin]	0	0	0	H	$-\frac{1213}{768}$	$+\frac{143}{2048}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$-\frac{5}{2}$	0	0	$+\frac{43}{8}$	$-\frac{25}{6}$	$+\frac{1069}{3072}$
	[cos − sin]	„	„	„	0	$+\frac{37}{384}$	$+\frac{487}{3840}$
	[cos]	$-\frac{5}{2}$	0	0	H	$-\frac{521}{256}$	$+\frac{2431}{10240}$
	[sin]	0	0	0	H	$-\frac{1637}{768}$	$+\frac{3397}{30720}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$-\frac{35}{8}$	0	0	$+\frac{57}{8}$	$-\frac{65}{16}$	$-\frac{19}{1024}$
	[cos − sin]	„	„	„	0	$+\frac{75}{128}$	$+\frac{179}{640}$
	[cos]	$-\frac{35}{8}$	0	0	H	$-\frac{445}{256}$	$+\frac{1337}{10240}$
	[sin]	0	0	0	H	$-\frac{595}{256}$	$-\frac{1527}{10240}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	-7	$-\frac{7}{8}$	0	$+\frac{73}{8}$	$-\frac{91}{48}$	$-\frac{1419}{1024}$
	[cos − sin]	„	„	„	0	$+\frac{641}{384}$	$+\frac{341}{640}$
	[cos]	-7	$-\frac{7}{8}$	0	H	$-\frac{29}{256}$	$-\frac{4367}{10240}$
	[sin]	0	0	0	H	$-\frac{1369}{768}$	$-\frac{9823}{10240}$

. . . *Table* $\left(\left(\frac{r}{a}\right) \ldots \left(\frac{r}{a}\right)^4\right) {}^{\cos}_{\sin} 3f$ . . .

	${}^{\cos}_{\sin} 3f$	$[\ ]^2$				$[\ ]^3$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$-\frac{7}{2}$	$+\frac{75}{8}$	$-\frac{659}{96}$	$+\frac{1687}{1152}$	$+1$	$-\frac{17}{2}$	$+\frac{1143}{64}$	$+\frac{1805}{128}$
	[cos − sin]	0	0	$-\frac{11}{480}$	$-\frac{13}{5760}$	0	0	0	$-\frac{69}{5120}$
	[cos]	H	H	$-\frac{551}{160}$	$+\frac{4211}{5760}$	H	H	H	$-\frac{72269}{10240}$
	[sin]	H	H	$-\frac{821}{240}$	$+\frac{11}{15}$	H	H	H	$-\frac{72131}{10240}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	-4	$+\frac{19}{2}$	$-\frac{79}{12}$	$+\frac{113}{72}$	$+1$	$-\frac{15}{2}$	$+\frac{927}{64}$	$-\frac{1415}{128}$
	[cos − sin]	0	0	$+\frac{1}{120}$	$+\frac{1}{36}$	0	0	0	$-\frac{1}{5120}$
	[cos]	H	H	$-\frac{263}{80}$	$+\frac{115}{144}$	H	H	H	$-\frac{56601}{10240}$
	[sin]	H	H	$-\frac{791}{240}$	$+\frac{37}{48}$	H	H	H	$-\frac{56599}{10240}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$-\frac{9}{2}$	$+\frac{33}{4}$	$-\frac{133}{32}$	$+\frac{11}{16}$	$+1$	-6	$+\frac{591}{64}$	$-\frac{349}{64}$
	[cos − sin]	0	0	$+\frac{13}{160}$	$+\frac{1}{30}$	0	0	0	$+\frac{137}{5120}$
	[cos]	H	H	$-\frac{163}{80}$	$+\frac{173}{480}$	H	H	H	$-\frac{27783}{10240}$
	[sin]	H	H	$-\frac{339}{160}$	$+\frac{157}{480}$	H	H	H	$-\frac{28057}{10240}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	-5	$+\frac{21}{4}$	$+\frac{17}{48}$	$-\frac{71}{72}$	$+1$	-4	$+\frac{183}{64}$	$+\frac{65}{64}$
	[cos − sin]	0	0	$+\frac{2}{15}$	$+\frac{1}{720}$	0	0	0	$+\frac{37}{1024}$
	[cos]	H	H	$+\frac{39}{160}$	$-\frac{709}{1440}$	H	H	H	$+\frac{1077}{2048}$
	[sin]	H	H	$+\frac{53}{480}$	$-\frac{79}{160}$	H	H	H	$+\frac{1003}{2048}$

. . . *Table* $\left(\left(\frac{r}{a}\right) \ldots \left(\frac{r}{a}\right)^4\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, 3f$ . . .

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, 3f$		$[\ ]^4$				$[\ ]^5$		
		e	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{5}{2}$	$-\frac{63}{4}$	$+\frac{757}{24}$	$-\frac{1979}{72}$	$+\frac{37}{8}$	$-\frac{635}{24}$	$+\frac{54765}{1024}$
	[cos − sin]	0	0	0	$-\frac{47}{5040}$	0	0	0
	[cos]	H	H	H	$-\frac{138577}{10080}$	H	H	H
	[sin]	H	H	H	$-\frac{15387}{1120}$	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+2$	$-\frac{23}{2}$	$+\frac{65}{3}$	$-\frac{5317}{288}$	$+\frac{25}{8}$	$-\frac{50}{3}$	$+\frac{98875}{3072}$
	[cos − sin]	0	0	0	$-\frac{17}{10080}$	0	0	0
	[cos]	H	H	H	$-\frac{2908}{315}$	H	H	H
	[sin]	H	H	H	$-\frac{31013}{3360}$	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{3}{2}$	$-\frac{57}{8}$	$+11$	$-\frac{2905}{384}$	$+\frac{15}{8}$	$-\frac{135}{16}$	$+\frac{13975}{1024}$
	[cos − sin]	0	0	0	$+\frac{57}{4480}$	0	0	0
	[cos]	H	H	H	$-\frac{793}{210}$	H	H	H
	[sin]	H	H	H	$-\frac{50923}{13440}$	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+1$	-3	$+\frac{37}{24}$	$+\frac{251}{144}$	$+\frac{7}{8}$	$-\frac{109}{48}$	$+\frac{535}{1024}$
	[cos − sin]	0	0	0	$+\frac{79}{5040}$	0	0	0
	[cos]	H	H	H	$+\frac{277}{315}$	H	H	H
	[sin]	H	H	H	$+\frac{1451}{1680}$	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right) \ldots \left(\frac{r}{a}\right)^4\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\ 3f$ . . .

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\ 3f$		$[\ ]^6$			$[\ ]^7$	
		e^3	e^5	e^7	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{61}{8}$	$-\frac{677}{16}$	$+\frac{56487}{640}$	$+\frac{4553}{384}$	$-\frac{84109}{1280}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{9}{2}$	$-\frac{189}{8}$	$+\frac{3807}{80}$	$+\frac{2401}{384}$	$-\frac{127253}{3840}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{9}{4}$	$-\frac{81}{8}$	$+\frac{2781}{160}$	$+\frac{343}{128}$	$-\frac{7889}{640}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{3}{4}$	$-\frac{27}{16}$	$-\frac{81}{160}$	$+\frac{245}{384}$	$-\frac{147}{128}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right) \ldots \left(\frac{r}{a}\right)^4\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 3f.$

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 3f$		$[\ ]^8$		$[\ ]^9$	$[\ ]^{10}$
		e^5	e^7	e^6	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{8551}{480}$	$-\frac{288221}{2880}$	$+\frac{26809}{1024}$	$+\frac{305593}{8064}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{128}{15}$	$-\frac{416}{9}$	$+\frac{59049}{5120}$	$+\frac{15625}{1008}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{16}{5}$	$-\frac{76}{5}$	$+\frac{19683}{5120}$	$+\frac{3125}{672}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{8}{15}$	$-\frac{26}{45}$	$+\frac{2187}{5120}$	$+\frac{625}{2016}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H

Table $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \begin{matrix}\cos\\ \sin\end{matrix}\ 3f \ldots$

	$\begin{matrix}\cos\\ \sin\end{matrix}\ 3f$	[ ]°			[ ]¹		
		e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$-\frac{1}{8}$	$-\frac{3}{32}$	$-\frac{9}{128}$	$+\frac{13}{8}$	$-\frac{41}{48}$	$-\frac{437}{3072}$
	[cos − sin]	,,	,,	,,	0	$+\frac{1}{384}$	$-\frac{11}{960}$
	[cos]	$-\frac{1}{8}$	$-\frac{3}{32}$	$-\frac{9}{128}$	H	$-\frac{109}{256}$	$-\frac{787}{10240}$
	[sin]	0	0	0	H	$-\frac{329}{768}$	$-\frac{2009}{30720}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	0	0	0	$+\frac{7}{8}$	$-\frac{5}{24}$	$-\frac{29}{1024}$
	[cos − sin]	,,	,,	,,	0	$+\frac{5}{384}$	$+\frac{23}{1280}$
	[cos]	0	0	0	H	$-\frac{25}{256}$	$-\frac{53}{10240}$
	[sin]	0	0	0	H	$-\frac{85}{768}$	$-\frac{237}{10240}$
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	0	0	0	$+\frac{3}{8}$	0	$+\frac{23}{1024}$
	[cos − sin]	,,	,,	,,	0	$-\frac{1}{128}$	$-\frac{11}{1280}$
	[cos]	0	0	0	H	$-\frac{1}{256}$	$+\frac{71}{10240}$
	[sin]	0	0	0	H	$+\frac{1}{256}$	$+\frac{159}{10240}$
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	0	0	0	$+\frac{1}{8}$	$+\frac{1}{48}$	$+\frac{55}{3072}$
	[cos − sin]	,,	,,	,,	0	$+\frac{1}{384}$	$+\frac{1}{384}$
	[cos]	0	0	0	H	$+\frac{3}{256}$	$+\frac{21}{2048}$
	[sin]	0	0	0	H	$+\frac{7}{768}$	$+\frac{47}{6144}$
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{1}{8}$	$+\frac{7}{16}$	$+\frac{63}{64}$	$+\frac{1}{8}$	$+\frac{17}{48}$	$+\frac{837}{1024}$
	[cos − sin]	,,	,,	,,	0	$+\frac{65}{384}$	$+\frac{353}{640}$
	[cos]	$+\frac{1}{8}$	$+\frac{7}{16}$	$+\frac{63}{64}$	H	$+\frac{67}{256}$	$+\frac{7009}{10240}$
	[sin]	0	0	0	H	$+\frac{71}{768}$	$+\frac{1361}{10240}$

... *Table* $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}\right) \substack{\cos \\ \sin} 3f$ ...

$\substack{\cos \\ \sin} 3f$		$[\ ]^2$				$[\ ]^3$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$-\frac{5}{2}$	$+\frac{13}{2}$	$-\frac{341}{96}$	$+\frac{187}{576}$	$+1$	-9	$+\frac{1143}{64}$	$-\frac{365}{32}$
	[cos − sin]	0	0	$+\frac{1}{120}$	$-\frac{1}{2880}$	0	0	0	$+\frac{9}{1024}$
	[cos]	H	H	$-\frac{567}{320}$	$+\frac{467}{2880}$	H	H	H	$-\frac{11671}{2048}$
	[sin]	H	H	$-\frac{1709}{960}$	$+\frac{13}{80}$	H	H	H	$-\frac{11689}{2048}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	-2	$+\frac{9}{2}$	$-\frac{41}{24}$	$+\frac{43}{288}$	$+1$	$-\frac{17}{2}$	$+\frac{951}{64}$	$-\frac{973}{128}$
	[cos − sin]	0	0	$+\frac{1}{120}$	$+\frac{23}{1440}$	0	0	0	$+\frac{27}{5120}$
	[cos]	H	H	$-\frac{17}{20}$	$+\frac{119}{1440}$	H	H	H	$-\frac{38893}{10240}$
	[sin]	H	H	$-\frac{103}{120}$	$+\frac{1}{15}$	H	H	H	$-\frac{38947}{10240}$
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$-\frac{3}{2}$	$+\frac{21}{8}$	$-\frac{19}{32}$	$+\frac{23}{192}$	$+1$	$-\frac{15}{2}$	$+\frac{687}{64}$	$-\frac{535}{128}$
	[cos − sin]	0	0	$-\frac{1}{80}$	$-\frac{1}{64}$	0	0	0	$-\frac{81}{5120}$
	[cos]	H	H	$-\frac{97}{320}$	$+\frac{5}{96}$	H	H	H	$-\frac{21481}{10240}$
	[sin]	H	H	$-\frac{93}{320}$	$+\frac{13}{192}$	H	H	H	$-\frac{21319}{10240}$
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	-1	$+\frac{5}{4}$	$-\frac{7}{48}$	$+\frac{23}{288}$	$+1$	-6	$+\frac{423}{64}$	$-\frac{125}{64}$
	[cos − sin]	0	0	$+\frac{1}{120}$	$+\frac{13}{1440}$	0	0	0	$+\frac{81}{5120}$
	[cos]	H	H	$-\frac{11}{160}$	$+\frac{2}{45}$	H	H	H	$-\frac{9919}{10240}$
	[sin]	H	H	$-\frac{37}{480}$	$+\frac{17}{480}$	H	H	H	$-\frac{10081}{10240}$
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$-\frac{1}{2}$	$+\frac{3}{4}$	$+\frac{61}{96}$	$+\frac{191}{144}$	$+1$	-4	$+\frac{255}{64}$	$+\frac{17}{64}$
	[cos − sin]	0	0	$+\frac{109}{480}$	$+\frac{31}{45}$	0	0	0	$+\frac{1553}{5120}$
	[cos]	H	H	$+\frac{69}{160}$	$+\frac{1451}{1440}$	H	H	H	$+\frac{2913}{10240}$
	[sin]	H	H	$+\frac{49}{240}$	$+\frac{51}{160}$	H	H	H	$-\frac{193}{10240}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\ 3f$ . . .

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\ 3f$	$[\ ]^4$				$[\ ]^5$		
		e	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{7}{2}$	$-\frac{179}{8}$	$+\frac{2009}{48}$	$-\frac{8893}{288}$	$+\frac{67}{8}$	$-\frac{2279}{48}$	$+\frac{274345}{3072}$
	[cos − sin]	0	0	0	$+\frac{11}{1260}$	0	0	0
	[cos]	H	H	H	$-\frac{311167}{20160}$	H	H	H
	[sin]	H	H	H	$-\frac{103781}{6720}$	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+4$	-24	$+\frac{241}{6}$	$-\frac{455}{18}$	$+\frac{85}{8}$	$-\frac{1355}{24}$	$+\frac{98525}{1024}$
	[cos − sin]	0	0	0	$+\frac{1}{315}$	0	0	0
	[cos]	H	H	H	$-\frac{15923}{1260}$	H	H	H
	[sin]	H	H	H	$-\frac{5309}{420}$	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{9}{2}$	-24	$+\frac{545}{16}$	$-\frac{71}{4}$	$+\frac{105}{8}$	$-\frac{125}{2}$	$+\frac{94825}{1024}$
	[cos − sin]	0	0	0	$-\frac{2}{105}$	0	0	0
	[cos]	H	H	H	$-\frac{7463}{840}$	H	H	H
	[sin]	H	H	H	$-\frac{7447}{840}$	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+5$	-22	$+\frac{607}{24}$	$-\frac{98}{9}$	$+\frac{127}{8}$	$-\frac{3065}{48}$	$+\frac{243805}{3072}$
	[cos − sin]	0	0	0	$+\frac{8}{315}$	0	0	0
	[cos]	H	H	H	$-\frac{1711}{315}$	H	H	H
	[sin]	H	H	H	$-\frac{191}{35}$	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{11}{2}$	$-\frac{141}{8}$	$+\frac{415}{24}$	$-\frac{5021}{1152}$	$+\frac{151}{8}$	$-\frac{2809}{48}$	$+\frac{63343}{1024}$
	[cos − sin]	0	0	0	$+\frac{16319}{40320}$	0	0	0
	[cos]	H	H	H	$-\frac{19927}{10080}$	H	H	H
	[sin]	H	H	H	$-\frac{32009}{13440}$	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 3f$ . . .

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 3f$	$[\ ]^6$			$[\ ]^7$	
		e^3	e^5	e^7	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+17$	$-\frac{2949}{32}$	$+\frac{57213}{320}$	$+\frac{12085}{384}$	$-\frac{32419}{192}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+\frac{47}{2}$	$-\frac{239}{2}$	$-\frac{33951}{160}$	$+\frac{17969}{384}$	$-\frac{301973}{1280}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{251}{8}$	$-\frac{4611}{32}$	$+\frac{36249}{160}$	$+\frac{8547}{128}$	$-\frac{391951}{1280}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+\frac{163}{4}$	$-\frac{2577}{16}$	$+\frac{1089}{5}$	$+\frac{35413}{384}$	$-\frac{709471}{1920}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{207}{4}$	$-\frac{2625}{16}$	$+\frac{30483}{160}$	$+\frac{47621}{384}$	$-\frac{262731}{640}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, 3f$.

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, 3f$	$[\ ]^8$		$[\ ]^9$	$[\ ]^{10}$
		e^5	e^7	e^6	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{26371}{480}$	$-\frac{1710983}{5760}$	$+\frac{471527}{5120}$	$+\frac{604279}{4032}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+\frac{2611}{30}$	$-\frac{39893}{90}$	$+\frac{790053}{5120}$	$+\frac{532345}{2016}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{10531}{80}$	$-\frac{7363}{12}$	$+\frac{1258449}{5120}$	$+\frac{98715}{224}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+\frac{23029}{120}$	$-\frac{35614}{45}$	$+\frac{385095}{1024}$	$+\frac{44377}{63}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{65153}{240}$	$-\frac{2725199}{2880}$	$+\frac{2850411}{5120}$	$+\frac{2190757}{2016}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H

Table $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) {\cos \atop \sin} 4f \ldots$

${\cos \atop \sin} 4f$		$[\ ]^0$		$[\ ]^1$			$[\ ]^2$		
		e^4	e^6	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{15}{16}$	$+\frac{9}{32}$	$-\frac{229}{48}$	$+\frac{2735}{768}$	$+\frac{2719}{30720}$	$+\frac{29}{4}$	$-\frac{101}{6}$	$+\frac{681}{64}$
	[cos − sin]	"	"	0	$+\frac{151}{3840}$	$+\frac{1423}{92160}$	0	0	$+\frac{37}{2880}$
	[cos]	$+\frac{15}{16}$	$+\frac{9}{32}$	H	$+\frac{6913}{3840}$	$+\frac{479}{9216}$	H	H	$+\frac{15341}{2880}$
	[sin]	0	0	H	$+\frac{1127}{640}$	$+\frac{3367}{92160}$	H	H	$+\frac{1913}{360}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{35}{16}$	$+\frac{7}{32}$	$-\frac{179}{24}$	$+\frac{2303}{384}$	$-\frac{1031}{3072}$	$+\frac{37}{4}$	$-\frac{83}{4}$	$+\frac{2639}{192}$
	[cos − sin]	"	"	0	$-\frac{17}{1920}$	$-\frac{1193}{15360}$	0	0	$+\frac{5}{576}$
	[cos]	$+\frac{35}{16}$	$+\frac{7}{32}$	H	$+\frac{5749}{1920}$	$-\frac{529}{2560}$	H	H	$+\frac{3961}{576}$
	[sin]	0	0	H	$+\frac{961}{320}$	$-\frac{1981}{15360}$	H	H	$+\frac{989}{144}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{35}{8}$	0	$-\frac{529}{48}$	$+\frac{6385}{768}$	$-\frac{20161}{30720}$	$+\frac{23}{2}$	$-\frac{139}{6}$	$+\frac{43}{3}$
	[cos − sin]	"	"	0	$-\frac{1159}{3840}$	$-\frac{25297}{92160}$	0	0	$-\frac{23}{720}$
	[cos]	$+\frac{35}{8}$	0	H	$+\frac{15383}{3840}$	$-\frac{4289}{9216}$	H	H	$+\frac{10297}{1440}$
	[sin]	0	0	H	$+\frac{2757}{640}$	$-\frac{17593}{92160}$	H	H	$+\frac{10343}{1440}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{63}{8}$	0	$-\frac{187}{12}$	$+\frac{1769}{192}$	$-\frac{311}{7680}$	$+14$	$-\frac{137}{6}$	$+\frac{171}{16}$
	[cos − sin]	"	"	0	$-\frac{1091}{960}$	$-\frac{13571}{23040}$	0	0	$-\frac{71}{720}$
	[cos]	$+\frac{63}{8}$	0	H	$+\frac{3877}{960}$	$-\frac{1813}{5760}$	H	H	$+\frac{953}{180}$
	[sin]	0	0	H	$+\frac{207}{40}$	$+\frac{6319}{23040}$	H	H	$+\frac{3883}{720}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 4f \ldots$

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\ 4f$	$[\ ]^3$				$[\ ]^4$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$-\frac{9}{2}$	$+\frac{399}{16}$	$-\frac{5415}{128}$	$+\frac{58251}{2048}$	$+1$	$-\frac{31}{2}$	$+\frac{239}{4}$	$-\frac{6719}{72}$
	[cos − sin]	0	0	0	$+\frac{411}{71680}$	0	0	0	0
	[cos]	H	H	H	$+\frac{509799}{35840}$	H	H	H	H
	[sin]	H	H	H	$+\frac{1019187}{71680}$	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	-5	$+\frac{209}{8}$	$-\frac{2783}{64}$	$+\frac{31065}{1024}$	$+1$	$-\frac{29}{2}$	$+\frac{215}{4}$	$-\frac{5993}{72}$
	[cos − sin]	0	0	0	$+\frac{209}{35840}$	0	0	0	0
	[cos]	H	H	H	$+\frac{271871}{17920}$	H	H	H	H
	[sin]	H	H	H	$+\frac{543533}{35840}$	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$-\frac{11}{2}$	$+\frac{411}{16}$	$-\frac{5025}{128}$	$+\frac{53459}{2048}$	$+1$	-13	$+\frac{353}{8}$	$-\frac{1151}{18}$
	[cos − sin]	0	0	0	$-\frac{541}{71680}$	0	0	0	0
	[cos]	H	H	H	$+\frac{467631}{35840}$	H	H	H	H
	[sin]	H	H	H	$+\frac{935803}{71680}$	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	-6	$+\frac{93}{4}$	$-\frac{933}{32}$	$+\frac{7929}{512}$	$+1$	-11	$+\frac{253}{8}$	$-\frac{346}{9}$
	[cos − sin]	0	0	0	$-\frac{459}{17920}$	0	0	0	0
	[cos]	H	H	H	$+\frac{4329}{560}$	H	H	H	H
	[sin]	H	H	H	$+\frac{138987}{17920}$	H	H	H	H

... *Table* $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) \begin{matrix}\cos \\ \sin\end{matrix} 4f \ldots$

$\begin{matrix}\cos \\ \sin\end{matrix} 4f$		$[\ ]^5$				$[\ ]^6$		
		e	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{7}{2}$	$-\frac{585}{16}$	$+\frac{47435}{384}$	$-\frac{3517045}{18432}$	$+\frac{33}{4}$	$-\frac{147}{2}$	$+\frac{15075}{64}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+3$	$-\frac{239}{8}$	$+\frac{18875}{192}$	$-\frac{155175}{1024}$	$+\frac{25}{4}$	$-\frac{215}{4}$	$+\frac{10831}{64}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{5}{2}$	$-\frac{365}{16}$	$+\frac{26845}{384}$	$-\frac{1884125}{18432}$	$+\frac{9}{2}$	-36	$+\frac{855}{8}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+2$	$-\frac{63}{4}$	$+\frac{3989}{96}$	$-\frac{239275}{4608}$	$+3$	-21	$+\frac{435}{8}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) \begin{matrix}\cos\\ \sin\end{matrix}\, 4f$ . . .

	$\begin{matrix}\cos\\ \sin\end{matrix}\, 4f$	$[\ ]^7$			$[\ ]^8$	
		e^3	e^5	e^7	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{787}{48}$	$-\frac{103937}{768}$	$+\frac{13103351}{30720}$	$+\frac{1423}{48}$	$-\frac{37729}{160}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{269}{24}$	$-\frac{34601}{384}$	$+\frac{4312189}{15360}$	$+\frac{297}{16}$	$-\frac{13897}{96}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{343}{48}$	$-\frac{41503}{768}$	$+\frac{4923079}{30720}$	$+\frac{32}{3}$	$-\frac{1184}{15}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{49}{12}$	$-\frac{5243}{192}$	$+\frac{548261}{7680}$	$\frac{16}{3}$	$-\frac{176}{5}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) \begin{matrix}\cos \\ \sin\end{matrix}\, 4f.$

$\begin{matrix}\cos \\ \sin\end{matrix}\, 4f$		$[\ ]^9$		$[\ ]^{10}$	$[\ ]^{11}$
		e^5	e^7	e^6	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{64653}{1280}$	$-\frac{4051137}{10240}$	$+\frac{47603}{576}$	$+\frac{84728011}{645120}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{18749}{640}$	$-\frac{1157699}{5120}$	$+\frac{25799}{576}$	$+\frac{1440343}{21504}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{19683}{1280}$	$-\frac{1161297}{10240}$	$+\frac{3125}{144}$	$+\frac{19487171}{645120}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{2187}{320}$	$-\frac{115911}{2560}$	$+\frac{625}{72}$	$+\frac{1771561}{161280}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H

Table $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) {}^{\cos}_{\sin} 4f \ldots$

${}^{\cos}_{\sin} 4f$		[ ]°		[ ]¹			[ ]²		
		e^4	e^6	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{1}{16}$	$+\frac{1}{16}$	$-\frac{73}{48}$	$+\frac{421}{768}$	$+\frac{4231}{30720}$	$+4$	$-\frac{33}{4}$	$+\frac{331}{96}$
	[cos − sin]	”	”	0	$-\frac{19}{3840}$	$-\frac{1}{10240}$	0	0	$-\frac{7}{1440}$
	[cos]	$+\frac{1}{16}$	$+\frac{1}{16}$	H	$+\frac{1043}{3840}$	$+\frac{1057}{15360}$	H	H	$+\frac{2479}{1440}$
	[sin]	0	0	H	$+\frac{177}{640}$	$+\frac{2117}{30720}$	H	H	$+\frac{1243}{720}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	0	0	$-\frac{17}{24}$	$+\frac{25}{384}$	$+\frac{47}{15360}$	$+\frac{11}{4}$	$-\frac{29}{6}$	$+\frac{21}{16}$
	[cos − sin]	”	”	0	$-\frac{7}{1920}$	$-\frac{361}{46080}$	0	0	$-\frac{1}{1440}$
	[cos]	0	0	H	$+\frac{59}{1920}$	$-\frac{11}{4608}$	H	H	$+\frac{1889}{2886}$
	[sin]	0	0	H	$+\frac{11}{320}$	$+\frac{251}{46080}$	H	H	$+\frac{1891}{2880}$
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	0	0	$-\frac{13}{48}$	$-\frac{29}{768}$	$-\frac{197}{6144}$	$+\frac{7}{4}$	$-\frac{29}{12}$	$+\frac{1}{3}$
	[cos − sin]	”	”	0	$+\frac{11}{3840}$	$+\frac{407}{92160}$	0	0	$+\frac{1}{288}$
	[cos]	0	0	H	$-\frac{67}{3840}$	$-\frac{637}{46080}$	H	H	$+\frac{97}{576}$
	[sin]	0	0	H	$-\frac{13}{640}$	$-\frac{1681}{92160}$	H	H	$+\frac{95}{576}$
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	0	0	$-\frac{1}{12}$	$-\frac{5}{192}$	$-\frac{149}{7680}$	$+1$	-1	$+\frac{1}{24}$
	[cos − sin]	”	”	0	$-\frac{1}{960}$	$-\frac{11}{7680}$	0	0	$-\frac{1}{360}$
	[cos]	0	0	H	$-\frac{13}{960}$	$-\frac{1}{96}$	H	H	$+\frac{7}{360}$
	[sin]	0	0	H	$-\frac{1}{80}$	$-\frac{23}{2560}$	H	H	$+\frac{1}{45}$
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	0	0	$-\frac{1}{48}$	$-\frac{7}{768}$	$-\frac{209}{30720}$	$+\frac{1}{2}$	$-\frac{1}{3}$	0
	[cos − sin]	”	”	0	$+\frac{1}{3840}$	$+\frac{31}{92160}$	0	0	$+\frac{1}{720}$
	[cos]	0	0	H	$-\frac{17}{3840}$	$-\frac{149}{46080}$	H	H	$+\frac{1}{1440}$
	[sin]	0	0	H	$-\frac{3}{640}$	$-\frac{329}{92160}$	H	H	$-\frac{1}{1440}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right)\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 4f$ . . .

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 4f$		$[\;]^3$				$[\;]^4$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$-\frac{7}{2}$	$+\frac{307}{16}$	$-\frac{3717}{128}$	$+\frac{30571}{2048}$	$+1$	-16	$+\frac{239}{4}$	$-\frac{3019}{36}$
	[cos − sin]	0	0	0	$-\frac{261}{71680}$	0	0	0	0
	[cos]	H	H	H	$+\frac{267431}{35840}$	H	H	H	H
	[sin]	H	H	H	$+\frac{535123}{71680}$	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	-3	$+\frac{123}{8}$	$-\frac{1305}{64}$	$+\frac{8547}{1024}$	$+1$	$-\frac{31}{2}$	$+\frac{433}{8}$	$-\frac{9793}{144}$
	[cos − sin]	0	0	0	$+\frac{27}{35840}$	0	0	0	0
	[cos]	H	H	H	$+\frac{74793}{17920}$	H	H	H	H
	[sin]	H	H	H	$+\frac{149559}{35840}$	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$-\frac{5}{2}$	$+\frac{183}{16}$	$-\frac{1611}{128}$	$+\frac{8035}{2048}$	$+1$	$-\frac{29}{2}$	$+\frac{365}{8}$	$-\frac{7111}{144}$
	[cos − sin]	0	0	0	$+\frac{243}{71680}$	0	0	0	0
	[cos]	H	H	H	$+\frac{70367}{35840}$	H	H	H	H
	[sin]	H	H	H	$+\frac{140491}{71680}$	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	-2	$+\frac{31}{4}$	$-\frac{215}{32}$	$+\frac{819}{512}$	$+1$	-13	$+\frac{283}{8}$	$-\frac{1147}{36}$
	[cos − sin]	0	0	0	$-\frac{81}{17920}$	0	0	0	0
	[cos]	H	H	H	$+\frac{7146}{8960}$	H	H	H	H
	[sin]	H	H	H	$+\frac{14373}{17920}$	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$-\frac{3}{2}$	$+\frac{75}{16}$	$-\frac{393}{128}$	$+\frac{1251}{2048}$	$+1$	-11	$+\frac{199}{8}$	$-\frac{655}{36}$
	[cos − sin]	0	0	0	$+\frac{243}{71680}$	0	0	0	0
	[cos]	H	H	H	$+\frac{11007}{35840}$	H	H	H	H
	[sin]	H	H	H	$+\frac{21771}{71680}$	H	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, 4f$ . . .

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, 4f$		$[\ ]^5$				$[\ ]^6$		
		e	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{9}{2}$	$-\frac{757}{16}$	$+\frac{58801}{384}$	$-\frac{1316215}{6144}$	$+13$	$-\frac{459}{4}$	$+\frac{11205}{32}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+5$	$-\frac{405}{8}$	$+\frac{29485}{192}$	$-\frac{1800725}{9216}$	$+\frac{63}{4}$	$-\frac{267}{2}$	$+\frac{12255}{32}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{11}{2}$	$-\frac{833}{16}$	$+\frac{55135}{384}$	$-\frac{2975165}{18432}$	$+\frac{75}{4}$	$-\frac{595}{4}$	$+\frac{12513}{32}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+6$	$-\frac{205}{4}$	$+\frac{11935}{96}$	$-\frac{184955}{1536}$	$+22$	$-\frac{317}{2}$	$+\frac{5945}{16}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{13}{2}$	$-\frac{765}{16}$	$+\frac{37925}{384}$	$-\frac{1493005}{18432}$	$+\frac{51}{2}$	$-\frac{321}{2}$	$+\frac{2613}{8}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 4f \ldots$

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 4f$	$[\ ]^7$			$[\ ]^8$	
		e^3	e^5	e^7	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{1471}{48}$	$-\frac{190507}{768}$	$+\frac{22796399}{30720}$	$+\frac{1027}{16}$	$-\frac{119323}{240}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+\frac{959}{24}$	$-\frac{119119}{384}$	$+\frac{13442527}{15360}$	$+\frac{2141}{24}$	$-\frac{53003}{80}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{2443}{48}$	$-\frac{284557}{768}$	$+\frac{29629663}{30720}$	$+\frac{2893}{24}$	$-\frac{40387}{48}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+\frac{763}{12}$	$-\frac{81193}{192}$	$+\frac{7623959}{7680}$	$+\frac{1273}{8}$	$-\frac{61219}{60}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{3751}{48}$	$-\frac{352919}{768}$	$+\frac{29213303}{30720}$	$+\frac{4943}{24}$	$-\frac{23591}{20}$
	[cos − sin]	0	0	0	0	0
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1}\ldots\left(\frac{r}{a}\right)^{-5}\right)\begin{matrix}\cos\\\sin\end{matrix}\,4f.$

$\begin{matrix}\cos\\\sin\end{matrix}\,4f$		$[\ ]^9$		$[\ ]^{10}$	$[\ ]^{11}$
		e^5	e^7	e^6	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{159263}{1280}$	$-\frac{9671209}{10240}$	$+\frac{65773}{288}$	$+\frac{86564393}{215040}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+\frac{117099}{640}$	$-\frac{6819921}{5120}$	$+\frac{25435}{72}$	$+\frac{210276803}{322560}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{333513}{1280}$	$-\frac{18298713}{10240}$	$+\frac{75947}{144}$	$+\frac{131087143}{129024}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+\frac{115617}{320}$	$-\frac{5856453}{2560}$	$+\frac{110099}{144}$	$+\frac{27485931}{17920}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{627003}{1280}$	$-\frac{28615329}{10240}$	$+\frac{155681}{144}$	$+\frac{1454039059}{645120}$
	[cos − sin]	0	0	0	0
	[cos]	H	H	H	H
	[sin]	H	H	H	H

Table $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right)^{\cos}_{\sin} 5f \ldots$

$^{\cos}_{\sin} 5f$		$[\ ]^0$		$[\ ]^1$		$[\ ]^2$		
		e^6	e^7	e^4	e^6	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$-\frac{21}{32}$	$-\frac{21}{64}$	$+\frac{1961}{384}$	$-\frac{4109}{1280}$	$-\frac{277}{24}$	$+\frac{575}{24}$	$-\frac{24781}{1920}$
	[cos − sin]	”	”	0	$-\frac{1091}{46080}$	0	0	$-\frac{209}{40320}$
	[cos]	$-\frac{21}{32}$	$-\frac{21}{64}$	H	$-\frac{29803}{18432}$	H	H	$-\frac{52061}{8064}$
	[sin]	0	0	H	$-\frac{146833}{92160}$	H	H	$-\frac{2032}{315}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$-\frac{7}{4}$	$-\frac{7}{16}$	$+\frac{1131}{128}$	$-\frac{25429}{3840}$	$-\frac{97}{6}$	$+\frac{817}{24}$	$-\frac{2461}{120}$
	[cos − sin]	”	”	0	$-\frac{887}{46080}$	0	0	$-\frac{1}{105}$
	[cos]	$-\frac{7}{4}$	$-\frac{7}{16}$	H	$-\frac{61207}{18432}$	H	H	$-\frac{1149}{112}$
	[sin]	0	0	H	$-\frac{304261}{92160}$	H	H	$-\frac{17219}{1680}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$-\frac{63}{16}$	$-\frac{21}{64}$	$+\frac{5509}{384}$	$-\frac{21763}{1920}$	$-\frac{263}{12}$	$+\frac{2135}{48}$	$-\frac{2641}{96}$
	[cos − sin]	”	”	0	$+\frac{6067}{46080}$	0	0	$+\frac{13}{2016}$
	[cos]	$-\frac{63}{16}$	$-\frac{21}{64}$	H	$-\frac{103249}{18432}$	H	H	$-\frac{6931}{504}$
	[sin]	0	0	H	$-\frac{528379}{92160}$	H	H	$-\frac{27737}{2016}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$-\frac{63}{8}$	0	$+\frac{8501}{384}$	$-\frac{10543}{640}$	$-\frac{347}{12}$	$+\frac{2557}{48}$	$-\frac{3673}{120}$
	[cos − sin]	”	”	0	$+\frac{32251}{46080}$	0	0	$+\frac{281}{5040}$
	[cos]	$-\frac{63}{8}$	0	H	$-\frac{145369}{18432}$	H	H	$-\frac{30797}{2016}$
	[sin]	0	0	H	$-\frac{791347}{92160}$	H	H	$-\frac{154547}{10080}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) {}^{\cos}_{\sin} 5f$ . . .

	${}^{\cos}_{\sin} 5f$	$[\ ]^3$			$[\ ]^4$			
		e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{93}{8}$	$-\frac{417}{8}$	$+\frac{77733}{1024}$	$-\frac{11}{2}$	$+\frac{207}{4}$	$-\frac{3715}{24}$	$+\frac{58583}{288}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{113}{8}$	-62	$+\frac{94081}{1024}$	-6	$+\frac{109}{2}$	$-\frac{484}{3}$	$+\frac{6917}{32}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{135}{8}$	$-\frac{1119}{16}$	$+\frac{103311}{1024}$	$-\frac{13}{2}$	$+\frac{443}{8}$	$-\frac{1885}{12}$	$+\frac{238195}{1152}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{159}{8}$	$-\frac{1191}{16}$	$+\frac{101103}{1024}$	-7	$+54$	$-\frac{3379}{24}$	$+\frac{1558}{9}$
	[cos − sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) \begin{matrix}\cos\\ \sin\end{matrix}\, 5f$ . . .

$\begin{matrix}\cos\\ \sin\end{matrix}\, 5f$		$[\ ]^5$				$[\ ]^6$			
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+1$	$-\frac{49}{2}$	$+\frac{9575}{64}$	$-\frac{441125}{1152}$	$+\frac{9}{2}$	$-\frac{561}{8}$	$+\frac{11415}{32}$	$-\frac{109167}{128}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+1$	$-\frac{47}{2}$	$+\frac{8975}{64}$	$-\frac{412175}{1152}$	$+4$	$-\frac{121}{2}$	$+\frac{1213}{4}$	$-\frac{11619}{16}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+1$	-22	$+\frac{7999}{64}$	$-\frac{177925}{576}$	$+\frac{7}{2}$	$-\frac{201}{4}$	$+\frac{7755}{32}$	$-\frac{9065}{16}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+1$	-20	$+\frac{6695}{64}$	$-\frac{138775}{576}$	$+3$	$-\frac{159}{4}$	$+\frac{2859}{16}$	$-\frac{12591}{32}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) \begin{matrix}\cos\\ \sin\end{matrix}\ 5f$ . . .

	$\begin{matrix}\cos\\ \sin\end{matrix}\ 5f$	$[\ ]^7$			$[\ ]^8$			
		e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{103}{8}$	$-\frac{3941}{24}$	$+\frac{780003}{1024}$		$+\frac{359}{12}$	$-\frac{32885}{96}$	$+\frac{1453367}{960}$
	[cos − sin]	0	0	0		0	0	0
	[cos]	H	H	H		H	H	H
	[sin]	H	H	H		H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{83}{8}$	$-\frac{259}{2}$	$+\frac{1829093}{3072}$		$+\frac{133}{6}$	$-\frac{2999}{12}$	$+\frac{263873}{240}$
	[cos − sin]	0	0	0		0	0	0
	[cos]	H	H	H		H	H	H
	[sin]	H	H	H		H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{65}{8}$	$-\frac{4667}{48}$	$+\frac{1331743}{3072}$		$+\frac{379}{24}$	$-\frac{8255}{48}$	$+\frac{141719}{192}$
	[cos − sin]	0	0	0		0	0	0
	[cos]	H	H	H		H	H	H
	[sin]	H	H	H		H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{49}{8}$	$-\frac{3283}{48}$	$+\frac{293853}{1024}$		$+\frac{32}{3}$	$-\frac{328}{3}$	$+\frac{6674}{15}$
	[cos − sin]	0	0	0		0	0	0
	[cos]	H	H	H		H	H	H
	[sin]	H	H	H		H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right)^1 \ldots \left(\frac{r}{a}\right)^4\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 5f.$

	$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 5f$	$[\ ]^9$		$[\ ]^{10}$		$[\ ]^{11}$	$[\ ]^{12}$
		e^4	e^6	e^5	e^7	e^6	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{7887}{128}$	$-\frac{848529}{1280}$	$+\frac{11263}{96}$	$-\frac{1402147}{1152}$	$+\frac{9742939}{46080}$	$+\frac{409821}{1120}$
	[cos − sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{5431}{128}$	$-\frac{578003}{1280}$	$+\frac{1817}{24}$	$-\frac{37429}{48}$	$+\frac{5931103}{46080}$	$+\frac{236641}{1120}$
	[cos − sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{3563}{128}$	$-\frac{184221}{640}$	$+\frac{4433}{96}$	$-\frac{66859}{144}$	$+\frac{3389017}{46080}$	$+\frac{101971}{896}$
	[cos − sin]	o	o	o	o	o	o
	[cos]	o	H	H	H	H	H
	[sin]	o	H	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{2187}{128}$	$-\frac{107163}{640}$	$+\frac{625}{24}$	$-\frac{71875}{288}$	$+\frac{1771561}{46080}$	$+\frac{1944}{35}$
	[cos − sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H

Table $\left(\left(\frac{r}{a}\right)^{-1}\ldots\left(\frac{r}{a}\right)^{-5}\right)\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}5f\ldots$

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 5f$		[ ]°		[ ]¹		[ ]²		
		e^6	e^7	e^4	e^6	e^3	e^6	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$-\frac{1}{32}$	$-\frac{5}{128}$	$+\frac{167}{128}$	$-\frac{247}{960}$	$-\frac{31}{6}$	$+\frac{859}{96}$	$-\frac{18683}{6720}$
	[cos − sin]	”	”	0	$+\frac{151}{46080}$	0	0	$+\frac{13}{6720}$
	[cos]	$-\frac{1}{32}$	$-\frac{5}{128}$	H	$-\frac{2341}{18432}$	H	H	$-\frac{1867}{1344}$
	[sin]	0	0	H	$-\frac{12007}{92160}$	H	H	$-\frac{779}{560}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	0	0	$+\frac{209}{384}$	$+\frac{43}{1280}$	$-\frac{19}{6}$	$+\frac{55}{12}$	$-\frac{77}{96}$
	[cos − sin]	”	”	0	$+\frac{37}{46080}$	0	0	$-\frac{1}{2016}$
	[cos]	0	0	H	$+\frac{317}{18432}$	H	H	$-\frac{809}{2016}$
	[sin]	0	0	H	$+\frac{1511}{92160}$	H	H	$-\frac{101}{252}$
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	0	0	$+\frac{73}{384}$	$+\frac{211}{3840}$	$-\frac{43}{24}$	$+\frac{193}{96}$	$-\frac{103}{960}$
	[cos − sin]	”	”	0	$-\frac{47}{46080}$	0	0	$-\frac{17}{20160}$
	[cos]	0	0	H	$+\frac{497}{18432}$	H	H	$-\frac{109}{2016}$
	[sin]	0	0	H	$+\frac{2579}{92160}$	H	H	$-\frac{1073}{20160}$
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	0	0	$+\frac{7}{128}$	$+\frac{49}{1920}$	$-\frac{11}{12}$	$+\frac{35}{48}$	$+\frac{11}{480}$
	[cos − sin]	”	”	0	$+\frac{19}{46080}$	0	0	$+\frac{1}{1120}$
	[cos]	0	0	H	$+\frac{239}{18432}$	H	H	$+\frac{1}{84}$
	[sin]	0	0	H	$+\frac{1157}{92160}$	H	H	$+\frac{37}{3360}$
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	0	0	$+\frac{5}{384}$	$+\frac{1}{128}$	$-\frac{5}{12}$	$+\frac{5}{24}$	$+\frac{1}{96}$
	[cos − sin]	”	”	0	$-\frac{1}{9216}$	0	0	$-\frac{1}{2016}$
	[cos]	0	0	H	$+\frac{71}{18432}$	H	H	$+\frac{5}{1008}$
	[sin]	0	0	H	$+\frac{73}{18432}$	H	H	$+\frac{11}{2016}$

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 5f$ . . .

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix} 5f$		$[\ ]^3$			$[\ ]^4$			
		e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos+sin]	$+\frac{59}{8}$	$-\frac{501}{16}$	$+\frac{39963}{1024}$	$-\frac{9}{2}$	$+\frac{337}{8}$	$-\frac{5683}{48}$	$+\frac{26119}{192}$
	[cos−sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos+sin]	$+\frac{45}{8}$	$-\frac{177}{8}$	$+\frac{24501}{1024}$	-4	$+36$	$-\frac{565}{6}$	$+\frac{3475}{36}$
	[cos−sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos+sin]	$+\frac{33}{8}$	$-\frac{29}{2}$	$+\frac{13281}{1024}$	$-\frac{7}{2}$	$+\frac{59}{2}$	$-\frac{3349}{48}$	$+\frac{17803}{288}$
	[cos−sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos+sin]	$+\frac{23}{8}$	$-\frac{139}{16}$	$+\frac{6263}{1024}$	-3	$+23$	$-\frac{1145}{24}$	$+\frac{1703}{48}$
	[cos−sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos+sin]	$+\frac{15}{8}$	$-\frac{75}{16}$	$+\frac{2535}{1024}$	$-\frac{5}{2}$	$+\frac{135}{8}$	$-\frac{715}{24}$	$+\frac{20945}{1152}$
	[cos−sin]	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \begin{matrix}\cos\\ \sin\end{matrix}\, 5f$ . . .

	$\begin{matrix}\cos\\ \sin\end{matrix}\, 5f$	$[\ ]^5$				$[\ ]^6$			
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+1$	-25	$+\frac{9575}{64}$	$-\frac{103325}{288}$	$+\frac{11}{2}$	-86	$+\frac{13593}{32}$	$-\frac{60877}{64}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+1$	$-\frac{49}{2}$	$+\frac{8999}{64}$	$-\frac{363365}{1152}$	$+6$	$-\frac{183}{2}$	$+\frac{3465}{8}$	$-\frac{29175}{32}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+1$	$-\frac{47}{2}$	$+\frac{8095}{64}$	$-\frac{298655}{1152}$	$+\frac{13}{2}$	$-\frac{759}{8}$	$+\frac{13509}{32}$	$-\frac{26213}{32}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+1$	-22	$+\frac{6935}{64}$	$-\frac{114085}{576}$	$+7$	$-\frac{383}{4}$	$+\frac{6275}{16}$	$-\frac{2751}{4}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+1$	-20	$+\frac{5615}{64}$	$-\frac{80455}{576}$	$+\frac{15}{2}$	$-\frac{375}{4}$	$+\frac{11055}{32}$	$-\frac{8595}{16}$
	[cos − sin]	0	0	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, 5f$ . . .

$\begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, 5f$		$[\;]^7$			$[\;]^8$		
		e^2	e^4	e^6	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{149}{8}$	$-\frac{3771}{16}$	$+\frac{3242339}{3072}$	$+\frac{1201}{24}$	$-\frac{54151}{96}$	$+\frac{4603927}{1920}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+\frac{175}{8}$	$-\frac{6461}{24}$	$+\frac{1183371}{1024}$	$+\frac{188}{3}$	$-\frac{4115}{6}$	$+\frac{16781}{6}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{203}{8}$	$-\frac{896}{3}$	$+\frac{3712093}{3072}$	$+\frac{463}{6}$	$-\frac{38771}{48}$	$+\frac{46693}{15}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+\frac{233}{8}$	$-\frac{5149}{16}$	$+\frac{3701999}{3072}$	$+\frac{281}{3}$	$-\frac{22135}{24}$	$+\frac{99181}{30}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{265}{8}$	$-\frac{16135}{48}$	$+\frac{1169805}{1024}$	$+\frac{2695}{24}$	$-\frac{24485}{24}$	$+\frac{160513}{48}$
	[cos − sin]	0	0	0	0	0	0
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H

. . . *Table* $\left(\left(\frac{r}{a}\right)^{-1} \ldots \left(\frac{r}{a}\right)^{-5}\right) \genfrac{}{}{0pt}{}{\cos}{\sin} 5f.$

	$\genfrac{}{}{0pt}{}{\cos}{\sin} 5f$	$[\ ]^9$		$[\ ]^{10}$		$[\ ]^{11}$	$[\ ]^{12}$
		e^4	e^6	e^5	e^7	e^6	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{14987}{128}$	$-\frac{394607}{320}$	$+\frac{11989}{48}$	$-\frac{161685}{64}$	$+\frac{22980061}{46080}$	$+\frac{2122251}{2240}$
	[cos − sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+\frac{19863}{128}$	$-\frac{2029257}{1280}$	$+\frac{8359}{24}$	$-\frac{983731}{288}$	$+\frac{33554587}{46080}$	$+\frac{40401}{28}$
	[cos − sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{25791}{128}$	$-\frac{2521323}{1280}$	$+\frac{1423}{3}$	$-\frac{1282973}{288}$	$+\frac{47722543}{46080}$	$+\frac{2391591}{1120}$
	[cos − sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+\frac{32907}{128}$	$-\frac{1516377}{640}$	$+\frac{7597}{12}$	$-\frac{67409}{12}$	$+\frac{66363649}{46080}$	$+\frac{1726833}{560}$
	[cos − sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{41355}{128}$	$-\frac{353187}{128}$	$+\frac{79711}{96}$	$-\frac{987035}{144}$	$+\frac{18100973}{9216}$	$+\frac{3906009}{896}$
	[cos − sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H

The foregoing tables are read as follows: for instance, in the Table $(x^0, x^1, x^2, x^3) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} f$, p. 208, *et seq.*, the third and fourth lines of the x^0-compartment show that

$$
\begin{aligned}
\cos f = & -e \\
& + \left(\tfrac{1}{2} - \tfrac{9}{16}e^2 + \tfrac{25}{384}e^4 - \tfrac{49}{18432}e^6\right) 2\cos g \\
& + \left(\tfrac{1}{2}e - \tfrac{2}{3}e^3 + \tfrac{3}{16}e^5 - \tfrac{1}{45}e^7\right) 2\cos 2g \\
& + \&c.
\end{aligned}
$$

$$
\begin{aligned}
\sin f = & \left(\tfrac{1}{2} - \tfrac{7}{16}e^2 + \tfrac{17}{384}e^4 - \tfrac{271}{18432}e^6\right) 2\sin g \\
& + \left(\tfrac{1}{2}e - \tfrac{7}{12}e^3 + \tfrac{1}{6}e^5 - \tfrac{19}{720}e^7\right) 2\sin 2g \\
& + \&c.
\end{aligned}
$$

and the first and second lines give the sum and difference respectively of the corresponding coefficients of the cosine and sine series.

Addition, 28*th Dec.* 1860.—The tables have been verified by me, on the proof-sheets; the x-tables, the cosine lines of the $\left(\frac{r}{a}\right)$-tables, and a portion of the sine lines of the same tables, in the manner explained at the commencement of the memoir; but for the remainder of the sine lines it was found easier to employ the following mode of verification; viz., the equation $\frac{r}{a} = 1 + x$, gives

$$\left\{\left(\frac{r}{a}\right)^4 - 4\left(\frac{r}{a}\right)^3 + 6\left(\frac{r}{a}\right)^2 - 4\left(\frac{r}{a}\right)\right\} \sin jf = (-1 + x^4)\sin jf;$$

and as regards the terms up to e^7, the limit of the tables,

$$\left\{\left(\frac{r}{a}\right)^{-5} - 5\left(\frac{r}{a}\right)^{-4} + 10\left(\frac{r}{a}\right)^{-3} - 10\left(\frac{r}{a}\right)^{-2} + 5\left(\frac{r}{a}\right)^{-1}\right\} \sin jf = (1 - x^5 + 5x^6 - 15x^7)\sin jf,$$

which equations afford the verification referred to. In going over the earlier sheets I omitted to see that the fractions were in their least terms, and it may happen that, in some instances, they are not so.

The expression for the true anomaly f itself has been repeatedly calculated to a much greater extent, in particular by SCHUBERT (*Astronomie Théorique, Pet.* 1822), as far as e^{20}. The easiest way of obtaining it seems to be by means of the equation

$$\frac{df}{dg} = \left(\frac{r}{a}\right)^{-2} \sqrt{1 - e^2};$$

and, for the convenience of reference, I here give it as far as e^7, viz.,

$$f = g + \Sigma\, [\sin]^i \sin i g,$$

where, as in the other sine series, i is to be taken as well negatively as positively, and $[\sin]^{-i} = -[\sin]^i$. Or, what is the same thing,

$$f = g + \Sigma\, [\sin]^i\, 2 \sin i g,$$

where i has only positive values. And the coefficients are

	e	e^2	e^3	e^4	e^5	e^6	e^7
$[\sin]^1 =$	1		$-\frac{1}{8}$		$+\frac{5}{192}$		$+\frac{107}{9216}$
$[\sin]^2 =$		$+\frac{5}{8}$		$-\frac{11}{48}$		$+\frac{17}{384}$	
$[\sin]^3 =$			$+\frac{13}{24}$		$-\frac{43}{128}$		$+\frac{95}{1024}$
$[\sin]^4 =$				$+\frac{103}{192}$		$-\frac{451}{960}$	
$[\sin]^5 =$					$+\frac{1097}{1920}$		$-\frac{5957}{9216}$
$[\sin]^6 =$						$+\frac{1223}{1920}$	
$[\sin]^7 =$							$+\frac{47273}{64512}$

The expression for $\frac{r}{a}$, as far as e^{13}, is given in SCHUBERT'S work, above referred to, and that of $\log \left(\frac{r}{a}\right)$, as far as e^9, was calculated by ORIANI. See the Introduction to DELAMBRE'S *Tables du Soleil*, Paris, 1806.

VIII. *A Memoir on the Problem of the Rotation of a Solid Body.*

By A. Cayley, Esq.

Read May 11, 1860.

The present memoir was written for the sake of the further elaboration of the analytical theory of the Rotation of a Solid Body, upon principles similar to those of my "Memoir on the Problem of Disturbed Elliptic Motion," *Mem. R. Ast. Soc.* vol. xxvii. pp. 1–29 (1858); the like elements are adopted, and the course of the investigation corresponds precisely to that of the memoir just referred to. The formulæ for the variations of the elements in the two problems (the motion being in each case referred to a fixed plane of reference and origin of angles therein) are found to be (as it is known they should be) identical in form; an investigation, in the present memoir, of the transformation of the system to the case of a variable plane of reference and departure-point as an origin of angles in such plane, would have been a mere repetition of that contained in the former memoir, and it was therefore unnecessary to give it. A point in the present memoir to which attention may be called is the definition of the angle g (varying uniformly with the time, but used as an element) which corresponds to the mean anomaly in elliptic motion. Besides the ultimate system of formulæ for the variations of the elements in terms of the differential coefficients of the Disturbing Function with respect to the elements, it appears to me that the intermediate formulæ for the variations in terms of the differential coefficients with respect to the co-ordinates (which are in the ordinary investigation altogether passed over) are not without interest, and that it is possible that they might be employed with advantage in the integration of the equations of motion for the purposes of physical astronomy,

I.

In the theory of elliptic motion, where the elements are

a, the mean distance,
e, the excentricity,
g, the mean anomaly,
ϖ, the departure of pericentre,
θ, the longitude of node,
σ, the departure of node,
φ, the inclination,

and if, besides, we have

n, the mean motion ($n^2 a^3 =$ sum of the masses),

and Ω denote the disturbing function taken with LAGRANGE'S sign ($\Omega = -\mathrm{R}$, if R be the disturbing function of the *Mécanique Céleste*), then the formulæ for the variations of the elements are

$$
\begin{aligned}
da &= \frac{2}{na}\frac{d\Omega}{dg}dt,\\
de &= \frac{1-e^2}{na^2e}\frac{d\Omega}{dg}dt - \frac{\sqrt{1-e^2}}{na^2e}\frac{d\Omega}{d\varpi}dt,\\
dg &= -\frac{2}{na}\frac{d\Omega}{da}dt - \frac{1-e^2}{na^2e}\frac{d\Omega}{de}dt,\\
d\varpi &= \frac{\sqrt{1-e^2}}{na^2e}\frac{d\Omega}{de}dt,\\
d\varphi &= -\frac{\cot\varphi}{na^2\sqrt{1-e^2}}\frac{d\Omega}{d\sigma}dt - \frac{\operatorname{cosec}\varphi}{na^2\sqrt{1-e^2}}\frac{d\Omega}{d\theta}dt,\\
d\sigma &= \frac{\cot\varphi}{na^2\sqrt{1-e^2}}\frac{d\Omega}{d\varphi}dt,\\
d\theta &= \frac{\operatorname{cosec}\varphi}{na^2\sqrt{1-e^2}}\frac{d\Omega}{d\varphi}dt,
\end{aligned}
$$

where $\Omega = \Omega\,(a, e, g, \varpi, \varphi, \sigma, \theta)$.

And if in these equations we write

$$h = -\frac{1}{a}\,(\text{sum of the masses}) = -\,n^2 a^2,$$

$$k = \quad n a^2 \sqrt{1-e^2};$$

then attending to the equation $d n = -\frac{3}{2}\frac{n}{a}\, d a$, we have

$$d h \;= n^2 a\, d a,$$

$$d k = \frac{1}{2}\, n a \sqrt{1-e^2}\, d a - \frac{n a^2 e}{\sqrt{1-e^2}},$$

and thence

$$\frac{d\Omega}{d a} = n^2 a \frac{d\Omega}{d h} + \tfrac{1}{2}\, n a \sqrt{1-e^2}\, \frac{d\Omega}{d k},$$

$$\frac{d\Omega}{d e} = \qquad\qquad - \frac{n^2 a}{\sqrt{1-e^2}} \frac{d\Omega}{d k};$$

and the formulæ are very easily transformed into

$$d h = \quad 2 n \frac{d\Omega}{d g}\, d t,$$

$$d g = -\,2 n \frac{d\Omega}{d h}\, d t,$$

$$d k = \qquad \frac{d\Omega}{d \varpi}\, d t,$$

$$d \varpi = \qquad \frac{d\Omega}{d k}\, d t,$$

$$d \varphi = -\frac{\cot\varphi}{k} \frac{d\Omega}{d\sigma}\, d t - \frac{\operatorname{cosec}\varphi}{k} \frac{d\Omega}{d\theta}\, d t,$$

$$d \sigma = \frac{\cot\varphi}{k} \frac{d\Omega}{d\varphi}\, d t,$$

$$d \theta = \frac{\operatorname{cosec}\varphi}{k} \frac{d\Omega}{d\varphi}\, d t,$$

where $\Omega = \Omega\,(h, g, k, \varpi, \varphi, \sigma, \theta)$.

This is the system of formulæ which will be obtained in the sequel for the variation of the elements in the problem of rotation, the new meanings of the symbols being explained *post*, Art. IV.

And if in either of the two problems, instead of the angles ϕ, σ, θ, which refer to a fixed plane of reference and origin of angles therein, we have the angles Φ, Σ, Θ, referring to a variable plane of reference and departure-point as an origin of angles in such plane, the position of these in respect to the fixed plane of reference and origin of angles therein being determined by

θ', the longitude of node,
σ', the departure of node,
ϕ', the inclination,

and if S' denote $\Theta - \sigma'$, then the system is

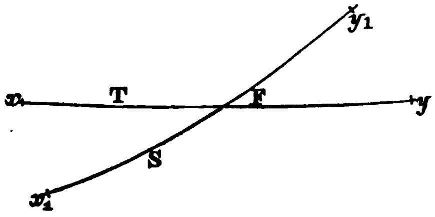
$$\begin{aligned}
dh &= \quad 2n \frac{d\Omega}{dh} dt, \\
dg &= -2n \frac{d\Omega}{dh} dt, \\
dk &= \quad \frac{d\Omega}{d\varpi} dt, \\
d\varpi &= \quad -\frac{d\Omega}{dk} dt, \\
d\Phi &= -\frac{\cot\Phi}{k}\frac{d\Omega}{d\Sigma} dt - \frac{\operatorname{cosec}\phi}{k}\frac{d\Omega}{d\Theta} dt - \qquad (\cos S' d\phi' + \sin S' \sin\phi' d\theta'), \\
d\Sigma &= \quad \frac{\cot\Phi}{k}\frac{d\Omega}{d\Phi} dt \qquad + \operatorname{cosec}\Phi\,(\sin S' d\phi' - \cos S' \sin\phi' d\theta'), \\
d\Theta &= \quad \frac{\operatorname{cosec}\Phi}{k}\frac{d\Omega}{d\Phi} dt \qquad + \cot\Phi\,(\sin S' d\phi' - \cos S' \sin\phi' d\theta'),
\end{aligned}$$

where $\Omega = \Omega\,(h, g, k, \varpi, \Phi, \Sigma, \Theta, \phi', \sigma', \theta')$, or what is the same thing, $\Omega = \Omega\,(h, g, k, \varpi, \Phi, \Sigma, \Theta)$. As already remarked, it is not necessary to repeat in the present memoir the transformation to this set of formulæ.

II.

Considering, now, the problem of rotation, let the axes $x\,y\,z$ denote axes fixed in space, and the axes $x_{,}y_{,}z_{,}$ denote the principal axes of the body; and if, to fix the ideas, $x\,y$ is called the ecliptic and $x_{,}\,y_{,}$ the equator (the ecliptic $x\,y$ being considered as a fixed circle of the sphere and the origin of longitudes x as a fixed point therein), then we may write

T, the longitude of node,
S, the departure of node,
F, the inclination.



And if, as usual, p, q, r are the angular velocities round the principal axes, or axes of $x_{,}$, $y_{,}$, and $z_{,}$ from $y_{,}$ to $z_{,}$, $z_{,}$ to $x_{,}$, and $x_{,}$ to $y_{,}$ respectively, so that

$$\begin{aligned} p\,dt &= -\sin S \sin F\,d\mathrm{T} + \cos S\,d\mathrm{F}, \\ q\,dt &= \cos S \sin F\,d\mathrm{T} + \sin S\,d\mathrm{F}, \\ r\,dt &= -d\mathrm{S} + \cos F\,d\mathrm{T}; \end{aligned}$$

and if A, B, C are the moments of inertia, then the *Vis Viva* function or sum of the elements of mass, each into the half square of its velocity, is

$$\mathbf{T} = \frac{1}{2}(\mathrm{A}p^2 + \mathrm{B}q^2 + \mathrm{C}r^2);$$

and if Ω be the disturbing function (taken with LAGRANGE'S sign), then the equations of motion

$$\frac{d}{dt}\frac{d\mathbf{T}}{d\mathrm{T}'} - \frac{d\mathbf{T}}{d\mathrm{T}} = \frac{d\Omega}{d\mathrm{T}}, \text{ \&c.}$$

give as usual

$$\begin{aligned} -\left[\mathrm{A}\frac{dp}{dt} + (\mathrm{C}-\mathrm{B})qr\right]\sin S \sin F + \left[\mathrm{B}\frac{dq}{dt} + (\mathrm{A}-\mathrm{C})rp\right]\cos S \sin F + \left[\mathrm{C}\frac{dr}{dt} + (\mathrm{B}-\mathrm{A})pq\right]\cos F &= \frac{d\Omega}{d\mathrm{T}}, \\ \left[\mathrm{A}\frac{dp}{dt} + (\mathrm{C}-\mathrm{B})qr\right]\cos S \qquad + \left[\mathrm{B}\frac{dq}{dt} + (\mathrm{A}-\mathrm{C})rp\right]\sin S \qquad &= \frac{d\Omega}{d\mathrm{F}}, \\ -\left[\mathrm{C}\frac{dr}{dt} + (\mathrm{B}-\mathrm{A})pq\right] \qquad &= \frac{d\Omega}{d\mathrm{S}}, \end{aligned}$$

or, as these equations may be written,

$$A\frac{dp}{dt}+(C-B)\,qr=-\sin S\left(\operatorname{cosec} F\frac{d\Omega}{dT}+\cot F\frac{d\Omega}{dS}\right)+\cos S\frac{d\Omega}{dF},$$

$$B\frac{dq}{dt}+(A-C)\,rp=\cos S\left(\operatorname{cosec} F\frac{d\Omega}{dT}+\cot F\frac{d\Omega}{dS}\right)-\sin S\frac{d\Omega}{dF},$$

$$C\frac{dr}{dt}+(B-A)\,pq=-\frac{d\Omega}{dS};$$

which, with the equations for p, q, r, determine the motion of the body.

III.

First, to integrate the equations of motion when the disturbing forces are neglected; we have, as usual, the integral equations

$$Ap^2+Bq^2+Cr^2=h,$$

$$A^2p^2+B^2q^2+C^2r^2=k^2;$$

where h, k are constants of integration, viz., h is the constant of *Vis Viva*, and k is the constant of the principal area.

Moreover, if the coefficients α, β, &c. are those which belong to the transformation from $x\,y\,z$ to $x_{\prime}y_{\prime}z_{\prime}$ viz., if in the table

	$x_{\prime}$	$y_{\prime}$	$z_{\prime}$
x	α	β	γ
y	α'	β'	γ'
z	α''	β''	γ''

the values of the coefficients are

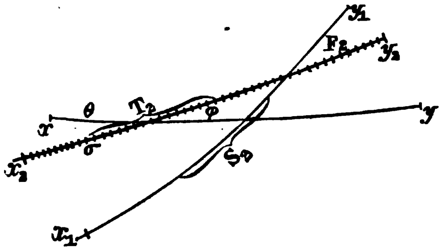
	$x_{\prime}$	$y_{\prime}$	$z_{\prime}$
x	$\cos S\cos T+\sin S\sin T\cos F$	$\sin S\cos T-\cos S\sin T\cos F$	$\sin T\sin F$
y	$\cos S\sin T-\sin S\cos T\cos F$	$\sin S\sin T+\cos S\cos T\cos F$	$-\cos T\sin F$
z	$-\sin S\sin F$	$\cos S\sin F$	$\cos F$

Then we have also, as usual, the integral equations

$$\begin{aligned} \mathrm{A}p\alpha + \mathrm{B}q\beta + \mathrm{C}r\gamma &= k \sin\theta \sin\varphi, \\ \mathrm{A}p\alpha' + \mathrm{B}q\beta' + \mathrm{C}r\gamma' &= - k\cos\theta\sin\varphi, \\ \mathrm{A}p\alpha'' + \mathrm{B}q\beta'' + \mathrm{C}r\gamma'' &= k\cos\varphi; \end{aligned}$$

where θ, φ are constants of integration which determine the position of the principal plane (or invariable plane, in the undisturbed motion). Considering now a new system of axes $x_2 y_2 z_2$, where x_2 and z_2 are in the principal plane, and x_2 is in the first instance considered as an arbitrary fixed point therein (afterwards when the plane is treated as variable, x_2 is assumed to be a departure-point), let the position of the new set of axes in reference to the axes $x\,y\,z$ be determined by

θ, the longitude of the node,
σ, the departure of the node,
φ, the inclination;



so that the relation between the co-ordinates $x\,y\,z$ and $x_2 y_2 z_2$ is given by the table

	x_2	y_2	z_2
x	a	b	c
y	a'	b'	c'
z	a''	b''	c''

where the values of the coefficients are given by

	x_2	y_2	z_2
x	$\cos\sigma\cos\theta + \sin\sigma\sin\theta\cos\varphi$	$\sin\sigma\cos\theta - \cos\sigma\sin\theta\cos\varphi$	$\sin\theta\sin\varphi$
y	$\cos\sigma\sin\theta - \sin\sigma\cos\theta\cos\varphi$	$\sin\sigma\sin\theta + \cos\sigma\cos\theta\cos\varphi$	$-\cos\theta\sin\varphi$
z	$-\sin\sigma\sin\varphi$	$\cos\sigma\sin\varphi$	$\cos\varphi$

and let the position of the axes $x_{\prime}, y_{\prime}, z_{\prime}$, in reference to the new axes $x_2\, y_2\, z_2$, be determined in a similar manner by

T_2, the longitude of node,
S_2, the departure of node,
F_2, the inclination;

so that we have the table

	$x_{\prime}$	$y_{\prime}$	$z_{\prime}$
x_2	α_2	β_2	γ_2
y_2	α'_2	β'_2	γ'_2
z_2	α''_2	β''_2	γ''_2

where the values of the coefficients are given by

	$x_{\prime}$	$y_{\prime}$	$z_{\prime}$
x_2	$\cos S_2 \cos T_2 + \sin S_2 \sin T_2 \cos F_2$	$\sin S_2 \cos T_2 - \cos S_2 \sin T_2 \cos F_2$	$\sin T_2 \sin F_2$
y_2	$\cos S_2 \sin T_2 - \sin S_2 \cos T_2 \cos F_2$	$\sin S_2 \sin T_2 + \cos S_2 \cos T_2 \cos F_2$	$-\cos T_2 \sin F_2$
z_2	$-\sin S_2 \sin F_2$	$\cos S_2 \sin F_2$	$\cos F_2$

The values of α_2, β_2, γ_2 are

$$a\,\alpha + a'\alpha' + a''\alpha'',$$
$$a\,\beta + a'\beta' + a''\beta'',$$
$$a\,\gamma + a'\gamma' + a''\gamma'',$$

with similar expressions for α_2', β_2', γ_2' and α_2'', β_2'', γ_2''; the last-mentioned three integrals are thus transformable into the form

$$\begin{aligned} A p \alpha_2 + B q \beta_2 + C r \gamma_2 &= 0, \\ A p \alpha_2' + B q \beta_2' + C r \gamma_2' &= 0, \\ A p \alpha_2'' + B q \beta_2'' + C r \gamma_2'' &= k, \end{aligned}$$

which are, in fact, the equations which show that the plane $x_2\, y_2$ is the principal plane, or plane of maximum area.

The equations just obtained, attending to the values of α_2'', β_2'', γ_2'', give

$$\begin{aligned} A p &= -k \sin S_2 \sin F_2, \\ B q &= k \cos S_2 \sin F_2, \\ C r &= k \cos F_2; \end{aligned}$$

and, since the expressions for p, q, r, in terms of T_2, S_2, F_2, must be similar to those in terms of T, S, F, we have

$$\begin{aligned} p\, dt &= -\sin S_2 \sin F_2\, d T_2 + \cos S_2\, d F_2, \\ q\, dt &= \cos S_2 \sin F_2\, d T_2 + \sin S_2\, d F_2, \\ r\, dt &= -d S_2 + \cos S_2\, d T_2; \end{aligned}$$

from which equations,

$$\sin F_2\, d T_2 = (-p \sin S_2 + q \cos S_2)\, dt;$$

and substituting for $\sin S_2$, $\cos S_2$, the values $\frac{-Ap}{k \sin F_2}$, $\frac{Bq}{k \sin F_2}$, this becomes

$$k^2 \sin^2 F_2\, d T_2 = k\,(A p^2 + B q^2)\, dt;$$

or, what is the same thing,

$$(k^2 - C^2 r^2)\, d T_2 = k\,(h - C r^2)\, dt.$$

The two equations $A p^2 + B q^2 + C r^2 = h$, $A^2 p^2 + B^2 q^2 + C^2 r^2$ may be considered as determining p, q, in terms of r, and the equation $C \frac{dr}{dt} + (B - A)\, p q = 0$, then gives

$$dt = \frac{-C\, dr}{(B - A)\, p q},$$

whence also the equation for $d T_2$ becomes

$$d T_2 = \frac{k\,(h - C r^2)}{k^2 - C^2 r^2} \, \frac{-C\, dr}{(B - A)\, p q}.$$

Instead of the time t, I consider a function $g = nt + \text{const.}$, n being for the present an arbitrary constant quantity which may be a function of the constants h and k; we have thus

$$dg = n\frac{-\mathrm{C}\,dr}{(\mathrm{B}-\mathrm{A})\,pq},$$

and the integral equation is

$$g = n\int\frac{-\mathrm{C}\,dr}{(\mathrm{B}-\mathrm{A})\,pq}.$$

The equation for $d\mathrm{T}_2$ gives, in like manner,

$$\mathrm{T}_2 = \varpi + \int\frac{k\,(h-\mathrm{C}r^2)}{k^2-\mathrm{C}^2r^2}\,\frac{-\mathrm{C}\,dr}{(\mathrm{B}-\mathrm{A})\,pq};$$

where it is assumed that the integrals are each of them taken from $r = r_0$, r_0 being an arbitrary constant value, say a function of h and k. The quantity g is analogous to the mean anomaly in elliptic motion, or rather it will become so when the significations of n and r_0 are fixed; it is considered as implicitly involving a constant of integration, and, consequently, no constant of integration is added to the integral: as regards T_2, the constant of integration is ϖ, which denotes the initial value (corresponding to $r = r_0$) of the angle T_2.

IV.

Recapitulating the integral equations, we have

$$\mathrm{A}p^2 + \mathrm{B}q^2 + \mathrm{C}r^2 = h,$$

$$\mathrm{A}^2p^2 + \mathrm{B}^2q^2 + \mathrm{C}^2r^2 = k^2,$$

$$\mathrm{A}p = -k\sin\mathrm{S}_2\sin\mathrm{F}_2,$$

$$\mathrm{B}q = k\cos\mathrm{S}_2\sin\mathrm{F}_2,$$

$$\mathrm{C}r = k\cos\mathrm{F}_2,$$

$$g = n\int\frac{-\mathrm{C}\,dr}{(\mathrm{B}-\mathrm{A})\,pq},$$

which equations give r, p, q, and thence S_2 and F_2 in terms of g and the constants h and k.

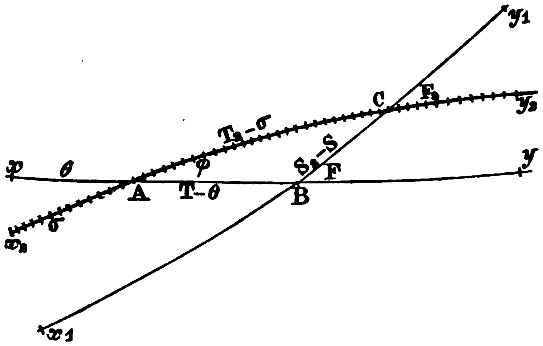
Moreover

$$T_2 = \varpi + \int \frac{k(h - Cr^2)}{k^2 - C^2r^2} \, \frac{-C\,dr}{(B - A)\,pq},$$

which gives T_2 in terms of g and the constants h, k, ϖ; and then T, S, F are given in terms of T_2, S_2, F_2, and the constants θ, σ, ϕ, as follows, viz., we have a spherical triangle A B C, the sides whereof a, b, c, and the opposite angles A, B, C, are respectively,

Sides	$S_2 - S$,	$T_2 - \sigma$,	$T - \theta$,
Opposite angles	ϕ ,	$180^\circ - F$,	F_2 ,

as appears by the figure.



The above values of p, q, r, give

$$h - Cr^2 = \left(k^2 - C^2r^2\right)\left(\frac{\sin^2 S_2}{A} + \frac{\cos^2 S_2}{B}\right),$$

which is an equation which will be useful in the sequel.

The coefficient n, and the value r_0, which is the inferior limit of the integrals, are thus far considered as arbitrary functions of h and k. As already remarked, g (which is a variable quantity, $= nt + c$) is used as an element, and the elements are h, k, g, ϖ, σ, θ, ϕ.

As to the signification of the different elements, it is proper to remark that it is not for the purposes of the present memoir necessary to completely fix the significations of the quantities n and r_0; the only conditions imposed on these quantities in the sequel are that n shall be a function of h

only, and that r_0 shall be a function of h and k, satisfying an equation of the form $r_0 = f\left(\frac{h - C r_0^2}{k^2 - C^2 r_0^2}\right)$, where f denotes an arbitrary function. The values of n and r_0 might, in accordance with these conditions, be fixed more definitively by reference to the cone rolling and sliding on the principal plane, used in the theories of POINSOT and JACOBI for the representation of the undisturbed motion, but to do this would require a further discussion of the integral equations, and it is a point which is not here entered into.

This being premised, we have (corresponding to the orbit in the theory of elliptic motion) the principal plane, with a departure point therein, the positions whereof, in respect to the fixed plane of reference, are determined by θ, the longitude of node; ϕ, the inclination; and σ, the departure of node. The precise signification of ϖ depends upon that of r_0, and the signification of r_0 being assumed to be completely determined, that of ϖ will be so likewise; ϖ is to be considered as an angle measured in the principal plane from the departure point, and determining in that plane a line (or, treating the plane as an orbit, a point) which I call the rotation pericentre, or simply the pericentre; or ϖ is the departure of the pericentre; and then g is an angle varying uniformly with the time, used for expressing in terms of the time the angle T, which determines the position, in regard to the principal plane and departure point therein, of the node of the plane of $x_{\prime}$ $y_{\prime}$, or *equator*, upon the principal plane,—such node corresponding with the moving body in the theory of elliptic motion. We may, in fact, say T, the departure of the last-mentioned node, $= \varpi$, the departure of pericentre, $+ (T - \varpi)$ the rotation true anomaly of such node; $T - \varpi$ being a function of g, the rotation mean anomaly of such node. The elements are then as follows, viz. :—

h, the constant of *Vis Viva*,
k, the constant of areas,
g, the rotation mean anomaly,
ϖ, the departure of rotation pericentre,
θ, the longitude of node of principal plane,
σ, the departure of ditto,
ϕ, the inclination of principal plane;

where θ, σ, ϕ determine the position of the principal plane and departure point therein, in respect to a fixed plane of reference and departure point therein; but it has been already remarked that the position of the principal plane and departure point therein might be, by the analogous quantities Θ, Σ, Φ, determined in reference to an arbitrarily varying plane of reference and departure point therein, and that the expressions for the variations of the elements would then be of the form given for this case in Art. I.

In the problem of the Rotation of the Earth, the principal plane is sensibly coincident with the plane of the equator, and on this account there is no actual physical representation of various quantities occurring in the mathematical problem. But a complete discussion of the mathematical problem does not thereby become unnecessary for the purposes of physical astronomy.

V.

If, now, the Disturbing Function is taken into account, the equations are to be integrated by the method of the variation of the elements. I use dg to denote that part of the variation of g $(= t\,dn + dc$, if $g = nt + c)$, which depends on the variation of the constants, and in like manner for dp, dq, dr, &c. I assume, moreover, that x_2 is a departure point in the principal plane; this gives $d\sigma - \cos\phi\, d\theta = 0$; and we see that the equations which lead to the expressions for the variations of the elements are

$$\begin{aligned}
A\,dp &= -\sin S\left(\operatorname{cosec} F\,\frac{d\Omega}{dT} + \cot F\,\frac{d\Omega}{dS}\right)dt + \cos S\,\frac{d\Omega}{dF}\,dt,\\
B\,dq &= \cos S\left(\operatorname{cosec} F\,\frac{d\Omega}{dT} + \cot F\,\frac{d\Omega}{dS}\right)dt + \sin S\,\frac{d\Omega}{dF}\,dt,\\
C\,dr &= -\frac{d\Omega}{dS}\,dt,\\
dT &= 0,\\
dS &= 0,\\
dF &= 0,\\
d\sigma &- \cos\phi\,d\theta = 0,
\end{aligned}$$

where, of course, $\Omega = \Omega\,(T, S, F)$.

The variations $d\,h$ and $d\,k$ are obtained from the equations

$$\tfrac{1}{2}\,d\,h = \mathrm{A}\,p\,d\,p + \mathrm{B}\,q\,d\,q + \mathrm{C}\,r\,d\,r,$$

$$k\,d\,k = \mathrm{A}^2 p\,d\,p + \mathrm{B}^2 q\,d\,q + \mathrm{C}^2 r\,d\,r,$$

expressions which will presently be resumed and reduced; S_2 and F_2 may be considered as given functions of h, k, r, and the variations $d\,\mathrm{S}_2$ and $d\,\mathrm{F}_2$ can be thus obtained. The expressions for T, S, F, in terms of T_2, S_2, F_2, σ, θ, ϕ, and the equations $d\,\mathrm{T} = 0$, $d\,\mathrm{S} = 0$, $d\,\mathrm{F} = 0$, $d\,\sigma - \cos\phi\,d\,\theta = 0$, then lead to the expressions for $d\,\theta$, $d\,\sigma$, $d\,\phi$, and to an equation $-d\,\mathrm{S}_2 + \cos\mathrm{F}_2\,d\,\mathrm{T}_2 = 0$; $d\,\mathrm{T}_2$ is thus also given in terms of $d\,h$, $d\,k$, $d\,r$. And this being so, the two equations

$$g = \int \frac{-\mathrm{C}\,d\,r}{(\mathrm{B}-\mathrm{A})\,p\,q},$$

$$\mathrm{T}_2 = \varpi + \int \frac{k\,(h - \mathrm{C}\,r^2)}{k^2 - \mathrm{C}^2 r^2}\;\frac{-\mathrm{C}\,d\,r}{(\mathrm{B}-\mathrm{A})\,p\,q},$$

lead to the expressions of $d\,g$, $d\,\varpi$, in terms of $d\,h$, $d\,k$, $d\,r$; the form of these expressions is simplified by partially fixing, in the manner already referred to, the significations of the quantities n and r_0, considered as functions of h and k. In this manner ($d\,h$, $d\,k$, $d\,r$, being given linear functions of $\frac{d\,\Omega}{d\,\mathrm{T}}\,d\,t$, $\frac{d\,\Omega}{d\,\mathrm{S}}\,d\,t$, $\frac{d\,\Omega}{d\,\mathrm{F}}\,d\,t$), we obtain $d\,h$, $d\,k$, $d\,g$, $d\,\varpi$, $d\,\theta$, $d\,\sigma$, $d\,\phi$, all of them expressed in the same form; that is, in terms of the differential coefficients of the disturbing function Ω, with respect to the co-ordinates T, S, F. We may then express the disturbing function Ω in terms of the elements, and transform the equations so as to obtain expressions for the variations of the elements in terms of the differential coefficients of Ω with respect to the elements.

VI.

Proceeding to carry out the foregoing plan, the equation

$$\tfrac{1}{2}\,d\,h = \mathrm{A}\,p\,d\,p + \mathrm{B}\,q\,d\,q + \mathrm{C}\,r\,d\,r,$$

putting for $\mathrm{A}\,d\,p$, $\mathrm{B}\,d\,q$, $\mathrm{C}\,d\,r$, their values, gives

$$\tfrac{1}{2}\,d h = (-p \sin \mathrm{S} + q \cos \mathrm{S})\left(\operatorname{cosec} \mathrm{F}\,\frac{d\,\Omega}{d\,\mathrm{T}} + \cot \mathrm{F}\,\frac{d\,\Omega}{d\,\mathrm{S}}\,dt\right)$$

$$+\ (p \cos \mathrm{S} + q \sin \mathrm{S})\,\frac{d\,\Omega}{d\,\mathrm{F}}\,d\,t - r\,\frac{d\,\Omega}{d\,\mathrm{S}}\,d\,t,$$

which may also be written

$$\tfrac{1}{2}\,d h = \frac{h\,(k - \mathrm{C}r^2)}{k^2 - \mathrm{C}^2 r^2}\left\{- \sin(\mathrm{S}_2 - \mathrm{S}) \sin \mathrm{F}_2\,\frac{d\,\Omega}{d\,\mathrm{F}}\,d\,t\right.$$

$$\left.+ \cos(\mathrm{S}_2 - \mathrm{S}) \sin \mathrm{F}_2\left(\operatorname{cosec} \mathrm{F}\,\frac{d\,\Omega}{d\,\mathrm{T}} + \cot \mathrm{F}\,\frac{d\,\Omega}{d\,\mathrm{S}}\right) d\,t - \cos \mathrm{F}_2\,\frac{d\,\Omega}{d\,\mathrm{S}}\,d\,t\right\}$$

$$+ \frac{k\,(\mathrm{B}-\mathrm{A})\,p\,q}{k^2 - \mathrm{C}^2 r^2}\left\{\ \cos(\mathrm{S}_2 - \mathrm{S}) \sin \mathrm{F}_2\,\frac{d\,\Omega}{d\,\mathrm{F}}\,d\,t\right.$$

$$\left.+ \sin(\mathrm{S}_2 - \mathrm{S}) \sin \mathrm{F}_2\left(\operatorname{cosec} \mathrm{F}\,\frac{d\,\Omega}{d\,\mathrm{T}} + \cot \mathrm{F}\,\frac{d\,\Omega}{d\,\mathrm{S}}\right) d\,t\right\}$$

$$+ \frac{r\,(\mathrm{C}\,h - k^2)}{k^2 - \mathrm{C}^2 r^2}\ \frac{d\,\Omega}{d\,\mathrm{S}}\,d\,t.$$

In fact, if, in this expression, we consider first the terms involving $\frac{d\,\Omega}{d\,\mathrm{S}}\,d\,t$, the coefficient is

$$\frac{-\,\mathrm{C}\,r\,(h - \mathrm{C}\,r^2) + r\,(\mathrm{C}\,h - k^2)}{k^2 - \mathrm{C}^2 r^2} = \frac{-\,r\,(k^2 - \mathrm{C}^2 r^2)}{k^2 - \mathrm{C}^2 r^2} = -\,r.$$

Next, the terms involving $\frac{d\,\Omega}{d\,\mathrm{F}}\,d\,t$, the coefficient is

$$\Big\{-\,(h - \mathrm{C}\,r^2) \sin(\mathrm{S}_2 - \mathrm{S}) + (\mathrm{B} - \mathrm{A})\,p\,q \cos(\mathrm{S}_2 - \mathrm{S})\Big\}\,\frac{1}{k \sin \mathrm{F}_2}$$

$$= \Big[\Big\{-\,(h - \mathrm{C}\,r^2) \sin \mathrm{S}_2 + (\mathrm{B} - \mathrm{A})\,p\,q \cos \mathrm{S}_2\Big\} \cos \mathrm{S}$$

$$+ \Big\{\ (h - \mathrm{C}\,r^2) \cos \mathrm{S}_2 + (\mathrm{B} - \mathrm{A})\,p\,q \sin \mathrm{S}_2\Big\} \sin \mathrm{S}\Big]\,\frac{1}{k \sin \mathrm{F}_2};$$

or, putting for sin S_2, cos S_2, their values $\frac{-Ap}{k\sin F_2}$, $\frac{Bq}{k\cos F_2}$, the coefficient is

$$\Big[\Big\{-(h-Cr^2)(-Ap)+(B-A)pq\,.\,Bq\Big\}\cos S$$

$$+\Big\{\;(h-Cr^2)\,Bq+(B-A)\,pq\,(-Ap)\Big\}\sin S\Big]\,\frac{1}{k^2\sin^2 F_2},$$

and the quantities in {} being respectively $p\,(k^2-C^2r^2)$ and $q\,(k^2-C^2r^2)$, and since $k^2-C^2r^2=k^2\sin^2F_2$, the coefficient is

$$=p\cos S+q\sin S.$$

In like manner, for the terms involving $\left(\operatorname{cosec}F\frac{d\Omega}{dT}+\cot F\frac{d\Omega}{dS}\right)dt$, the coefficient is

$$\Big[(h-Cr^2)\cos(S_2-S)+(B-A)\,pq\sin(S_2-S)\Big]\,\frac{1}{k\sin F_2}$$

$$=\Big[\Big\{(h-Cr^2)\cos S_2+(B-A)\,pq\sin S_2\Big\}\cos S$$

$$+\Big\{(h-Cr^2)\sin S_2-(B-A)\,pq\cos S_2\Big\}\sin S\Big]\,\frac{1}{k\sin F_2},$$

which is

$$=-p\sin S+q\cos S,$$

and the foregoing transformed expression for $\frac{1}{2}\,dh$ is thus seen to be correct.

The equation

$$k\,dk=A^2p\,dp+B^2q\,dq+C^2r\,dr,$$

substituting for $A\,dp$, $B\,dq$, $C\,dr$, their values, becomes

$$k\,dk=Ap\Big\{-\sin S\left(\operatorname{cosec}F\frac{d\Omega}{dT}+\cot F\frac{d\Omega}{dS}\right)dt+\cos S\frac{d\Omega}{dF}\,dt\Big\}$$

$$+Bq\Big\{\quad\cos S\left(\operatorname{cosec}F\frac{d\Omega}{dT}+\cot F\frac{d\Omega}{dS}\right)dt+\sin S\frac{d\Omega}{dF}\,dt\Big\}$$

$$+Cr\Big\{-\frac{d\Omega}{ds}\,dt\qquad\qquad\qquad\qquad\qquad\qquad\Big\}.$$

But, from the equations $A p = - k \sin S_2 \sin F_2$, $B q = k \cos S_2 \sin F_2$, $C r = k \cos F_2$, we deduce

$$\begin{aligned} - A p \sin S + B q \cos S &= k \cos (S_2 - S) \sin F_2, \\ A p \cos S + B q \sin S &= - k \sin (S_2 - S) \sin F_2, \\ C r &= k \cos F_2, \end{aligned}$$

and we thence find

$$d k = - \sin (S_2 - S) \sin F_2 \frac{d \Omega}{d F} d t + \cos (S_2 - S) \sin F_2 \left(\operatorname{cosec} F \frac{d \Omega}{d T} + \cot F \frac{d \Omega}{d S} \right) d t - \cos F_2 \frac{d \Omega}{d S} d t ;$$

so that $d h$ and $d k$ are now determined.

VII.

The expressions for $d h$ and $d k$ being thus obtained, and $d r$ being also given by the equation $C d r = - \frac{d \Omega}{d S} d t$, we may now express $d F_2$ and $d S_2$ in terms of $d h$, $d k$, $d r$. In fact, the equations

$$C r = k \cos F_2,$$

$$h - C r^2 = (k^2 - C^2 r^2) \left(\frac{\sin^2 S_2}{A} + \frac{\cos^2 S_2}{B} \right),$$

give immediately

$$k \sin F_2 d F_2 = - C d r + \cos F_2 d k,$$

$$\sin S_2 \cos S_2 \left(\frac{1}{A} - \frac{1}{B} \right) d S_2 = \frac{C (C h - k^2) r}{(k^2 - C^2 r^2)^2} d r + \frac{\frac{1}{2} d h}{k^2 - C^2 r^2} - \frac{(h - C r^2) k d k}{(k^2 - C^2 r^2)^2} ;$$

or, multiplying by $k^2 - C^2 r^2$ $(= k^2 \sin^2 F_2)$, and replacing $\cos S_2 \sin F_2 \sin^2 F_2$ by $- A B p q$, and dividing, the last equation becomes

$$d S_2 = \frac{- C (C h - k^2) r d r}{(k^2 - C^2 r^2) (B - A) p q} - \frac{\frac{1}{2} d h}{(B - A) p q} + \frac{(h - C r^2) k d k}{(k^2 - C^2 r^2) (B - A) p q} ;$$

and thence also

$$\cos F_2\, d\, S_2 = \frac{-C^2\,(C\,h - k^2)\,r^2\,d\,r}{k\,(k^2 - C^2\,r^2)\,(B - A)\,p\,q} - \frac{\frac{1}{2}\,C\,r\,d\,h}{k\,(B - A)\,p\,q} + \frac{C\,(h - C\,r)\,r\,d\,k}{(k^2 - C^2\,r^2)\,(B - A)\,p\,q},$$

which is the value of $d\,T_2$, as given by the equation (not yet demonstrated) $d\,T_2 = \cos S_2\, d\,F_2$.

The expression for $d\,F_2$, substituting for $d\,r$ and $d\,k$, their values, gives

$$k \sin F_2\, d\,F_2 = \cos F_2 \sin F_2 \cos (S_2 - S) \left(\operatorname{cosec} F \frac{d\,\Omega}{d\,T} + \cot F \frac{d\,\Omega}{d\,S}\right) d\,t$$

$$- \cos F_2 \sin F_2 \sin (S_2 - S) \frac{d\,\Omega}{d\,F}\, d\,t$$

$$- \cos^2 F_2 \frac{d\,\Omega}{d\,S}\, d\,t + \frac{d\,\Omega}{d\,S}\, d\,t\,;$$

or, combining the last two terms, and reducing,

$$d\,F_2 = \frac{1}{k}\left\{\cos F_2 \cos (S_2 - S) \left(\operatorname{cosec} F \frac{d\,\Omega}{d\,T} + \cot F \frac{d\,\Omega}{d\,S}\right) d\,t - \cos F_2 \sin (S_2 - S) \frac{d\,\Omega}{d\,F}\, d\,t + \sin F_2 \frac{d\,\Omega}{d\,S}\, d\,t\right\}.$$

We might, in like manner, transform the expression for $d\,S_2$, but it is somewhat more simple to obtain the new form by differentiation of the equation $\tan S_2 = -\frac{A\,p}{B\,q}$; this, in fact, gives

$$d\,S_2 = \frac{A\,B\,(p\,d\,q - q\,d\,p)}{A^2\,p^2 + B^2\,q^2} = \frac{1}{k^2 \sin^2 F_2}\,(A\,p\,.\,B\,d\,q - B\,q\,.\,A\,d\,p),$$

and the value of the numerator being

$$- k \sin S_2 \sin F_2 \left\{\cos S \left(\operatorname{cosec} F \frac{d\,\Omega}{d\,T} + \cot F \frac{d\,\Omega}{d\,S}\right) d\,t + \sin S \frac{d\,\Omega}{d\,F}\, d\,t\right\}$$

$$- k \cos S_2 \sin F_2 \left\{-\sin S \left(\operatorname{cosec} F \frac{d\,\Omega}{d\,T} + \cot F \frac{d\,\Omega}{d\,S}\right) d\,t + \cos S \frac{d\,\Omega}{d\,F}\, d\,t\right\},$$

we find

$$d\,\mathrm{S}_2 = -\frac{1}{k \sin \mathrm{F}_2}\left\{\sin(\mathrm{S}_2-\mathrm{S})\left(\operatorname{cosec}\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{T}}+\cot\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{S}}\right)d\,t+\cos(\mathrm{S}_2-\mathrm{S})\frac{d\,\Omega}{d\,\mathrm{F}}d\,t\right\};$$

the last-mentioned expressions for $d\,\mathrm{F}_2$ and $d\,\mathrm{S}_2$ will be used for obtaining $d\,\theta$, $d\,\sigma$, $d\,\varphi$.

VIII.

I form now the equations

$$\begin{aligned}
-\sin\mathrm{S}\sin\mathrm{F}\,d\,\mathrm{T}+\cos\mathrm{S}\,d\,\mathrm{F} &= p\,d\,t-\sin\mathrm{S}_2\sin\mathrm{F}_2\,d\,\mathrm{T}_2+\cos\mathrm{S}_2\,d\,\mathrm{F}_2\\
&\quad+\alpha_2\,(-\sin\sigma\sin\varphi\,d\,\theta+\cos\sigma\,d\,\varphi)\\
&\quad+\alpha_2'(\ \ \cos\sigma\sin\varphi\,d\,\theta+\sin\sigma\,d\,\varphi)\\
&\quad+\alpha_2''(\ \ -d\,\sigma+\cos\varphi\,d\,\theta\qquad),\\
\cos\mathrm{S}\sin\mathrm{F}\,d\,\mathrm{T}+\sin\mathrm{S}\,d\,\mathrm{F} &= q\,d\,t+\cos\mathrm{S}_2\sin\mathrm{F}_2\,d\,\mathrm{T}_2+\sin\mathrm{S}_2\,d\,\mathrm{F}_2\\
&\quad+\beta_2\,(-\sin\sigma\sin\varphi\,d\,\theta+\cos\sigma\,d\,\varphi)\\
&\quad+\beta_2'(\ \ \cos\sigma\sin\varphi\,d\,\theta+\sin\sigma\,d\,\varphi)\\
&\quad+\beta_2''(\ \ -d\,\sigma+\cos\varphi\,d\,\theta\qquad),\\
-d\,\mathrm{S}+\cos\mathrm{F}\,d\,\mathrm{T} &= r\,d\,t-d\,\mathrm{S}_2+\cos\mathrm{F}_2\,d\,\mathrm{T}_2\\
&\quad+\gamma_2\,(-\sin\sigma\sin\varphi\,d\,\theta+\cos\sigma\,d\,\varphi)\\
&\quad+\gamma_2'(\ \ \cos\sigma\sin\varphi\,d\,\theta+\sin\sigma\,d\,\varphi)\\
&\quad+\gamma_2''(\ \ -d\,\sigma+\cos\varphi\,d\,\theta\qquad);
\end{aligned}$$

which will presently be useful, but which require some explanation. As the equations stand, on the left hand, $d\,\mathrm{T}$, $d\,\mathrm{S}$, $d\,\mathrm{F}$, denote the entire variations of T, S, F, treating not only the constants, but also the time, as variable; but on the right hand, $d\,\mathrm{T}_2$, $d\,\mathrm{S}_2$, $d\,\mathrm{F}_2$, denote the variations of T_2, S_2, F_2, depending only on the variation of the constants; if, on the left hand and the right hand respectively, $d\,\mathrm{T}$, $d\,\mathrm{S}$, $d\,\mathrm{F}$, and $d\,\mathrm{T}_2$, $d\,\mathrm{S}_2$, $d\,\mathrm{F}_2$, were used to denote either the entire variations of T, S, F, and T_2, S_2, F_2, or else the variations of these quantities depending only on the variations of the constants, then the terms $p\,d\,t$, $q\,d\,t$, $r\,d\,t$, would have to be omitted, and the equations would still be true. And it is to be noticed that, omitting the terms pdt, qdt, rdt, the equations express the relations existing between $d\,\mathrm{T}$, $d\,\mathrm{S}$, $d\,\mathrm{F}$, $d\,\mathrm{T}_2$, $d\,\mathrm{S}_2$, $d\,\mathrm{F}_2$, $d\,\theta$, $d\,\sigma$, $d\,\varphi$, in virtue of the integral relations implied by the existence of the spherical triangle, the sides and angles whereof are $S_2 - S$, $T_2 - \sigma$, $T - \theta$, and ϕ, $180° - F$, F_2.

For the present purpose we are to omit the terms $p\,dt$, $q\,dt$, $r\,dt$, and to write $dT = 0$, $dS = 0$, $dF = 0$. We have thus

$$\begin{aligned}
-(-\sin S_2 \sin F_2\, dT_2 + \cos S_2\, dF_2) = {} & \alpha_2\,(-\sin\sigma\sin\phi\, d\theta + \cos\sigma\, d\phi) \\
& + \alpha_2'(\ \cos\sigma\sin\phi\, d\theta + \sin\sigma\, d\phi) \\
& + \alpha_2''(\ d\sigma + \cos\phi\, d\theta\ ),
\end{aligned}$$

$$\begin{aligned}
-(\ \cos S_2 \sin F_2\, dT_2 + \sin S_2\, dF_2) = {} & \beta_2\,(-\sin\sigma\sin\phi\, d\theta + \cos\sigma\, d\phi) \\
& + \beta_2'(\ \cos\sigma\sin\phi\, d\theta + \sin\sigma\, d\phi) \\
& + \beta_2''(\ -d\sigma + \cos\phi\, d\theta\ ),
\end{aligned}$$

$$\begin{aligned}
-(-d S_2 + \cos F_2\, dT_2) = {} & \gamma_2\,(-\sin\sigma\sin\phi\, d\theta + \cos\sigma\, d\phi) \\
& + \gamma_2'(\ \cos\sigma\sin\phi\, d\theta + \cos\sigma\, d\phi) \\
& + \gamma_2''(\ d\sigma + \cos\phi\, d\theta\ ).
\end{aligned}$$

Hence we have

$$\begin{aligned}
-\sin\sigma\sin\phi\, d\theta + \cos\sigma\, d\phi = {} & -\alpha_2\,(-\sin S_2 \sin F_2\, dT_2 + \cos F_2\, dS_2) \\
& -\beta_2\,(\ \cos S_2 \sin F_2\, dT_2 + \sin F_2\, dS_2) \\
& -\gamma_2\,(\ \cos F_2\, dT_2 - dS_2\ ),
\end{aligned}$$

which, attending to the values of α_2'', β_2'', γ_2'', may be written,

$$\begin{aligned}
-\sin\sigma\sin\phi\, d\theta + \cos\sigma\, d\phi = {} & -\alpha_2\,(\alpha_2''\, dT_2 + \cos S_2\, dF_2) \\
& -\beta_2\,(\beta_2''\, dT_2 + \sin S_2\, dF_2) \\
& -\gamma_2\,(\gamma_2''\, dT_2 - dS_2\ ) \\
= {} & -(\alpha_2 \cos S_2 + \beta_2 \sin S_2)\, dF_2 + \gamma_2\, dS_2,
\end{aligned}$$

since the coefficient of dT_2 vanishes; and, in like manner,

$$\begin{aligned}
\cos\sigma\sin\phi\, d\theta + \sin\sigma\, d = {} & -\alpha_2'\,(\alpha_2''\, dT_2 + \cos S_2\, dF_2) \\
& -\beta_2'\,(\beta_2''\, dT_2 + \sin S_2\, dF_2) \\
& -\gamma_2'\,(\gamma_2''\, dT_2 - dS_2\ ) \\
= {} & -(\alpha_2' \cos S_2 + \beta_2' \sin S_2)\, dF_2 + \gamma_2'\, dS_2;
\end{aligned}$$

and so also

$$\begin{aligned}-d\sigma+\cos\varphi\,d\theta &= -\alpha_2''(\alpha_2''\,d\,T_2+\cos S_2\,d\,F_2)\\ &\quad -\beta_2''(\beta_2''\,d\,T_2+\sin S_2\,d\,F_2)\\ &\quad -\gamma_2''(\gamma_2''\,d\,T_2-d\,S_2\qquad)\\ &= d\,T_2-(\alpha_2''\cos S_2+\beta_2''\sin S_2)\,d\,F_2+\gamma_2''\,d\,S_2.\end{aligned}$$

But we have

$$\begin{aligned}\alpha_2\ \cos S_2+\beta_2\ \sin S_2 &= \cos T_2, & \gamma_2 &= \sin T_2\sin F_2,\\ \alpha_2'\ \cos S_2+\beta_2'\ \sin S_2 &= \sin T_2, & \gamma_2' &= -\cos T_2\sin F_2,\\ \alpha_2''\cos S_2+\beta_2''\sin S_2 &= 0\ , & \gamma_2'' &= \cos F_2,\end{aligned}$$

and the foregoing equations thus become

$$\begin{aligned}-\sin\sigma\sin\varphi\,d\theta+\cos\sigma\,d\varphi &= -\cos T_2\,d\,F_2+\sin T_2\sin F_2\,d\,S_2,\\ \cos\sigma\sin\varphi\,d\theta+\sin\sigma\,d\varphi &= -\sin T_2\,d\,F_2-\cos T_2\sin F_2\,d\,S_2,\\ -d\sigma+\cos\varphi\,d\theta &= d\,T_2\quad+\cos F_2\,d\,S_2.\end{aligned}$$

The last equation, making x_2 a departure point, or putting $-d\sigma+\cos\varphi\,d\theta=0$, gives

$$d\,T_2=-\cos F_2\,d\,S_2,$$

an equation above referred to. The other two equations give

$$\begin{aligned}d\varphi &= -\cos(T_2-\sigma)\,d\,F_2+\sin(T_2-\sigma)\sin F_2\,d\,S_2,\\ \sin\varphi\,d\theta &= -\sin(T_2-\sigma)\,d\,F_2-\cos(T_2-\sigma)\sin F_2\,d\,S_2,\end{aligned}$$

which, with the equation,

$$d\sigma=\cos\varphi\,d\theta,$$

give $d\theta$, $d\sigma$, $d\varphi$, in terms of $d\,S_2$ and $d\,F_2$.

IX.

Proceeding to substitute for $d\,\mathrm{S}_2$, $d\,\mathrm{F}_2$, their values, we find

$$\begin{aligned}k\,d\phi = &-\cos(\mathrm{T}_2-\sigma)\Big\{\cos\mathrm{F}_2\cos(\mathrm{S}_2-\mathrm{S})\Big(\operatorname{cosec}\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{T}}+\cot\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{S}}\Big)dt-\cos\mathrm{F}_2\cos(\mathrm{S}_2-\mathrm{S})\frac{d\,\Omega}{d\,\mathrm{F}}dt+\sin\mathrm{F}_2\frac{d\,\Omega}{d\,\mathrm{S}}dt\Big\}\\ &-\sin(\mathrm{T}_2-\sigma)\Big\{\sin(\mathrm{S}_2-\mathrm{S})\Big(\operatorname{cosec}\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{T}}+\cot\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{S}}\Big)dt+\cos(\mathrm{S}_2-\mathrm{S})\frac{d\,\Omega}{d\,\mathrm{F}}dt\Big\},\end{aligned}$$

or reducing

$$\begin{aligned}k\,d\,\phi = &-\Big[\sin(\mathrm{T}_2-\sigma)\sin(\mathrm{S}_2-\mathrm{S})+\cos(\mathrm{T}_2-\sigma)\cos(\mathrm{S}_2-\mathrm{S})\cos\mathrm{F}_2\Big]\Big(\operatorname{cosec}\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{T}}+\cot\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{S}}\Big)dt\\ &-\Big[\sin(\mathrm{T}_2-\sigma)\cos(\mathrm{S}_2-\mathrm{S})-\cos(\mathrm{T}_2-\sigma)\sin(\mathrm{S}_2-\mathrm{S})\cos\mathrm{F}_2\Big]\frac{d\,\Omega}{d\,\mathrm{F}}dt\\ &-\Big[\cos(\mathrm{T}_2-\sigma)\sin\mathrm{F}_2\Big]\frac{d\,\Omega}{d\,\mathrm{S}}dt;\end{aligned}$$

and, in like manner,

$$\begin{aligned}k\sin\phi\,d\,\theta = &-\sin(\mathrm{T}_2-\sigma)\Big\{\cos\mathrm{F}_2\cos(\mathrm{S}_2-\mathrm{S})\Big(\operatorname{cosec}\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{T}}+\cot\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{S}}\Big)dt-\cot\mathrm{F}_2\cos(\mathrm{S}_2-\mathrm{S})\frac{d\,\Omega}{d\,\mathrm{F}}dt+\sin\mathrm{F}_2\frac{d\,\Omega}{d\,\mathrm{S}}dt\Big\}\\ &+\cos(\mathrm{T}_2-\sigma)\Big\{\sin(\mathrm{S}_2-\mathrm{S})\Big(\operatorname{cosec}\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{T}}+\cot\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{S}}\Big)dt+\cos(\mathrm{S}_2-\mathrm{S})\frac{d\,\Omega}{d\,\mathrm{F}}dt\Big\},\end{aligned}$$

or reducing

$$\begin{aligned}k\sin\phi\,d\,\theta = &\Big[\cos(\mathrm{T}_2-\sigma)\sin(\mathrm{S}_2-\mathrm{S})-\sin(\mathrm{T}_2-\sigma)\cos(\mathrm{S}_2-\sigma)\cos\mathrm{F}_2\Big]\Big(\operatorname{cosec}\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{T}}+\cot\mathrm{F}\frac{d\,\Omega}{d\,\mathrm{S}}\Big)dt\\ &+\Big[\cos(\mathrm{T}_2-\sigma)\cos(\mathrm{S}_2-\mathrm{S})+\sin(\mathrm{T}_2-\sigma)\sin(\mathrm{S}_2-\mathrm{S})\cos\mathrm{F}_2\Big]\frac{d\,\Omega}{d\,\mathrm{F}}dt\\ &+\Big[-\sin(\mathrm{T}_2-\sigma)\sin\mathrm{F}_2\Big]\frac{d\,\Omega}{d\,\mathrm{S}}dt.\end{aligned}$$

These expressions for $d\,\phi$ and $d\,\theta$ are in a form which is convenient for some purposes, but they may be further reduced by means of the spherical triangle. In fact, in the expression for $d\,\phi$, the three coefficients in [ ] are respectively,

$$\begin{aligned}
\sin b \sin a + \cos b \cos a \cos C &= \sin B \sin A - \cos B \cos A \cos c \\
&= \sin F \sin \phi + \cos F \cdot \cos \phi \cos (T - \theta), \\
\sin b \cos a - \cos b \sin a \cos C &= \cos A \sin c \\
&= \cos \phi \sin (T - \theta), \\
\cos b \sin C &= \cos B \sin A + \sin B \cos A \cos c \\
&= - \cos F \sin \phi + \sin F \cos \phi \cos (T - \theta);
\end{aligned}$$

And we have thus

$$\begin{aligned}
k\,d\phi = &- \left(\sin F \sin \phi + \cos F \cos \phi \cos (T - \theta)\right) \left(\operatorname{cosec} F \frac{d\Omega}{dT} + \cot F \frac{d\Omega}{dS}\right) dt \\
&- \cos \phi \sin (T - \theta) \frac{d\Omega}{dF} dt \\
&- \left(- \cos F \sin \phi + \sin F \cos \phi \cos (T - \theta)\right) \frac{d\Omega}{dS} dt;
\end{aligned}$$

and the right-hand side is

$$\begin{aligned}
&- \left(\sin F \sin \phi + \cos F \cos \phi \cos (T - \theta)\right) \operatorname{cosec} F \frac{d\Omega}{dT} dt \\
&- \cos \phi \sin (T - \theta) \frac{d\Omega}{dF} dt \\
&+ \operatorname{cosec} F \cos \phi \cos (T - \theta) \frac{d\Omega}{dS} dt,
\end{aligned}$$

or we have, finally,

$$\begin{aligned}
k\,d\phi = &- \cos \phi \cos (T - \theta) \left(\cot F \frac{d\Omega}{dT} + \operatorname{cosec} F \frac{d\Omega}{dS}\right) dt \\
&- \sin \phi \frac{d\Omega}{dT} dt \\
&- \cos \phi \sin (T - \theta) \frac{d\Omega}{dF} dt.
\end{aligned}$$

In like manner, in the original expression for $k \sin \phi \, d\theta$, the three coefficients in [] are

$$\begin{aligned}
\cos b \sin a - \sin b \cos a \cos C &= \cos B \sin c \\
&= -\cos F \sin (T - \theta), \\
\cos b \cos a + \sin b \sin a \cos C &= \cos c \\
&= \cos (T - \theta), \\
-\sin b \sin C \quad &= -\sin B \sin c \\
&= -\sin T \sin (T - \theta),
\end{aligned}$$

and thence

$$\begin{aligned}
k \sin \phi \, d\theta = &-\cos F \sin (T - \theta) \left(\operatorname{cosec} F \frac{d\Omega}{dT} + \cot F \frac{d\Omega}{dS}\right) dt \\
&+ \cos (T - \theta) \frac{d\Omega}{dF} dt \\
&- \sin F \sin (T - \theta) \frac{d\Omega}{dS} dt,
\end{aligned}$$

and the right-hand side is

$$\begin{aligned}
&- \cot F \sin (T - \theta) \frac{d\Omega}{dT} dt \\
&+ \cos (T - \theta) \frac{d\Omega}{dF} dt \\
&- \operatorname{cosec} F \sin (T - \theta) \frac{d\Omega}{dS} dt,
\end{aligned}$$

or we have, finally,

$$\begin{aligned}
k \sin \phi \, d\theta = &- \sin (T - \theta) \left(\cot F \frac{d\Omega}{dT} + \operatorname{cosec} F \frac{d\Omega}{dS}\right) dt \\
&+ \cos (T - \theta) \frac{d\Omega}{dF} dt.
\end{aligned}$$

The expression for $d\sigma$ can be obtained from either of those for $d\theta$, by the equation $d\sigma = \cos \phi \, d\theta$, and we have thus the values of $d\sigma$, $d\theta$, $d\phi$.

X.

It remains to find the expressions for $d g$ and $d \varpi$. We have

$$g = n \int \frac{-C\,dr}{(B - A)\,p q},$$

$$T_2 = \varpi + \int \frac{k\,(h - C r^2)}{k^2 - C^2 r^2}\,\frac{-C\,dr}{(B - A)\,p q},$$

where p, q, are considered as functions of r, h, k, and where, besides, n and the inferior limit r_0 of the integrals are functions of h and k. We may write

$$dg = \frac{-n C}{(B - A)\,p q}\,dr + \mathfrak{A}\,dh + \mathfrak{B}\,dk,$$

$$d T_2 = d\varpi + \frac{k\,(h - C r^2)}{k^2 - C^2 r^2}\,\frac{-C\,dr}{(B - A)\,p q}\,dr + \mathfrak{C}\,dh + \mathfrak{D}\,dk,$$

where $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, are functions which contain integrals with respect to r, and there is not any algebraical relation between them, except the equation $\mathfrak{B} = -2 n \mathfrak{C}$, which will be obtained presently. I retain, therefore, $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, in the formulæ. The first equation gives the value of $d g$: from the second equation we have

$$d\varpi = d T_2 + \frac{C k\,(h - C r^2)}{(k^2 - C^2 r^2)\,(B - A)\,p q}\,dr - \mathfrak{C}\,dh - \mathfrak{D}\,dk,$$

or substituting for $d T_2$ its value $\cos S_2\,d F_2$, the expression for which has been obtained above, we have

$$\begin{aligned} d\varpi = {} & \frac{-C^2\,(C h - k^2)\,r^2\,dr}{k\,(k^2 - C^2 r^2)\,(B - A)\,p q} - \frac{\frac{1}{2}\,C r\,dk}{k\,(B - A)\,p q} + \frac{C\,(h - C r^2)\,r\,dk}{(k^2 - C^2 r^2)\,(B - A\,p q)} \\ & + \frac{C k^2\,(h - r^2)\,dr}{k\,(k^2 - C^2 r^2)\,(B - A)\,p q} - \mathfrak{C}\,dh - \mathfrak{D}\,dk, \end{aligned}$$

or reducing, this is

$$d\varpi = \frac{C h\,dr}{k\,(B - A)\,p q} + \left[\frac{-\frac{1}{2}\,C r}{k\,(B - A)\,p q} - \mathfrak{C}\right] dh + \left[\frac{C\,(h - C r^2)\,r}{(k^2 - C^2 r^2)\,(B - A)\,p q} - \mathfrak{D}\right] dk,$$

which might be retained in this form. I obtain, however, a different form as follows, viz., we have

$$d\varpi = \cos F_2\, d\,S_2 + \frac{C\,k\,(h - C\,r^2)\,d\,r}{(k^2 - C^2 r^2)\,(B - A)\,p\,q} - \mathfrak{C}\,d\,h - \mathfrak{B}\,d\,k;$$

or using the other form of $d\,S_2$, and substituting also for $d\,r$ its value from the equation $C\,d\,r = -\frac{d\,\Omega}{d\,S}\,d\,t$, we have

$$d\varpi = -\frac{1}{k}\cot F_2 \left\{\sin(S_2 - S)\left(\operatorname{cosec} F\,\frac{d\,\Omega}{d\,T} + \cot F\,\frac{d\,\Omega}{d\,S}\right) d\,t + \cos(S_2 - S)\,\frac{d\,\Omega}{d\,F}\,d\,t\right\}$$

$$- \frac{k\,(h - C\,r^2)}{(k^2 - C^2 r^2)\,(B - A)\,p\,q}\,\frac{d\,\Omega}{d\,S}\,d\,t - \mathfrak{C}\,d\,h - \mathfrak{B}\,d\,k.$$

We have now

$$\mathfrak{B} = \frac{d}{dk}\,n\int \frac{-C\,d\,r}{(B - A)\,p\,q} = n\int dr\,\frac{d}{dk}\,\frac{-C}{(B - A)\,p\,q} + \frac{dn}{dk}\int \frac{-C\,d\,r}{(B - A)\,p\,q} + \frac{n\,C}{(B - A)\,p_0 q_0}\,\frac{d\,r_0}{d\,k},$$

$$\mathfrak{C} = \frac{d}{dh}\int \frac{k\,(h - C\,r^2) - C\,dr}{k^2 - C^2 r^2\,(B - A)\,p\,q} = \int dr\,\frac{d}{dh}\,\frac{-C\,k\,(h - C\,r^2)}{(k^2 - C^2 r^2)\,(B - A)\,p\,q} + \frac{C\,k\,(h - C\,r_0^2)}{(k^2 - C^2 r_0^2)\,(B - A)\,p_0 q_0}\,\frac{dr_0}{dh},$$

where p_0, q_0, are the values of p and q corresponding to $r = r_0$. If we assume that n is a function of h only, the term multiplied by $\frac{d\,h}{d\,k}$ will disappear, and by properly determining r_0 as a function of h and k, we can, as regards the terms which contain r_0, satisfy the equation $\mathfrak{B} = -2\,n\,\mathfrak{C}$; the condition for this is

$$\frac{d\,r_0}{d\,k} = \frac{-2\,k\,(h - C\,r_0^2)}{k^2 - C^2 r_0^2}\,\frac{d\,r_0}{d\,h},$$

which will be satisfied if r_0 is determined as a function of h and k by an equation of the form

$$r_0 = f\left(\frac{h - C\,r_0^2}{k^2 - C^2 r_0^2}\right),$$

where f denotes an arbitrary function.

It remains to show that, as regards the terms involving integrals, we have $\mathfrak{B} = -2n\mathfrak{C}$, and this will be the case if

$$\frac{d}{dk}\frac{-\mathrm{C}}{(\mathrm{B}-\mathrm{A})pq} = -2\frac{d}{dh}\frac{-\mathrm{C}k(h-\mathrm{C}r^2)}{(k^2-\mathrm{C}^2r^2)(\mathrm{B}-\mathrm{A})pq},$$

or, what is the same thing, if

$$\frac{d}{dk}\frac{1}{pq} = -2\frac{d}{dh}\frac{k(h-\mathrm{C}r^2)}{(k^2-\mathrm{C}^2r^2)pq} = \frac{-2k}{(k^2-\mathrm{C}^2r^2)}\frac{d}{dh}\frac{h-\mathrm{C}r^2}{pq},$$

in which equation p, q, are considered as functions of h, k, given by the equations

$$\mathrm{A}p^2 + \mathrm{B}q^2 + \mathrm{C}r^2 = h,$$

$$\mathrm{A}^2p^2 + \mathrm{B}^2q^2 + \mathrm{C}^2r^2 = k^2.$$

We find without difficulty

$$\frac{d\,.\,pq}{dh} = \frac{-\frac{1}{2}(\mathrm{A}^2p^2-\mathrm{B}^2q^2)}{\mathrm{AB}(\mathrm{B}-\mathrm{A})pq}, \qquad \frac{d\,.\,pq}{dk} = \frac{k(\mathrm{A}p^2-\mathrm{B}q^2)}{\mathrm{AB}(\mathrm{B}-\mathrm{A})pq},$$

and thence

$$\begin{aligned}\frac{d}{dh}\frac{h-\mathrm{C}r^2}{pq} &= \frac{1}{pq} + \frac{\frac{1}{2}(h-\mathrm{C}r^2)(\mathrm{A}^2p^2-\mathrm{B}^2q^2)}{\mathrm{AB}(\mathrm{B}-\mathrm{A})p^3q^3}\\ &= \frac{1}{\mathrm{AB}(\mathrm{B}-\mathrm{A})p^3q^3}\left[\mathrm{AB}(\mathrm{B}-\mathrm{A})p^2q^2 + \tfrac{1}{2}(\mathrm{A}p^2+\mathrm{B}q^2)(\mathrm{A}^2p^2-\mathrm{B}^2q^2)\right]\\ &= \frac{1}{\mathrm{AB}(\mathrm{B}-\mathrm{A})p^3q^3}\,\tfrac{1}{2}(\mathrm{A}p^2-\mathrm{B}q^2)(\mathrm{A}^2p^2)+(\mathrm{B}^2q^2)\\ &= \frac{\frac{1}{2}(k^2-\mathrm{C}^2r^2)(\mathrm{A}p^2-\mathrm{B}q^2)}{\mathrm{AB}(\mathrm{B}-\mathrm{A})p^3q^3},\end{aligned}$$

and the right-hand side of the equation in question is therefore

$$= \frac{-k(\mathrm{A}p^2-\mathrm{B}q^2)}{\mathrm{AB}(\mathrm{B}-\mathrm{A})p^3q^3},$$

which is obviously also the value of the left-hand side. Hence, under the assumed relations $\left(n \text{ a function of } h \text{ only, and } r_0 = f\left(\frac{h-\mathrm{C}r_0^2}{k^2-\mathrm{C}^2r_0^2}\right)\right)$, we have the above-mentioned equation $\mathfrak{B} = -2n\mathfrak{C}$.

XI.

It will be convenient to recapitulate the various formulæ for the variations, as follows; we have

$$\tfrac{1}{2}\,dh = (-p\sin S + q\cos S)\left(\operatorname{cosec} F\frac{d\Omega}{dT} + \cot F\frac{d\Omega}{dS}\right)dt$$

$$+ (p\cos S + q\sin S)\frac{d\Omega}{dF}\,dt - r\frac{d\Omega}{dS}\,dt,$$

and

$$\tfrac{1}{2}\,dh = \frac{k(h-Cr^2)}{k^2-C^2r^2}\left\{\cos(S_2-S)\sin F_2\left(\operatorname{cosec} F\frac{d\Omega}{dT}+\cot F\frac{d\Omega}{dS}\right)dt - \sin(S_2-S)\sin F_2\frac{d\Omega}{dF}\,dt - \cos F_2\frac{d\Omega}{dS}\,dt\right\}$$

$$+\frac{k(B-A)pq}{k^2-C^2r^2}\left\{\sin S_2-S)\sin F_2\left(\operatorname{cosec} F\frac{d\Omega}{dT}+\cot F\frac{d\Omega}{dS}\right)dt + \cos(S_2-S)\sin F_2\frac{d\Omega}{dF}\,dt\right\}$$

$$+\frac{r(Ch-k^2)}{k^2-C^2r^2}\frac{d\Omega}{dS}\,dt;$$

$$k\,dk = (-Ap\sin S + Bq\cos S)\left(\operatorname{cosec} F\frac{d\Omega}{dT} + \cot F\frac{d\Omega}{dS}\right)dt$$

$$+ (Ap\cos S + Bq\sin S)\frac{d\Omega}{dF}\,dt - Cr\frac{d\Omega}{dS}\,dt,$$

and

$$dk = \cos(S_2-S)\sin F_2\left(\operatorname{cosec} F\frac{d\Omega}{dT}+\cot F\frac{d\Omega}{dS}\right)dt - \sin(S_2-S)\sin F_2\frac{d\Omega}{dF}\,dt - \cos S_2\frac{d\Omega}{dS}\,dt;$$

$$k\,d\varphi = -\cos\varphi\cos(T-\theta)\left(\cot F\frac{d\Omega}{dT} + \operatorname{cosec} F\frac{d\Omega}{dS}\right)dt$$

$$-\sin\varphi\frac{d\Omega}{dT}\,dt - \cos\varphi\sin(T-\theta)\frac{d\Omega}{dF}\,dt,$$

and

$$k\,d\varphi = -\Big[\sin(T_2-\sigma)\sin(S_2-S) + \cos(T_2-\sigma)\cos(S_2-S)\cos F_2\Big]\left(\operatorname{cosec} F\frac{d\Omega}{dT} + \cot F\frac{d\Omega}{dS}\right)dt$$

$$-\Big[\sin(T_2-\sigma)\cos(S_2-S) - \cos(T_2-\sigma)\sin(S_2-S)\cos F_2\Big]\frac{d\Omega}{dF}\,dt$$

$$-\Big[\qquad -\cos(T_2-\sigma)\sin F_2 \qquad\Big]\frac{d\Omega}{dS}\,dt;$$

$$k \sin\varphi\, d\theta = -\sin(\mathrm{T}-\theta)\left(\cot\mathrm{F}\frac{d\Omega}{d\mathrm{T}} + \operatorname{cosec}\mathrm{F}\frac{d\Omega}{d\mathrm{S}}\right)dt$$

$$+\cos(\mathrm{T}-\theta)\,\frac{d\Omega}{d\mathrm{F}}\,dt,$$

and

$$k\sin\varphi\, d\theta = \Big[\cos(\mathrm{T}_2-\sigma)\sin(\mathrm{S}_2-\mathrm{S}) - \sin(\mathrm{T}_2-\sigma)\cos(\mathrm{S}_2-\mathrm{S})\cos\mathrm{F}_2\Big]\left(\operatorname{cosec}\mathrm{F}\frac{d\Omega}{d\mathrm{T}} + \cot\mathrm{F}\frac{d\Omega}{d\mathrm{S}}\right)dt$$

$$+\Big[\cos(\mathrm{T}_2-\sigma)\cos(\mathrm{S}_2-\mathrm{S}) + \sin(\mathrm{T}_2-\sigma)\sin(\mathrm{S}_2-\mathrm{S})\cos\mathrm{F}_2\Big]\frac{d\Omega}{d\mathrm{F}}\,dt$$

$$+\Big[\qquad -\sin(\mathrm{T}_2-\sigma)\sin\mathrm{F}_2 \qquad\Big]\frac{d\Omega}{d\mathrm{S}}\,dt;$$

$$d\sigma = \cos\varphi\, d\theta;$$

$$dg = \frac{-n\mathrm{C}}{(\mathrm{B}-\mathrm{A})pq}\,dr + \mathfrak{A}\,dh + \mathfrak{B}\,dk,$$

and

$$dg = \frac{n}{(\mathrm{B}\ \mathrm{A})pq}\frac{d\Omega}{d\mathrm{S}}\,dt + \mathfrak{A}\,dh + \mathfrak{B}\,dk;$$

$$d\varpi = \frac{\mathrm{C}h\,dr}{k(\mathrm{B}-\mathrm{A})pq} + \left[\frac{-\frac{1}{2}\mathrm{C}r}{k(\mathrm{B}-\mathrm{A})pq} - \mathfrak{C}\right]dh + \left[\frac{\mathrm{C}(h-\mathrm{C}r^2)r}{(k^2-\mathrm{C}^2r^2)(\mathrm{B}-\mathrm{A})pq} - \mathfrak{D}\right]dk,$$

and

$$d\varpi = -\frac{1}{k}\cot\mathrm{F}_2\left\{\sin(\mathrm{S}_2-\mathrm{S})\left(\operatorname{cosec}\mathrm{F}\frac{d\Omega}{d\mathrm{T}} + \cot\mathrm{F}\frac{d\Omega}{d\mathrm{S}}\right)dt + \cos(\mathrm{S}_2-\mathrm{S})\frac{d\Omega}{d\mathrm{F}}\,dt\right\}$$

$$+\frac{\mathrm{B}\sin^2\mathrm{S}_2 + \mathrm{A}\cos^2\mathrm{S}_2}{k(\mathrm{B}-\mathrm{A})\sin^2\mathrm{F}_2\sin\mathrm{S}_2\cos\mathrm{S}_2}\frac{d\Omega}{d\mathrm{S}}\,dt - \mathfrak{C}\,dh - \mathfrak{D}\,dk;$$

where it will be remembered that $\mathfrak{B} = -2n\mathfrak{C}$. The different forms for the variations of the same element are, or may be, each of them useful. The first expressions for dg and $d\varpi$ respectively are to be considered as giving these variations in terms of dr, dh, dk; and the second expressions are those obtained by substituting for these quantities their values, but in the terms multiplied by the integral expressions $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, which, on account of these multipliers, do not unite with the other terms, dh, dk, are retained as standing for their values. The following equations may be added,

$$dT_2 = d\varpi - \frac{Ck(h - Cr^2)}{(k^2 - C^2r^2)(B - A)pq}\,dr + \mathfrak{C}\,dh + \mathfrak{D}\,dk,$$

$$dF_2 = \frac{-Ck\sin F_2}{k^2 - C^2r^2}\,dr + \frac{k\sin F_2\cos F_2}{k^2 - C^2r^2}\,dk,$$

$$dS_2 = \frac{-C(Ch - k^2)r}{(k^2 - C^2r^2)(B - A)pq}\,dr - \frac{\frac{1}{2}\,dh}{(B - A)pq} + \frac{(h - Cr^2)\,k\,dk}{(k^2 - C^2r^2)(B - A)pq},$$

which give dF_2, dS_2, in terms of dr, dh, dk; and I call to mind, also, the equations

$$Ap = - k\sin S_2\sin F_2, \qquad Bq = k\cos S_2\sin F_2, \qquad Cr = k\cos F_2.$$

XII.

To find the differential coefficients of Ω with respect to the elements, I proceed as follows; considering the function first under the form $\Omega = \Omega\,(T, S, F)$, the total differential is

$$\frac{d\Omega}{dT}\,dT + \frac{d\Omega}{dS}\,dS + \frac{d\Omega}{dF}\,dF,$$

which must be equal to the total differential of Ω considered under the form $\Omega = \Omega\,(h, k, g, \varpi, \sigma, \theta, \phi)$; that is, it must be

$$= \frac{d\Omega}{dh}\,dh + \frac{d\Omega}{dk}\,dk + \frac{d\Omega}{dg}\,(n\,dt + dg) + \frac{d\Omega}{d\varpi}\,d\varpi + \frac{d\Omega}{d\theta}\,d\theta + \frac{d\Omega}{d\sigma}\,d\sigma + \frac{d\Omega}{d\phi}\,d\phi;$$

where, as elsewhere, dg denotes only the part of the variation of g, which depends on the variation of the constants; so that the total variation of g is represented by $n\,dt + dg$. The value of $d\Omega$,

$$= \frac{d\Omega}{dT}\,dT + \frac{d\Omega}{dS}\,dS + \frac{d\Omega}{dF}\,dF,$$

is to be obtained in a form comparable with the last-mentioned expression, by means of the formulæ *suprà*, Art. VIII., which, for shortness, I represent as follows:—

$$\begin{aligned} -\sin S \sin F\, dT + \cos S\, dF &= dP, \\ \cos S \sin F\, dT + \sin S\, dF &= dQ, \\ -dS + \cos F\, dT \qquad &= dR; \end{aligned}$$

these equations give

$$\begin{aligned} dF &= \qquad \cos S\, dP + \sin S\, dQ, \\ dT &= \operatorname{cosec} F\,(-\sin S\, dP + \cos S\, dQ), \\ dS &= \cot F\,(-\sin S\, dP + \cos S\, dQ) - dR; \end{aligned}$$

and we then have

$$\begin{aligned} d\Omega = \; & (\cos S\, dP + \sin S\, dQ)\,\frac{d\Omega}{dS} \\ & + -(\sin S\, dP + \cos S\, dQ)\left(\operatorname{cosec} F\,\frac{d\Omega}{dT} + \cot F\,\frac{d\Omega}{dS}\right) \\ & - dR\,\frac{d\Omega}{dS}; \end{aligned}$$

and substituting for dP, dQ, dR, their values, the resulting expression may, for shortness, be represented by

$$d\Omega = d_1\Omega + d_2\Omega + d_3\Omega,$$

where

$$\begin{aligned} d_1\Omega = \; & (\;p\cos S + q\sin S)\,\frac{d\Omega}{dF}\,dt \\ & + (-p\sin S + q\cos S)\left(\operatorname{cosec} F\,\frac{d\Omega}{dT} + \cot F\,\frac{d\Omega}{dS}\right)dt \\ & - r\,\frac{d\Omega}{dS}\,dt. \end{aligned}$$

$$\begin{aligned} d_2\Omega = & \left\{\cos(S_2-S)\sin F_2\left(\operatorname{cosec} F\,\frac{d\Omega}{dT} + \cot F\,\frac{d\Omega}{dS}\right) - \sin(S_2-S)\sin F_2\,\frac{d\Omega}{dF} - \cos F_2\,\frac{d\Omega}{dS}\right\} dT_2 \\ & + \left\{ \sin(S_2-S)\left(\operatorname{cosec} F\,\frac{d\Omega}{dT} + \cot F\,\frac{d\Omega}{dS}\right) + \cos(S_2-S)\,\frac{d\Omega}{dF} \right\} dF_2 \\ & + \frac{d\Omega}{dS}\,dS_2, \end{aligned}$$

which, for shortness, I represent by

$$d_2 \Omega = \mathrm{X}\, d\mathrm{T}_2 + \mathrm{Y}\, d\mathrm{F}_2 + \mathrm{Z}\, d\mathrm{S}_2;$$

and

$$d_3 \Omega = \left\{\begin{array}{l} (\alpha_2 \cos \mathrm{S} + \beta_2 \sin \mathrm{S}) \,(-\sin\sigma \sin\phi\, d\theta + \cos\sigma\, d\phi) \\ + (\alpha_2' \cos \mathrm{S} + \beta_2' \sin \mathrm{S}) \,(\cos\sigma \sin\phi\, d\theta + \sin\sigma\, d\phi) \\ + (\alpha_2'' \cos \mathrm{S} + \beta_2'' \sin \mathrm{S}) \,(-d\sigma + \cos\phi\, d\theta) \end{array}\right\} \frac{d\Omega}{d\mathrm{F}}$$

$$+ \left\{\begin{array}{l} (-\alpha_2 \sin \mathrm{S} + \beta_2 \cos \mathrm{S}) \,(-\sin\sigma \sin\phi\, d\theta + \cos\sigma\, d\phi) \\ + (-\alpha_2' \sin \mathrm{S} + \beta_2' \cos \mathrm{S}) \,(\cos\sigma \sin\phi\, d\theta + \sin\sigma\, d\phi) \\ (-\alpha_2'' \sin \mathrm{S} + \beta_2'' \cos \mathrm{S}) \,(-d\sigma + \cos\phi\, d\theta) \end{array}\right\} \left(\operatorname{cosec} \mathrm{F} \frac{d\Omega}{d\mathrm{T}} + \cot \mathrm{F} \frac{d\Omega}{d\mathrm{S}}\right)$$

$$- \left\{\begin{array}{l} \gamma_2 \,(-\sin\sigma \sin\phi\, d\theta + \cos\sigma\, d\phi) \\ + \gamma_2' \,(\cos\sigma \sin\phi\, d\theta + \sin\sigma\, d\phi) \\ + \gamma_2'' \,(-d\sigma + \cos\phi\, d\theta) \end{array}\right\} \frac{d\Omega}{d\mathrm{S}},$$

or substituting for α_2, &c. their values, and reducing

$$d_3 \Omega = \left\{\begin{array}{l} \Big([\cos(\mathrm{S}_2-\mathrm{S}) \sin(\mathrm{T}_2-\sigma) - \sin(\mathrm{S}_2-\mathrm{S}) \cos(\mathrm{T}_2-\sigma) \cos \mathrm{F}_2] \sin\phi - \sin(\mathrm{S}_2-\mathrm{S}) \sin \mathrm{F}_2 \cos\phi\Big)\, d\theta \\ + \Big(\cos(\mathrm{S}_2-\mathrm{S}) \cos(\mathrm{T}_2-\sigma) + \sin(\mathrm{S}_2-\mathrm{S}) \sin(\mathrm{T}_2-\sigma) \cos \mathrm{F}_2\Big)\, d\phi + \sin(\mathrm{S}_2-\mathrm{S}) \sin \mathrm{F}_2\, d\sigma \end{array}\right\} \frac{d\Omega}{d\mathrm{F}}$$

$$+ \left\{\begin{array}{l} \Big([\sin(\mathrm{S}_2-\mathrm{S}) \sin(\mathrm{T}_2-\sigma) + \cos(\mathrm{S}_2-\mathrm{S}) \cos(\mathrm{T}_2-\sigma) \cos \mathrm{F}_2] \sin\phi + \cos(\mathrm{S}_2-\mathrm{S}) \sin \mathrm{F}_2 \cos\phi\Big)\, d\theta \\ + \Big(\sin(\mathrm{S}_2-\mathrm{S}) \cos(\mathrm{T}_2-\sigma) - \cos(\mathrm{S}_2-\mathrm{S}) \sin(\mathrm{T}_2-\sigma) \cos \mathrm{F}_2\Big)\, d\phi - \cos(\mathrm{S}_2-\mathrm{S}) \cos \mathrm{F}_2\, d\sigma \end{array}\right\} \left(\operatorname{cosec} \mathrm{F} \frac{d\Omega}{d\mathrm{T}} + \cot \mathrm{F} \frac{d\Omega}{d\mathrm{S}}\right)$$

$$- \left\{\Big(-\cos(\mathrm{T}_2-\sigma) \sin \mathrm{F}_2 \sin\phi + \cos \mathrm{F}_2 \cos\phi\Big)\, d\theta + \sin(\mathrm{T}_2-\sigma) \sin \mathrm{F}_2\, d\phi - \cos \mathrm{F}_2\, d\sigma\right\} \frac{d\Omega}{d\mathrm{S}}.$$

Hence, comparing $d_1 \Omega + d_2 \Omega + d_3 \Omega$ with the other expression for $d\Omega$, and observing that $d\mathrm{T}_2$, $d\mathrm{S}_2$, $d\mathrm{F}_2$, do not involve $d\theta$, $d\sigma$, $d\phi$, we have

$$d_1 \Omega = \frac{d\Omega}{dg}\, n\, dt,$$

$$d_2 \Omega = \frac{d\Omega}{dh}\, dh + \frac{d\Omega}{dk}\, dk + \frac{d\Omega}{dg}\, dg + \frac{d\Omega}{d\varpi}\, d\varpi,$$

$$d_3 \Omega = \frac{d\Omega}{d\theta}\, d\theta + \frac{d\Omega}{d\sigma}\, d\sigma + \frac{d\Omega}{d\phi}\, d\phi.$$

XIII.

The first equation gives

$$n\frac{d\Omega}{dg} = -(p\sin S + q\cos S)\left(\operatorname{cosec} F\frac{d\Omega}{dT} + \cot F\frac{d\Omega}{dS}\right)$$
$$+ (p\cos S + q\sin S)\frac{d\Omega}{dF} - r\frac{d\Omega}{dS},$$

and, comparing this with the first form of $\frac{1}{2}\,dh$, we have

$$dh = 2n\frac{d\Omega}{dg}dt.$$

The third equation gives

$$\frac{d\Omega}{d\theta} = \left\{[\sin(S_2-S)\sin(T_2-\sigma)+\cos(S_2-S)\cos(T_2-S)\cos F_2]\sin\phi+\cos(S_2-S)\sin F_2\cos\phi\right\}\left(\operatorname{cosec} F\frac{d\Omega}{dT}+\cot F\frac{d\Omega}{dS}\right)$$
$$+\left\{[\cos(S_2-S)\sin(T_2-\sigma)-\sin(S_2-S)\cos(T_2-\sigma)\cos F_2]\sin\phi-\sin(S_2-S)\sin F_2\cos\phi\right\}\frac{d\Omega}{dF}$$
$$+\left\{\cos(T_2-\sigma)\sin F_2\sin\phi-\cos F_2\cos\phi\right\}\frac{d\Omega}{dS},$$

$$\frac{d\Omega}{d\sigma} = -\cos(S_2-S)\cos F_2\left(\operatorname{cosec} F\frac{d\Omega}{dT}-\cot F\frac{d\Omega}{dS}\right)$$
$$+\sin(S_2-S)\sin F_2\frac{d\Omega}{dF}+\cos F_2\frac{d\Omega}{dS},$$

$$\frac{d\Omega}{d\phi} = \left\{\sin(S_2-S)\cos(T_2-\sigma)-\cos(S_2-S)\sin(T_2-\sigma)\cos F_2\right\}\left(\operatorname{cosec} F\frac{d\Omega}{dT}+\cot F\frac{d\Omega}{dS}\right)$$
$$+\left\{\cos(S_2-S)\cos(T_2-\sigma)+\sin(S_2-S)\sin(T_2-\sigma)\cos F_2\right\}\frac{d\Omega}{dF}$$
$$-\sin(T_2-\sigma)\sin F_2\frac{d\Omega}{dS},$$

and thence

$$\cot\phi\frac{d\Omega}{d\sigma}+\operatorname{cosec}\phi\frac{d\Omega}{d\theta} =$$
$$\left\{\sin(S_2-S)\sin(T_2-\sigma)+\cos(S_2-S)\cos(T_2-\sigma)\cos F_2\right\}\left(\operatorname{cosec} F\frac{d\Omega}{dT}+\cot F\frac{d\Omega}{dS}\right)$$
$$+\left\{\cos(S_2-S)\sin(T_2-\sigma)-\sin(S_2-S)\cos(T_2-\sigma)\cos F_2\right\}\frac{d\Omega}{dF}$$
$$+\cos(T_2-\sigma)\sin F_2\frac{d\Omega}{dS};$$

and we have, therefore,

$$d\varphi = -\frac{\cot\varphi}{k}\frac{d\Omega}{d\sigma}dt - \frac{\operatorname{cosec}\varphi}{k}\frac{d\Omega}{d\theta}dt,$$

$$d\sigma = \frac{\cot\varphi}{k}\frac{d\Omega}{d\varphi}dt,$$

$$d\theta = \frac{\operatorname{cosec}\varphi}{k}\frac{d\Omega}{d\varphi}dt.$$

The second equation, viz.

$$\frac{d\Omega}{dh}dh + \frac{d\Omega}{dk}dk + \frac{d\Omega}{dg}dg + \frac{d\Omega}{d\varpi}d\varpi = d_2\Omega = X\,dT_2 + Y\,dF_2 + Z\,dS_2$$

if we substitute for dg, dT_2, dF_2, dS_2, their values, becomes

$$\frac{d\Omega}{dh}dh + \frac{d\Omega}{dk}dk + \frac{d\Omega}{dg}\left(-\frac{nC}{(B-A)pq}dr + \mathfrak{A}\,dh + \mathfrak{B}\,dk\right) + \frac{d\Omega}{d\varpi}d\varpi$$

$$= X\left\{d\varpi - \frac{Ck(h-Cr^2)}{(k^2-C^2r^2)(B-A)pq}dr + \mathfrak{C}\,dh + \mathfrak{D}\,dk\right\}$$

$$+ Y\left\{-\frac{Ck\sin F_2}{k^2-C^2r^2}dr + \frac{k\sin F_2\cos F_2}{k^2-C^2r^2}dk\right\}$$

$$+ Z\left\{-\frac{C(Ch-k^2)r}{(k^2-C^2r^2)(B-A)pq}dr - \frac{\frac{1}{2}dh}{(B-A)pq} + \frac{(n-Cr^2)k\,dk}{(k^2-C^2r^2)(B-A)pq}\right\}.$$

The comparison of the terms involving $d\varpi$ gives at once $\frac{d\Omega}{d\varpi} = X$, or, substituting for X its value,

$$\frac{d\Omega}{d\varpi} = \cos(S_2-S)\sin F_2\left(\operatorname{cosec}F\frac{d\Omega}{dT} + \cot F\frac{d\Omega}{dS}\right)$$

$$-\sin(S_2-S)\sin F_2\frac{d\Omega}{dF} - \cos F_2\frac{d\Omega}{dS};$$

and comparing with the expression for dk, we find

$$dk = \frac{d\Omega}{d\varpi}dt.$$

The terms involving dr give

$$n\frac{d\Omega}{dg} = \frac{k(h-Cr^2)}{k^2-C^2r^2}X + \frac{k\sin F_2(B-A)pq}{k^2-C^2r^2}Y + \frac{(Ch-k^2)r}{k^2-C^2r^2}Z,$$

or, substituting for X, Y, Z, their values,

$$\begin{aligned} n\frac{d\Omega}{dg} = {} & \frac{k(h-Cr^2)}{k^2-C^2r^2}\left\{\cos(S_2-S)\sin F_2\left(\operatorname{cosec}F\frac{d\Omega}{dT}+\cot F\frac{d\Omega}{dS}\right)-\sin(S_2-S)\sin F_2\frac{d\Omega}{dF}-\cos F_2\frac{d\Omega}{dS}\right\} \\ & +\frac{k(B-A)pq}{k^2-C^2r^2}\left\{\sin(S_2-S)\sin F_2\left(\operatorname{cosec}F\frac{d\Omega}{dT}+\cot F\frac{d\Omega}{dS}\right)+\cos(S_2-S)\sin F_2\frac{d\Omega}{dF}\right\} \\ & +\frac{r(Ch-k^2)}{k^2-C^2r^2}\frac{d\Omega}{dS}, \end{aligned}$$

which agrees with the second form for $\frac{1}{2}\,dh$, and gives, as before,

$$dh = 2n\frac{d\Omega}{dg}dt;$$

the terms involving dh give

$$\frac{d\Omega}{dh} + \mathfrak{A}\frac{d\Omega}{dg} = \mathfrak{C}X - \frac{\frac{1}{2}}{(B-A)pq}Z,$$

and thence

$$-2n\frac{d\Omega}{dh} = 2n\mathfrak{A}\frac{d\Omega}{dg} - 2n\mathfrak{C}X + \frac{n}{(B-A)pq}Z;$$

or, substituting for $-2n\mathfrak{C}$ its value $\mathfrak{B}$, for X the value $\frac{d\Omega}{d\varpi}$ and for Z its value $\frac{d\Omega}{dS}$, we have

$$-2n\frac{d\Omega}{dh} = 2n\mathfrak{A}\frac{d\Omega}{dg} + \mathfrak{B}\frac{d\Omega}{d\varpi} + \frac{n}{(B-A)pq}\frac{d\Omega}{dS},$$

where, on the right hand, $\frac{d\Omega}{dg}$ and $\frac{d\Omega}{d\varpi}$ may be considered as standing for given functions of $\frac{d\Omega}{dT}, \frac{d\Omega}{dS}, \frac{d\Omega}{dF}$; for the present purpose, however, multiplying by dt, and substituting for $2n\frac{d\Omega}{dg}$ and $\frac{d\Omega}{d\varpi}$ the values dh and dk, we have

$$-2n\frac{d\Omega}{dh}dt = \mathfrak{A}\,dh + \mathfrak{B}\,dk + \frac{n}{(B-A)pq}\frac{d\Omega}{dS}dt,$$

which agrees with the foregoing value of $d\,g$, or we have

$$dg = -2n\frac{d\Omega}{dh}\,dt.$$

The comparison of the terms involving $d\,k$ gives

$$\frac{d\Omega}{dk} + \mathfrak{B}\frac{d\Omega}{dg} = \mathfrak{D}\mathrm{X} + \frac{k\sin \mathrm{F}_2\cos \mathrm{F}_2}{k^2-\mathrm{C}^2r^2}\mathrm{Y} + \frac{(h-\mathrm{C}r^2)\,k}{(k^2-\mathrm{C}^2r^2)(\mathrm{B}-\mathrm{A})pq}\mathrm{Z};$$

or, substituting and reducing

$$-\frac{d\Omega}{dk} = -2n\mathfrak{C}\frac{d\Omega}{dg} - \mathfrak{D}\frac{d\Omega}{d\varpi} - \frac{1}{k}\cot \mathrm{F}_2\left\{\cos(\mathrm{S}_2-\mathrm{S})\frac{d\Omega}{d\mathrm{F}} + \sin(\mathrm{S}_2-\mathrm{S})\left(\operatorname{cosec}\mathrm{F}\frac{d\Omega}{d\mathrm{T}} + \cot \mathrm{F}\frac{d\Omega}{d\mathrm{S}}\right)\right\}$$
$$-\frac{(h-\mathrm{C}r^2)\,k}{(k^2-\mathrm{C}^2r^2)(\mathrm{B}-\mathrm{A})\,p\,q}\frac{d\Omega}{d\mathrm{S}},$$

where, on the right-hand side, $\frac{d\Omega}{d\,g}$ and $\frac{d\Omega}{d\varpi}$ may be considered as standing for given functions of $\frac{d\Omega}{d\mathrm{T}}, \frac{d\Omega}{d\mathrm{S}}, \frac{d\Omega}{d\mathrm{F}}$; for the present purpose, however, multiplying by $d\,t$ and putting for $2\,n\,\frac{d\Omega}{d\,g}\,d\,t$ and $\frac{d\Omega}{d\varpi}\,d\,t$, the values $d\,h$ and $d\,k$, we have

$$-\frac{d\Omega}{dk}dt = -\mathfrak{C}\,dh - \mathfrak{D}\,dk - \frac{1}{k}\cot \mathrm{F}_2\left\{\cos(\mathrm{S}_2-\mathrm{S})\frac{d\Omega}{d\mathrm{F}}dt + \sin(\mathrm{S}_2-\mathrm{S})\left(\operatorname{cosec}\mathrm{F}\frac{d\Omega}{d\mathrm{T}} + \cot \mathrm{F}\frac{d\Omega}{d\mathrm{S}}\right)dt\right\}$$
$$-\frac{(h-\mathrm{C}r^2)\,k}{(k^2-\mathrm{C}^2r^2)(\mathrm{B}-\mathrm{A})\,p\,q}\frac{d\Omega}{d\mathrm{S}}\,dt,$$

which agrees with the foregoing value of $d\,\varpi$, or we have

$$d\varpi = -\frac{d\Omega}{dk}\,dt,$$

and we have thus the system of formulæ for the variation of the elements in the problem of rotation, given in Art. I.

IX. *On a Method for Determining Longitude by means of Observations on the Moon's greatest Altitude.* By William Spottiswoode, Esq., M.A., F.R.S., &c.

Read December 8, 1860.

THE object of the present investigations is to suggest a method whereby, at certain parts of the Moon's orbit and under favourable circumstances, the longitude may be determined by means of a simple sextant observation. The method does not attain to the same degree of accuracy as some others already in use; nor can it be successfully applied except when the Moon's motion in declination is considerable; and the observation, being restricted in time to the immediate vicinity of the greatest altitude, is very dependent on weather. For the traveller supplied with proper instruments the eclipses of *Jupiter's* satellites will doubtless always furnish one of the best means of determining the longitude; and for one carrying only a sextant the method of lunar distances is superior to all others; but the labour of the latter observations is such as to render them practicable only at rare intervals on a journey, while the complexity of the calculations almost precludes the possibility of obtaining satisfactory results until the data have been sent home. A method, therefore, in which both observation and calculation are simple may not be without use, either as supplementary to more elaborate processes or as a substitute when they are not practicable; and the following is suggested for the use of the traveller to check his dead reckoning, after the last of his chronometers has broken down, and when results from lunar distances are not attainable.

The present method consists in determining the Moon's declination at the time of her greatest altitude. This being ascertained, if the local time be known, we have merely to compare the Greenwich time, deduced from the *Nautical Almanac*, at which she has that declination; the difference in times is of course the longitude expressed in time. If, however, the local time be not known, a formula deduced from the same data and given below determines the hour-angle at the time of the greatest altitude. By means of this the correction to be applied to the Moon's declination at the time of observation, in order to reduce it to that on the meridian, may be calculated; and a comparison of the declinations at her meridian passage at the place and at Greenwich, will with the aid of the table of differences of declination in the *Nautical Almanac* give the longitude.

Let A = the moon's greatest altitude,
L = the latitude of the place of observation,
D = the moon's declination at the time of observation,
h = the hour-angle.

Then in the triangle P Z M, P being the Pole, Z the Zenith, and M the Moon,

$$PZ = 90^\circ - L, \quad PM = 90^\circ - D, \quad ZM = 90^\circ - A,$$

and the ordinary formula of spherical triangles gives

$$\sin A = \sin L \sin D + \cos L \cos D \cos h. \tag{1}$$

If A is a maximum,

$$\cos A \frac{dA}{dh} = 0 = \sin L \cos D \frac{dD}{dh} - \cos L \sin D \cos h \frac{dD}{dh} - \cos L \cos D \sin h, \tag{2}$$

or

$$(\tan L - \cos h \tan D) \frac{dD}{dh} - \sin h = 0. \tag{3}$$

By eliminating h from (1) and (3) we obtain a final equation for determining the Moon's declination in terms of the latitude and the observed greatest altitude. On the other hand, by eliminating D from the same equations we obtain an equation which will determine the hour-angle or the distance of the Moon from the meridian at the time of her attaining her greatest altitude. I proceed to perform the eliminations.

Writing for convenience

$$\sin A = a, \quad \sin L = l, \quad \frac{dD}{dh} = m \tag{4}$$

we have from (1) and (3),

$$m \tan \mathrm{L} - m \tan \mathrm{D} \frac{a - l \sin \mathrm{D}}{\cos \mathrm{L} \cos \mathrm{D}} - \left\{1 - \left(\frac{a - l \sin \mathrm{D}}{\cos \mathrm{L} \cos \mathrm{D}}\right)^2\right\}^{\frac{1}{2}} = 0, \qquad (5)$$

or, multiplying throughout by cos L $\cos^2$ D,

$$m\, l \cos^2 \mathrm{D} - m \sin \mathrm{D}\,(a - l \sin \mathrm{D}) - \cos \mathrm{D}\,\{(1 - l^2) \cos^2 \mathrm{D} - (a - l \sin \mathrm{D})^2\}^{\frac{1}{2}} = 0,$$

or

$$m\,(l - a \sin \mathrm{D}) - \cos \mathrm{D}\,(1 - \sin^2 \mathrm{D} - l^2 - a^2 + 2\, a\, l \sin \mathrm{D})^{\frac{1}{2}} = 0,$$

whence, transposing the first term, squaring, and dividing by $\cos^2$ D,

$$1 - \sin^2 \mathrm{D} - l^2 - a^2 + 2\, a\, l \sin \mathrm{D} = m^2 \frac{(l - a \sin \mathrm{D})^2}{\cos^2 \mathrm{D}},$$

and adding $-a^2 l^2$ to both sides, transposing, and taking the square root,

$$\begin{aligned} \sin \mathrm{D} - a\, l &= \left\{(1 - a^2)(1 - l^2) - m^2 \frac{(l - a \sin \mathrm{D})^2}{\cos^2 \mathrm{D}}\right\}^{\frac{1}{2}} \\ &= \left\{(1 - a^2)(1 - l^2) - m^2 \frac{[(1 - a^2)\, l - a\,(\sin \mathrm{D} - a\, l)]^2}{1 - (a\, l + \sin \mathrm{D} - a\, l)^2}\right\}^{\frac{1}{2}}. \qquad (6) \end{aligned}$$

Let

$$\left.\begin{aligned} \sin \mathrm{D} - a\, l &= u, \\ (1 - a^2)(1 - l^2) &= z, \\ -m^2 &= x, \\ \frac{[(1 - a^2)\, l - a\, u]^2}{1 - (a\, l + u)^2} &= \varphi(u), \\ z^{\frac{1}{2}} &= \mathrm{F}(z), \\ \sin^{-1}(u + a\, l) &= \psi(u), \end{aligned}\right\} \qquad (7)$$

then (6) takes the form

$$u = \mathrm{F}\,\{z + x\, \varphi(u)\},$$

whence, by Laplace's theorem,

$$\psi(u) = \psi\,\{\mathrm{F}(z)\} + \varphi\,\{\mathrm{F}(z)\} \frac{d\, \psi\,\{\mathrm{F}(z)\}}{d z} \frac{x}{1} + \frac{d}{d z}\left\{\Big(\varphi\,\{\mathrm{F}(z)\}\Big)^2 \frac{d\, \psi\,\{\mathrm{F}(z)\}}{d z}\right\} \frac{x^2}{1 \cdot 2} + \ldots$$

Now from the equations given above

$$\psi\{F(z)\} = \sin^{-1}\{F(z) + a\,l\} = \sin^{-1}(z^{\frac{1}{2}} + a\,l)$$

$$\varphi\{F(z)\} = \frac{\{(1-a^2)\,l - a\,z^{\frac{1}{2}}\}^2}{1-(a\,l+z^{\frac{1}{2}})^2},$$

$$\frac{d\psi\{F(z)\}}{dz} = \frac{1}{2}\,\frac{1}{z^{\frac{1}{2}}\{1-(a\,l+z^{\frac{1}{2}})^2\}^{\frac{1}{2}}},$$

$$\varphi\{F(z)\}\frac{d\psi\{F(z)\}}{dz} = \frac{1}{2}\,\frac{\{(1-a^2)\,l - a\,z^{\frac{1}{2}}\}^2}{z^{\frac{1}{2}}\{1-(a\,l+z^{\frac{1}{2}})^2\}^{\frac{3}{2}}},$$

$$\Big(\varphi\{F(z)\}\Big)^2\frac{d\psi\{F(z)\}}{dz} = \frac{1}{2}\,\frac{\{(1-a^2)\,l - a\,z^{\frac{1}{2}}\}^4}{z^{\frac{1}{2}}\{1-(a\,l+z^{\frac{1}{2}})^2\}^{\frac{5}{2}}},$$

$$\frac{d}{dz}\left\{\Big(\varphi\{F(z)\}\Big)^2\frac{d\psi\{F(z)\}}{dz}\right\} = -\frac{\{(1-a^2)\,l-a z^{\frac{1}{2}}\}^3 a}{z\{1-(al+z^{\frac{1}{2}})^2\}^{\frac{5}{2}}} + \frac{\{(1-a^2)\,l-a\,z^{\frac{1}{2}}\}^4}{2z^{\frac{1}{2}}\{1-(al+z^{\frac{1}{2}})^2\}^{\frac{5}{2}}}\left\{-\frac{1}{2z}+\frac{5}{2}\,\frac{al+z^{\frac{1}{2}}}{z^{\frac{1}{2}}\{1-(al+z^{\frac{1}{2}})^2\}}\right\}$$

$$= \frac{\{(1-a^2)\,l-az^{\frac{1}{2}}\}^3}{4z\{1-(al+z^{\frac{1}{2}})^2\}^{\frac{5}{2}}}\left\{-4a-\frac{\{(1-a^2)l-az^{\frac{1}{2}}\}}{z}+\frac{5(al+z^{\frac{1}{2}})\{(1-a^2)\,l-az^{\frac{1}{2}}\}}{\{1-(al+z^{\frac{1}{2}})^2\}}\right\}.$$

It will, however, be more convenient to use the moon's zenith distance and north polar distance, and the co-latitude, instead of the altitude, declination, and latitude. Let, therefore,

$$Z = 90^\circ - A, \quad C = 90^\circ - L, \quad \Delta = 90^\circ - D.$$

Then

$$\psi(u) = \cos^{-1}(al+u),$$

$$\psi\{F(z)\} = \cos^{-1}(\cos Z \cos C \pm \sin Z \sin C)$$

$$= Z \pm C,$$

$$\varphi\{F(z)\} = \frac{\sin^2 Z\,(\sin Z \cos C \mp \cos Z \sin C)^2}{1-(\cos Z \cos C \pm \sin Z \sin C)^2} = \sin^2 z,$$

$$\frac{d\psi\{F(z)\}}{dz} = \pm\frac{1}{2}\,\frac{1}{\sin Z \sin C \sin(Z \mp C)},$$

whence

$$\varphi\{F(z)\}\frac{d\psi\{F(z)\}}{dz} = \pm\frac{1}{2}\,\frac{\sin Z}{\sin C \sin(Z \mp C)},$$

and finally, as far as the second power of m,

$$\Delta = Z \mp C \pm \frac{m^2}{2} \frac{\sin Z}{\sin C \sin (Z \mp C)}.$$

Of the two signs given by the analysis, the lower belongs to the case of similar latitude and declination, *i.e.* both North or both South, and the upper to that of different, *i.e.* North declination and South latitude; or, South declination and North latitude.

Proceeding to the third term,

$$\frac{d}{dz}\left\{\left(\varphi\{F(z)\}\right)^2 \frac{d\psi\{F(z)\}}{dz}\right\} = -\frac{\sin^3 Z \sin^3 (Z \mp C)}{\sin^2 Z \sin^2 C \sin^5 (Z \mp C)}\left\{-4\cos Z \mp \frac{\sin Z \sin (Z \mp C)}{\sin Z \sin C} + \frac{5\cos(Z \mp C) \sin Z \sin (Z \mp C)}{\sin^2 (Z \mp C)}\right\}$$

$$= -\frac{\sin Z}{\sin^3 C \sin^3 (Z \mp C)}\{-4\cos Z \sin C \sin (Z \mp C) \mp \sin^2 (Z \mp C) + 5 \sin Z \sin C \cos (Z \mp C)\}$$

$$= -\frac{\sin Z}{\sin^3 C \sin^3 (Z \mp C)}\{\pm 4 \sin^2 C + \sin Z \sin C \cos (Z \mp C) \pm \sin^2 (Z \mp C)\},$$

but, owing to the smallness of m, there is no practical necessity for the use of this expression.

In order to determine the hour-angle at the time of greatest altitude, we must find $\sin \Delta$ and $\cos \Delta$ in terms of z and c, and substitute them in the equation derived from (1)

$$\cos h = \frac{\cos Z - \cos C \cos \Delta}{\sin C \sin \Delta}.$$

In this case,

$$u = \cos \Delta - a\,l, \quad \psi(u) = u + a\,l,$$

the other equations of the system (7) remaining as before. Then,

$$\psi\{F(z)\} = a\,l \pm z^{\frac{1}{2}} = \cos (Z \mp C),$$

$$\varphi\{F(z)\} = \sin^2 Z,$$

$$\frac{d\psi\{F(z)\}}{dz} = \pm \frac{1}{2} z^{-\frac{1}{2}} = -\frac{1}{2} \frac{1}{\sin Z \sin C},$$

Hence

$$\cos\Delta = \cos(Z\mp C) \mp \frac{m^2}{2}\frac{\sin Z}{\sin C}.$$

Understanding the meaning of the double sign, and writing the lower one only, we have, as far as the second power of m,

$$\cos^2\Delta = \cos^2(Z+C) + m^2\cos(Z+C)\frac{\sin Z}{\sin C},$$

$$\sin^2\Delta = \sin^2(Z+C) - m^2\cos(Z+C)\frac{\sin Z}{\sin C},$$

$$\sin\Delta = \sin(Z+C) - \frac{m^2}{2}\cot(Z+C)\frac{\sin Z}{\sin C},$$

hence

$$\cos h = \frac{\cos Z - \cos C\cos(Z+C) - \frac{m^2}{2}\sin Z\cot C}{\sin C\sin(Z+C) - \frac{m^2}{2}\sin Z\cot(Z+C)}$$

$$= \left\{\cos Z - \cos C\cos(Z+C) - \frac{m^2}{2}\sin Z\cot C\right\}$$

$$\times\frac{1}{\sin C\sin(Z+C)}\left\{1 + \frac{m^2}{2}\frac{\sin Z}{\sin C}\frac{\cos(Z+C)}{\sin^2(Z+C)}\right\}$$

$$= \frac{1}{\sin C\sin(Z+C)}\left\{\sin C\sin(Z+C) + \frac{m^2}{2}[\sin Z\cot(Z+C) - \sin Z\cot C]\right\}$$

$$= 1 + \frac{m^2}{2}\frac{\sin^2 Z}{\sin^2 C\sin^2(Z+C)}\{\sin C\cos(Z+C) - \cos C\sin(Z+C)\}$$

$$= 1 - \frac{m^2}{2}\frac{\sin^2 Z}{\sin^2 C\sin^2(Z+C)}.$$

The evaluation of Δ may also be effected by a geometrical method, as was pointed out to me by Mr. Main, Radcliffe Observer, Oxford; his process is here subjoined. Making

$$C+Z=\Delta+y,$$

y being a small quantity, then

$$\cos h = \frac{\cos Z - \cos C \cos (C + Z - y)}{\sin C \sin (C + Z - y)}$$

$$= \frac{\cos Z - \cos C \{\cos (C + Z) + y \sin (C + Z)\}}{\sin C \{\sin (C + Z) - y \cos (C + Z)\}}$$

$$= \frac{\cos Z (1 - \cos^2 C) + \sin C \cos C \sin Z - y \cos C \sin (C + Z)}{\sin C \{\sin (C + Z) - y \cos (C + Z)\}}$$

$$= \frac{\sin C \sin (C + Z) - y \cos C \sin (C + Z)}{\sin C \{\sin (C + Z) - y \cos (C + Z)\}}$$

$$= \frac{1 - y \cot C}{1 - y \cot (C + Z)}$$

$$= 1 - y \frac{\sin Z}{\sin C \sin (C + Z)} = 1 - \frac{h^2}{2},$$

or

$$h^2 = 2 y \frac{\sin Z}{\sin C \sin (C + Z)};$$

again, because h is small,

$$h \sin C \sin \Delta = - m \sin (\Delta - C)$$

$$= - m \sin (Z - y),$$

or

$$h \sin C \sin (Z + C - y) = - m (\sin Z - y \cos Z),$$

$$h \sin C \{\sin (Z + C) - x \cos (Z + C)\} = - m (\sin Z - y \cos Z),$$

$$h \sin C \sin (Z + C) = - m \sin Z,$$

whence

$$h^2 = m^2 \frac{\sin^2 Z}{\sin^2 C \sin^2 (Z + C)} = 2 x \frac{\sin Z}{\sin C \sin (Z + C)},$$

$$x = \frac{m^2}{2} \frac{\sin Z}{\sin C \sin (Z + C)},$$

$$\Delta = Z+C-\frac{m^2}{2}\frac{\sin Z}{\sin C\sin(Z+C)},$$

as was before found.

In order to make use of the above formulæ we have,

$$m^2 = \left(\frac{d\,\Delta}{d\,h}\right)^2 = \left(\frac{d\,\Delta}{d\,t}\,\frac{d\,t}{d\,h}\right)^2,$$

and, owing to the moon's motion in right ascension, the hour-angle is to the time, on the average, in the ratio of 1 : 1·04; hence

$$m^2 = \left(1{\cdot}04\,\frac{d\,\Delta}{d\,t}\right)^2.$$

The value of $d\Delta$ must be taken from the *Nautical Almanac*, for which purpose it is supposed that the longitude, and consequently the Greenwich time, is known approximately "by account"; dt must be expressed in terms of arc. And if we take one minute as the unit of time, $d\,t = 15' = 900''$. Lastly, the term involving m^2 must be divided by sin 1″. Hence, since 1·04 : 900 = ·00116, the final expression for Δ will be

$$\Delta \quad = Z+C-\frac{(\cdot 00116\,d\,\Delta)^2}{2\sin 1''}\,\frac{\sin Z}{\sin C\sin(Z+C)};$$

and that for h will be

$$\cos h = 1 - \frac{(\cdot 00116\,d\,\Delta)^2}{2\sin 1''}\,\frac{\sin^2 Z}{\sin^2 C\sin^2(Z+C)},$$

in both of which the last terms will be expressed in seconds of arc. The quantity $(\cdot 00116)^2 : 2\sin 1''$ may be represented by ·139.

The form which the values of Δ and of $\cos h$ have finally taken enables us to deduce a very simple expression for finding the meridian declination from an observed greatest altitude. For since

$$2(1-\cos h) = 4\sin^2\frac{h}{2} = m^2\,\frac{\sin^2 Z}{\sin^2 C\sin^2(Z+C)},$$

and h is small, we have approximately,

$$h = m\,\frac{\sin Z}{\sin C\sin(Z+C)};$$

also, if $\Delta_{,}$ be the meridian N.P.D., we have, on account of the smallness of h,

$$\Delta_{,} = \Delta + m\,h,$$

and consequently

$$\Delta_{,} = Z + C + \frac{m^2}{2} \frac{\sin Z}{\sin C \sin (Z+C)}.$$

By means of this formula the following tables have been calculated by Mr. GODWARD, Jun., of the *Nautical Almanac* Office. Table I. contains the corrections to be added to (Z + C) to obtain the meridian declination, when the Moon's motion in declination in one minute is 17″. Table II. or $\left(\frac{17''}{\Delta''}\right)^2$, gives the divisions of Table I. to obtain the corrections for any motion Δ″ in one minute.

As an example of the foregoing method, suppose that on 25 June, 1860, at $5^{h}\ 38^{m}$ P.M., in lat. 51° 45′ 36″ North, the true maximum altitude of the Moon's centre was found to be 33° 58′ 20″, required the longitude. In this case

$$Z = 56^\circ\ 1'\ 40''$$

$$C = 38^\circ\ 14'\ 24''$$

$$Z + C = 94^\circ\ 16'\ 4''$$

Tabular correction = 54″, Δ″ = 15″·67 $\qquad d = 1{\cdot}17$

$$\begin{array}{lrrr}
 & 94^\circ & 16' & 4'' \\
 & & & 46\ + \\ \hline
☽\text{'s meridian declination} = & 4 & 16 & 50 \\
☽\text{'s declination at } 5^{h}\text{ G.M.T.} & 4 & 5 & 29 \\ \hline
 & & 11 & 21 \\ \hline\hline
\text{G.M.T. of local mer. passage} & 5^{h} & 43^{m} & 28^{s} \\
\text{G.M.T. of Greenwich ditto} & 5 & 38 & 17 \\ \hline
\text{Longitude} = & & 5 & 11\ \text{W} \\ \hline\hline
\end{array}$$

$$\begin{array}{r}
1{\cdot}7)\ 54''\ (46'' \\
\underline{468} \\
72
\end{array}$$

$$\begin{array}{r}
1567)\ 681''\ (4346 \\
\underline{6268} \\
5420 \\
\underline{4701} \\
7190 \\
\underline{6268} \\
922
\end{array}$$

Altitude.	Tab. I.							LATITUDE.							Part I.	
	70°	68°	66°	64°	62°	60°	58°	56°	54°	52°	50°	48°	46°	44°	42°	40°
80°	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10″
78	—	—	—	—	—	—	—	—	—	—	—	—	—	—	13″	12
76	—	—	—	—	—	—	—	—	—	—	—	—	—	15″	15	14
74	—	—	—	—	—	—	—	—	—	—	—	—	18″	17	17	16
72	—	—	—	—	—	—	—	—	—	—	—	21″	20	19	19	17
70	—	—	—	—	—	—	—	—	—	—	24″	23	22	21	20	19
68	—	—	—	—	—	—	—	—	—	28″	26	25	24	22	22	20
66	—	—	—	—	—	—	—	—	32″	30	28	27	25	24	23	22
64	—	—	—	—	—	—	—	36″	34	32	30	28	27	25	25	23
62	—	—	—	—	—	—	41″	38	36	34	31	30	28	27	26	25
60	—	—	—	—	—	46″	43	40	38	35	33	31	30	28	28	26
58	—	—	—	—	52″	48	45	42	39	37	34	33	31	30	29	28
56	—	—	—	59″	54	50	47	43	41	38	36	34	33	31	31	29
54	—	—	67″	61	56	52	48	45	42	40	37	36	34	33	32	31
52	—	76″	69	63	58	53	50	46	44	41	39	37	36	34	34	32
50	86″	78	71	65	59	55	51	48	45	43	40	39	37	36	35	33
48	88	80	73	66	61	56	53	49	47	44	42	40	39	37	36	35
46	90	82	74	68	62	58	54	51	48	46	43	42	40	38	38	36
44	92	83	76	69	64	59	56	52	50	47	45	43	41	40	39	38
42	93	85	77	71	65	61	57	54	51	49	46	44	43	41	41	39
40	95	86	79	72	67	62	59	55	53	50	47	46	44	43	42	40
38	96	88	80	74	68	64	60	57	54	51	49	47	46	44	43	42
36	98	89	82	75	70	65	62	58	55	53	50	49	47	45	45	43
34	99	91	83	77	71	67	63	59	57	54	52	50	48	47	46	45
32	101	92	85	78	73	68	64	61	58	56	53	51	50	48	48	46
30	102	94	86	80	74	69	66	62	60	57	54	53	51	50	49	48
28	104	95	88	81	75	71	67	64	61	58	56	54	53	51	51	49
26	105	97	89	82	77	72	69	65	62	60	57	56	54	53	52	51
24	107	98	90	84	78	74	70	66	64	61	59	57	56	54	54	53
22	108	99	92	85	80	75	71	68	65	63	60	59	57	56	56	55
20	109	101	93	87	81	76	73	69	67	64	62	60	59	58	58	56
18	111	102	95	88	82	78	74	71	68	66	63	62	61	60	59	
16	112	104	96	89	84	79	76	72	70	67	65	64	63	61		
14	114	105	97	91	85	81	77	74	71	69	67	66	64			
12	115	106	99	92	87	82	79	75	73	71	69	67				
10	116	108	100	94	88	84	80	77	75	73	70					
8	118	109	102	95	90	85	82	79	77	74						
6	119	111	103	97	91	87	84	81	78							
4	121	112	105	98	93	89	86	82								
2	122	114	106	100	95	91	87									
0	124	115	107	101	96	92										

Altitude.	Tab. I. LATITUDE. Part II.				
	40°	30°	20°	10°	0°
	″	″	″	″	″
80°	10	9	7	7	7
70	19	16	14	14	15
60	26	23	21	22	23
50	33	30	29	30	
40	40	38	37		
30	48	46			
20	56				

Tab. II. DIVISORS OF TAB. I. **Tab. II.**

Arg. Diff. of Dec. for 1ᵐ, Δ″.

Δ″	*d*	Δ″	*d*	Δ″	*d*	Δ″	*d*
″ 5·0	11·6	″ 8·0	4·51	″ 11·0	2·39	″ 14·0	1·47
·1	11·2	·1	4·40	·1	2·35	·1	1·45
·2	10·8	·2	4·30	·2	2·31	·2	1·43
·3	10·4	·3	4·20	·3	2·27	·3	1·41
·4	10·0	·4	4·10	·4	2·23	·4	1·39
·5	9·6	·5	4·00	·5	2·19	·5	1·37
·6	9·2	·6	3·91	·6	2·15	·6	1·35
·7	8·9	·7	3·82	·7	2·11	·7	1·33
·8	8·6	·8	3·73	·8	2·07	·8	1·31
·9	8·3	·9	3·65	·9	2·04	·9	1·29
6·0	8·0	9·0	3·57	12·0	2·01	15·0	1·28
·1	7·7	·1	3·49	·1	1·97	·1	1·26
·2	7·4	·2	3·41	·2	1·94	·2	1·25
·3	7·2	·3	3·34	·3	1·91	·3	1·23
·4	7·0	·4	3·27	·4	1·88	·4	1·21
·5	6·8	·5	3·20	·5	1·85	·5	1·20
·6	6·6	·6	3·13	·6	1·82	·6	1·18
·7	6·4	·7	3·07	·7	1·79	·7	1·17
·8	6·2	·8	3·01	·8	1·76	·8	1·15
·9	6·0	·9	2·95	·9	1·73	·9	1·14
7·0	5·9	10·0	2·89	13·0	1·71	16·0	1·13
·1	5·7	·1	2·83	·1	1·68	·1	1·11
·2	5·6	·2	2·77	·2	1·65	·2	1·10
·3	5·4	·3	2·72	·3	1·62	·3	1·08
·4	5·3	·4	2·67	·4	1·60	·4	1·07
·5	5·1	·5	2·62	·5	1·58	·5	1·06
·6	4·9	·6	2·57	·6	1·55	·6	1·04
·7	4·8	·7	2·52	·7	1·53	·7	1·03
·8	4·7	·8	2·47	·8	1·51	·8	1·02
·9	4·6	·9	2·43	·9	1·49	·9	1·01

X. *On the Distribution of the Perihelia of the Parabolic and Hyperbolic Comets in relation to the Motion of the Solar System in Space.* By R. C. Carrington, Esq.

Read December 14, 1860.

Inquiries into the direction and amount of motion of the Solar System have hitherto been entirely based upon the residual small motions of what were once termed the fixed stars; and although the results for direction now exhibit a progressively increasing agreement and certainty, it is always desirable, where a case admits of it, to endeavour to obtain an independent result by approaching the subject from another point of view. It may not at first appear obvious how the distribution of comets can have any bearing on this matter; but one of the phenomena of meteors, which, in fact, suggested the idea to my mind, will most readily illustrate the connexion.

M. Coulvier-Gravier, in his work, *Recherches sur les Etoiles Filantes*, p. 171, first established the fact of there being a sensible horary variation in the number of meteors, the mean numbers per hour for the latitude of Paris, deduced by him from very extensive observations made in all seasons and over many years, being as follows:—

6 P.M.		10 P.M.		2 A.M.	
	3·3		4·5		7·1
7		11		3	
	3·5		5·0		7·6
8		12		4	
	3·7		5·8		8·0
9		1 A.M.		5	
	4·0		6·4		8·2
10		2		6	

The probable cause of this variation was explained some time since in the *Monthly Notices* for March 1857, by Mr. Bompas; but his paper, from having undergone considerable condensation, has, perhaps, lost somewhat of that clearness which is requisite to catch general attention. His explanation is, that if, as is probable, the greatest heights at which meteors appear bear only a small proportion to the Earth's diameter, and if the conclusion (from such paths as have admitted of calculation) may be accepted that their mean velocity is about double the orbital velocity of the Earth: then, firstly, if they be distributed equally in space, they would converge to the Earth *at rest* equally on all sides; but, secondly, if the Earth be in motion, they would converge to it more on the side towards which it is moving than the other, and, in the proportion of nearly two-thirds of the number, would have an apparent motion more or less opposed to that of the Earth, and apparently diverging from the point towards which the Earth is moving, with a gradual increase in number from 6 P.M. to 6 A.M.

This is more easily seen, perhaps, by first considering a simpler case in which the circumstances are not so materially different as to affect the general conclusion. Suppose the axis of the Earth's rotation to be perpendicular to its plane of revolution, and consider the position of the hemisphere in the poles whereof an observer in the equator will be situated at 6 P.M., midnight, and 6 A.M. Many meteors of velocities less than that of the Earth will manifestly be encountered by the third of these hemispheres, which would otherwise escape approach or contact, and many will fail to reach the first at all. Or suppose the velocity of the Earth to be greatly increased, and the result is plain. So, also, that the result will be similar, but different in degree, when we restore the actual condition of things.

Now, precisely as these little chance visitors of the Earth bear their testimony to the direction of the Earth's motion, and the rate of their horary variation of number depends on the relative velocities of the Earth and of the meteors, it appeared to me that the comets, if chance visitors of the Solar System, should bear analogous testimony to the direction of the motion of the Sun. Here, however, instead of a body which, by its bulk, encounters comparative motes whose number may be taken as infinite, and whose directions of motion may be taken as of every variety, we have the case of a predominating centre of attraction, of which the dimensions may be neglected. If the Sun moves in the direction from A to B, where A may be taken to be the bright star α *Columbæ*, and B is situated in the line joining π and μ *Herculis*, relatively this motion is equivalent to a corresponding general motion of the comets in the direction from B towards A, the Sun being at rest; or, taking one with another, to an excess of aphelia in the hemisphere of which B is the pole, and, therefore, to an excess of perihelia in the hemisphere of which A is the pole.

Several circumstances, however, exist which may, and appear to, prevent our arriving at an equally positive result. In the first place, can we always distinguish between a periodic and a non-periodic orbit, for with the former we have had nothing to do, and may not the unrecognised closed orbits unavoidably included in our data mar our conclusions? Secondly, has the search for comets been so conducted that our table of non-periodical comets offers a fair basis for any cosmical speculation? The preponderance of observatories and comet-seekers in the northern hemisphere, and the repeated occurrence of fine and cloudy seasons at the same time of the year, at once appear inimical to success. Thirdly, will the conditions of visibility from our particular planet affect the observed distribution on which we reason? Fourthly, is our catalogue sufficiently extensive to give any result which may not be upset by the addition of future orbits?

Of these sources of objection the gravest appears to be the preponderance of search from the northern hemisphere,—a circumstance which, as it is beyond the power of a CANNING to remedy by "calling a new world into existence to redress the balance of the old," must necessarily be felt in all speculations on the distribution of matter in space. Though prepared for such a result as has been arrived at, I thought the subject worth the moderate degree of labour required, and though the older and reliable result for the Sun's motion is not hereby confirmed, I wish to put on record a conception by which the subject may theoretically be approached in a new way. The proposition, then, stands thus:—

If the comets moving in unclosed orbits be chance visitors, and if the conditions of visibility on the whole affect the discovery of all alike, it would be expected that were the Sun at rest, the perihelia would be found equally scattered over the heavens; but if the Sun be in motion, as we now hold it to be, from *α Columbæ* towards *π* and *μ Herculis*, that there would be found a certain excess of perihelia in the hemisphere of which *α Columbæ* is the pole, and a certain condensation about that region; so that if for each orbit the angle were computed between *α Columbæ* and the comet's perihelion, the mean of these angles would be less than 90 degrees.

In the following tables I have, accordingly, classified all the orbits computed, beginning with the year 1585, placing by themselves, first, the orbits of short period, then those of long period, and four which were not sufficiently determined to be ranged in any class, and, lastly, the large group of the parabolic and hyperbolic orbits. The numbers in the first columns are purposely given that any person may easily satisfy himself, by comparing my tables with the well-known catalogues of ENCKE, HIND, or COOPER, that no partiality or exclusion has been practised. The four columns of Elements require no explanation beyond the remark that the distinction of Direct and Retrograde is avoided by not limiting the Inclinations to 90 degrees. There next follow the longitude and latitude of the perihelion referred to the equinox of the year, and then two columns (the numbers in which have been carefully checked by rough measurement on a globe and duplicate computation), containing the distance from and the angle around *α Columbæ*. Of the 133 orbits treated the perihelia of 61 lie in the hemisphere from which, and those of 72 in the hemisphere towards which, the Sun is moving, the mean angular distance between *α Columbæ* and each perihelion coming out 95° 39′,—a result which superior evidence compels us to regard as nugatory and vitiated by uncontrolled conditions, among which the principal is probably the unequal distribution of discoverers.

Apology may, perhaps, be required for the space occupied by the tables which follow; but I believe that a condensed table of classified orbits will be found of very general convenience for itself alone.

ADDITION TO THE ABOVE.

The Astronomer Royal, to whom the above paper was referred, has favoured the author with the following remarks:—

"I have grave doubts upon the validity of any inference from the places of perihelia or other elements of parabolic orbits, for the following reasons:—

"Every rest or movement, actually occurring with the Sun in motion, may be referred to the Sun at rest, by impressing upon every external object a movement equal and opposite to that which the Sun actually had.

"(1.) If such impress leaves every distant cometary mass at rest, the effect of the Sun's attraction will be to draw them directly towards himself, on whatever side they may be; and if, *when sensible velocity towards the Sun has been acquired* (but not before), some disturbing force acts transversely to its course, it will describe a parabola. But the chances are equal for the occurrence of distant cometary masses at rest in all directions from the Sun, and, consequently, the chances are equal for the occurrence of the perihelia in all directions.

"(2.) If such impress leaves to the distant cometary mass *any* motion in *any* direction, it will describe a hyperbola, and no other orbit, about the Sun. If the motion is from the Sun, or inclined less than 90 degrees to the direction from the Sun, it has passed perihelion, and will never be seen. If the motion is nearly towards the Sun, it is approaching to perihelion, and may be seen; but the range in the position of perihelion may be great.

"(3.) No circumstance can enable the mass to describe an elliptic orbit except this:—That when the mass has acquired a considerable velocity by falling towards the Sun, its velocity is then checked and turned aside by some external cause.

"If we suppose, as in case (2), that the impress does leave a motion, then the more probable it is that the cometary masses are in reality nearly at rest, the more probable it will be that the impressed motion leaves an apparent movement nearly meeting the Sun, which becomes a hyperbolic orbit.

"It appears to me, therefore, that the indication of the direction of the Sun's motion is to be sought, not from the position of the perihelion of a parabolic orbit, but from the direction of the asymptote of that branch of the hyperbola in which a comet is approaching to the Sun, without regard to the position of perihelion or the direction of recess from the Sun."

The author has to add upon the foregoing valuable criticism, that he has hitherto habitually regarded parabolic orbits simply as undiscriminated ellipses or hyperbolas, and that, in forming his last Table, his object was as much as possible to get rid of the ellipses and retain the hyperbolas. For by far the greater portion of these, however, the direction of approach to the Sun could not differ more than 2 or 3 degrees from the point opposed to the perihelion in the concluded parabola, the inclination of the asymptote of a hyperbola to its major axis being the angle whose tangent $=\sqrt{e^2-1}$, which for $e=1\cdot001$ is $=2^\circ 35'$, and which for $e=1$ (or the parabola) vanishes. For his purpose it appeared to him sufficient to take the original direction of approach parallel to the axis of the observed orbit, and the nugatory result of the inquiry did not lead him to reflect further on the matter. Admitting the force of the Astronomer Royal's objection to inference from actual parabolic orbits, he would point out that there appear to be two sources of apparent parabolic motion. The first is that of Mr. AIRY's case (1), which supposes that distant cometary masses have no other motion than one parallel and equal to that of our Sun, and probably resulting from a common cause, though the masses themselves are supposed to be at a distance at which the Sun's attraction is nearly insensible. In the approach to the Sun no etherial resistance is supposed. But a parabolic orbit may (2) result from an original hyperbolic one by the action of resistance of a medium; for, as shown by NEWTON, if V be the velocity at any distance R of a body solicited by an absolute central force μ, the orbit will be

An Ellipse	if	V^2	be	less	than $\frac{2\mu}{R}$
A Hyperbola	if	„		greater	„
A Parabola	if	„		equal to	„

As Mr. AIRY remarks, if the distance R be so great that $\frac{2\mu}{R}$ is at the limit of sensible magnitude, the ellipse becomes an impossible case, and the hyperbola is the only curve resulting from a sensible value of V. Now the author remarks that this is true for the outset only, and that if the value of V be small originally, and resistance is experienced in the approach to the Sun, the orbit may degrade to the parabola and even to the ellipse of great excentricity. In this second case it is not clear to him that the same objection would lie against the data supplied by parabolic elements: although it is true that a small correction is required in each case, the amount of which the data leave undetermined.

The above considerations may supply an additional motive to discuss still more rigorously than heretofore these very long orbits.

Elements of the Periodical Comets.

No.	Year.	ϖ − ν.	ν.	ι.	Period.	Name.
69	1786	182° 30′	334° 8′	13° 36′	3·3 y	Encke's Comet.
81	1795	182 2	334 39	13 42	„	
91	1805	182 27	334 20	13 34	„	
107	1819	182 26	334 33	13 37	„	
113	1822	182 47	334 25	13 20	„	
121	1825	182 47	334 28	13 21	„	
131	1829	182 48	334 30	13 21	„	
134	1832	182 49	334 32	13 21	„	
140	1835	182 49	334 35	13 21	„	
142	1838	182 51	334 37	13 22	„	
147	1842	182 51	334 39	13 21	„	
158	1845	183 25	334 20	13 8	„	
174	1848	183 25	334 22	13 9	„	
184	1852	183 28	334 23	13 8	„	
199	1855	183 27	334 26	13 8	„	
212	1858	183 29	334 29	13 4	„	
110	1819	350 5	77 14	9 1	4·8	Blanplain's Comet.
52	1766	177 2	74 11	8 2	5	Messier's Comet.
36	1743	6 25	86 54	1 54	5·4	By Clausen.
54	1770	224 17	132 0	1 35	5·5	Lexell's Comet.
152	1844	278 42	63 49	2 55	5·5	De Vico's.
	1855	..	..	..	..	(Seen by Goldschmidt ?)
161	1846	13 47	102 41	30 56	5·5	Brorsen's Comet.
202	1857	13 58	101 46	29 49	..	
109	1819	161 30	113 11	10 43	5·5	Winnecke's Comet.
208	1858	162 6	113 33	10 48	..	
65	1783	356 18	55 45	44 53	5·6	Pigott's Comet.

Elements of the Periodical Comets (*continued*).

No.	Year.	ϖ — ν.	ν.	ι.	Period.	Name.
181	1851	174° 32′	148° 27′	13° 56′	$6^{y}\cdot 4$	D'Arrest's Comet.
	1857	..	..	..	..	Observed at the Cape.
57	1772	216 15	254 0	18 18	6·6	Biela's Comet.
92	1806	218 17	251 15	13 39		
123	1826	218 18	251 28	13 34	..	Recognised.
136	1832	221 45	248 16	13 13		
160	1846	223 8	245 55	12 35	..	Doubled.
187	1852	223 11	245 51	12 34		
	1859	223 51	245 43	12 24	..	(Hubbard's Elements.)
151	1843	200 5	209 29	11 23	7·5	Faye's Comet.
180	1851	200 12	209 31	11 22		
213	1858	200 4	209 45	11 22		
75	1790	204 36	267 9	56 58	14	Bruhn's Comet.
207	1858	206 49	269 3	54 24		
164	1846	339 39	260 29	30 24	14	Peters's Comet.
186	1852	56 59	346 13	40 59	70	Westphal's Comet.
100	1812	199 18	253 1	73 57	71	Pons's Comet.
162	1846	12 54	77 33	85 6	73	De Vico's.
103	1815	65 33	83 29	44 30	74	Olbers's Comet.
171	1847	129 23	309 49	19 8	75	Brorsen's Comet.
5	1607	107 2	48 40	162 48	76	Halley's Comet.
17	1682	109 16	51 11	162 15	,,	
45	1759	110 40	53 50	162 23	,,	
141	1835	110 38	55 10	162 15	,,	
4	1596	59 56	330 21	128 2	249	Colla's Comet.
157	1845	75 46	337 49	131 18		
204	1857	180 56	200 51	32 47	256	Peters's Olcott Comet.

Elements of the Periodical Comets (continued).

No.	Year.	$\varpi - \nu$.	ν.	ι.	Period.	Name.
15	1678	159° 28′	163° 20′	2° 52′	?	Query. De Vico's Comet?
18	1683	86 47	173 18	96 12	190	
144	1840	156 39	236 50	120 47	340	
146	1840	133 35	248 56	57 57	350	
149	1843	82 33	1 12	142 19	376	
80	1793	69 54	2 0	51 31	420	(Period doubtful.)
165	1846	99 51	261 51	150 41	450	
99	1811	314 26	93 2	31 17	875	
94	1807	4 7	266 47	63 10	1714	
53	1769	329 7	175 4	40 46	2000	
211	1858	129 6	165 19	116 58	2100	Donati's Comet.
205	1857	124 50	14 58	123 57	2460	
130	1827	258 42	149 39	125 55	2600	
159	1846	337 58	111 8	47 26	2700	
98	1811	65 23	140 25	106 57	3000	The Great Comet.
122	1825	256 57	215 43	146 27	4400	
115	1822	181 4	92 45	127 21	5400	
177	1849	236 31	30 32	66 59	8400	
39	1746	165 0	335 0	6 0	—	
49	1763	88 40	356 17	72 34	—	
145	1840	138 16	186 4	79 51	—	
167	1847	254 22	21 50	48 40	—	
178	1850	180 31	92 53	68 12	—	
189	1853	199 13	40 58	122 11	—	
196	1854	187 1	238 8	14 9	—	

The Elements of the following Comets are not sufficiently determined:—

No.	Year.	
12	1668	
21	1689	
22	1695	
125	1826	The *third* comet of this year.

Distribution of the Perihelia of the Non-periodical Comets.

No.	Year.	$\varpi - \nu$.	ν.	i.	Log q.	Long.	Lat.	A P.*	E A P.
1	1585	331° 32′	37° 44′	6° 5′	0·0396	9° 24′	− 2° 54′	75° 20′	288° 11′
2	1590	307 40	165 37	150 30	9·7541	214 3	−22 56	92 11	141 35
3	1593	12 4	164 15	87 58	8·9499	164 41	+12 3	99 10	81 57
6	1618	24 55	293 25	21 28	9·7101	316 48	+ 8 52	113 21	248 42
7	..	287 21	75 44	37 12	9·5906	7 9	−35 15	50 17	264 37
8	1652	300 9	88 10	79 28	9·9281	70 42	−58 14	3 37	254 5
9	1661	33 22	81 54	33 1	9·6461	110 48	+17 26	79 49	32 13
10	1664	310 33	81 14	158 41	0·0110	128 40	−16 2	56 8	64 24
11	1665	156 8	228 2	103 55	9·0273	54 6	+23 8	82 54	338 24
13	1672	109 29	297 31	83 22	9·8435	99 26	+69 27	127 50	9 31
14	1677	99 12	236 49	100 57	9·4481	106 22	+75 44	134 26	9 32
16	1680	350 40	272 9	60 40	7·7940	267 33	− 8 8	113 58	190 41
19	1684	330 37	268 15	65 49	9·9823	255 16	−26 35	95 59	177 45
20	1686	86 26	350 35	31 22	9·5119	76 24	+31 18	88 43	358 48
23	1698	356 53	267 44	168 14	9·8397	270 47	− 0 38	121 4	195 2
24	1699	109 14	321 46	110 40	9·8716	187 6	+62 4	145 48	51 56
25	1701	165 0	298 41	138 21	9·7728	130 0	+ 9 54	79 31	52 8
26	1702	309 47	188 59	4 25	9·8108	138 51	− 3 24	71 49	66 34
27	1706	59 18	13 12	55 14	9·6292	57 3	+44 56	103 50	344 51
28	1707	27 8	52 47	88 36	9·9344	53 30	+27 7	87 1	338 14
29	1718	6 16	127 55	148 52	0·0109	122 33	+ 3 14	70 17	47 49
30	1723	331 22	14 14	130 0	9·9995	33 34	−21 32	48 18	298 44
31	1729	9 53	310 38	77 5	0·6067	312 52	+ 9 38	116 43	243 57
32	1737	99 33	226 22	18 21	9·3480	326 25	+18 5	117 0	261 21
33	..	138 43	123 54	39 14	9·9380	269 41	+24 40	146 18	198 31
34	1739	104 47	207 25	124 17	9·8284	92 19	+53 2	111 0	8 50
35	1742	328 3	185 38	113 1	9·8840	199 20	−29 9	80 14	130 24
37	1743	118 43	5 16	134 12	9·7173	237 6	+38 57	156 51	133 35
38	1744	151 27	45 45	47 9	9·3465	205 27	+20 31	126 44	110 48
40	1747	230 17	147 19	100 54	0·3421	314 30	−49 3	64 1	217 7
41	1748	17 28	232 52	94 32	9·9245	231 27	+17 25	135 11	141 46
42	..	245 39	33 8	67 3	9·7961	253 53	−57 2	65 30	177 9
43	1757	268 45	214 13	12 50	9·5283	122 56	−12 50	55 40	55 20

* A, ... α Columbæ. P, ... Comet's Perihelion. E, ... N. Pole of Ecliptic.

Distribution of the Perihelia of the Non-periodical Comets (continued).

No.	Year.	$\varpi - \nu$.	ν.	i.	Log q.	Long.	Lat.	A P.	E A P.
44	1758	36° 48′	230° 50′	68° 19′	9·3331	246° 17′	+33° 49′	154° 56′	154° 51′
46	1759	273 45	139 39	78 59	9·9023	68 35	−78 22	21 14	185 41
47	..	301 26	79 51	175 8	9·9850	138 19	− 4 9	70 30	65 44
48	1762	115 29	348 33	85 38	0·0039	159 28	+64 11	136 3	38 17
50	1764	104 50	120 5	127 6	9·7445	6 23	+50 27	123 7	313 32
51	1766	100 55	244 11	139 10	9·7036	139 53	+39 57	109 55	45 29
55	1770	260 19	108 42	148 34	9·7228	210 1	−30 56	82 33	139 17
56	1771	76 11	27 52	11 15	9·9559	103 47	+10 55	71 18	25 47
58	1773	314 5	121 6	61 14	0·0519	94 41	−39 2	21 1	35 50
59	1774	136 43	180 45	83 20	0·1562	354 31	+42 55	122 24	300 18
60	1779	62 10	25 4	32 31	9·8532	83 1	+28 23	85 51	3 28
61	1780	237 5	123 41	125 37	8·9836	261 42	−43 2	79 33	181 56
62	..	254 9	141 1	107 56	9·7120	273 42	−66 15	55 52	187 3
63	1781	156 10	83 1	81 43	9·8898	259 22	+23 34	146 10	180 25
64	..	61 19	77 23	152 47	9·9827	18 59	+23 39	95 17	307 5
66	1784	336 5	56 49	128 51	9·8499	72 22	−18 24	39 20	349 47
67	1785	205 40	264 12	70 14	0·0582	93 26	−24 3	34 54	23 10
68	..	127 4	64 34	92 28	9·6307	247 50	+52 52	172 6	120 13
70	1786	325 3	194 23	50 54	9·6129	170 36	−26 24	68 44	106 5
71	1787	99 7	106 52	131 44	9·5427	3 19	+47 28	122 8	309 16
72	1788	57 49	156 57	167 32	0·0265	99 45	+10 32	70 0	21 32
73	..	30 25	352 24	64 30	9·8793	6 35	+27 11	103 59	298 58
74	1790	114 26	172 50	150 16	9·8735	55 13	+26 51	86 39	338 39
76	..	119 28	33 11	116 8	9·9020	251 7	+51 25	172 23	138 16
77	1792	154 17	190 46	140 13	0·1116	31 5	+16 7	83 37	313 54
78	..	147 16	283 15	130 58	9·9851	126 6	+24 6	90 26	41 45
79	1793	239 47	108 29	119 39	9·6057	248 8	−48 41	73 21	172 21
82	1796	184 18	17 2	115 5	0·1982	195 12	− 3 54	100 12	114 12
83	1797	279 48	329 16	129 19	9·7215	44 1	−49 40	22 4	275 9
84	1798	342 50	122 9	43 52	9·6855	109 33	−11 48	51 5	39 16
85	..	215 4	249 31	137 34	9·8918	42 8	−22 49	43 46	306 18
86	1799	95 53	99 33	129 4	9·9243	0 16	+50 34	125 52	309 40
87	..	136 29	326 49	102 58	9·7964	158 51	+42 9	119 30	56 53
88	1801	220 39	44 28	158 40	9·4178	185 49	−13 43	87 2	111 5

Distribution of the Perihelia of the Non-periodical Comets (continued).

No.	Year.	$\varpi - \nu$.	ν.	i.	Log q.	Long.	Lat.	A P.	E A P.
89	1802	21° 54′	310° 16′	57° 1′	0·0391	322° 37′	+18° 14′	119° 38′	257° 12′
90	1804	332 4	176 50	56 44	0·0303	160 37	−23 4	66 2	95 47
93	1806	225 17	322 19	144 57	0·0341	102 44	−24 5	37 17	36 32
95	1808	253 46	322 59	134 17	9·5909	75 37	−43 25	14 12	348 26
96	..	131 56	24 28	140 43	9·7837	245 13	+28 6	149 1	154 59
97	1810	114 16	308 53	62 46	9·9864	83 27	+54 9	111 36	2 29
101	1813	350 52	60 48	158 46	9·8446	69 19	− 3 18	54 42	347 26
102	..	204 57	42 41	98 58	0·0850	218 32	−24 37	91 4	143 21
104	1816	304 21	323 15	43 5	8·6857	276 21	−34 20	87 11	193 47
105	1818	112 19	70 26	89 44	0·0784	249 47	+67 41	168 48	19 32
106	..	348 5	90 0	116 55	9·9320	95 27	−10 37	48 20	21 1
108	1819	13 23	273 43	80 45	9·5328	275 54	+13 12	134 6	202 18
111	1821	169 11	48 41	106 27	8·9630	231 47	+10 22	128 19	144 5
112	1822	344 43	177 27	126 23	9·7028	186 39	−12 15	88 33	110 46
114	..	239 7	97 40	141 47	9·9226	224 57	−32 4	85 52	151 3
116	1823	28 29	303 3	103 48	9·3551	295 41	+27 35	140 56	235 43
117	1824	334 3	234 19	125 26	9·7718	250 4	−20 53	101 18	170 49
118	..	85 15	279 16	54 37	0·0212	1 6	+54 21	128 30	313 6
119	1825	106 11	20 6	123 19	9·9490	262 15	+53 22	175 43	200 32
120	..	177 18	192 56	89 42	9·9462	12 55	+ 2 42	80 5	291 14
124	1826	279 16	197 38	40 3	0·3034	119 40	−39 25	31 20	72 25
126	..	13 42	44 6	25 57	9·9308	56 28	+ 5 57	66 7	334 32
127	..	279 36	235 8	90 38	8·4296	238 52	−80 23	41 43	174 52
128	1827	150 57	184 28	102 24	9·7046	11 16	+28 19	103 3	302 46
129	..	20 39	318 10	136 21	9·9075	302 55	+14 5	125 54	234 58
132	1830	5 50	206 22	21 16	9·9645	211 48	+ 2 7	113 2	126 12
133	..	26 54	337 53	135 14	9·1000	318 4	+18 35	122 29	252 53
135	1832	204 31	72 27	136 42	0·0732	234 5	−16 32	103 2	154 42
137	1833	259 50	323 1	7 21	9·6613	222 46	− 7 14	108 41	140 51
138	1834	49 45	226 49	5 57	9·7118	276 25	+ 4 32	125 33	200 26
139	1835	210 37	58 20	170 52	0·3100	208 2	− 4 38	105 18	125 40
143	1840	72 14	119 58	53 6	9·7913	181 53	+49 36	135 33	64 54
148	1842	240 32	207 50	106 26	9·7028	1 14	−56 38	40 24	236 20
150	1843	124 15	157 15	52 45	0·2085	295 37	+41 9	152 8	249 49

Distribution of the Perihelia of the Non-periodical Comets (*continued*).

No.	Year.	ϖ − ν.	ν.	*i.*	Log *q*.	Long.	Lat.	A P.	E A P.
153	1844	211° 15′	31° 39′	131° 24′	9·9321	189° 47′	−22° 54′	80° 48′	118° 35′
154	..	177 37	118 23	45 37	9·4001	296 43	+ 1 42	117 10	222 12
155	1845	114 35	336 45	46 51	9·9568	100 32	+41 34	100 27	15 28
156	..	205 27	347 7	56 24	0·0985	181 52	−20 58	78 34	111 12
163	1846	78 46	161 19	122 24	0·1387	91 39	+55 55	113 42	7 5
166	..	93 55	4 41	49 41	9·9194	100 43	+49 32	108 17	13 58
168	1847	32 20	173 58	100 26	0·3253	167 26	+31 44	114 58	69 36
169	..	55 26	76 43	147 21	0·1717	26 1	+26 23	95 13	313 17
170	..	91 36	338 18	96 33	0·2473	234 32	+83 16	153 20	6 28
172	..	276 30	190 56	107 49	9·5185	260 31	−71 4	51 32	180 12
173	1848	260 58	211 32	95 35	9·5050	0 4	−79 24	32 18	199 49
175	1849	208 1	215 13	85 3	9·9821	37 51	−27 54	41 41	296 45
176	..	33 11	202 33	67 10	0·0642	216 47	+30 18	139 47	113 32
179	1850	243 14	205 59	40 6	9·7524	82 35	−35 6	22 22	5 23
182	1851	88 3	223 9	37 44	9·9918	310 41	+37 42	141 48	261 17
183	..	294 20	44 26	74 0	9·1502	13 4	−61 9	32 56	234 48
185	1852	37 8	317 8	131 7	9·9567	290 40	+27 3	142 47	228 27
188	1853	275 49	69 33	159 46	0·0382	153 21	−20 8	64 9	87 16
190	..	170 27	140 31	61 31	9·4867	315 56	+ 8 23	114 59	244 31
191	..	277 53	220 3	119 1	9·2380	294 7	−60 1	59 34	198 54
192	1854	170 56	227 3	113 53	0·3106	50 45	+ 8 17	69 56	328 52
193	..	101 38	315 27	97 27	9·4426	167 39	+76 13	144 21	24 6
194	..	74 35	347 40	108 41	9·8117	298 23	+65 57	160 21	311 24
195	..	129 54	324 33	41 0	9·9030	102 29	+30 13	89 37	19 11
197	1855	324 33	189 39	128 41	0·3454	213 38	−26 55	87 5	139 37
198	..	22 37	260 19	156 53	9·7534	239 21	+ 8 41	128 41	153 17
200	..	325 32	51 34	169 49	0·0903	94 4	− 5 45	52 48	17 29
201	1857	121 34	313 10	87 56	9·8879	129 49	+58 31	122 26	28 8
203	..	134 3	23 41	121 0	9·5653	231 43	+38 3	153 9	123 48
206	..	95 5	139 19	142 10	0·0039	42 53	+37 40	100 7	330 50
209	1858	25 16	170 43	23 0	0·0827	194 12	+ 9 36	110 53	105 24
210	..	98 52	324 58	99 57	9·7358	192 53	+76 42	150 11	25 16
							133)	12720 52	
								95 39	

A LIST OF PERSONS

TO WHOM

THE MEDALS OR TESTIMONIALS OF THE SOCIETY

HAVE BEEN ADJUDGED.

1823.

June 13. CHARLES BABBAGE, Esq.

The Gold Medal.—For his Invention of an Engine for computing and printing Mathematical Tables.

Professor JOHANN FRIEDRICH ENCKE.

The Gold Medal.—For his Investigations relative to the Comet which bears his name.

CHARLES RUMKER, Esq.

The Silver Medal.—For his Re-discovery of ENCKE's Comet in 1822.

M. JEAN LOUIS PONS.

The Silver Medal.—For his Discovery of two Comets in 1822.

1826.

Feb. 7. J. F. W. HERSCHEL, Esq. and JAMES SOUTH, Esq.

The Gold Medal, each.—For their important Researches on the subject of Multiple Stars.

Feb. 10. Professor STRUVE.

The Gold Medal.—For his important Researches on the subject of Multiple Stars.

1827.

Feb. 2. FRANCIS BAILY, Esq.

The Gold Medal.—For his "New Tables for determining the places of 2881 Stars."

WILLIAM SAMUEL STRATFORD, Esq.

The Silver Medal.—For his Superintendence of the Computation of "New Tables for determining the places of 2881 Stars."

Feb. 5. Colonel MARK BEAUFOY.

The Silver Medal.—For his valuable Collection of Observations, particularly those of the Eclipses of *Jupiter's* Satellites.

1828.

Jan. 11. Sir Thomas Macdougall Brisbane, K.C.B.

The Gold Medal.—For his Establishment of an Observatory, and for an important series of Observations made at Paramatta.

James Dunlop, Esq.

The Gold Medal.—For his Observations of the Nebulæ of the Southern Hemisphere.

Feb. 4. Miss Caroline Herschel.

The Gold Medal.—For her recent Reduction to January 1800, of the Nebulæ discovered by Sir William Herschel.

1829.

Jan. 9. Rev. William Pearson.

The Gold Medal.—For his work, entitled "An Introduction to Practical Astronomy."

Professor Bessel.

The Gold Medal.—For his Zone Observations.

Professor Schumacher.

The Gold Medal.—For the Publication of his various Astronomical Tables, and the "Astronomische Nachrichten."

1830.

Jan. 8. Mr. William Richardson.

The Gold Medal.—For his Investigation of the Constant of Aberration.

Professor Encke.

The Gold Medal.—For the New Berlin Ephemeris.

1831.

Jan. 14. Captain Kater.

The Gold Medal.—For his Invention of the Vertical Floating Collimator.

Baron Damoiseau.

The Gold Medal.—For his Memoir upon the Theory of the Moon, and for his Lunar Tables.

1833.

Jan. 11. Professor Airy.

The Gold Medal.—For his Discovery of the long Inequality of *Venus* and the Earth.

1835.

Jan. 9. Lieutenant Johnson.

The Gold Medal.—For his Catalogue of 606 Southern Stars.

1836.

Jan. 8. Sir John F. W. Herschel.

The Gold Medal.—For his Catalogue of Nebulæ, printed in the "Philosophical Transactions" for 1833.

1837.
Jan. 13. Professor Rosenberger.
The Gold Medal.—For his Investigations relative to Halley's Comet.

1839.
Jan. 11. Hon. John Wrottesley.
The Gold Medal.—For his Catalogue of the Right Ascensions of 1318 Stars.

1840.
Jan. 10. M. Jean Plana.
The Gold Medal.—For his work, entitled "Théorie du Mouvement de la Lune."

1841.
Jan. 8. Professor Bessel.
The Gold Medal.—For his Observations and Researches on the Paralla of 61 *Cygni.*

1842.
Jan. 14. M. Hansen.
The Gold Medal.—For his Researches in Physical Astronomy.

1843.
Jan. 13. Francis Baily, Esq.
The Gold Medal.—For his Experiments to determine the Mean Density of the Earth in repetition of what is generally termed the "Cavendish Experiment."

1845.
Jan. 10. Captain William Henry Smyth, R.N.
The Gold Medal.—For his "Bedford Catalogue," forming the second part of his work entitled "Celestial Cycle."

1846.
Jan. 9. George Biddell Airy, Esq. Astronomer Royal.
The Gold Medal.—For his Reduction of the Observations of Planets made at the Royal Observatory, Greenwich, from 1750 to 1830.

1848.
Jan. 14. *Testimonials were awarded to,*

George Biddell Airy, Esq. Astronomer Royal.
For the Lunar Reductions recently made at Greenwich.

John Couch Adams, Esq.
For his Researches in the Problem of Inverse Perturbations leading to the Discovery of the Planet *Neptune.*

Professor Argelander.
For his Catalogue of Stars.

1848.
Jan. 14. **George Bishop**, Esq.
For the Foundation of an Observatory leading to various Astronomical Discoveries.

Lieut.-Col. **George Everest.**
For his Measurement of the Indian Arc.

Sir **John F. W. Herschel.**
For his Work on the Southern Hemisphere.

Professor **P. A. Hansen.**
For his Lunar Theory and Computation of Perturbations.

M. **Hencke.**
For his Discovery of two Planets, *Astræa* and *Hebe.*

John Russell Hind, Esq.
For his Discovery of two Planets, *Iris* and *Flora.*

M. **U. J. Le Verrier.**
For his Researches in the Problem of Inverse Perturbations leading to the Discovery of the Planet *Neptune.*

Sir **John Lubbock.**
For his Researches in the Theory of Perturbations.

M. **M. Weisse.**
For his Catalogue of Stars in **Bessel**'s Zones.

1849.
Feb. 9. **William Lassell**, Esq.
The Gold Medal. — For the Construction of his Equatoreal Instrument and for the Discoveries made with it.

1850.
Feb. 8. M. **Otto von Struve.**
The Gold Medal. — For his Paper on the Constant of Precession.

1851.
Feb. 15. Dr. **Annibale de Gasparis.**
The Gold Medal. — For the Discovery of three Planets, *Hygeia, Parthenope*, and *Egeria.*

1852.
Feb. 13. **Dr. C. A. F. Peters.**
The Gold Medal.—For his Papers on the Parallax of the Fixed Stars, and on the Constant of Nutation.

1853.
Feb. 11. **John Russell Hind, Esq.**
The Gold Medal.—For the Discovery of eight Planets, and other Astronomical Discoveries.

1854.
Feb. 10. **M. Charles Rumker.**
The Gold Medal.—For his Catalogue of 12,000 Stars, and for other Astronomical Services.

1855.
Feb. 9. **Rev. W. R. Dawes.**
The Gold Medal.—For his Astronomical Labours generally.

1856.
Feb. 8. **Robert Grant, Esq. M.A.**
The Gold Medal.—For his "History of Physical Astronomy."

1857.
Feb. 13. **M. Schwabe.**
The Gold Medal.—For his Discovery of the Periodicity of the Solar Spots.

1858.
Feb. 12. **Rev. Robert Main, M.A.**
The Gold Medal.—For his various Contributions to the *Memoirs* of the Society.

1859.
Feb. 11. **R. C. Carrington, Esq.**
The Gold Medal.—For his "Redhill Catalogue of 3735 Circumpolar Stars."

1860.
Feb. 10. **Professor P. A. Hansen.**
The Gold Medal.—For his Lunar Tables.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY,

CONTAINING

PAPERS,

ABSTRACTS OF PAPERS,

AND

REPORTS OF THE PROCEEDINGS

OF

THE SOCIETY,

FROM NOVEMBER 1859, *TO JULY* 1860.

VOL. XX.

Pursuant to a Resolution of the Council submitted to the Society and approved of by them at the Annual Meeting in 1859, *the* Monthly Notices *are printed with the* Memoirs, *as well as in the accustomed octavo form.*

Monthly Notices

OF THE

ROYAL ASTRONOMICAL SOCIETY.

⁂ *The paging of the octavo edition is repeated.*

Vol. XX. *November* 11, 1859. No. 1.

The Rev. R. Main, President, in the Chair.

Lieut. Wm. Chimmo, H. M. S. Seagull,

was balloted for and duly elected a Fellow of the Society.

The Astronomische Nachrichten.

A new volume of this important periodical, the fifty-second, is about to commence. The price of a volume is fifteen shillings, payable in advance. There are generally published two volumes in the course of a year. Subscriptions are received by Mr. Williams, Assistant-Secretary of the Society. It may be mentioned that the *Astronomische Nachrichten* admits into its columns contributions in any other European language besides the staple German.

The Astronomer Royal has lately circulated the following Notice:—

"The construction of the meridional instrument, and the general arrangements in this observatory, offer great advantages for the meridional observations of the small planets; and it has been my wish to maintain these observations with the utmost practicable regularity. This, however, can be done only with the aid of a very approximate ephemeris.

"In general, ephemerides of the small planets are not wanting; but their errors are frequently so large as to make it nearly impossible for observers to assure themselves that the object which they see in the meridional telescope is the planet which they are seeking. A great amount of labour is thus wasted. And, finally, I have been compelled to lay down, as a general rule, that small planets are not to be sought on the meridian unless we have a pretty accurate knowledge of the error of the ephemeris.

"It is much to be wished that information on the magnitude of the error could be furnished to me before the meridional observations commence. As good extrameridional determinations are usually obtained before meridional instruments can be used, I venture to request that, as soon as you have secured one or two certain observations of a planet, you will have the kindness to communicate to me a comparison of the observed places with the corresponding places in any published ephemeris. This would probably enable me to observe the planet on the meridian through its entire meridional apparition.

"*Royal Observatory, Greenwich,*
November, 1859."

Measures of Saturn and his Rings. By Captain W. S. Jacob, Director of the Madras Observatory.

Measures of the Satellites of Saturn with the Madras Equatoreal. By Captain W. S. Jacob.

Measures of Saturn and his Satellites. By Captain W. S. Jacob.*

The first of these papers contains a series of measures of *Saturn* and his rings, executed at the Madras Observatory in the month of January, 1856. The second paper contains a series of measures of the satellites of *Saturn*, executed at the same observatory, and extending from November 6, 1856, to April 10, 1858. In the third paper the author has given a very elaborate discussion of these measures, and has deduced from them corrections to the elements of the orbits of the respective satellites. The materials which form the basis of investigation are divided into two groups, from each of which he derives a corresponding set of elements. The first group consists of measures executed in 1856 and in the early part of 1857. An abstract of the results for this group will be found in Vol. xviii. of the *Monthly Notices* (page 1 *et seq.*). The second group consists of measures executed in 1857–8. Upon these data the author thus remarks:—

"The observations of 1857–8 are on the whole slightly inferior to those of the previous season, the weather being on the average less favourable, and the observer not quite in so good working condition, owing to the state of his health; but the distance-measures of *Titan* are at least equally good, and those near elongation more numerous, so that the mean result should be more trustworthy; also there were two nights of such exceptional fineness as to admit of distance-measures being taken of *Enceladus*."

The following are the elements of the five exterior satellites as deduced from the observations of 1857–8. The epoch of mean longitude is 1858, Jan. 0·0.

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* These papers by Captain Jacob, as well as those which follow, and also the papers of which abstracts are given at pages 61 and 68 of vol. xix. of the *Monthly Notices*, are published *in extenso* in the twenty-eighth volume of the *Memoirs* of the Society, at the expense of the India Government.—R. G.

	Japetus.	Titan.	Rhea.	Dione.	Tethys.
	° ′	° ′	° ′	° ′	° ′
ι	294 26·1	260 26·5	215 14·2	245 19·2	47 15·2
ι	18 49·4	27 33·8	28 13·7	28 10·4	28 10·4
☊	142 46·2	167 54·5	166 49·6	167 38·0	167 38·0
ϖ	353 30	257 27	69 34	330 10	158 47
e	·027960	·029236	·00252	·00470	·00874
a	514″·34	176″·61	76″·12	54″·48	42″·92
μ	4°·538031	22°·577030	79°·690190	131°·534944	190°·69775

The author is disposed to regard the elements deduced from the first series of observations as more trustworthy than those deduced from the second, since in the former case the position of the planet was more favourable for determining ι, ι and ☊. He remarks, however, that in the case of *Titan* the second set of elements present a better agreement with those of Bessel than the first do. They are, in fact, almost identical with the results given by that astronomer in No. 242 of the *Astronomische Nachrichten*, allowing for the annual motion of ϖ and ☊.

The author, referring to the considerable differences observable in the values of ι for *Rhea* and *Tethys*, suggests that it is probably due to the mutual action of the satellites, which must occasion inequalities of long period in several pairs of those whose mean motions are nearly commensurate. Thus the mean motions of *Japetus* and *Titan* are nearly as 1 to 5; of *Titan* and *Rhea*, 2:7; of *Dione* and *Enceladus*, *Tethys* and *Mimas* both, 1:2. As regards the values of *a* for *Titan*, derived from the two sets of observations, and the effect of their discordance upon the value of *Saturn's* mass, the author makes the following remarks:—

"The most unsatisfactory difference is that in the values of *a* for *Titan*, amounting to 0″·29, being rather more than I am inclined to admit as mere error of observation, considering the number of measures and the care with which they were made; being also more than twice the sum of the probable errors; as the observations extend over so many revolutions, the *casual* perturbations would also probably be eliminated, being of an order not exceeding that of errors of observation."

The author suggests that the discordance may arise from two distinct causes; 1st, the long equation between *Japetus* and *Titan*; and, secondly, errors in the tables of *Saturn*, especially in the eccentricity or place of perihelion, which, by affecting the distance of the planet from the earth, would alter the reduction of the observed elongation to the mean distance. He accordingly concludes that until we have obtained corrections to the tables of *Saturn*, or until we have good measures of *Titan's* elongation at opposite points of the planet's orbit, we shall be unable to determine with certainty the value of the mass of the latter. He is of opinion that in the present state of the researches on the subject the mass of the planet may be assumed to be included between $\frac{1}{3500}$ and $\frac{1}{3476}$.

Measures of Jupiter and his Satellites, taken with the Madras Equatoreal. By Captain W. S. Jacob.

These measures of the third and fourth satellites of *Jupiter* were executed during the latter half of the year 1857. They were undertaken with the view of obtaining a test for estimating the value to be assigned to the measures of the Saturnian system made by the same observer with the same instrument; for by comparing the mass of *Jupiter* deduced from the auhor's measures with the value as already very approximatelty determined by Airy and others, a fair opinion might be formed respecting the degree of accuracy attributable to the measures. Moreover, as there was reason to suppose that the elements of *Sat.* IV. were not sufficiently exact, measures of that satellite were taken in other positions besides near the greatest elongations and approximate corrections of the elements obtained. The mean value of the Jovian mass as deduced from the measures of both satellites is $\frac{1}{1047\cdot54}$, agreeing almost exactly with Airy's value, $\frac{1}{1046\cdot77}$ (*Mem. Ast. Soc.* vol. x.) and with that assigned by Bessel in the *Astronomische Nachrichten* (No. 443), viz. $\frac{1}{1047\cdot87}$.

Occultations of Stars, observed at Madras. By Captain W. S. Jacob.

These observations extend from February 20, 1856, to December 29, 1857. They contain several interesting cases of the projection of the star on the moon's disk.

Notes on the Zodiacal Light, as observed at Madras in the Years 1856–8, *also on the Brightness of certain Stars.* By Captain W. S. Jacob.

The observations which form the groundwork of these notes extend from the 24th of January, 1856, to the 10th of April, 1858. Their number, however, is inconsiderable, the author's attention having been mainly occupied with other matters. We quote the following remarks:—

"The light at its brightest was considerably fainter than the brighter portions of the milky way, and shaded off so gradually that its limit could hardly be guessed at within several degrees. Colour white, much like that of the milky way, with at times a very slight bluish, and at others a yellowish hue,—tints most probably due to our atmosphere. The great variations in extent and intensity, sometimes occurring in the course of a few minutes, are most probably also dependent on the transparency of our atmosphere; but these variations interfere considerably with any attempt to determine its exact figure or dimensions. The outline generally appeared of a parabolic or probably elliptical form, and it would seem to be excentric as regards the sun, and also inclined, though but slightly, to the ecliptic. In one point it would seem to extend to or beyond the earth's orbit, viz. at about 100° or 105°; indeed, from the middle of November till the end of February, the elongation in a clear sky appears to exceed 90°. At the part opposite to this there are scarcely any observations, but in April and August the elongation appears not much to exceed 60°."

Observations of Eclipses of Jupiter's Satellites. By Captain W. S. Jacob.

These observations extend from September 25, 1856, to January 16, 1858. They relate to occultations of the satellites by their primary as well as eclipses.

Observations of Comet VIII., 1858, *Iris, Lutetia, and Mnemosyne.* By Norman Pogson, Esq., Director of the Hartwell Observatory.

(*Communicated by Dr. Lee.*)

Comet VIII., 1858 (Tuttle's).

Greenwich Mean Time.	Mag.	Apparent R.A.	Log. Par. × Δ	Apparent N.P.D.	Log. Par. × Δ	Comparisons.
h m s		h m s		° ′ ″		
Oct. 8 13 2 55		22 19 54·26	+9·510	69 56 4·3	−0·729	8 with g &

Iris.

Greenwich Mean Time.	Mag.	Apparent R.A.	Log. Par. × Δ	Apparent N.P.D.	Log. Par. × Δ	Comparisons.
1858						
July 19 13 2 19		19 45 25·90	+8·993	105 32 46·2	−0·894	14 with e
21 12 53 6	9·3	19 43 16·69	+9·000	105 34 3·8	−0·894	8 with e
21 13 17 44		19 43 16·26	+9·135	105 34 4·8	−0·892	10 with e
Aug. 1 12 46 50	9·4	19 31 49·21	+9·235	105 42 58·4	−0·889	12
5 12 49 58	8·5	19 28 3·59	+9·309	105 47 5·0	−0·885	7 wi
6 12 31 21	8·3	19 27 10·17	+9·266	105 48 2·7	−0·887	6 wi
7 12 24 37	8·7	19 26 17·82	+9·260	105 49 1·9	−0·888	12

Lutetia.

Greenwich Mean Time.				Mag.	Apparent R.A.			Log. Par. × Δ	Apparent N.P.D.			Log. Par. × Δ	Comparisons.
1859	h	m	s		h	m	s		°	′	″		
une 29	12	19	21	10·1	19	39	5·19	−8·948	114	0	34·1	−0·916	5 with *d*
29	12	56	3		19	39	3·38	−8·436	114	0	42·1	−0·919	5 with *d*
ly 11	12	4	59	9·5	19	28	12·46	−8·192	114	51	0·0	−0·920	11 with *a*
12	12	31	49	9·7	19	27	13·46	+8·586	114	55	3·5	−0·920	6 with *a*

*Mnemosyne.**

1859													
ct. 3	12	9	27	10·9	23	58	3·20	+8·941	83	31	27·6	−0·784	10 with *k*
4	10	17	42	10·7	23	57	28·69	−8·903	83	40	18·0	−0·785	10 with *l*
5	10	47	37	11·2	23	56	51·01	−8·411	83	50	1·3	−0·788	7 with *l*
5	11	29	28		23	56	49·37	+8·600	83	50	18·1	−0·783	4 with *l*

The observation of Tuttle's comet was taken with the equatoreal of the Radcliffe Observatory and difference-micrometer. The comet was also observed simultaneously by Mr. Quirling, with the heliometer, and I regret not being able to append his position, as intended, for comparison of the two methods.

The observations of *Iris* were made with the five-foot Smythian telescope, at my residence, in South Parade, Oxford; and by means of a sidereal chronometer kindly lent me by the Royal Geographical Society. The first of the two places recorded on July 21st was determined by Mr. Quirling, using the Bogus-awski method or difference-micrometer, for comparison with my ing-micrometer observation taken immediately afterwards, and presents a most satisfactory agreement therewith.

Lutetia and *Mnemosyne* were observed with the Hartwell quatoreal and ring-micrometer, except on October 5th, when the irst position was taken with the wire-micrometer. The planet was, however, much too faint to bear the requisite illumination, and the second place is consequently the most trustworthy.

The ring-micrometer, though much used abroad, has been ut little employed in this country for planetary observations; nd indeed a general prejudice appears to prevail against it, so s to deter amateurs from giving it a fair trial. But as the inor planets are for the most part too faint for the ordinary lumination requisite to render spider-lines visible, or yet to uffer the loss of light caused by any kind of double-image icrometer, if early positions before, and late ones after oppotion, are to be obtained, we are inevitably thrown upon eticules, the Boguslawski method, or the ring-micrometer. or comet observations I much prefer the Boguslawski or ifference-micrometer, as it is undoubtedly far more certain to t the precise moment when so diffused an object as a faint omet is bisected by a thick wire, or the straight edge of a bar, an by a segment of a circle. But for a minute planet the ng is less troublesome, as one comparison-star is sufficient, hile the difference-micrometer requires two, and therefore ore reduction. That the calculations and the long refraction rrection ever necessary are tedious must be admitted, but ese are considerations which ought not to outweigh the imrtance of procuring positions of an object beyond the reach meridian instruments or illuminated fields. In order to give me idea of the degree of accuracy attainable in the determinon of the radii of the outer and inner edges of a ring, I beg ve to record the measures recently made here, of a new crometer with which we have provided ourselves in case of ed, together with the stars employed, and probable errors of gle and mean results. It must of course be borne in mind, t in using different pairs of determining stars, the errors of actual result depend far more upon the catalogue differences of N.P.D. than upon the observation, which is one of extreme accuracy. Bessel's incomparable *Verzeichniss von* 53 *Sternen der Plejaden* (*Ast. Unters.* vol. i. p. 237) affords the best and readiest choice of suitable pairs throughout the heavens; and if the night be slightly cloudy or foggy so much the better, the brighter, *i.e.*, the standard stars of the group can be employed, instead of the 8th or 9th magnitude stars preferable in a clear sky. Our results are these:—

Outer edge of the Hartwell equatoreal ring-micrometer.

	Determining pair of Stars.	Authority.	Radius.	Measures.	Probable errors of one measure.	Probable errors of mean result.
1859			″		″	″
April 6	2665 Gr. & ✱ 19′ *np* 3ˢ	Rad. Obs.	595·83	10	±0·48	±0·15
Oct. 17	18 Tauri & 22 Tauri	Bessel	596·30	10	±0·43	±0·14
	Adopted mean		596·1	20	±0·45	±0·10

Inner edge of the Hartwell equatoreal ring-micrometer.

May 9	2678 Gr. & 2687 Gr.	Rad. Obs.	453·82	10	±0·44	±0·14
Oct. 8	Celœno & 22 Tauri	Bessel	453·42	10	±0·48	±0·15
	Adopted mean		453·6	20	±0·47	±0·11

Similarly for the outer edge of the 5-foot telescope ring-micrometer.

1857						
Aug. 2	1669 Gr. & R Urs. Maj.	Rad. Obs.	828·06	5		
1858						
Feb. 28	Electra & Merope	Bessel	826·35	5		
	Adopted mean		827·2	10	±0·76	±0·24

Inner edge of the 5-foot telescope ring-micrometer.

1857						
Aug. 2	1669 Gr. & R. Urs. Maj.	Rad. Obs.	660·67	4		
1858						
Feb. 28	Anonyma 25 & 40 Pleiadum	Bessel	660·38	5		
	Adopted mean		660·5	9	±0·65	±0·22

The magnifying power employed being only fifty, these probable errors are remarkably small, but any one may with ease fix the value of his ring-radii within similar limits. Too much care cannot however be bestowed upon the determination, for a very minute error in the adopted radius will render the difference of N.P.D. faulty, and worse than useless, except when the chords described are very small. The assumed positions of the comparison-stars used for the preceding observations, reduced to the epoch 1859·0, were as follows:—

Reference.	Authority.	Mag.	Mean R.A.			Mean N.P.D.		
			h	m	s	°	′	″
a	A Z 239·50 = A Z 240·39	8·7	19	28	1·35	114	51	32·4
b	A Z 230·161	9·5	19	28	25·76	106	4	12·3
c	37403 Lalande	6·5	19	35	28·75	105	47	24·5
d	A Z 240·53	8·5	19	38	43·06	113	43	21·4
e	A Z 249·9	8·0	19	42	23·67	105	17	14·2
f	A Z 249·10	9·0	19	42	26·77	105	46	26·5
g	Maedler's General Cat. (33 Pegasi)	...	22	16	52·65	69	51	55·2
h	B Z 196	9·0	22	21	4·23	70	2	37·6
k	Weisse 23·1011 = 46931 Lalande	9·3	23	49	48·82	83	19	29·9
l	Nautical Almanac (ω Piscium)	4·0	23	52	4·35	83	55	2·2

Hartwell, November 10*th*, 1859.

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* This is the name of the planet discovered by Dr. Luther, at Bilk, on the f September last, see *Monthly Notices*, vol. xix. p. 327.—R. G.

On some lately discovered Double Stars.
By M. Otto Struve.

The double stars on which this paper treats have been discovered by me since the publication of my catalogue in 1850, the greater part on occasion of certain researches I had undertaken concerning stars with large proper motion. They are only 9 in number, and bear in my catalogue the Nos. 531–539, in the order of their discovery. The approximate mean positions correspond to the epoch 1860·0.

No. 531, P. M. 242. $\alpha = 3^{h}\,58^{m}{\cdot}2$; $\delta = +\,37^{\circ}\,42'$.

Argelander, in his excellent catalogue of 560 stars, has first pointed out the probability that this star forms a binary system of wide aperture with the next following of his catalogue, 50 *Persei*, on account of the equality in direction and quantity of their large proper motions. Our observations have confirmed this supposition in so far as they prove that there is but a very slight difference in their proper motions, which perhaps might be explained by orbital motion. My last observations give for the epoch 1859·15, $\Delta\alpha = +\,1^{m}\;2^{s}{\cdot}250$; $\Delta\delta = -\,1'\,57''{\cdot}77$. Argelander has for 1830·0, $\Delta\alpha = +\,1^{m}\;2^{s}{\cdot}22$; $\Delta\delta = -\,1'\,56''{\cdot}1$. Hence the change in the relative positions of the two stars in the course of about 30 years would only be $+0^{s}{\cdot}03$ in R.A. and $+1''{\cdot}67$ in Decl.; and supposing that the annual proper motion of 50 *Persei* was exactly determined by the comparison of Argelander's observations with Bradley's to $+0^{s}{\cdot}0157$ in R.A. and $-0''{\cdot}208$ in Decl., that of P. M. 242 would follow $+0^{s}{\cdot}0167$ in R.A. and $-0''{\cdot}152$ in Decl.

This pair of stars can pretend to a higher interest still, by the fact that P. M. 242 is in itself a double star of Herschel's first class; being composed of two stars (6.7th and 8.9th magnitude) of 3″ to 4″ distance. The physical connexion of these two stars is proved by our observations continued through 8 years, for until now a change in their relative position cannot yet be made out with certainty. The last measures give,—

1859·15 $e = 3''{\cdot}48$, $P = 150^{\circ}{\cdot}4$.

A fourth star (7.8th magnitude) is south proceeding to P. M. 242 at a distance of nearly 4′. This star does not belong to the system, for the changes observed in its position with regard to P. M. 242 correspond exactly to the proper motion of the latter. This star, therefore, would be a very qualified object of comparison for determining the relative parallax of P. M. 242, for which a considerable amount is indicated by the proper motion and by the probable physical connexion with 50 *Persei* at 15′ distance.

No. 532, β Aquilæ, $\alpha = 19^{h}\,48^{m}{\cdot}4$; $\delta = +\,6^{\circ}\,4'$.

This well-known fundamental star has a small companion (11.12th magnitude), at the distance of about 12″, which participates in its large proper motion of nearly half a second a-year. It is, therefore, a physical double star. Micrometrical measures continued through 8 years do not yet indicate any change in the relative position due to orbital motion. My last observations give,

1859·65 $e = 12''{\cdot}34$, $P = 17^{\circ}{\cdot}0$.

The observations of this pair of stars belong to the most difficult, from the splendour of the principal star and the faintness of the companion.

No. 533, 4 Delphini, $\alpha = 20^{h}\,32^{m}{\cdot}3$; $\delta = +9^{\circ}\,36'$.

In appearance this double star is analogous to *β Aquilæ*, being composed of two stars (4.5th and 11.12th magnitude), 10″ distant, and the companion being exactly north, but contrary to *β Aquilæ*. The observations continued through 8 years prove that the small star does not participate in the proper motion (of $0''{\cdot}33$ a-year) of the principal stars. Hence it follows that they are not physically connected, but form only for the present time an optical double star. My last measures are,—

1859·62 $e = 10''{\cdot}09$, $P = 359^{\circ}{\cdot}4$.

This position, combined with the proper motion of 4 *Delphini*, shows that within 30 or 40 years the distance of the small stars will exceed 16″, so that then, according to the limits adopted by my father, this pair will no longer have the right to figure in the lists of double stars.

Argelander mentions, in a note to his catalogue, that there is a star of the 9th magnitude, $14^{s}{\cdot}3$ following 4 *Delphini*, and $0'{\cdot}7$ to the south. This star, estimated by me of the 8th magnitude, has been repeatedly joined by micrometrical measures with the bright stars. Until now there is not the least trace of change in their relative position, whence we must conclude that we have here another binary system of very wide aperture of the orbit. My observations give the mean relation:

1855·02 $e = 214''{\cdot}72$; $P = 100^{\circ}\,45'{\cdot}0$
or Δ R.A. $= +\,14^{s}{\cdot}263$; Δ Decl. $= -40''{\cdot}05$.

This Δ R.A. compared with that given in Argelander's Catalogue, is in itself a sufficient proof that both stars are transported on the celestial sphere by the same proper motion.

No. 534 $\alpha = 17^{h}\,58^{m}{\cdot}9$; $\delta = +21^{\circ}\,25'$.

This double star has been discovered on looking out for No. 341 of our catalogue, to which it precedes 52^{s} nearly upon the parallel. The magnitudes of the two components are 7.8 and 9.10. Until now this pair offers no particular interest, but must be noticed to avoid mistakes with the following pair. For the mean of three observations we have found:

1852·95 $e = 2''{\cdot}14$; $P = 270^{\circ}{\cdot}7$.

No. 535, δ Equulei = Σ 2777, $\alpha = 21^{h}\,7^{m}{\cdot}7$; $\delta = +9^{\circ}\,27'$.

In the introduction to the *Positiones Mediæ Stellarum Duplicium*, p. 232, my father has proved that the small star discovered by W. Herschel in the neighbourhood of *δ Equulei* changes its relative position according to the proper motion of the principal stars; whence it follows, that the two stars form only optically double stars. At present even the designation of optical double stars in the limits introduced by my father, would no longer be quite exact, as the distance exceeds already 32″. A very careful calculation, executed by M. Schiaparelli, a young Italian astronomer residing at Pulkowa, on the Dorpat observations, combined with the whole series of measures I have made here until now, confirms entirely my father's conclusions. On considering the long series of measures made on this star at Dorpat and Pulkowa, it will rather excite surprise when I mention that only in 1852 I happened to discover that the principal star was in itself a close double star. However, this late discovery explains itself sufficiently by the circumstance that the observation of the distant companion required only low magnifying powers, and could be made in ordinary atmospheric condition, without expecting the best definition, which is a necessary condition for seeing the division of the principal star even, as it appears, at the epoch of the greatest distance of the two stars.

The two components of *δ Equulei* are nearly of the same magnitude. Therefore, until now we have no certain indications to discern the two opposite directions. As this star might pretend to a higher interest, I will give here the entire list of my observations on it, which, however, is only of very moderate length, for the reason that a definition, as it is wanted for this object, does not frequently occur in the course of a year.

1852·63 $e=0''\cdot37$ $P=16^\circ\cdot3$ } the two stars distinctly separated
1852·66 $0\cdot34$ $12\cdot6$ }
1853·71 oblong, distance does not exceed $0''\cdot2$, $P=191^\circ\cdot7$
1854·69 } Simple
1856·58 }
1857·67 oblong, distance of centres by estimation $0''\cdot16$, $P=199^\circ\cdot8$
1857·67 ,, ,, ,, $0\cdot18$ $200\cdot8$
1858·59 the two stars sometimes distinctly separated, distance by estimation $0''\cdot30$, $P=8^\circ\cdot9$
1859·65 oblong, distance by estimation $0''\cdot30$, $P=6^\circ\cdot6$.

Though in the last observation, the distance was found by estimation equal to that I had estimated the year before, I am inclined to think that in the interval it has decreased a little, on the ground that in the present year I never have been able to separate distinctly the two stars.

The preceding observations show, 1, that the two stars form a binary system; 2, that there is a very rapid motion in the orbit; 3, that apparently the plane of the orbit is not much inclined to the line drawn from the sun; but on account of the near equality of the two stars it cannot yet be decided if the same star, after the period of being simple, has emerged again on the same or on the opposite side of the other star. Evidently we have here an object analogous to 42 *Comæ Ber.*, with this difference that in the present instance the axis of the orbit appears to be still smaller, and again the period of revolution even more rapid than in the other case.

No. 536, Arg. 533 $\alpha = 22^h\,51^m\cdot5$; $\delta = +8^\circ\,37'$.

According to Argelander this star has a proper motion of about half a second a-year. It is composed of two stars 7th and 7.8th magnitude. For 1852–53 the distance between them did not amount to $0''\cdot4$. In the present year the star appeared simple in a very good condition of the atmosphere. These short statements are sufficient to show that here again we have to do with an interesting binary system with rapid orbital motion.

No. 537, $\alpha=21^h\,51^m\cdot5$; $\delta= +59^\circ\,13'$.

This double star (8th and 10.11th magnitude) has been entered in our lists only for its neighbourhood to our No. 458, to avoid mistakes. The relative position was found,

1856·73 $e=2''\cdot59$, $P=11^\circ\cdot3$.

No. 538, σ Coronæ = Σ 2032 $\alpha = 16^h\,9^m\cdot4$; $\delta = +34^\circ\,13'$.

Besides the two stars that form this well-known binary system and the third star *c* of the 10.11th magnitude, which my father has proved (*Pos. Med.* p. 229) not to belong to the system, there is at the distance of about 20″ from the principal couple a very faint object. Observations continued through six years prove that it likewise does not belong to the system, as it is not affected with the proper motion. The mean of four measures I have found to be,

1854·43 $e = 20''\cdot46$, $P = 231^\circ\cdot6$.

The observations of this star are, on account of its faintness, the most difficult I ever have made with our refractor, and as by the effect of proper motion it approaches to the principal pair, in a few years hence it will perhaps be quite impossible to see it. According to our mode of estimation, the 13th magnitude must be assigned to it, as it is very near the limit of the penetrating power of the Pulkowa refractor. In this regard for the next time it is a fair object for testing the power of other telescopes in comparison with ours.

No. 539, Σ 1516 $\alpha = 11^h\,6^m\cdot0$; $\delta = +74^\circ\,14'$.

This double star had been observed 23 times by my father at Dorpat, and at least 12 times by me at Pulkowa, before I remarked last year that there was at the distance of only 8″ a third star of 10.11th magnitude. If this star has been simply overlooked, or if the late discovery is to be attributed to variability of light, future observations must decide. Though the proper motion of the principal star is more than $0''\cdot4$ a-year in space, the interval between my extreme observations is yet so short as not to admit of a decision in regard to the existence of a physical connexion, but next year this point might be settled. In the mean of three observations we have the following relation,—

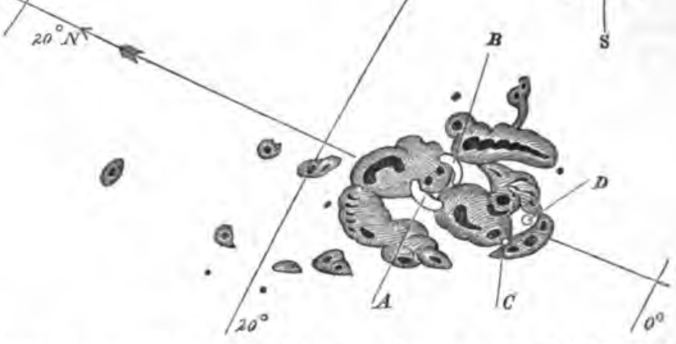
1858·86 $e = 8''\cdot01$; $P = 291^\circ\cdot6$.

The measures of the principal pair obtained since the publication of the *Positiones Mediæ* have been submitted, in combination with the ancient measures, to a new computation by M. Oom, of the Royal Observatory of Lisbon, at present living at Pulkowa. The results of his computation have entirely confirmed my father's conclusions, that the changes observed in the course of 28 years in the relative positions of the two stars find a complete explanation in the proper motion of the principal star, but the new formula does but very little diminish the discordance of the results obtained in 1823 by transit observations.

Pulkowa, October, 1859.

Description of a Singular Appearance seen in the Sun on September 1, 1859. By R. C. Carrington, Esq.

While engaged in the forenoon of Thursday, Sept. 1, in taking my customary observation of the forms and positions of the solar spots, an appearance was witnessed which I believe to be exceedingly rare. The image of the sun's disk was, as usual with me, projected on to a plate of glass coated with distemper of a pale straw colour, and at a distance and under a power which presented a picture of about 11 inches diameter. I had secured diagrams of all the groups and detached spots, and was engaged at the time in counting from a chronometer and recording the contacts of the spots with the cross-wires used in the observation, when within the area of the great north group (the size of which had previously excited general remark), two patches of intensely bright and white light broke out, in the positions indicated in the appended diagram by the letters A and B, and of the forms of the spaces left white. My first impression was that by some chance a ray of light had penetrated a hole in the screen attached to the object-glass, by



which the general image is thrown into shade, for the brilliancy was fully equal to that of direct sun-light; but, by at once interrupting the current observation, and causing the image to move by turning the R.A. handle, I saw I was an unprepared witness of a very different affair. I thereupon noted down the time by the chronometer, and seeing the outburst to be very rapidly on the increase, and being somewhat flurried by the surprise, I hastily ran to call some one to witness the exhibition with me, and on returning within 60 seconds, was mortified to find that it was already much changed and enfeebled. Very shortly afterwards the last trace was gone, and although I maintained a strict watch for nearly an hour, no recurrence took place. The last traces were at C and D, the patches having travelled considerably from their first position and vanishing as two rapidly fading dots of white light. The instant of the first outburst was not 15 seconds different from $11^h 18^m$ Greenwich mean time, and $11^h 23^m$ was taken for the time of disappearance. In this lapse of 5 minutes, the two patches of light traversed a space of about 35,000 miles, as may be seen by the diagram, which is given exactly on a scale of 12 inches to the sun's diameter. On this scale the section of the earth will be very nearly equal in area to that of the detached spot situated most to the north in the diagram, and the section of *Jupiter* would about cover the area of the larger group, without including the outlying portions. It was impossible, on first witnessing an appearance so similar to a sudden conflagration, not to expect a considerable result in the way of alteration of the details of the group in which it occurred; and I was certainly surprised, on referring to the sketch which I had carefully and satisfactorily (and I may add fortunately) finished before the occurrence, at finding myself unable to recognise any change whatever as having taken place. The impression left upon me is, that the phenomenon took place at an elevation considerably above the general surface of the sun, and, accordingly, altogether above and over the great group in which it was seen projected. Both in figure and position the patches of light seemed entirely independent of the configuration of the great spot, and of its parts, whether nucleus or umbra. The customary observation was shortly resumed, and the diagram engraved, as well as the larger drawing exhibited at the Meeting on Nov. 11, was deduced from an exact reduction of the recorded times.

It has been very gratifying to me to learn that our friend Mr. Hodgson chanced to be observing the sun at his house at Highgate on the same day, and to hear that he was a witness of what he also considered a very remarkable phenomenon. I have carefully avoided exchanging any information with that gentleman, that any value which the accounts may possess may be increased by their entire independence.

(Mr. Carrington exhibited at the November Meeting of the Society a complete diagram of the disk of the sun at the time, and copies of the photographic records of the variations of the three magnetic elements, as obtained at Kew, and pointed out that a moderate but very marked disturbance took place at about $11^h 20^m$ A.M., Sept. 1st, of short duration; and that towards four hours after midnight there commenced a great magnetic storm, which subsequent accounts established to have been as considerable in the southern as in the northern hemisphere. While the contemporary occurrence may deserve noting, he would not have it supposed that he even leans towards hastily connecting them. "One swallow does not make a summer.")

Mr. Carrington regrets having omitted previously to communicate the following passage in a letter from M. Schwabe, dated March, which is plainly intended for others as well as himself:—"The remarks and drawings of *Jupiter* in the *Monthly Notices* by Dawes, Murray, and Lassell, have much interested me, as I also have long followed the changes which take place in his bands. Although none of the observations in the *Monthly Notices* coincide in time with those which I have obtained, yet I recognise a good accordance in general, especially in the two figures given by Sir W. Murray.* Yet I do not find indicated in those two figures, that, as I observe, the whole surface of *Jupiter* is covered with delicate parallel dark lines, which are the most conspicuous in the dark belts. The most delicate ones are to be found in the bright equatoreal zone, the zone of Schroeter, in the middle of which, or on the equator itself, they appear rather more distinctly, although this middle line is not always there. . . . In the figures of the *Monthly Notices* there are representations of bright round beads which I do not observe under that character. My impression is that I see longish bright places between the abovenamed parallel lines, in which, or through which, the bright surface of the planet is discernible. I much wish to learn whether Messrs. Dawes, Murray, or Lassell, or either of them, see also those delicate parallel lines. The large and almost black spots which were seen a short time ago, also dissolved into similar but much broader and darker parallel lines. . . ."

On a curious Appearance seen in the Sun.

By R. Hodgson, Esq.

"While observing a group of solar spots on the 1st September, I was suddenly surprised at the appearance of a very brilliant star of light, much brighter than the sun's surface, most dazzling to the protected eye, illuminating the upper edges of the adjacent spots and streaks, not unlike in effect the edging of the clouds at sunset; the rays extended in all directions; and the centre might be compared to the dazzling brilliancy of the bright star *α Lyræ* when seen in a large telescope with low power. It lasted for some five minutes, and disappeared instantaneously about 11.25 A.M. Telescope used, an equatoreal refractor 6 inches aperture, carried by clockwork; power, a single convex lens, 100, with a pale neutral-tint sunglass; the whole aperture was used with a diagonal reflector.

"The phenomenon was of too short duration to admit of a micrometrical drawing, but an eye-sketch was taken, from which the enlarged diagram† has been made; and from a photograph taken at Kew the previous day, the size of the group appears to have been about $2^m 8^s$, or (say) 60,000 miles.

"The magnetic instruments at Kew were simultaneously disturbed to a great extent.

"*Nov. 11, 1859.*"

Results of the Meridional Observations of Small Planets; Occultations of Stars by the Moon; and Phenomena of Jupiter's Satellites, observed at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

Hebe ⑥.

Mean Solar Time of Observation.		Mean Solar Time of Observation.			Apparent R.A.			Apparent N.P.D.		
		h	m	s	h	m	s	°	′	″
1859, Oct.	3	13	7	54·5	1	56	42·67	104	44	20·57
	19	11	54	13·9	1	45	54·83	107	26	37·50
	26	11	21	36·8	1	40	48·24	108	4	20·58
	29	11	7	45·5	1	38	44·34	108	13	25·82

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* Lassell (?).—Ed.

† This well-executed diagram was exhibited at the meeting, and excited much interest. It may still be seen at the Apartments of the Society. R. G.

Psyche ⑯.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, July 4	13 35 45·8	20 25 52·01	106 32 45·99
7	13 21 57·8	20 23 51·43	106 41 11·19
12	12 58 40·2	20 20 12·82	106 56 29·95
13	12 53 58·2	20 19 26·59	106 59 52·42
19	12 25 34·8	20 14 37·89	107 20 12·86
Aug. 2	11 18 52·5	20 2 56·40	108 10 42·11
27	9 25 2·0	19 47 21·15	109 27 51·29
31	9 8 2·9	19 46 5·45	109 36 54·74
Sept. 10	8 27 25·6	19 44 47·04	109 54 25·76

Parthenope ⑪.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, Oct. 22	14 58 42·8	5 2 43·71	73 23 41·52
26	14 41 39·2	5 1 23·55	73 30 20·96

Melpomene ⑱.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, July 1	12 1 32·9	18 39 34·00	98 50 50·02
4	11 46 39·0	18 36 27·31	99 2 51·57
5	11 41 41·0	18 35 25·02	99 7 19·37
6	11 36 43·0	18 34 22·74	99 11 51·77
7	11 31 45·1	18 33 20·66	99 16 35·32
11	11 11 56·9	18 29 15·44	99 37 7·11
12	11 7 1·3	18 28 15·52	99 42 41·44
15	10 52 17·9	18 25 19·39	100 0 2·50
16	10 47 25·0	18 24 22·30	100 6 6·06
19	10 32 52·6	18 21 37·18	100 25 2·49
26	9 59 40·3	18 15 55·33	101 13 7·73
Aug. 12	8 44 54·0	18 7 58·16	103 22 56·00

July 6.—The observation of N.P.D. has been altered by increasing the reading of each microscope-micrometer one revolution.

Massilia ⑳.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, July 1	14 9 48·0	21 7 52·91	105 24 56·68
7	14 5 13·7	21 7 14·43	105 27 33·09
11	13 46 44·6	21 4 28·56	105 38 50·40
12	13 42 4·3	21 3 43·99	105 41 51·96
13	13 37 22·8	21 2 58·36	105 44 57·50
Aug. 1	12 5 33·9	20 45 48·88	106 54 18·04
2	12 0 39·2	20 44 49·99	106 58 8·15
4	11 50 50·1	20 42 52·34	107 5 56·09
5	11 45 55·7	20 41 53·72	107 9 44·86
12	11 11 44·3	20 35 12·57	107 35 53·32
15	10 57 19·4	20 32 28·70	107 46 25·26
10	10 38 2·8	20 29 1·48	107 59 37·28
27	10 0 34·2	20 22 59·18	108 22 51·71
30	9 46 52·3	20 21 4·67	108 30 10·94
31	9 42 21·3	20 20 29·43	108 32 32·44
Sept. 6	9 15 47·6	20 17 30·77	108 44 28·56
21	8 13 41·7	20 14 22·97	109 0 7·48
24	8 2 1·7	20 14 30·71	109 0 51·09
29	7 43 8·0	20 15 16·66	109 0 20·53
Oct. 1	7 35 46·2	20 15 46·72	108 59 29·60

Proserpine ㉖.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, Oct. 19	13 2 39·6	2 54 31·83	73 59 24·47
31	12 4 50·5	2 43 51·89	74 33 51·28

Euterpe ㉗.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, July 4	10 18 6·8	17 7 40·59	112 36 47·23
6	10 8 37·3	17 6 2·58	112 35 53·64

Lutetia ㉑.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, July 7	12 30 15·6	19 32 0·69	114 34 34·00
12	12 5 50·1	19 27 14·03	114 54 52·53
Aug. 2	10 24 45·3	19 8 40·30	115 59 19·47
4	10 15 34·4	19 7 21·01	116 3 6·19
5	10 11 1·9	19 6 44·33	116 4 54·38
16	9 23 13·0	19 2 9·75	116 17 31·01
19	9 10 54·9	19 1 39·26	116 18 56·60

Fides (57).

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, Aug. 15	12 31 24·7	22 6 55·80	105 13 9·10
22	11 57 31·3	22 0 32·66	105 41 35·10
24	11 47 49·6	21 58 42·52	105 49 19·84
Sept. 24	9 23 0·4	21 35 42·64	106 59 30·59
Oct. 1	8 53 9·9	21 33 23·13	106 58 29·28

Mnemosyne (57).

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, Sept. 29	11 27 51·6	0 0 37·19	82 53 7·95
Oct. 1	11 18 42·8	23 59 19·97	83 11 59·18
4	11 5 2·8	23 57 27·39	83 40 34·42

The observations of N.P.D. have been corrected for parallax, excepting in the case of *Mnemosyne*, the distances of which from the earth were not at hand.

Occultations of Stars by the Moon.

Day of Obs.	Phenomenon.	Moon's Limb.	Mean Solar Time.	Observer.
			h m s	
1859, April 13	37 Sextantis disapp.	Dark	8 44 35·0	E.
13	37 Sextantis disapp.	Dark	8 44 36·1	C.
Sept. 21	μ^2 Cancri disapp.	(*a*) Bright	13 30 1·0	E.
21	μ^2 Cancri reapp.	(*b*) Dark	14 20 52·4	E.
Oct. 14	19 Tauri disapp.	(*c*) Bright	11 44 9·4	C.
14	20 Tauri disapp.	(*d*) Bright	12 1 18·6	C.

(*a*) Uncertain to about a second of time.
(*b*) Instantaneous; the moon's dark limb was distinctly visible.
(*c*) Star exceedingly faint when close to the moon.
(*d*) Very doubtful, the sky being cloudy.

The initials E. and C. are those of Mr. Ellis and Mr. Criswick.

The observations of the occultation of *Saturn* by the moon on May 8, and that of the disappearance of a small star which was occulted about the same time, have been already published in the *Monthly Notices*, vol. xix., No. 7.

Eclipses, Occultations, and Transits of Jupiter's Satellites.

Day of Observation.	Satellite.	Phenomenon.	Mean Solar Time.			Observer.
1859.			h	m	s	
Jan. 10	II	Ecl. reapp.	5	11	15·2	L
19	I	Ingress, 1st cont.	5	30	17·4	C
19	I	„ central bis.	5	31	47·2	C
19	I	„ total ingr.	5	34	16·7	C
19	I	Egress, 1st app.	7	43	25·5	C
19	I	„ last cont.	7	47	54·8	C
22	II	Ingress, 1st cont.	10	44	49·4	J C
22	II	„ total ingr.	10	50	18·5	J C
22	II	Egress, 1st app.	13	20	53·9	J C
22	II	„ last cont.	13	26	23·0	J C
26	I	Ingress, central bis.	7	22	14·7	E
31	II	Occ. disapp.	8	12	33·9	E
Feb. 3	III	Occ. disapp. 1st cont.	8	37	5·7	L
3	III	„ central bis.	8	39	30·3	L
3	III	„ total disapp.	8	42	24·8	L
7	II	Occ. disapp. central bis.	10	38	48·4	E
10	I	Ecl. reapp.	11	37	11·7	D
21	III	Ingress, 1st cont. (*a*)	6	9	20·6	C
21	III	„ central bis. (*a*)	6	12	50·0	C
21	III	„ total ingr. (*a*)	6	15	19·6	C
23	II	Ingress, total ingr.	10	6	22·5	E
23	II	Egress, last cont.	12	41	57·1	E
Mar. 4	II	Occ. disapp. 1st cont. (*b*)	7	27	10·4	W C
4	II	„ central bis. (*b*)	7	28	40·3	W C
4	II	„ total disapp. (*b*)	7	30	54·8	W C
5	I	Occ. disapp. central bis.	8	21	7·3	E
18	III	Occ. reapp. last cont. (*c*)	11	2	25·6	D
21	I	Ecl. reapp.	10	14	35·4	E
22	II	Ecl. reapp.	7	9	5·5	D
April 6	I	Ecl. reapp.	8	34	54·2	D
13	I	Ecl. reapp. (*d*)	10	32	44·0	E
21	I	Egress, central bis.	8	39	40·6	E
Aug. 20	IV	Occ. disapp. total dis. (*e*)	16	0	31·9	D
Oct. 19	III	Ecl. disapp. (*f*)	13	16	41·7	E
19	III	Ecl. reapp. (*g*)	16	21	15·3	L
19	I	Ecl. disapp. (*g*)	17	14	57·0	L
22	I	Egress, last cont.	12	33	13·5	C
26	IV	Ecl. reapp. (*h*)	10	31	39·0	E

(*a*), Cloudy. (*b*), Planet exceedingly faint at first; the time of total disappearance is, therefore, the most reliable. (*c*), Planet excessively tremulous; observation very unsatisfactory. (*d*), Observation not good; *Jupiter* very low and tremulous. (*e*), Exceedingly doubtful; planet behind thin cloud, and very tremulous; observation uncertain to at least a minute. (*f*), At the time noted, the last faint trace of the satellite disappeared. (*g*), Satisfactory. (*h*), Unsatisfactory, *Jupiter* being very near the horizon; the observation uncertain to a quarter of a minute.

The initials D, E, C, L, J C, and W C, are those of Mr. Dunkin, Mr. Ellis, Mr. Criswick, Mr. Lynn, Mr. Carpenter, and Mr. Christy.

Catalogue of Multiple Stars observed in Upper India in 1855–6.

By Charles Gubbins, Esq.

These observations were made at the Civil Station of Ally- rh, situated in the Upper Doab, between the Ganges and Jumna, about 600 feet above the level of the sea. Its geographical position was found to be

	°	′	″
Latitude N.	27	53	56
Longitude E.	78	7	10

This was determined by triangulation from the church not far distant, which had been one of the stations of the second Trigonometrical Survey of India.

The instrument employed in these observations was a small achromatic telescope of 3½ inches aperture, mounted equatoreally. It was supported on a mahogany tripod, terminating in brass caps with levelling screws, which were received on three metal plates, imbedded in circles of masonry on a slightly raised platform. The hour and declination circles were each 7 inches in diameter.

The instrument was furnished with a position-micrometer, to which were adapted two eye-pieces, with powers 40 and 80. Of these the latter, giving a field of 2^m of time, was almost exclusively used. The time was determined with a chronometer which was regulated, in the beginning of the operations, by observations of the eclipses of *Jupiter's* satellites, and subsequently by observing the distances of known stars from the meridian, and comparing the results with those contained in Dietrichsen's *Almanac*, the only substitute which the author possessed for the *Nautical Almanac*.

The Catalogue contains the approximate places of 122 double stars, with micrometrical measures of their angles of position and distances. The nomenclature used by the author is that of the Catalogue contained in Sir John Herschel's *Results of Astronomical Observations made at the Cape of Good Hope*, the only astronomical work on the subject of double stars to which he had access at the time of his observations.

The author states, that although he commenced his observations in the autumn of 1855, it was not until the beginning of the following year that he had acquired sufficient mastery over his instrument to be able to turn it to the best possible account; and by the middle of summer he was interrupted by a severe illness, which eventually compelled his return to England.

The following is a synopsis of stars which appear from the author's measures to indicate a change of angle of position. This conclusion, he states, being also supported by Captain Jacob's measures:—

Nomenclature of Star.	
617	Direct.
3447	Do.
p, 6, Eridani	Retrograde.
39 Eridani	Direct.
Σ 1121	Retrograde.
Δ 70	Do.
4608	Direct.
k Centauri	Retrograde.
54 Hydræ	Retrograde.
B 7080	Do.

Accompanying the Catalogue are charts of two clusters of stars (H vii, 17, and H viii, 38), which, the author states, were constructed with the utmost care. "The latter," he adds, "may be depended on for accuracy in all its leading features; and the great number of cross-measurements and eye-allineations will afford facilities for correcting minor errors. Of the other I cannot speak so confidently, as the great majority of its stars were too faint to be seen with light enough to show the wires;

and the measurements of angles can therefore only be regarded as approximations."

The author's paper has been deposited in the archives of the Society. It may hereafter afford useful corroborative evidence in cases where a change of relative position in any of the objects to which it refers shall have been suspected.

Two letters from the Rev. J. B. Kearney, of King's School, Canterbury, to Sir John Herschel, were read. In the first of these, dated February 9, 1858, Mr. Kearney expresses his opinion that the star *Capella* is diminishing in brightness. He had watched the star attentively with the naked eye for fifteen years, and felt a strong suspicion that it was waning in lustre. In the years subsequent to 1843 its superior brightness* attracted his attention. In the second letter, dated September 9, 1859, he states that the star had resumed its original brightness. We extract the following passage:—

"My favourite star *Capella* has reasserted her superiority. From the receipt of your letter, dated February 25, 1858, I have carefully watched *Capella*, and compared it with *α Lyræ* and *Arcturus*. From February '58 to April '58, *Capella* and *α Lyræ* appeared about equal. From August '58 to November '58, I fancied that *Capella* gained on *α Lyræ* after November 30, '58. I also fancied that *Capella* lost ground compared with *α Lyræ* till August 19, '59, when, at 8 o'clock, *Capella* was brighter, beyond all possibility of doubt, than either *Arcturus* or *α Lyræ*. At 10 o'clock *Capella* was magnificent, very far surpassing all its rivals. It flashed all manner of colours, blue and crimson being perhaps the most striking. *Capella* was very decidedly A 1 on August 26 and September 1, particularly the latter, when it seemed as brilliant as on August 19, except as regards change of colour. By the way, the colour of *Capella* seems permanently much less blue than it used to be. To-night (9 o'clock) I consider *Capella slightly* first, *Arcturus* a good second, and *α Lyræ*, though *very* bright, third. *Arcturus* seemed A 1 both on Monday and Tuesday last; but it was very damp, so that perhaps *Capella* may not have had a fair chance. I think it may be laid down as certain that both *Arcturus* and *Capella* have lately gained on *α Lyræ*."

Sir John Herschel communicated an interesting series of delineations of the surface of the sun taken between the 27th of August last, and the 12th of September, by the Rev. F. Howlett, a near neighbour of his at Hawkhurst and a gentleman of much skill as a draughtsman, as well as a careful microscopist. Sir John believed that they would interest the members of the Society, and hoped that by the reception they met with Mr. Howlett might be encouraged to continue his record. Some nuclei of very curious forms are represented in these drawings, and excited considerable interest at the meeting.

Sir John Herschel deposited with the Society three MS. olumes of observations of the solar spots made by the late M. Pastorff. These volumes were originally presented by the uthor to Sir John Herschel. They were now transferred to he Society on the understanding that they should continue to e considered as belonging to Sir John Herschel during his ifetime; but, after his decease, should become the property of he Society. In the meantime, Fellows of the Society intersted in the subject of those observations would always have he opportunity of consulting them. The thanks of the Society ere awarded to Sir John Herschel for this valuable donation.

MISCELLANEOUS INTELLIGENCE.

Professor Wolf, of Zurich, mentions, in a recent letter, that, in consequence of the liberal aid afforded by a gentleman of his neighbourhood, who has recently succeeded to a considerable property, and who evinces great interest in his researches, such fresh energy has been given to the project of suitably mounting the instruments of the Zurich Polytechnic Institute, that he hopes now shortly to be in a position to take observations of precision. The tenth number of his *Mittheilungen über die Sonnenflecken* is in the press.

M. C. Bulard has been recently appointed by the Prince Napoleon to arrange and duly furnish a small Astronomical Observatory at Algiers. It is proposed that his attention be principally given to physical details of the heavenly bodies, for the delineation of which M. Bulard possesses remarkable aptitude. He is at present provided with a transit instrument and two silvered speculum reflecting telescopes. M. Bulard will gladly receive publications on astronomical subjects and communications addressed to him as

Astronome de l'Observatoire à Alger.

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*Suspected Existence of a Zone of Asteroids revolving between Mercury and the Sun.**

A few weeks since the eminent Director of the Imperial Observatory at Paris addressed a letter to M. Faye, on the subject of the motions of *Mercury*, so forcibly reminding the astronomical public of the period preceding the discovery of *Neptune*, that it will not be matter of surprise that in the minds of many an expectation has been in some degree raised that the event of the year 1846 may be shortly repeated at the other extremity of the planetary system. Some conversation took place at the evening-meeting of the Society, at which the following account of M. Le Verrier's letter was read. The original appeared in the *Comptes Rendus* of the French Institute for the 12th of September.

Substance of M. Le Verrier's Letter to M. Faye.

"In my studies on the planetary motions, I have, as you know, met with many difficulties. The outstanding discordances in the case of *Uranus* were ultimately explained by the discovery of *Neptune*. The difficulties presented by the sun have been considerable; and they led me, first, to revise at great length the processes and principles on which were based the concluded positions of the fundamental stars. I next revised with care the whole theory of the inequalities of the earth; and in the comparison of the theoretical and observed positions, I have discussed not less than 9000 observations from various sources.

"The theory of the sun once placed beyond doubt, it has been possible for me to resume with confidence the study of the movements of *Mercury;* and it is to this portion of my recent labours that on this occasion I particularly call your attention. For this inquiry we possess, for the last century and a half, records of 21 observations of transits of the planet over the sun's disk, which admit of comparisons of the distances of the centres of the sun and *Mercury* at definite instants, liable to error not exceeding a second of arc, and consequently far exceeding in precision the meridian observations of the planet at the earlier date. In my former examinations of the ancient observations of *Mercury*, I had, in 1842, recognised considerable discordances, but into the discussion of which I abstained from entering until the theory of the sun should be more fully and precisely established. Now that this is done, and the positions of the sun given by my latest tables are introduced, systematic discordances remain between the theoretical and observed positions of *Mercury*, which can only be annihilated by such violent suppositions as that Lalande, Cassini, Bouguer, and others, committed errors of whole minutes of time in recording contacts, and errors which have a progressive variation of increase or decrease, wholly impossible. But what is most remarkable, I find that by simply increasing by 38 seconds the secular movement of the perihelion of the planet, all the 21 transits are at once satisfied to such a degree of exactness that the greater part exhibit residues of half-seconds, and a few of seconds only. This empirical correction is accordingly so successful as to show at once that the theory of *Mercury* requires this modification; and it remains to inquire whether any modification of the masses of the exterior planets employed in the calculation of *Mercury's* perturbations will produce the required increase, or whether by its inadmissibility it is necessary to search further." M. Le Verrier then states that if the error is assumed to lie in the mass of *Venus*, a correction of $\frac{1}{10}$th of that mass would be required to meet the case; and he shows such a correction to be inadmissible, from the error it would, per contra, introduce into the theory of the Earth. M. Le Verrier commends the difficulty to the serious consideration of astronomers, and propounds, as an explanation to which he himself inclines, the possible existence of a planet hitherto undetected between *Mercury* and the sun. In this case, inasmuch as no discordance of similar amount of the motion of the node of the orbit of *Mercury* is found to exist, the unseen planet must be assumed to have its orbit but little inclined to that of *Mercury*. There will be conditions connecting its supposed mass and distance from the sun, similar to those which it was necessary to set out with in the research after *Neptune;* it being necessary, in order to produce the desired inequality, to increase the mass the less the distance from sun of the hypothetical planet is taken, and *vice versâ;* and for a distance a little less than half that of *Mercury*, the sought mass will require to be equal to that of *Mercury*—a very admissible assumption.

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* Sir John Herschel, in a paper read January 10, 1840, on the Variability *α Orionis*, published in vol. xi. of the *Memoirs* of the Society, remarks, in foot-note, page 269, "I have a strong impression that *Capella* within my collection has increased in brightness. M. Struve is of the same opinion." R. G.

* Drawn up for the use of the readers of the *Monthly Notices* by a Member of the Council.—R. G.

But if the improbability of the existence of such a planet undetected, with its necessary brilliancy, be urged, M. Le Verrier remarks that it may be replaced with equal success as an explanatory hypothesis by a series of corpuscles circulating round the sun interior to *Mercury*, whose united action would be equivalent in perturbating effect, while their individual minuteness might explain their non-detection.

M. Le Verrier asks, "May not some of the larger of these corpuscles, in their occasional transits over the solar disk, yet be detected? Astronomers, already so attentive to the phenomena exhibited on the disk, will doubtless, in these reflections of mine, find a new motive for carefully examining the motions of the smallest and least-defined spots."

The ideas of Professor W. Thomson, of Glasgow, respecting a zone of meteors in the neighbourhood of the sun, thrown out as a possible source of supply of the solar heat and energy, in his paper "On the Mechanical Energies of the Solar System," published in the *Philosophical Magazine* for December, 1854, may be read in connexion with the above.

In a letter to Mr. Warren De La Rue Dr. Steinheil states that he is progressing favourably with the 13-inch glass mirror he undertook to make for him, and that in the meantime he has completed a 12-inch mirror, which is quite perfect in its definition. With parallel rays the foci of the several zones do not differ more than $\frac{1}{30}$th of a line. Dr. Steinheil's price for a 12-inch glass reflector with diagonal mirror and eye-piece is 1000 florins.

For disposal, a 5-foot telescope, 4-inch object-glass, by Dollond, with eight astronomical eye-pieces, diagonal reflector, Barlow lens, and day eye-piece, spider-line micrometer, and illuminating apparatus, with long mahogany polar axis, circles of 15 inches, counterpoise, &c. &c. There is also a mahogany tripod-stand and cradle, with steadying rods. This telescope was formerly the property of Admiral Sheriff, and afterwards was mounted in Mr. Drew's observatory, Southampton.

For further particulars, apply to Mr. Williams, Assistant Secretary, Somerset House.

The planet discovered by Dr. Luther on the 22d of September has been called *Mnemosyne.* The following elements of its orbit, extracted from No. 1223 of the *Astronomische Nachrichten,* have been calculated by M. Tiele:—

Epoch 1860, Jan. 1·0.

		°	′	″	
M	……	335	26	7·0	
☊	……	200	8	58·7	Mean Equinox of the Epoch.
ϖ	……	53	24	35·7	
ι	……	15	4	3·9	
φ	……	6	5	32·8	
μ	……	633″·0918			
Log *a*	……	0·4990266			

These elements are founded on the Berlin observations of Sept. 25, Oct. 11, and Nov. 2, executed by Dr. Förster. M. Tiele computed from them an ephemeris of the planet for the month of November.

RECENT PUBLICATIONS.

Places of 5345 Stars, observed from 1828 to 1854, at the Armagh Observatory. By Rev. T. R. Robinson, D.D. F.R.S., &c. &c. Printed at the expense of Her Majesty's Government, on the recommendation of the Royal Society. 8vo, pp. 847, Dublin, 1859.

In an Introduction extending to 67 pages Dr. Robinson has given a description of the instruments and methods of observation employed by him, with accompanying remarks suggested by his long experience as an observer, which cannot fail to be of great value to the practical astronomer. The Armagh Observatory was founded, in 1793, by Primate Robinson, but it continued to be inadequately provided with instruments, and little work was consequently performed, until the present Archbishop of Armagh, Lord John George Beresford remedied this defect by his gift of a transit instrument, a mural circle, and a reflector of 15 inches aperture, equatoreally mounted. The first of these instruments was erected in 1827 and the last in 1835.

The Observatory is situated on a hill in the immediate vicinity of Armagh, about 100 feet above the general level of the district. In the *Philosophical Magazine* for August, 1846, Dr. Robinson has called attention to a periodical motion of this hill, as indicated by variations of the transit instrument in level, and of the index correction of the circle, and he has given an explanation of its origin. The existence of such a variation, whatever be its cause, combined with the fact of similar variations having been subsequently established at other observatories, and more especially the liability of such variations to sudden change, appear to Dr. Robinson "to furnish a strong argument against any system of observing which assumes an invariable position of the instruments for any considerable period." He accordingly considers it absolutely necessary to verify the principal corrections within a few hours of the time of observation.

The author remarks that the tremors caused by the traffic on the Ulster Railroad, the nearest part of which to the Observatory is about 700 yards distant, has been found sensibly to affect the observations.* The most important disturbance arising from this cause is "the motion of the circle if a train passes *after* the bisection, but *before* the microscopes are read." This appears to the author to arise from the friction at the pivots being released, in consequence of the jar caused by the vibration of the ground. The disturbance here referred to has been known to amount to as much as $4''$. The author accordingly was in the habit of rejecting all observations made under such circumstances.

Latitude.—This was determined, first, by direct and reflected observations of stars; secondly, by the reflection of the N.P.D. wire of the circle; thirdly, by the vertical collimator of the late Captain Kater. The first of these methods gave $125^{\circ}\ 38'\ 47''\cdot317$ for the horizontal point; the second method gave $128^{\circ}\ 38'\ 47''\cdot151$. The third method is referred to by the author merely from his wish to put on record his experience of the instrument used, and to point out what he conceives to be its defects. Forty-seven sets of observations made from 1835 to 1839 gave $47''\cdot158$. The close accordance of this result with that furnished by the second method is very remarkable, considering that the probable error of a set of observations amounts to $\pm 1''\cdot044$, being three times as great as by the second method, and that two of the observations differ no less than $5''\cdot76$. The author cites this fact as a striking proof of the precision of a mean *when the errors are casual.* After some further interesting details, he concludes this part of his subject with the remark that he does not feel disposed to change the value of the latitude communicated by him some years ago to the *Nautical Almanac*, viz. $54^{\circ}\ 21'\ 12''\cdot70$.

Longitude.—This was determined: 1st, by chronometers; 2d, by occultations; 3d, by moon-culminating stars. In 1838, Mr. Edward Dent transported fifteen of his best chronometers from Greenwich to the Dublin and Armagh Observatories. The mean of thirty comparisons, going and returning, gave $-26^{m}\ 35^{s}\cdot44$ for the longitude of Armagh.† Eight comparisons of the author's own chronometer with Greenwich gave also $35^{s}\cdot44$. The difference of longitude between Armagh and Dublin was determined by Sir W. Hamilton and the author, by means of rocket signals, and found to be $-1^{m}\ 14^{s}\cdot26$. This result, combined with the difference of longitude between Greenwich and Dublin, as indicated by Dent's chronometers, viz. $-25^{m}\ 21^{s}\cdot08$, gave $-26^{m}\ 35^{s}\cdot34$ for the longitude of Armagh. The investigation of the longitude by the second method was founded on five phases of the solar eclipses of 1826 and 1836, and twenty-two occultations of stars. The definitive value of the longitude thus deduced is $-26^{m}\ 35^{s}\cdot47$. The method of moon-culminating stars was in high repute when the author commenced his observations, and he at first gave it much attention; but he soon found that, however useful it might be to a traveller, it is destitute of the precision requisite for a fixed observatory. He, on the whole, assumes as the true value of the longitude, until new determinations are obtained by electric signals, the mean result of the chronometers and occultations, viz.—

$$-26^{m}\ 35^{s}\cdot74$$

The Transit Instrument.—This instrument was constructed by Thomas Jones, and was erected in 1827. It has an object-glass of $3^{in}\cdot8$ aperture and a focal length of 63 inches. The author has given valuable details respecting his investigation of the qualities of the different parts of this instrument, especially the pivots, and his mode of observing with it.

Corrections of the Transit.—The existence of the periodic changes of the transit already referred to, and others of a more irregular character, induced the author to reject the system of leaving an instrument out of perfect adjustment, and applying to the observations the necessary corrections, as determined by observations of standard stars. This process, he adds, assumes the permanence of the instrument to an extent which he cannot admit. He, therefore, either kept the transit perfectly adjusted, or determined the error within a few hours of the observations which it affected.

Clock.—The clock is by Earnshaw. The author speaks favourably of its qualities. It was originally furnished with a gridiron pendulum, which was replaced in 1830 by a mercurial pendulum, the work of Mr. Sharp, sen., of Dublin. On the 5th of July, 1832, the author applied the compensation barometers, described by him in a paper published in vol. v. of the *Memoirs* of this Society (*On the dependence of a clock's rate on the height of the barometer*). He has given some interesting tables exhibiting the rate of the clock for each of the three different sorts of pendulum.

Mural Circle.—This instrument was also constructed by Jones. Its diameter is 56 inches, and its telescope a duplicate of the transit. The author refers to vol. ix. of the *Memoirs* of this Society for a detailed description of it, and in his remarks on the errors of the instrument he refers to another paper by him "On Correcting Errors of the Astronomical Circle" (*Trans. R. I. A.* vol. xv.), wherein he investigates the effect of equidistant readings in correcting ellipticity, the deformation of a circle by its own weight, and the change of figure due to unequal temperature or irregular strength.

Meteorological Instruments.—The thermometer is by Troughton. The barometer is a standard by Newman, $0^{in}\cdot57$ in bore, similar to that of the Royal Society.

Observed Places.—To have published the entire observations in detail would have swelled the work to an enormous and useless extent. The author has, therefore, only given the result of each observation, as the correction of an assumed place, and the mean place deduced from the individual results. On receiving the instruments, he resolved to devote them to a re-observation of Bradley's stars. On the whole, the stars of the *Fundamenta* are tolerably represented. The author's observations in right ascension contain 2703 out of 2933; those in north polar distance, 2562 out of 2954. As Bradley's stars were disposed of, others were selected from Piazzi or Groombridge to employ the vacant time. It was the author's intention that every star should be observed at least five times; but he was unable to carry this plan generally into effect.

With very few exceptions the transit observations till March 1837 were made by the author, and afterwards by his first assistant, Mr. Neil M'Neil Edmondson, to whose accuracy as an observer and computer, and devotion to his duty, the author bears the highest testimony. The circle observations were made by the author till 1849, and subsequently by his second assistant, Mr. W. Hautenville Rambaut.

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* The author here refers to a paper by him on this subject, which is published in vol. v. of the *Proceedings* of the Royal Irish Academy.

† See a paper by the author on the subject published in vol. xix. of the *Transactions of the Royal Irish Academy.*

The Catalogue.—The epoch is 1840·0. The precessions are those of the British Association Catalogue, when the star is found in it, reduced to 1840. For stars not found in that Catalogue the precessions are computed by the same constants.

Proper Motion.—The author does not consider it necessary to discuss the general question of proper motion, which has recently been so thoroughly done by Mr. Main and Mr. Airy. He has, however, given in the Introduction an interesting list of proper motions, as determined by a comparison of his own results with those of Bradley, Piazzi, Lalande, and Groombridge. He is of opinion that differences of $0^{s}{\cdot}5$ and $5''{\cdot}0$ imply a real change of place, even when they result from a single observation. He has, therefore, taken these as limits, and whenever the correction of any Catalogue equals or exceeds them, he has considered the quotient of it by the number of years between the epoch of the Catalogue of comparison and the mean date of his own observations to be the proper motion.

The author concludes his Introduction by remarking that many small stars have large proper motions. He considers the most probable explanation of this circumstance to be, that they are nearer to us than other stars which, although brighter, have less apparent motion. He cites, in support of this opinion, the case of many physically double stars, the constituent members of which, although unequally bright, are known to be situate nearly at the same distance. The great reflectors of Lord Rosse have also established the existence of a similar difference of brightness in the stars of which clusters are composed.

The Introduction is followed by the individual observations, conveniently arranged in double columns. For each star the division error of the circle is given, with an appropriate symbol indicating whether the observation is one above pole, below pole, or by reflection. This part of the volume occupies 646 pages.

The volume closes with the Catalogue. The right ascensions are given on the left-hand page, and the north polar distances on the opposite page. The numbers on the right-ascension page are arranged in ten columns. The first column contains the number of the star in the Catalogue; the second, its usual designation in those cases wherein it is not anonymous; the third, its magnitude; the fourth, its right ascension; the fifth, the amount of annual precession; the sixth, the secular variation of precession; the next three columns exhibit the differences between the right ascension given by the author and the corresponding results of Bradley, Piazzi, or some other observer. The last column contains the number of individual observations. The numbers on the opposite page of the Catalogue, relating to the north polar distances, are drawn up in only eight columns, the repetition of the name and magnitude of the star being obviously unnecessary. In other respects the arrangement is the same as for the right ascensions.

We shall conclude with expressing an assurance that the readers of the *Monthly Notices* will cordially congratulate the author on having brought his arduous and important labours to so satisfactory a close.

The U.S. Naval Astronomical Expedition to the Southern Hemisphere, during the years 1849–50–51–52. Vol. III., *Observations to determine the Solar Parallax.* By Lieut. J. M. Gillis, LL.D., Superintendent. Washington, 1856.*

Allusion has frequently been made in the *Monthly Notices* to the expedition despatched to Chili by the American Government for the purpose of determining the solar parallax, by observations of *Venus* and *Mars*, in accordance with a method proposed by Dr. Gerling of Marburg in No. 502 of the ***Astronomische Nachrichten***. The merit of suggesting this expedition is due to Lieut. Gillis, who offered to execute, at some place in Chili, the observations which would be required, in conjunction with corresponding observations made at other places in both hemispheres, for carrying Dr. Gerling's method into effect. Lieut. Gillis, who was already favourably known as an observer, was accordingly appointed Superintendent of the operations of the Expedition, which sailed from the United States in the autumn of 1849. A temporary observatory was erected at Santiago. The instruments were: an equatoreally mounted refractor of 6·4 inches aperture and 103·7 inches focal length, by Fitz of New York; and a meridian circle reading to half seconds of arc by four micrometer microscopes, and furnished with a telescope of 51 lines aperture and 73 inches focal length. The Expedition returned to the United States towards the close of 1852 after an absence of thirty-nine months.

The first two volumes of the work, containing an account of the operations of this expedition, so creditable to the enlightened enterprise of the Government of the United States, is devoted to the geography, climate, natural history, &c., of Chili. The present volume contains a complete account of the observations for determining the solar parallax, which formed the main object of the expedition, including the subsequent discussion of them by Dr. Gould, and a statement of the definitive results deducible from them.

The introduction to the contents of the volume commences with an account of the origin and progress of the expedition by Lieut. Gillis. This is followed by a description of the instruments and methods of observation. The next subject is the discussion, by Dr. B. A. Gould, of all the observations available for determining the value of the solar parallax. The author of this part of the work has prefaced his labours by a valuable historical sketch of the subject. He then proceeds to a general statement of the problem. The observations of *Mars* and *Venus* at the Santiago Observatory consist of four series of micrometric measures executed on 217 days, including two oppositions of the former planet, and two inferior conjunctions of the latter, besides a large number of corresponding meridian observations of both planets. Dr. Gould regrets that the number of corresponding micrometric comparisons made at the observatories in the northern hemisphere proved totally inadequate to admit of an investigation of the solar parallax conformably to the original plan of procedure. Under these circumstances he considered that "the only course remaining for him was to make use of the largest possible number of observations of every kind in each hemisphere, and to trust to the accumulation of observations for eliminating the inaccuracies due to erroneous assumptions in the adopted positions of the comparison-stars, and to the unavoidably large mean error of the meridian declinations." He accordingly has given a complete discussion of the subject, adapted to the data available to him. The entire investigation and calculations occupy 228 pages of the Introduction. We have only room for a statement of the final results, which consist of new values of the semi-diameters of *Mars* and *Venus*, and of the solar parallax. They are, for the semi-diameters of the two planets,

$$\text{Semi-diameter of Mars} = 4''{\cdot}6639 + 1''{\cdot}9681 = 6''{\cdot}63$$
$$\text{Semi-diameter of Venus} = 8{\cdot}6625 - 0{\cdot}3118 = 8{\cdot}35$$

These results are derived exclusively from observations made at Santiago and Washington.

The value of the solar parallax is derived solely from the series of observations of *Mars* which include the first opposition of the planet. It is found to be

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* This volume, although bearing the date 1856, was published only in year 1858.

$$= 8''.5712 - 0''.0762 = 8''.4950$$

The author accordingly concludes that we may assume with advantage

The sun's horizontal parallax = 8″·5000,

being less by 0″·07 than the value commonly adopted, and corresponding to a distance of 24,266 terrestrial semi-diameters, 96,160,000 statute miles, 15,475,000 myriameters.

The investigation of the solar parallax by Dr. Gould closes the introductory part of the volume, which comprises 288 pages. The remainder of the volume is occupied with a detailed statement of the observations made at Santiago by Lieut. Gilliss, and the corresponding observations made by Lieut. Maury at the Washington Observatory, by Mr. Bond at the Harvard College Observatory, and by Mr. Maclear at the Royal Observatory, Cape of Good Hope.

The *Nautical Almanac* for 1863 has recently been published. The contents and arrangement are generally the same as for preceding years. Accompanying it is a Supplement containing ephemerides of the minor planets and of *Neptune* for the year 1860.

Annales de l'Observatoire Royal de Bruxelles. Par le Directeur A. Quetelet. Tome xiv. 4to. Bruxelles, 1859.

In the Introduction to this volume M. Quetelet has given a brief sketch of the history of the Brussels Observatory since its establishment in 1835. The astronomical operations, which commenced with measures of double stars, were interrupted in the year 1840, by the necessity of attending to the phenomena of terrestrial magnetism and other points relating to the physical constitution of the earth, which it was then proposed to study simultaneously at the principal observatories throughout Europe. In 1848, the astronomical observations were resumed, and have been regularly prosecuted down to the present time. Those for the years 1848–9 have been reduced and published in tome xii. of the *Annales*. The observations for the succeeding five years have not yet been reduced. It is intended that they shall be included in tome xiii., which, consequently, has not yet been published.

The present volume contains observations with the transit instrument and the mural circle, made in the years 1855–6. They consist chiefly of observations of stars, but there are also transit observations of the sun, moon, and principal planets, and also of some of the minor planets. From the star-observations of each year a provisional catalogue is formed, the epoch of reduction being the beginning of the year. The number of stars in each catalogue is upwards of a thousand. It is intended, as soon as the nine annual catalogues subsequent to 1848 have been completed, to construct from the totality of the results a single catalogue reduced to some common epoch. The volume concludes with an account of the meteorological and magnetical observations executed at the observatory during the years 1855–6.

Bericht über die Cometen von den Jahren 975, 1264, *und* 1556. Von P. Augustin Reslhuber, Director der Sternewarte zu Kremsmünster. Linz, 1857.

In this *brochure* the author briefly notices the researches of Dunthorne, Hind, Hoek, and others, who have investigated the subject of the identity of these famous comets. According to Hoek, to whom the most recent examination of the question is due, there exists very little probability of the reapparition of the comet of 1556 in our own day.

Tables of Victoria computed with regard to the Perturbations of Jupiter and Saturn. By F. Brünnow, Ph. Dr., Professor of Astronomy in the University of Michigan, and Director of the Observatory at Ann Arbor, 4to, New York, 1859.

Dr. Brünnow has devoted much attention to the improvement of the theory of this planet, and at length, when the proper time appeared to him to have arrived, has undertaken the laborious task of computing tables of its motion. In No. 1077 of the *Astronomische Nachrichten* he has given the expressions for the perturbations of the planet produced by *Jupiter* and *Saturn*, together with the elliptical elements as deduced from five oppositions of the planet in 1850, 1852, 1853, 1854, and 1856. Subsequent corrections of the elements, deduced from the opposition of 1857, were found to be of trifling magnitude.

The perturbations were calculated from the formulæ published by Professor Hansen in No. 799 of the *Astronomische Nachrichten*. The tables give, therefore, the perturbations of the mean anomaly, those of the logarithm of the elliptic radius vector, and those of the perpendicular to the plane of the orbit. Subsidiary tables are given for computing the heliocentric rectangular co-ordinates referred to the equator, and the mean equinox of 1857·0.

Dr. Brünnow, by his self-imposed labours in the construction of tables of *Flora* and *Victoria*, has earned the unqualified thanks of all astronomers. It is to be hoped that his example will be speedily followed by others of his contemporaries, and that ere long the computation of ephemerides of the minor planets, or, at any rate, of those most favourable for the purpose, will be effected by the aid of tables constructed beforehand.

Observations of Donati's Comet made at the Observatory of Edward J. Cooper, Esq., with the large Equatoreal.

This pamphlet, which has been printed for private circulation, among astronomers, contains differential measures for determining the position of the comet, executed by Mr. Graham at the Markree Observatory, and extending from September 18, 1858, to October 16 in the same year. Appended to these are a series of physical observations of the comet by Mr. Cooper and Mr. Graham. There are some well-executed drawings, illustrative of the text, which may be advantageously compared with the delineations of the same sort made at other observatories.

Vol. XX. *December* 9, 1859. No. 2.

The Rev. R. Main, President, in the Chair.

The Rev. Percival Alfred Fothergill, B.A.,
was balloted for and duly elected a Fellow of the Society.

Unexpected Reappearance of U *Geminorum.* By Norman Pogson, Esq., Director of the Hartwell Observatory.

(*Communicated by Dr. Lee.*)

M. Hermann Goldschmidt, in a letter of the 30th ultimo, announces the premature apparition of this singular star, which must have passed its maximum on Nov. 12th, full 27 days before the time indicated by computation. The following extract from his interesting and important communication will show how zealously he *toils* at his celestial recreations:—

"On the evening of Nov. 18th, I looked for it, and it was brighter than a star I estimated 10·11 magnitude, but to which Dr. Winnecke assigned (*h*) 10·3 magnitude. On the next evening I was surprised to find it less bright, and nearly equal to this star. I was obliged to compare with the list of stars given by Dr. Winnecke, for I want the *Monthly Notices* in which you gave the magnitudes of your comparison stars. On the 20th, 10 hours, it was equal to the stars *d* and *e*. On the 21st, it was on the extreme visibility of my four inches: it even diminished one hour after. On the 22d, *it was painfully seen;* 0·1 magnitude less bright than the small star *b*. On the 23d it had quite vanished, and since my instrument shows 12·4 magnitude, after your scale, 12·5 can be obtained by favourable circumstances. I must say that I was constantly for two hours and more in comparing, and favoured by a beautiful sky. I made use of eye-pieces of 36 and 80 magnifying power. At instants I could see the small stars next the variable; *a* and *b* in Dr. Winnecke's list. You may safely trust these observations *laboriously obtained.* I send a diagram of the comparison stars to avoid all mistake. The scale of Dr. Winnecke is in the mean 0·3 magnitude higher than my estimations."

Now, applying these differences to my own standards (*Monthly Notices*, vol. xvii. p. 202),* for comparison with previous observations, we get, for about 11 hours each evening,—

1859. Nov. 18	U Geminorum	0·3 mag. brighter than *e*	=	10·7	magnitude.
19	,,	0·1 mag. less than *e*	=	11·1	,,
20	,,	Equal to *g*		12·1	,,
21	,,	A limit of favourable vision		12·5	,,
22	,,	0·1 mag. less than *l*		13·1	,,
23	,,	Invisible	Under	12·5	,,

Well might M. Goldschmidt say of his comparison on the 22d, that "the star was painfully seen," for I estimate *l* (Winnecke's *b*) 13·0, Mr. Baxendell 13·3; and besides his "two hours and more in comparing, favoured by a beautiful sky," he must possess an exquisite eye and telescope (by Steinheil), to peer so far beyond the limit of his aperture as to have glimpsed it at all! But some of his extremely faint planetary discoveries have proved this long since. I can only say that, ordinarily, the star *l* is the limit of the Hartwell equatoreal, and that, with a larger but inferior telescope at Oxford I never saw it at all!

Knowing that U *Geminorum* was not due till Dec. 9th, and being anxious to push forward other work, I am only able to add to my friend's valuable observations one useful record, viz., that U *Geminorum* was invisible and under 13·0 magnitude on Nov. 7th. Now, whatever other anomalies have been displayed by this truly wonderful star, it has hitherto kept in pretty close conformity with a uniform period. My latest elements of variation, viz., Epoch = 1859, Feb. 19·6; Period = 96·879 days; furnish for the six observed maxima, from which they were deduced, the residual errors (Observation—Calculation), +1·4; +2·5; −2·5; +7·3; +1·2, and +0·4 days respectively, the equation, 7·3 days out, having but small weight in regard to the others; but here is an irregularity of 27 days at once, more than a fourth of the entire period! For the future it must be more closely watched.

Hartwell Observatory, Dec. 8, 1859.

On the Variability of α *Cassiopeiæ.* By W. R. Birt, Esq.

Some doubts having been expressed relative to the variability of α *Cassiopeiæ*, which I had the honour to discover in the year 1831, have induced me to give particular attention to the star during the preceding summer and autumn, the result of a very careful series of *eye-comparisons* fully confirming my previous conclusions as to the fact of the varying intensity of its light, although within narrow limits.

Sir John Herschel's determination of the magnitudes of γ, α, and β, in the table appended to his Cape Observations are as follows:—

γ 2·52, α 2·57, β 2·63;

In the course of my observations, extending from March 31 to December 1, 1859, I have seen α brighter than γ, and I have also observed it less than β, the most usual estimation of its magnitude being 2·57, that assigned to it by Sir John, this places it nearly midway between γ and β.

It is not my intention, in the *present communication*, either to attempt anything like a re-determination of the period, or to compare any of my observations during the past summer and autumn with those that I made in 1831, or others that are on record by various observers since, my object being to point out for the information of astronomers engaged with variable stars the epochs when I observed the star either brighter than the γ or less than the β. I have, however, attempted in the following tables to exhibit the *great irregularity* of the phases of brilliancy, indicating at the same time, in Table III., the progression from max. to min. and the reverse; the epochs of the *true* maxima, as well as brilliancy, as shown by a curve in most cases differing considerably from those greater brightnesses of the star which appear to be connected with a very remarkable feature which has been presented in the course of these observations: it is the alternation of brightness which the star has exhibited, even in the course of a single evening, nearly the same irrěgularity appertains to this feature as well as to the increase and decrease of brilliancy. The epochs of the principal instances are as under:—

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* Comparison star *h* is certainly under-estimated in my list, and must refore be rejected.

1859, June 24 to July 7, August 19 to Sept. 1, Nov. 19 to 28.

On these occasions the light of the star has apparently been in a state of great fluctuation, nothing at all settled; one evening bright, the other dim; sometimes settled for a few hours, then suddenly brightening up or decreasing in its brilliancy; while on other occasions the course of the observations has been such as to induce the belief that for some little time, at least, the star has been steady at its brilliancy, very gradually increasing or decreasing in brightness. During the alternations above alluded to, the γ and β have not appeared to participate, indicating that the alternations have rather occurred in connexion with the star itself than have been produced by fluctuations in our own atmosphere. On one or two occasions, but very rarely indeed, a suspicion has crossed my mind that both γ and β have exhibited a slight change of lustre, but nothing at all in any way comparable with the variations of α, the extent of which I estimate to be in accordance with Sir John Herschel's scale, about 0·3 of a magnitude.

Table I.

Epochs of greater brilliancy, the star nearly as bright or brighter than the γ.

1859.	G.M.T. d h m	Mag.	Interval days.	Remarks.
May	5 13 0 5 14 0	2·53	49·84	Sky very favourable.
June	24 10 0 24 10 15	2·50	5·00	Strong twilight.
June	29 10 15	2·45	15·98	Strong twilight.
July	15 9 45	2·50	5·01	Very clear, moon present.
July	20 10 0	2·47	29·95	Clear, moon present.
Aug.	19 9 0	2·52	11·00	Moon low.
Aug.	30 9 0	2·52	22·02	
Sept.	21 9 20	2·50		

Table II.

Epochs of minima or inferior brightness, the star generally less than β.

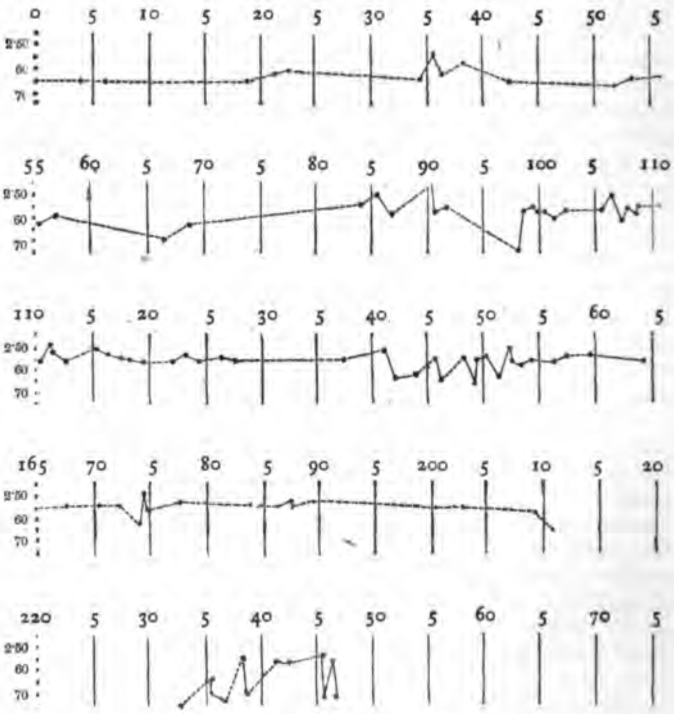
1859.	G.M.T. d h m	Mag.	Interval days.	Remarks.
April	19 10 15	2·65	32·98	Very clear.
May	22 9 45	2·65	14·11	Fine strong twilight.
June	5 12 30	2·70	31·92	Clear.
July	7 10 40	2·74	43·99	Moon just set.
Aug.	20 10 20	2·65	4·10	
Aug.	24 12 45	2·63	2·84	Clear, Milky way bright.
Aug.	27 9 0	2·65	25·07	
Sept.	21 10 40	2·65	36·02	
Oct.	27 11 0	2·67	22·83	
Nov.	19 7 0	2·74		

Table III.

Maxima and minima of α Cassiopeiæ, the maxima determined by the projection of a curve with intervals of increase and decrease.

1859.	Epoch of Min. d h m	Mag.	Interval days.	Epoch of Max. d h m	Mag.	Interval days.*
April	19 10 15	2·65	16·13	May 5 13 30	2·53	16·85
May	22 9 45	2·65	4·10	May 26 12 15	2·60	10·01
June	5 12 30	2·70	23·91	June 29 10 15	2·45	8·01
July	7 10 40	2·74	12·98	July 20 10 0	2·47	35·11
Aug.	24 12 45	2·65	18·08	Sept. 11 14 40	2·57	9·83
Sept.	21 10 40	2·65	14·83	Oct. 6 6 30	2·55	21·19
Oct.	27 11 0	2·67		No observation.		

Curve of the variation of the brightness of α Cassiopeiæ from March 31 to Dec. 2, 1859.



o = March 31, 1859. Brightness of γ Cass. = 2·52, of β Cass. 2·63.

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* Intervals between the maxima and *succeeding* minima.

Ephemeris of the Variable Stars for 1860. By Norman Pogson, Esq., Director of the Hartwell Observatory.

(*Communicated by Dr. Lee.*)

Star.	Probable Mag.	Mean Place, 1860. R.A.			N.P.D.		Times of Maxima.	Authority.
		h	m	s	°	′		
T Piscium	9·5	0	24	46	76	14	May 7, September 27 ...	S.
α Cassiopeiæ	2·0	0	32	35	34	14	Jan. 19, April 8, June 26, Sept. 13, Nov. 31	A.
S Piscium	9·0	1	10	15	81	49	June?	—
R Piscium	7·5	1	23	25	87	51	Early in June	—
R Arietis	8·0	2	8	10	65	36	January 16, July 21	B.
ο Ceti	2·0	2	12	17	93	37	September 6·5	A.
R Tauri	8·0	4	20	38	80	9	July 23	W.
S Tauri	10·0	4	21	32	80	22	February 17	W.
R Orionis	9·0	4	51	22	82	5	May 26	W.
α Orionis	1·0	5	47	36	82	37	June, December	—
R Geminorum	7·3	6	58	56	67	5	January 15	P.
R Can. Min.	8·0	7	1	0	79	46	April?	—
S Can. Min.	7·5	7	25	7	81	23	August 22	W.
S Geminorum	9·2	7	34	38	66	13	March 15, and January 3, 1861	P.
T Geminorum	8·9	7	40	54	65	55	October 13	P.
U Geminorum	9·1	7	46	48	67	38	March 13, June 18, September 23, December 29	P.
R Cancri	6·0	8	8	51	77	51	April 8	A.
U Cancri	9·0	8	27	45	70	37	October 29	W.
S Hydræ	8·5	8	46	16	86	24	August 26; not observable ...	W.
T Cancri	9·5	8	48	40	69	37	Uncertain, Minimum (12 mag.), July 1; not observable ...	W.
T Hydræ	6·5	8	48	51	98	37	July 18	W.
R Leonis	5·0	9	40	2	77	55	Feb. 12, and Jan. 2, 1861 (Minimum, Aug. 18; 10 mag. ...	P.
R Urs. Maj.	7·0	10	34	41	20	29	September 2 (Minimum, May 12)	P.
R Comæ	8·0	11	57	4	70	26	October?	W.
R Virginis	6·5	12	31	24	82	14	January 5·6, May 29·9, Oct. 22·2	A.
S Urs. Maj.	7·5	12	37	48	28	8	June 2 (Minima, Feb. 25, Oct. 7)	P.
U Virginis	7·5	12	44	0	83	41	April 8, November 6 ...	W.
V Virginis	7·0	13	20	36	92	28	Possibly in March and November	P.
R Hydræ	4·0	13	22	4	112	33	August 31, irregular ...	B.
S Virginis	6·0	13	25	42	96	28	April 25 (Minimum; 11 mag.; Feb. 21)	P.
R Boötis	8·0	14	31	1	62	39	May 3, December 22 ...	W.
S Serpentis	8·0	15	15	7	75	11?	April 10	A.
R Coronæ	6·2	15	42	49	61	25	October 15; very irregular ...	B.
R Serpentis	6·5	15	44	15	74	26	September 10	A.
R Herculis	8·5	15	59	38	71	15	April 7	B.
S Scorpii	9·5	16	9	17	112	33	April; maximum brilliancy very uncertain	P.
R Scorpii	9·0	16	9	19	112	36	July; uncertain	P.
S Ophiuchi	9·3	16	26	12	106	52	January 19, September 7 ...	P.
S Herculis	7·5	16	45	32	74	49	March 16	B.
R Ophiuchi	7·6	16	59	44	105	54	January 4, October 31 ...	P.

Star.	Probable Mag.	Mean Place, 1860.					Times of Maxima.	Authority.
		R.A.			N.P.D.			
		h	m	s	°	′		
α Herculis	3·0	17	8	16	75	27	March 11, May 16, July 22, Sept. 26	A.
T Herculis	7·9	18	3	48	59	0	January 6, June 14, November 21	K.
R Scuti	5·0	18	40	1	95	50	April 2, June 14, Aug. 24, Nov. 3	A.
13 Lyræ	4·3	18	51	4	46	14	Jan. 19, March 6, April 21, June 7, July 23, Sept. 8, Oct. 24, Dec. 10	B.
R Aquilæ	6·5	18	59	38	81	59	June 8 … …	W.
R Sagittarii	8·2	19	8	28	109	33	Not until February, 1861 …	P.
R Cygni	8·0	19	33	4	40	7	August 3; Minimum (14·0) Mar. 1	P.
χ Cygni	5·0	19	45	11	57	27	March 10 … …	A.
U Capricorni	10·5	20	40	22	105	18	Not until February, 1861 …	P.
R Vulpeculæ	8·0	20	58	10	66	44	February 15, July 11, December 5	W.
T Capricorni	9·0	21	14	17?	105	45	April 25 … …	S.
β Pegasi	2·0	22	57	0	62	41	Jan. 19, April 14, May 28, July 10, Aug. 23, Oct. 5, Nov. 17, Dec. 31	Au.
R Pegasi	8·5	22	59	37	80	14	April 2 … …	A.
R Aquarii	7·0	23	37	15	106	3	June 24 … …	A.
R Cassiopeiæ	6·0	23	51	18	39	23	June 25; Minimum (13 mag.) Jan. 9.	P.

Minima of the Short Period Variable Stars during 1860 *in G.M.T. and for the Earth's Mean Distance from each Star.*

λ *Tauri.*

Max. 4·0; Min. 4·5. Variation rapid.

	d	h	m		d	h	m		d	h	m		d	h	m
Jan.	31	13	8	Feb.	16	8	37	Oct.	6	13	56	Oct.	22	9	25
Feb.	4	12	0		20	7	29		10	12	48		26	8	17
	8	10	52	Sept.	28	16	12		14	11	41	Dec.	24	15	20
	12	9	44	Oct.	2	15	4		18	10	33		28	14	12

Algol.

Maximum 2·3; Minimum 4·0. Whole variation completed in seven hours.

	d	h	m		d	h	m		d	h	m		d	h	m
Jan.	2	16	35	Mar.	11	12	8	July	18	12	47	Nov.	1	14	55
	5	13	24		14	8	57		21	9	36		4	11	44
	8	10	13		31	13	50	Aug.	7	14	29		7	8	33
	11	7	2	April	3	10	39		10	11	18		10	5	22
	22	18	17		20	15	32		30	13	0		21	16	37
	25	15	6		23	12	21	Sept.	2	9	49		24	13	26
	28	11	55		26	9	10		19	14	42		27	10	15
	31	8	44	May	13	14	3		22	11	31		30	7	4
Feb.	14	16	48		16	10	52		25	8	20	Dec.	11	18	19
	17	13	37	June	5	12	34	Oct.	9	16	24		14	15	8
	20	10	26		8	9	23		12	13	13		17	11	57
	23	7	15		11	14	16		15	10	2		20	8	46
Mar.	8	15	19		28	11	5		18	6	51		23	5	35

ζ *Geminorum.*

Max. 3·8; Min. 4·5. Increasing $5^d 6^h$.

	d	h	m		d	h	m		d	h	m		d	h	m
Jan.	30	5	48	Feb.	19	13	24	April	20	12	12	Dec.	20	7	24
Feb.	9	9	36	April	10	8	24	Oct.	30	12	24		30	11	12

S *Cancri.*

R.A. $8^h 35^m 56^s$; N.P.D. 70° 28′.

Max. 8·0; Min. 10·5. Variation rapid.

	d	h	m		d	h	m		d	h	m		d	h	m
Jan.	2	13	5	Feb.	28	10	46	Sept.	14	14	41	Nov.	10	12	23
	21	12	18	Mar.	18	10	0	Oct.	3	13	55		29	11	37
Feb.	9	11	32	April	6	9	14		22	13	9	Dec.	18	10	51

β *Lyræ.*

Principal Minimum (4·3); First Max. (3·4), $3^d 2^h$ after.

Second Min. (3·8), $6^d 9^h$ after.; Second Max. (3·4), $9^d 10^h$ after.

	d	h	m		d	h	m		d	h	m		d	h	m
Jan.	26	5	11	May	8	11	12	Sept.	14	12	44	Oct.	23	6	0
Apr.	12	15	42		21	8	58		27	10	30	Dec.	26	18	46
	25	13	27	Sept.	1	14	59	Oct.	10	8	15				

η *Aquilæ.*

Max. 3·6; Min. 4·4. Increasing $2^h 9^d$.

	d	h	m		d	h	m		d	h	m		d	h	m
Feb.	16	18	9	June	10	13	55	Aug.	21	8	15	Nov.	8	6	50
Mar.	23	15	20	July	16	11	5		28	12	29	Dec.	14	4	
Apr.	28	12	30		23	15	19	Oct.	3	9	40				

δ *Cephei.*

Max. 3·7; Min. 4·8. Increasing $1^d 14^h$.

	d	h	m		d	h	m		d	h	m		d	h	m
Jan.	4	14	33	Mar.	30	11	15	July	26	12	44	Oct.	20	9	27
	15	8	8	April	15	13	38	Aug.	11	15	7	Nov.	5	11	50
	20	16	56	May	1	16	1		22	8	42		16	5	25
	31	10	31		12	9	37	Sept.	7	11	5		21	14	13
Feb.	16	12	54		28	12	0		23	13	28	Dec.	2	7	48
Mar.	3	15	17	June	13	14	23	Oct.	4	7	4		7	16	36
	14	8	52	July	10	10	21		9	15	51		18	10	11

The minima of all the short-period variable stars, except λ *Tauri*, are computed from Prof. Argelander's elements. The corrections to be applied to the observed times, before comparing with the ephemeris, may be found by the formulæ given in the *Monthly Notices*, vol. xvii. p. 27.

The initials in the last column of the first table indicate the authorities for the elements of variation employed, viz. Argelander, Auvers, Baxendell, Krüger, Pogson, Schönfeld, and Winnecke. When no authority is given, the month specified must be regarded as little better than a mere conjecture as to the time of return. I am under especial obligation to the valuable information contained in Dr. Winnecke's paper, in *Ast. Nach.* No. 1224; as also to Mr. Baxendell, for his communication of the dates for λ *Tauri* and the other stars bearing his initial.

For stars given under my own signature I have deduced fresh elements, from all observations up to the present time. The total number of stars of undoubted variability now amounts to ninety-nine; sixty-two of which are sufficiently under control to be given in an ephemeris.

V *Virginis* is an interesting variable star, discovered by M. Hermann Goldschmidt, but at present necessarily very uncertain both as regards time and light. The position of S *Scorpii* is a mere estimation, though probably not a bad one; that of its neighbour R *Scorpii* is more trustworthy; both have unfortunately been incorrectly given on several previous occasions. The N.P.D. of S *Serpentis* differs 20' from that given by Dr. Winnecke, and I cannot say which is correct. The R.A. of T *Capricorni* is also doubtful by some few seconds.

Hartwell Observatory, December 20, 1859.

Ephemerides of the Telescopic Appearances of the Lunar Craters *Geminus, C, and Bernouilli, March and April,* 1860. By W. R. Birt, Esq.

Geminus.

A walled crater (20) on Rev. T. W. Webb's index-map, 54 miles broad; eastern wall, 12,300 feet, western, 16,700 feet high, on the authority of Beer and Mädler. At about 4·14 days of the moon's age this crater is seen as a deep excavation below the surface. Under the morning illumination it presents a clear elliptical outline; at mid-day it undergoes a remarkable change, the northern portion being brightly luminous, the southern dusky, which is separated from the northern by a narrow rille. In the evening the elliptical form again comes out, by the strong contrast of light and shadow.

Morning Illumination.

G.M.T.	Moon's Age.	Form.	South Wall.	North Wall.
March.				
d h m	d			
26 8 0	4·25	Elliptical.	Terraced.	
28 8 0	6·25	,,	,,	Rim distinct.
29 8 0	7·25	,,	Rille across the floor.	
30 8 0	8·25	Divided.		

Mid-day Illumination.

G.M.T.	Moon's Age.	Form.	North Wall.
March.			
31 8 0	9·25	Half lucid; half obscure.	Bright.
		Rille across the floor forming the southern boundary of the northern lucid portion. The crater is bounded by a dark ribbon-like fringe on the eastern wall.	
April.			
4 10 0	13·34		

Evening Illumination.

G.M.T.	Moon's Age.	Form.	Eastern Wall.	Western Wall.
April.				
5 10 0	14·34	Elliptical.	In shadow.	Bright.
6 10 0	15·34	A small crater, extreme south.	,,	,,
7 10 0	16·34	A spot on north terrace.	Terraced.*	Terraced.*
7 15 0	16·54		Shadows advancing.	

* These terraces are on the S.W. and N.E. parts of the wall.

C.

A small crater, situated *close* to the south-western terrace of *Geminus*, becoming rather *intensely* bright as the lunar day advances, and exhibiting a streak of light at first rather broad, but narrower and *forked* under the mid-day illumination. It is marked on Beer and Mädler's map, but not noticed by Webb.

Morning Illumination.

G.M.T.	Moon's Age.	Greatest Libration.*	Aspect.
March.			
d h m	d	d	
26 8 0	4·25	−2·05	Southern rim in shadow.
28 8 0	6·25	−0·05	Streak *broad*.
29 8 0	7·25	+0·95	,, ,,
30 8 0	8·25	+1·95	Crater brightly luminous.

Mid-day Illumination.

G.M.T.	Moon's Age.	Greatest Libration.*	Aspect.
31 8 0	9·25	+2·95	Apparently some distance from Geminus.
April.			
4 10 0	13·34	+7·04	Very bright; the streak *narrow* and forked.

Evening Illumination.

G.M.T.	Moon's Age.	Greatest Libration.*	Aspect.
5 10 0	14·34	+8·04	Eastern wall in shadow.

The crater afterwards becomes very *insignificant*, as compared with its mid-day aspect.

Bernouilli.

A crater north-west of *Geminus* (21) in Webb's map, and characterised by a *dark* floor, offering, in this respect, a fine contrast to the *lucid* portion of *Geminus* under the mid-day illumination. This dark floor appears in some way to be connected with a *dark* space seen to great advantage under the mid-day illumination, and therefore more of the character of a selenological "marking" than an appearance dependent on hypsometrical relations. This dark space extends to the margin of the crater *Messala*, and appears to be remarkably persistent during the three illuminations.

Epoch of greatest libration, N.E., March $28^{\mathrm{d}}\ 9^{\mathrm{h}}\ 10^{\mathrm{m}}$
,, ,, ,, N.W., April 9 19 26

Morning Illumination.

G.M.T.	Moon's Age.	Aspect.
March.		
d h m	d	
26 8 0	4·25	Outline of wall distinct; a dark spot at the S.W. point.*
28 8 0	6·25	The dark spot distinct; the N.E. wall bright.

Mid-day Illumination.

G.M.T.	Moon's Age.	Aspect.
31 8 0	9·25	Floor very dark; ring bright, and generally visible.
April.		
4 10 0	13·34	

* The dark and lucid spots are the same.

Evening Illumination.

April. d h m	d	
5 10 0	14·34	Eastern part of floor *darker*; two ridges extend from the southern part of the wall.
6 10 0	15·34	Eastern wall in shadow; western luminous, ridges very apparent.
7 10 0	16·34	Strong contrast of light and shadow; lucid spot at S.W. point.*
7 15 0	16·54	Shadow extending over floor of crater opening in the wall between the two ridges.

For indicating the most favourable epochs for catching *minute details* the distance in time from the epoch of perigee is here added. This should be given with every observation, as well as the data pointed out by Mr. Webb.

Epoch of Perigee, April 4^{d}·0^{h}.

	G.M.T. d	h	m	Perigee. d		G.M.T. d	h	m	Perigee. d
March	26	8	·0	−8·67	April	4	10	0	+0·42
	28	8	0	−6·67		5	10	0	+1·42
	29	8	0	−5·67		6	10	0	+2·42
	30	8	0	−4·67		7	10	0	+3·42
	31	8	0	−3·67		7	15	0	+3·62

The following notices of the Great Comet of 1858 and of Occultations have been communicated to the Society by the Board of Trade through Admiral Fitzroy.

The Great Comet of 1858.

Captain J. W. Callow, commanding the "Charles" in the Southern Ocean, reports as follows:—

"October 11. At noon, lat. 36° 17′ S., long. 22° 52′ W. At 7^{h} 15^{m} P.M. saw a comet, and at 7^{h} 25^{m} P.M. found it bore by compass N.W. by W. ½ W., its altitude at the same time being 12° 21′. At 7^{h} 42′ its altitude was 6° 47′; and its angular distance from *Venus* at 7^{h} 53^{m} was 26° 23′: the length of the tail being 8 or 9°. At setting it bore W. 10° N. (by compass?). Variation by observation 9° 42′ W. The nucleus of the comet was about the size of a star of the first magnitude, of a yellowish light, and distinctly visible through cumulo-stratus clouds, through which the stars in its neighbourhood were not seen. The tail, which traversed the constellation of *Scorpio*, made an angle of about 15° with the horizon. Barometer 30$^{in.}$17, thermometer 55°·0.

"October 12. At noon, lat 35° 47′ S.; long. 21° 40′ W. At 7^{h} 18^{m} P.M. in lat.† 35° 50′ S. and long. 21° 8′ W., saw the comet bearing N.W. by W. ¼ W. at an altitude of 13° 57′; its angular distance from the moon was 37° 47′; from *Venus*, 22° 40′; and the distance of *Venus* from the moon was 19° 13′. Two stars, one of which belonged to the bow of *Sagittarius*, were visible through the luminous vapour of the tail; one of these stars was 2° 0′ distant from the nucleus, as near as could be observed with the aid of a sextant. The nucleus was very bright, and visible after sunset before any stars in the same region of the heavens; being also seen through cumulus and cumulo-stratus clouds. Barometer 30$^{in.}$12, thermometer 56°.

"October 13. This evening being cloudy, the comet was only seen for a short time, and there was no opportunity for taking observations.

"October 14 and 15. Cloudy on both evenings, comet not visible.

"October 16. At 7^{h} 25^{m} P.M. in lat. 35° 22′ S. and long. 13° 51′ W., the comet's distance from *Antares* was 9° 28′; its altitude being 22° 18′; that of *Antares*, 29° 0′. The distance of *Antares* from *Venus* was 1° 19′; I have given the distance between *Venus* and the star, because about this time they are nearly in conjunction at Greenwich; and the distance may be some guide as to the difference of parallax between that at the ship's position and at Greenwich. The comet very luminous to-night; its nucleus is in size and appearance like *Saturn*, the tail very distinct and fully 10° long. Barometer 29$^{in.}$53, thermometer 61°·0.

"October 17. At 7^{h} 30^{m} P.M. in lat. 35° 25′ S. and long. 10° 9′ W., the distance of the comet from *Antares* was 6° 56′; its altitude, 24° 0′; that of *Antares* being 26° 12′. The tail of the comet appears longer than it was last night, and its nucleus is very large and bright. Barometer 29$^{in.}$70, thermometer 58°.

"October 18. The comet seen for a short time to-night; it appears to be still increasing in brightness.

"October 19. The comet very distinct; its tail now forms an angle of about 60° with the horizon, and its nucleus appears to be approaching *Venus*.

"October 20 and 21. The comet only visible for a short time on these nights.

"October 22. Comet distinctly visible, its position has greatly changed, and the tail is now nearly perpendicular to the horizon.

"October 24. Comet seen for a short time; the nucleus does not appear to have diminished in brightness yet.

"October 26. Comet distinctly visible for a short time; the tail appeared longer and larger, though not so luminous as before. The nucleus is very bright.

"October 28. To-night the comet was seen very brilliantly for a short time. The nucleus was close to the tail of *Scorpio*, and the tail of the comet terminated in the bow of *Sagittarius*.

"October 29. Comet not so distinct to-night, though it does not appear to have diminished in size.

"November 1. Comet seen only for a short time.

"November 4. Comet visible for a short time this evening, it is now in the constellation *Corona Australis*; its shape is somewhat altered, the head being more pointed, and the tail like a double bow.

"November 6, 7, and 8. The comet was seen each evening; it has now greatly decreased in brilliancy.

"The comet was seen very indistinctly with the naked eye on the 11th, and was not visible afterwards without the aid of a telescope; it was seen for the last time with the assistance of a 2·5-inch glass on November 16th."

This comet was also seen by Captain Trivett of the Hudson Bay Company's steamer, "Labouchere," for the first time on the 5th of October, in lat. 5° 20′ S. and long. 29° 55′ W.; and (its last appearance) on November 1st, in lat. 38° 45′ S. and long. 55° 10′ W. The following positions* are given by Captain Trivett, as deduced from sextant observations.

		G.M.T. h m	Comet's R.A. h m s	Comet's Decl. ° ′ ″
October	7	9 7	14 35 55	+13 41 35
	8	9 8	14 47 15	10 7 38
	9	9 11	14 59 27	+ 6 41 43
	13	9 16	15 47 11	− 4 3 41
	14	9 17	15 56 56	−10 34 51

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* The dark and lucid spots are the same.

† The latitudes and longitudes given at the times of observation are derived rom those given at noon; supposing the latitude and longitude to vary as the ime.

* It is not stated what corrections were applied to the observations used in deducing these positions.

On board the Arctic discovery yacht "Fox," Captain M'Clintock, the comet was seen on September 13th and 23d, and on October 8th and 9th, in (about) lat. 72° N. and long. 94° W.

Captain James Clarke of the ship "Marco Polo," likewise reports having seen the comet (for the first time) in lat. 53° 0′ S. and long. 56° 46′ W., on October 11 about 8 P.M., at which time its tail was 2° long. The comet afterwards increased in size, but the weather being very unsettled it is uncertain when it began again to decrease; it was seen for the last time on November 5th, in lat. 5° 57′ S. and long. 27° 17′ W. The following are the observations for position:

"October 15. $10^h\ 35^m\ 0^s$ G.M.T. The comet bore W. by S. by compass from the ship; its altitude was 17° 40′; the distance from the moon (query? centre, or bright limb) 59° 50′, and from *Venus* 12° 32′. The tail this night was 3° long. Barometer $29^{in}\cdot80$, thermometer $42^{\circ}\cdot5$ in air. Variation 6° E.

"October 22. $9^h\ 24^m\ 0^s$ G.M.T. The comet 3° 4′ distant from *Venus*, and 8° 51′ from *Antares*; its altitude 29° 55′, and that of *Venus* 26° 51′. Barometer $30^{in}\cdot14$, thermometer $63^{\circ}\cdot0$.

"November 5. $9^h\ 58^m\ 0^s$ G.M.T. The comet now much fainter than when first seen, its distance from *Venus* is 17° 46′, its altitude 16° 25′, that of *Venus* being 8° 37′. Barometer $30^{in}\cdot04$, thermometer $78^{\circ}\cdot0$."

Occultations.

Captain David Smith observed the following occultations on board the ship "Hanover," the first on the outward passage to India, and the second on the return voyage to England.

"1858, July 23. In lat. 34° 14′ S. and long. 77° 12′ E., observed the star *φ Sagittarii* disappear at the moon's dark limb at $17^h\ 5^m\ 0^s$ apparent time.

"1859, May 8. In lat. 50° 33′·5 N. and long. 0° 28′ E., observed the planet *Saturn's* remote limb disappear at $8^h\ 27^m\ 17^s$ apparent time at ship."

Occultation of the Pleiades, 1859, *December* 8, *observed at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

Greenwich Mean Solar Time of Observation.

Phenomenon.	Moon's Limb.	Hour and Minute. h m	Seconds Observed by M	D	C	T
23 Tauri, disapp.	Dark	4 7	3·8	4·5	4·3	
16 Tauri, reapp.	Bright	4 10	...	...	46·8	
17 Tauri, reapp.	Bright	4 25	60·7	60·7	58·8	
η Tauri, disapp.	Dark	4 32	54·5	54·9	55·1	
23 Tauri, reapp.	Bright	4 48	...	48·0	44·6	
28 Tauri, disapp.	Dark	5 18	16·1	17·0	16·7	
η Tauri, reapp.	Bright	5 24	36·0	31·1	26·4	
27 Tauri, 1st disapp.	...	5 34	47·4	35·4	...	
27 Tauri, 2d disapp.	...	5 35	...	30·3	...	
27 Tauri, reapp.	...	5 38	...	31·8	...	
28 Tauri, reapp.	Bright	5 59	...	...	40·9	38·8

Notes by Mr. Main [M]. Observed with the Sheepshanks' equatoreal. The reappearance of 17 *Tauri* uncertain to a second or two; that of η *Tauri* was observed with a very low power, and the star was exceedingly faint when first seen, but it appeared to be in contact with the limb. 27 *Tauri* appeared to run along the limb at a part where it was slightly unilluminated, and to vanish at the central of three flat detached patches of illumination. Returning to the telescope two minutes after the disappearance, the star was again visible.

By Mr. Dunkin [D]. Observed with the Altazimuth. The disappearance of 27 *Tauri* was a most singular phenomenon; the star appeared to move a considerable time along the moon's limb, and disappeared behind a prominence at the first time noted; in a few seconds it reappeared, and finally disappeared at the second time noted. The other observations were generally satisfactory.

By Mr. Criswick [C]. Observed with the North Equatoreal. 16 *Tauri* was separated from the moon's limb when first seen, and the observation is not, therefore, satisfactory. 27 *Tauri* was not occulted at all, though it passed so close to some of the illuminated peaks of the dark limb as hardly to be distinguishable from them.

By Mr. Talmage [T]. Observed with the Sheepshanks' equatoreal. The star was very faint; it was slightly separated from the limb when first seen.

Occultations of the Pleiades observed at Highbury, December 8, 1859. By T. W. Burr, Esq.

Local Sidereal Time.

		h m s	
η Tauri (Alcyone)	Immersion	21 40 13·5	Dark limb.
	Emersion	22 31 56	Bright limb.
23 Tauri (Merope)	Emersion	21 56 26	Bright limb.
27 Tauri (Atlas)	Immersion	22 40 55	Bright limb.
	Emersion	22 45 7	

The disappearance of *Alcyone* was instantaneous, and an excellent observation as to time. The reappearance is a little doubtful, the eye not having been directed to the exact point. The phenomena attending the occultation of 27 *Tauri* were very interesting. It approached the upper rugged limb of the moon, and continued to graze along the mountain ridges, and to look like a double star with several isolated illuminated peaks, but at last vanished, and was hidden for the time stated. 20 *Tauri* also grazed the lower limb of the moon, and I lost sight of it for a few seconds; but this may have arisen from the brightness of the lunar surface with which it was in contact, and my attention being drawn off to watch *Alcyone*, which was approaching immersion.

Telescope employed, 3⅛ inches aperture, by Ross, focal length 52 inches, power 128. Latitude 51° 33′ 45″·1 N.; Longitude $23^s\cdot8$ W.

Dec. 15, 1859.

Occultations of Stars by the Moon. Observed by Captain Noble (previously unreported).

1859, Wednesday, April 13.

37 *Sextantis.*

The star disappeared instantaneously at $10^h\ 10^m\ 39^s\cdot2$ L.S.T.

and reappeared also instantaneously at $10\ 58\ 41\cdot0$ L.S.T.

Saturday, July 16.

ι Capricorni.

The disappearance took place at $20^h\ 38^m\ 57^s$ L.S.T.

and the reappearance at $21\ 42\ 13$ L.S.T. (subject to an error of 1^s)

The star entered into contact in a remarkably sluggish manner; the moon's limb apparently exerting a repulsive power upon it, as though, so to speak, the moon and star were ***similarly electrified:*** but although an inordinate time elapsed before the disappearance took place, yet nothing in the nature of a projection of the star upon the moon's limb was seen.

Wednesday, September 21.

μ^2 Cancri.

The star disappeared at $1^{\mathrm{h}}\ 31^{\mathrm{m}}\ 32^{\mathrm{s}}{\cdot}9$ L.S.T.
reappearance not seen.

The atmospheric tremor was very great.

Tuesday, November 8.

101 *Piscium.*

The star disappeared instantaneously at $1^{\mathrm{h}}\ 14^{\mathrm{m}}\ 17^{\mathrm{s}}{\cdot}7$ L.S.T.

The emersion is untrustworthy on account of the overpowering brightness of the moon.

Thursday, December 8.

The *Pleiades.*

s Tauri disappeared instantaneously at $21^{\mathrm{h}}\ 40^{\mathrm{m}}\ 10^{\mathrm{s}}{\cdot}6$ L.S.T.
and reappeared at 22 31 53? L.S.T. } emersion very doubtful.

27 *Tauri* was not occulted at all at this station, the moon's limb passing within a few seconds of the star, but never covering it.

28 Tauri disappeared instantaneously at $22^{\mathrm{h}}\ 25^{\mathrm{m}}\ 52^{\mathrm{s}}{\cdot}5$ L.S.T.
and reappeared about 23 6 33·0 L.S.T.

The telescope employed was my Ross Equatoreal of 4·2 inches aperture, with a power in all cases of 115, adjusted on the respective stars.

Observatory, Forest Lodge, Maresfield, December 8, 1859.

New Double Stars discovered by Mr. Alvan Clark.

(*Communicated by the Rev. W. R. Dawes.*)

(*Second series.*)

Currt No.	Designation.	R.A. 1860.	N.P.D. 1860.	Mag.	Dist.	Date of discovery.
		h m s	° ′		″	
13	Lalande 1980	1 0 51	45 32·4	8, 8	0·45	1858, Nov. 29
14	Lalande 2634	1 20 7	47 56·6	8, 9	0·8	1858, Oct. 13
15	99 Herculis	18 1 43	59 27·4	5½, 10½	1·7	1859, July 10
16	∗ Vulpeculæ	19 52	63 7	7½, 8	0·4	1858, Aug. 30
17	Cygni 153 B B.A.C. 6959	20 8 38	38 57·4	6, 11½	3·8	1859, July 19
18	44 Cygni	20 25 41	53 32·0	6½, 11½	2·8	1859, July 6
19	A.Z. xxiv. 11	21 10 50	26 10·2	7¼, 7¼+	0·9	1859, July 8
20	75 Cygni	21 34 41	47 21·6	5½, 11	2·9	1859, July 19

All the double stars in this series may be considered as good test-objects for telescopes of from 6 to 8 or 9 inches aperture. But it is not merely as such that they are interesting: they become especially so from the fact that they are all situated in the northern hemisphere, and all but one at more than 30° from the equator. They consequently attain a good altitude at Pulkova; notwithstanding which they have escaped the acute search of M. Otto Struve with the 15-inch refractor, in addition to the previous one by his father with the Dorpat telescope of 9·6 inches aperture. Either, therefore, they have recently undergone considerable change; or, if not, it appears that objects of great difficulty and delicacy may be detected with very perfect telescopes of smaller size, which have escaped the most diligent scrutiny with far larger instruments. The whole of these objects were discovered by Mr. Clark with his own object-glasses of 8 and 8¼ inches aperture, and five of them in my own observatory, during his visit to me last summer; since which I have met with *seven others*, of similar character and situation; and it should be mentioned that none of the objects recorded are below Struve's eighth magnitude for the larger star; all smaller, of which many have been found, having been systematically rejected. It appears, therefore, that there is still much occupation for telescopes of moderate dimensions, even in this department of astronomy, which might reasonably have been supposed to have been long since exhausted. The *distances* stated in the list are from my own recent measurements.

Haddenham, Thame,
November, 1859.

Remarkable Solar Spot. By the Rev. W. R. Dawes.

Having turned my 8¼-inch refractor upon the sun, about noon on the 22d of October, I noticed that near the south-eastern edge there was an unusually large mass of bright streaks, or *faculæ*. With the whole aperture and power 234 the edge was at times extremely quiet and very sharply defined. During the best views I satisfied myself that *a bright streak, which formed the very edge, projected irregularly beyond the circular contour of the edge;* reminding me of a ridge of low hills often seen at the enlightened limb of the moon. Nearly parallel to this, and about two or three seconds from it, ran another streak nearly as bright; and the space between them was observed to be darker than is the ordinary limb of the sun, but not so black as the *umbra* of a spot. I supposed it to be the *penumbra* of a large spot just entered on the disk, the umbra of which was as yet hidden by the inner bright streak.

At 2ʰ P.M. I went again to the telescope. The state of the air was less favourable; but during the most tranquil moments I satisfactorily made out an excessively narrow *black line*, a little broken in two or three places, as if by irregularities in the inner bright streak, the top or outer edge of which was projected upon it. Clouds soon covered the sky, and an extraordinary storm of hail, sleet, and snow, came on.

The 23d was cloudy. On the 24th a fine spot was visible, with a remarkably extensive penumbra. At about 1 o'clock I made a sketch of it. On the 26th I saw it again; and finding the *umbra* had changed its form in a very extraordinary way, I carefully sketched it under tolerably favourable circumstances. I subsequently saw it well, and sketched it carefully, on the 27th, 29th, and 31st of October, and on the 2d and 3d of November. The changes were more extraordinary than I ever before observed, and some of the forms which the umbra assumed were curiously grotesque. The accompanying tracings from my sketches are as faithful representations of their outlines as my eye and hand could produce.*

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* These delineations were exhibited at the meeting, and attracted much attention.—ED.

The last transient view which I had of the spot was on November 4th, in the afternoon; and it passed off in the night of that day.

It appeared to me very remarkable, that in so violent and extensive a disturbance of the luminous envelopes, I could not find in any part of the *umbra* a *really black nucleus*. In spots of not one-tenth part of the area I have, in numerous instances, perceived a decidedly *black* portion, not always central, but comparatively well defined, and totally devoid of any light visible through the darkening glass rendered necessary by the solar illumination of our own atmosphere. The umbra of this large spot was, however, of various depths of shade in different parts. It appears probable that the upward force by which the luminous envelopes were thrown aside must, in this instance, have acted upon them *very obliquely*.

To obtain ocular demonstration of the bright streaks being really *elevated ridges or waves* of the exterior luminous envelope, is, of course, a very rare occurrence; but in the present case the evidence was as complete as could be desired. The combination of circumstances was most fortunate; a bright streak of unusual size being precisely at the sun's edge, and the state of the air permitting the use of a large aperture and high power with full advantage.

The same spot has now come round again; but beyond the richness of its vicinity in *faculæ*, there is nothing remarkable in the group into which it has resolved itself.

Haddenham, Thame,
21 November, 1859.

Physical Observations of Jupiter's Satellites.
By W. Lassell, Esq.

I take the liberty of drawing the attention of telescopic observers to the fourth satellite of *Jupiter*, which, after a considerable interval, has this year begun to pass across the disk of the planet. I saw it last night from about $11^h\,30^m$ to $12^h\,30^m$ P.M. near the following and south limb. At first I took it be a shadow of one of the other satellites; but from the peculiarity of its aspect, was induced to refer to the *Nautical Almanac*, which showed it to be the body of the fourth satellite. On viewing it again, I thought it scarcely so *black* as the shadows ordinarily appear, yet so very dark that a cursory glance would certainly have taken it for a shadow; and it was difficult to conceive that any object so dark in its present aspect could ever appear as a bright one. I remember, many years ago, having noticed it as a dusky spot; but I fancy never so dark as now. The atmosphere was not fine enough to see anything new upon the face of the planet; but reflecting upon the very different phenomena now presented from those which marked the disk twenty years ago, one cannot but be filled with wonder at the stupendous chronic changes which must be taking place, even though they should be wholly or principally confined to the atmosphere of the planet.

I had an opportunity, a few minutes after midnight, of noticing the striking contrast between the bodies of the fourth and first satellite, the latter having then caught my attention as a very brilliant spot near the preceding limb. The observation of this contrast prepares one for the belief that the darkest solar spots might, if removed from the insupportable splendour of the sun's disk, appear as very bright and shining objects.

Bradstones, Sandfield Park, Liverpool, Dec. 8, 1859.

Physical Observations of Jupiter. By Sir W. Keith Murray.

"I enclose a few diagrams of *Jupiter*,* selected out of twenty-one taken since Sept. 26th between 15^h and 19^h G.M.T. The atmosphere has seldom been very favourable until this morning, when several conspicuous white spots were seen on the second belt, two of them being nearly vertical.

"The principal belt had quite a flocculent cloudy appearance; indeed, it presents a very mottled aspect this season; more so than I have noticed before. I hope other observers who possess sufficient means may have been favoured with more propitious weather than I have, as the thermometer has generally ranged from 25° to 33° Fahr. at the hours of observation.

"I have reason to think a small star has been occulted by *Saturn*, as I saw it near the planet on several successive mornings to the apparent east, but cloudy weather prevailed; it being seen till this evening, where I observe it now to the apparent west, and in such a position that I think the planet must have occulted it. It was less bright than *Dione* or *Tethys*.

"*Ochtertyre, Crieff, Nov.* 14, 1859."

Extract of a Letter dated Dec. 9th, from W. S. Jacob, Esq., to one of the Secretaries.

"Adverting to p. 15 of the *Monthly Notice* for November, I beg to refer you . . to my drawings of *Jupiter*, as seen at Madras in Dec. 1857 and Jan. 1858, copies of which were sent to the Society at the time. . . You will there see, I fancy, the same thing as M. Schwabe describes,—the shading of the belts in fine dark nearly parallel streaks, very different from anything I have seen described or delineated by any one else, whether by Dawes, Lassell, or Smyth (?); indeed it takes first-rate definition to bring out such fine markings, such definition as is rather rare in this climate, I believe. Dawes's round beads I have also never seen, but only occasional irregular blotches, more or less elongated (like cirrus clouds), here also agreeing with M. Schwabe."

On the Position of Venus during the Total Eclipse of July 18, 1860. By Prof. Chevallier.

"I have not observed any notice that *Venus* will be within a very few hours of her inferior conjunction with the sun during the total eclipse on the 18th of July, 1860. This will afford an excellent opportunity, within the track of the moon's shadow, of making observations on the character of the cusps of *Venus*, as affected by her atmosphere, and possibly of her diameter too.

"And, as the eclipse of the sun will be large over the whole of western Europe and in part of North America, the partial darkness may facilitate observations of *Venus* at the time, even in fixed observatories.

"I do not know whether it is worth while to notice that *β Leonis* will have very nearly the same N.P.D. as *Venus*, and nearly four hours greater Right Ascension.

"*Durham, Dec.* 8, 1859."

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* These diagrams were exhibited at the meeting.—Ed.

On the Physical Constitution of Comets. By O. G. Downes, Esq.

The author proposes to inquire how far the appearances and motions of comets may be due to the operation of known laws on a body similar in constitution to the earth. It is argued that if our earth were to move in a long elliptical orbit, the degree of cold to which it would be exposed in aphelion would be sufficient to solidify not only the aqueous matter on the surface, but the atmosphere itself; and the author proceeds to consider how the redissolution of the atmosphere, as the body again approaches the sun, would give rise to phenomena such as are presented by a comet. The author remarks that his hypothesis indicates that comets belong to the same family as planets, and that with continuously contracting orbits they will, in the fulness of time, take their place with the other planets in the solar system.

Description of an Equatoreal recently erected at Hopefield Observatory, Haddenham, Bucks. By the Rev. W. R. Dawes.

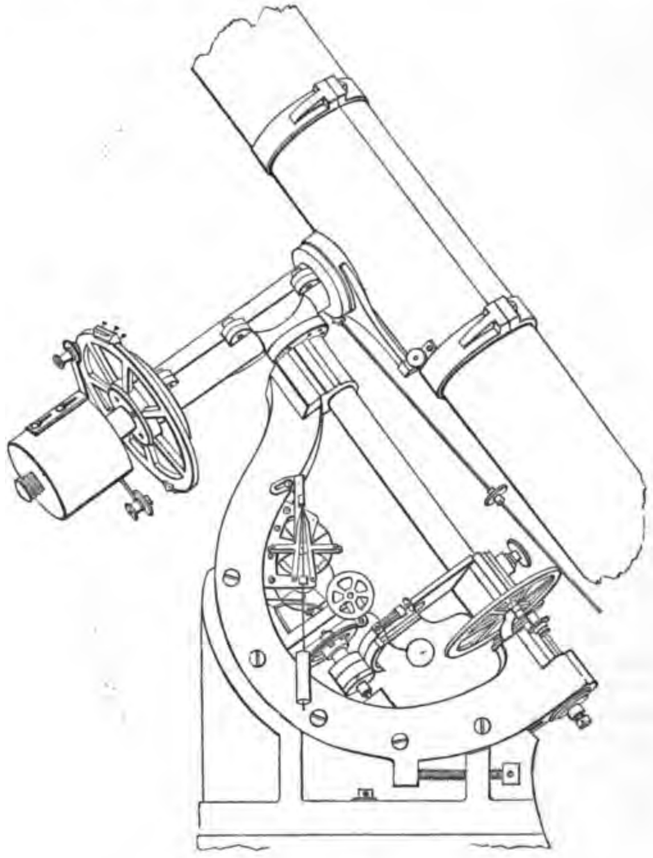
My observatory was furnished, in May last, with an equatoreally-mounted telescope by Messrs. Alvan Clark and Sons, of Boston, U. S., which in several important points differs from any other in this country; and I therefore hope that a brief description of it may not prove uninteresting to the Royal Astronomical Society.

The accompanying picture of it by Mr. Alvan Clark shows its general form, which combines great firmness and compactness with considerable elegance of design. The massive part of its structure is of cast-iron, the base of which is firmly bolted down to a stone pier. The semicircular form of the upper part affords a secure position for most of the wheel-work of the driving-clock, of which the going-weight descends in a groove on the east side of the pier, and is not seen in the drawing. The space between the polar axis and the semicircular bed-piece is occupied at its lower part by the hour-circle. Immediately above this is a sector, which clamps on to the axis, and the wheel-work of the clock occupies the upper portion. The sector has a radius of rather more than 9 inches, and an arc of 30°, or two hours of right ascension. This arc has a face of an inch and a half in breadth, between which and a cylinder 7 inches in circumference there is just room enough for two thin bands of sheet-brass, each of about three fourths of an inch in width, to pass side by side. These bands are both keyed by the end into one groove in the cylinder, at such a distance that they cannot overlap or interfere with each other. They are then bent round the cylinder in opposite directions, the end of one being fastened to one extremity of the arc of the sector, and the end of the other at the other extremity of the arc to a piece of brass which is acted upon by a screw and nut, for giving to both the bands a due degree of tension. The sector and cylinder thus move together without friction, irregularity, or lost time.

Upon the same arbor with the cylinder is the wheel, 15 nches in circumference, in the racked edge of which the lriving-screw works. This arrangement gives the screw about he same driving-power as if it acted on the edge of a wheel learly 40 inches in diameter, fixed on the polar axis.

I have every reason to be satisfied with the going of the riving-clock; and the cylindrical bob of the pendulum being crewed on to its steel rod, the rate is capable of adjustment to he greatest nicety. Great care has been bestowed by the makers upon the accurate dividing of the wheel-work; and I have much pleasure in acknowledging that its performance fully bears out my expectations, founded on the character given by the Messrs. Bond of the clock-work applied by the same makers to the great Munich equatoreal in Harvard Observatory, which has been so successfully employed for the purposes of telescopic photography. While the speed of the clock is regulated by the vibrations of the half-seconds pendulum, the action of the pendulum on the wheel-work is rendered smooth and equable by an ingenious application of Bond's *Spring-governor;* and so perfectly successful is this contrivance, that with the thread of the micrometer bisecting a star, and a power of 800 or 1000 on the telescope, no interruption or jerk from the escapement is perceivable.

For producing a slow motion in right ascension, the driving-screw is mounted on a brass frame, which, being carried by a fine screw under the observer's control, acts as a slipping-piece through nearly five minutes of time.



Equatoreal Mounting of a Telescope,
aperture 8¼ inches, focal length 110 inches,
by Alvan Clark and Sons, Boston, U.S.;
Erected in the Observatory of the
Rev. W. R. Dawes, Haddenham, Bucks.

A firm clamp, close to the cradle of the telescope, fixes the declination-axis, and is accessible to the observer both at the eye-end, and also during the setting of the declination-circle. A slow-motion screw acts on an arm extending from the clamp to the bottom of the cradle to which the screw is attached.

To permit the adjustment of the polar-axis to the latitude and meridian of the place, the upper part of the cast-iron bed-piece is made with a groove which receives loosely a projecting keel on the portion bolted down to the pier. The form both of the groove and of the keel being semicircular, the upper portion is moved upon the lower by the stout screw which is seen in the drawing, and the polar-axis is thus easily raised to the required angle. The adjustment to the meridian is performed by the screws on each side of the groove in the upper piece pressing against the keel in the lower, which has play enough in the groove to allow of a moderate degree of azimuthal motion.

To facilitate the finding of objects in Mr. Clark's "Two-eye-piece Micrometer," when their distance exceeds the field of one of the eye-lenses, the finder is furnished at its eye-end with a small position-circle divided into degrees. The thick wires of the finder being placed in the direction of the objects to be measured, the reading of the position-circle indicates the approximate setting for the micrometer, whereby the two objects may be immediately found by their respective eye-lenses. The aperture of the finder being two inches, it will show a star of the 9¼ magnitude of Struve's scale.

The object-glass of the telescope has a clear aperture of 8¼ inches, and a focal length of about 110 inches. The materials were furnished by Chance and Co., of Birmingham. The figure is excellent to the circumference, and the dispersion but little over-corrected. Its performance fully supports the character of Mr. Alvan Clark's object-glasses, and I believe it to be capable of everything which can be performed by such an aperture. It clearly divides γ^2 *Andromedæ*, and shows the smallest companions among the stars of the Pulkova Catalogue.

Haddenham, near Thame,
November, 1859.

The Astronomer Royal called the attention of the Society to the approaching total solar eclipse of 1860, July 18. His remarks were suggested, partly by the circumstance that the Superintendent of the *Nautical Almanac* had that evening presented to the Society copies of his new map of the path of the shadow through Spain and Algeria, and partly by the appearance in the French *Comptes Rendus* of suggestions by M. Faye as to the apparatus which might be used with advantage in the observations.

In the first place, a selection is to be made of the place of observation. The extreme facility now given by the French railways for the journey as far as the southern frontier of France (either at Bayonne or at Perpignan), and the continuation of the Spanish royal road, on which it is understood that the travelling is very good, through Vittoria and Burgos (both in the band of total obscuration), would probably determine amateurs in general to fix on one of these cities. But it ought to be considered that the heat in these places, at the season of the eclipse, would be very great; and the accommodation for travellers in Spain is said to be most uncomfortable to Englishmen. Those who might prefer the eastern coast of Spain would find it easy to go from Perpignan to Barcelona; but there would remain a long journey (on which he could give no information) to the point crossed by the eclipse-shadow. To himself it appeared that, if conveyance by sea could be obtained, it might be best to adopt the port of Santander, which is nearly in the centre of the shadow-path.

Adverting then to the phenomena of the eclipse, it was at once assumed that the appearances specially requiring attention are the "red prominences." Upon the locality and origin of these there is much obscurity. Illustrating his remarks by a small model, the Astronomer Royal endeavoured to point out what is known and what is not known. He could assert with the utmost certainty that, in the total eclipse of 1851, as the eclipse proceeded, red projections were lost on one side of the moon's black disk, and new red projections appeared on the other side, exactly as if they were attached to the sun, and were covered on one side and uncovered on the other side by the motion of the moon. On the other hand, it was not remarked by any of the observers that the red prominences which were visible throughout the totality (of which the most remarkable was that known as the "boomerang") altered their elevation in a degree corresponding properly to the motion of the moon. Moreover the appearance of the "boomerang" was not exactly the same at all observing stations; at some, instead of red, it was white; at some it was partially connected with the floating balloon, while at others no such connexion was seen. Apparently this had led some philosophers to conceive that, though the light which formed these prominences came in the first instance from the sun, the appearances remarked might in reality derive their peculiarities from some action of the surface of the moon upon the light which grazed over it. Cosmically, it is of the utmost importance to determine whether these appearances are attached to the sun or to the moon. This suggested the remarks which followed.

The second point, then, which requires the observer's attention is the mode of defining the angular position of any prominence upon the moon's edge at least twice during the obscuration. The telescope being firmly planted, if its motion were equatoreal the polar point or equatoreal point would be zero; if its motion were altazimuthal the vertical point would be zero. These would be equally good, and all that is then wanted is a mode of referring any prominence to the vertical line or the polar line passing through the centre of the moon's disk. M. Faye, in treating of this, had condemned all attempts at determining the angular position by anything like a toothed circle in the field of view surrounding the moon; first, because observations so made had hitherto been discordant, and, secondly, because they implied the use of a power so low that the whole disk could be seen. He proposed, instead, to use a wire or straight bar in the field, which could be made a tangent to the moon's limb at the point where the prominence is seen. By means of the model it was shown that if a prominence were seen on these parts, which are nearly 90° from the points of first and last totality-contact, its angular position, if it belonged to the moon, would not be altered; but if it belonged to the sun, it would be altered about 6° in the course of the obscuration. M. Faye conceived that this could be well determined by a tangent-wire. The speaker suggested for consideration whether, if the field of view were armed with six intersecting wires, dividing its circumference into twelve parts of 30° each, such a change of angle as 6° might not be very well estimated by the eye.

The third point is, the method of measuring the elevation of the prominences. By means of the model it was shown that the prominences to which the second head alludes would not sensibly change their elevation during the totality; but that those situated near the points of first and last totality-contact, if they belonged to the sun, would greatly change their elevation. Observers must prepare themselves with some system of measuring these elevations, at least approximately. Probably it would be very difficult to apply anything like screw-motion; but perhaps an estimation might be made, by the use of parallel wires at a known distance, or some other contrivance.

The fourth point requiring the observer's consideration is the mode of making a record. The difficulties to be overcome here are of three kinds: first, the shortness of time; secondly, the extreme darkness, which makes it difficult to read anything without artificial light; thirdly, the difficulty of keeping an assistant to his duty during the strange and appalling appearances of a total eclipse. The speaker recounted a history which he had heard from M. Arago of the preparations made in detail at one station for the eclipse of 1841, but which failed totally from the impossibility of maintaining discipline. In regard to the use of artificial light, the Astronomer Royal believed that, in the eclipse of 1851, he had lost much time and important results from attempting to refer to it, and he would earnestly dissuade observers from any use of a lamp. If an assistant were writing, or reading a chronometer, he must have a lamp. But without relying on an assistant, some arrangements like the following may be considered. M. Faye had proposed to register the position of a rotating wire (for angular position of prominences) by scratches upon a circle of soft tin. This gives no record of the order of succession of the registers. The man of cool temperament and good memory might trust to his recollection of the estimations of angle, &c. Or it might be advisable to acquire the habit of writing in the dark, and to devote one leaf of a small pocket-book to every record, however brief.

The fifth point is the method of registering time. The aid of a trustworthy assistant, with a chronometer and lamp before him, would remove all difficulties from this and other parts of the operation. Even in those parts where the observer and the chronometer-reader had both to make records, the use of a numerical reference (No. 5, No. 6, &c.) would connect both without risk. Or some mechanical device like the following might be used. The totality occupies less than 4^m of time. Suppose, then, a barrel to be made to revolve in about 5^m or 6^m of time (no accuracy is required). An impression might be made on this, without any sensible friction, by the touch of a camel's-hair pencil carrying oily ink (watery ink might dry up in the arid atmosphere). Upon this barrel, then, the time of every observation might be registered without confusion; and, the 60^s of chronometer last preceding and first following the totality being recorded in the same way, the zero and scale of time would be obtained.

The Astronomer Royal acknowledged his obligations to Mr. Carrington for remarks long since made by him on the importance of attempting to secure accuracy in these observations.

Finally, the Astronomer Royal urged strongly on the members of the Society that he had no intention to recommend to them the exclusive adoption of any of the methods which he had sketched out. The experience of two total eclipses had made him acquainted with the phenomena and with the difficulty of observing them properly; and his whole intention now was, after conveying some idea of the points to be observed, to impress strongly upon intending observers that they must have carefully considered the apparatus for observation, and must have fitted themselves with it, some months before the occurrence of the eclipse.

Dr. Lee read to the meeting and illustrated by large diagrams a paper of an interesting character by Mr. Pogson on the variable stars R *Ursæ Majoris*, S *Ursæ Majoris*, R *Cygni*, R *Cassiopeiæ*, and U *Geminorum*. The paper was read originally before the Mathematical Section at the Meeting of the British Association in September last, and was fully printed in the *Aberdeen Free Press* of September 16; but as many of the members had not had the advantage of hearing or seeing the paper, Mr. Pogson's results were looked upon with great interest.

MISCELLANEOUS INTELLIGENCE.

Le Verrier's ***Bulletin*** of the 8th December, 1859, contains some remarks by M. Faye on the recent experiments of M. Fizeau, considered in relation to the movement of translation of the Solar System. M. Fizeau had announced that in his experiments a variation of 30 kilometers per second in the velocity of a refracting apparatus placed in the path of a ray of polarised solar light was shown by a displacement of nearly a degree in the position of the plane of polarisation, when observed alternately in the direction of the ray and in the contrary direction. The experiments are published in the *Comptes Rendus*, 14th November, 1859. M. Faye considers that the experiments (although the author doubts their sufficient exactness for the purpose) ought to indicate the motion of translation of the solar system; but on discussing them in reference to this end, he finds that the influence of the annual motion of the earth is alone sensible, and that the introduction into the question of the motion of translation of the solar system causes in some of the observations discordances too great and too regular to leave any room for doubt; and he infers that, if the experiments of M. Fizeau are as exact as they appear to be, it must be concluded that the movement of translation towards λ *Herculis* does not exist; or if, on the contrary, this motion is to be taken as proved, and M. Otto Struve's velocity is not much too great (the recent researches of the Astronomer Royal on the proper motion of the Solar System seem to show that the velocity is to be increased rather than diminished), it must be concluded that the experiments of M. Fizeau are affected by some systematic error, or that there is some important lacuna in the theory adopted by him.

The Editor is requested to notify that the records of appearance seen on the sun by M. Pastorff, which are now in the custody of the Society, terminate abruptly in the year 1833, and that at present no reference is possible respecting the statement of M. Wartmann, that in the years 1834, 1836, and 1837, Pastorff saw little bodies form over the disk in comparatively short spaces of time,—a result quoted by Mr. Herrick, of Newhaven, U.S., in a letter to M. Leverrier. The statement will receive attention.

Humboldt Foundation.

The Committee, consisting of Prof. Magnus and others, state that their desire is to establish a fund called the Humboldt Foundation: a memorial of gratitude designed to promote scientific talent wherever it appears, in all those branches in which Humboldt developed his scientific energy, viz. in works of natural history and distant travels. The application of the fund to be intrusted to the Royal Prussian Academy of Sciences at Berlin, who have agreed to plan, in conjunction with the Committee, and according to the sum collected, the statutes of the foundation, and to superintend its proper administration. Contributions to be sent to the house of Mendelsohn and Co., Berlin, or to any member of the Committee.

Wanted to purchase, a good second-hand telescope with a maximum power of about 300 or 350, and two or three lower powers and a terrestrial eye-piece; apply to the Assistant-Secretary.

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Rev. R. MAIN, President, in the Chair.

Thomas Heelis, Esq., Princess Street, Manchester, was balloted for and duly elected a Fellow of the Society.

Evening Illumination of Fracastorius (Lunar Crater).

By W. R. Birt, Esq.

I obtained a fine view last evening, at $11^h\ 10^m$ G.M.T., of the crater *Fracastorius* described by Webb as an *imperfect* ring forming a bay of the *Mare Nectaris*, and marked on his index-map (372). There are several similar objects on the lunar surface demanding close attention, from the probability of portions of their walls having been destroyed since the formation of the craters. The terminator last evening just grazed the *outer* portion of the ring on the western side, and the angle of illumination was such as to show very distinctly every inequality of the floor and its surrounding *enclosure*. The ring was seen to be *complete*. The following is the entry I made at the time of observation :—"The north-eastern edge of the crater (northern ?) is perceptible *as a series of low points or peaks, casting very short shadows.*"

These low peaks, seen under such an illumination as I had last evening, completely separate the crater from the mare, and, combined with the difference of level, the floor of the crater appearing as if slightly depressed below the level of the mare, would indicate that whatever force might have *broken* down the portion of the wall *towards* the mare (if such has been the case), the floor in this instance *has not been altered* so as to produce the appearance of a filled or partially filled crater, giving rise to that of a bay. Several instances of partially filled craters may be found on the moon. The shadows of the eastern peaks were long, sharp, and well defined, one being interrupted by the brilliant point of a projecting crag. The unevenness of the floor was well seen. Epoch 1860, Jan. $12^d\cdot47$ G.M.T. Moon's age, $19^d\cdot73$. Libration, N.W.—$3^d\cdot76$. Epoch Jan. $16^d\ 5^h\ 34^m$. Perigee $+2^d\cdot84$. Epoch Jan. $9^d\ 15^h\ 0^m$.

Letter from the Rev. Father Secchi to the Astronomer Royal, received 1860, *January* 11.

"I take this opportunity to inform you that I have been progressing in my researches on the polarisation of light reflected by the moon. The principal result deserving attention is the almost perfect equality of polarisation which takes place in all the smooth parts of the moon, and that under any inclination of the plane touching the reflecting surface. The strongest indeed is in the *Mare Serenitatis*, but the difference compared with the other *maria* is quite insignificant; on the contrary, the tops of the mountains are not polarised at all in the neighbourhood of the Caucasus, Riphœi, and Apennini. In the neighbourhood of Tycho, where there is a great alternation of flat and mountainous region, the light is more polarised than at the above-mentioned mountains, and less than in the *maria*. The means of measuring the quantity of polarisation is by a pile of glass-plates as a polarometer in combination with a very sensible polariscope of *Arago*. The error in the angle is seldom one degree; but as I have not yet graduated my pile, I can only tell you the more or less, and I will give you afterwards the absolute numbers of the quantity of polarised light. This seems to me a very remarkable thing, as I have said in the *Comptes Rendus*, that we find equally polarised light on the whole smooth surface of the moon, whatever be the inclination of the incident and reflected rays. The only thing which practically gives the same result is the glass paper used in the arts. Can this kind of surface have influence in the *corona* seen during eclipses? this is not improbable. I have made this experiment:—Through a telescope I have looked at the image of the sun made in the focus of an object-glass, and have imitated the eclipse, putting in the way of the luminous rays two objects: 1st, a smooth cylinder; 2d, a cylinder covered with glass (or emery) paper, and I have seen great difference in the outline of the termination of the edges; the latter seemed to have a bright fringe and some scattered rays starting from the edges. If the moon is so constituted, no wonder if a similar effect be produced and the *corona* so generated. But as I have never seen the *corona*, I cannot say whether the appearance observed by me was equal to it or not. The experiment might be repeated by those who have seen it and can judge better.

"*Jupiter* now has a very remarkable appearance: the principal belt is decidedly red, and several bands of green and white alternate in such a way that I have never seen such colours before. I will send you some drawings."

On Dr. Sœmmerring's Observations of the Solar Spots in the Years 1826, 1827, 1828, *and* 1829. *By* R. C. Carrington, Esq.

When I visited North Germany in the year 1856, one object which I had in view was to obtain personal information of the observations of the solar spots made by Dr. von Sœmmerring; as I thought it probable, from the account given of them by Professor Thilo, in a dissertation published in the year 1828, that records made by a man of Sœmmerring's eminence would exhibit a degree of accuracy which would repay the labour of reduction; and, when reduced, would put me in immediate possession of an ancient series which might enable me to obtain a more exact value of the time of rotation of the sun on its axis. I started under the impression that these records extended over a much longer period than I have since found to be the case, and that they were deposited in the Library of Göttingen,—a supposition also erroneous, as, on my arrival at Göttingen, I found they were at Frankfort, but could not learn with sufficient certainty in whose immediate possession, to induce me to change my plan of travel. On my return home, I wrote respecting them to Dr. Argelander of Bonn, who very obligingly communicated my wish to Dr. Wilhelm Sœmmerring, the son; and, by his intervention, the volumes were in the course of a few months transmitted to me in England, with the permission to retain them till I had extracted from them the desired information. My time and that of my assistant were so fully taken up in the preparation for the press of my Circumpolar Star Catalogue to the end of the year 1857, that I took no immediate steps with the volumes entrusted to me; and when that work was off my hands, it appeared to me to be desirable that the reductions of my own solar observations should first be brought up to date, and that the additional acquaintance with the subject which would then be gained would probably be of advantage in the proposed treatment of the older and less exact records. I was still thus engaged when, in July 1858, my astronomical plans were interrupted by an event which has been previously alluded to, and which prevented my doing more at the time than giving the Society an account in November 1858 of some results thus derived from my own records. In the past year, 1859, I was first able to make and to get made by Mr. Henry Criswick a transcript of Sœmmerring's diagrams, and subsequently to effect their reduction by the assistance of Mr. Carl Moritz von Bose. The foregoing explanation of the long detention of his father's valuable MSS. is mainly due to the present Dr. Sœmmerring, and will, I hope, be pardoned by the readers of this paper.

Some account of Dr. Sœmmerring's observations to the year 1827 may be found in the tract above referred to, which was published at Frankfort on the occasion of Sœmmerring's attaining his Jubiläum, in which also are given two plates exhibiting the distribution of the spots during the last six months of 1826 and the first six months of 1827, and an account of the process of reduction followed by Dr. Thilo. He finds his results to be generally confirmatory of those of Lalande respecting the comparative paucity in the neighbourhood of the equator and the general limits north and south of the principal zones of action. But I do not find in this tract, which, indeed, was intended to be complimentary, any material addition to the subject, or any indication that a period of maximum energy was approaching, as the subsequent labours of Schwabe showed to have occurred. This publication gave rise to another by Dr. Schweigger of Halle in the year 1829, "Uber die Natur der Sonne mit Beziehung auf v. Sœmmerring's Sonnenbeobachtungen," but to which I need not now refer.

The records of Sœmmerring I found to consist of diagrams of the sun's disk, contained in two thin stitched books, apparently put together by himself, and two larger bound volumes, the drawings in which could scarcely be other than copies of those made at the telescope, to judge by the size of the volumes and the nicety of the execution of details. Excepting a few diagrams on a larger scale (and not always the same) taken principally in April, May, June, and July, 1826, and which I rejected entirely from the insufficiency or total want of indication of the direction of diurnal motion; the disk of the sun was uniformly represented by a circle of the diameter of 90 millimeters, crossed by two parallel vertical lines and two parallel horizontal lines, the intersection of which formed a square in the centre whose sides were each 30 millimeters, and 30 millimeters distant from the zenith, nadir, and E and W horizontal points. Fig. 2 of Table II. in Dr. Thilo's tract may be taken as a faithful specimen with the correction that for N and S (north and south) should be read N and Z (nadir and zenith), for the evidence of the record is conclusive that the upper point of the circle in all the drawings is the nadir point of the disk, allowing for such error as the representations are liable to. I conclude that the telescope employed had a motion round axes intended to be vertical and horizontal, and that in its field were cross-wires dividing the disk roughly into vertical and horizontal third parts, and that the positions recorded were estimated by the eye, with the assistance of the square of wires screwed approximately into the centre of the visible disk of the sun. Apart from the excessive trouble of this method of observation, especially when the sun is high in the heavens, it is liable among other grave objections to this, that the fixed square divides the disk into inexact thirds by reason of the variation of the sun's diameter, even supposing the observer could, by the mechanism of the instrument, place it and keep it in the desired position. The errors of this class are, however, insignificant in comparison with those of estimation and of that of the north point deduced from the given nadir point and the time, for, during the two first years, the time of observation only purports to be given to the nearest hour or half-hour, and for the two last years to the nearest quarter. In naming this last defect as decidedly the weakest part of the records, and which leads me to doubt if a record of position was a material part of the observer's plan, I may name as a set-off that the records are for the most part so continuous, and the positions of the same spots on successive days so numerous, that a computer at once sees that the method of means will afford him considerable aid, and reduce the error of the concluded position of a group very materially below that of a single observation. This circumstance, in fact, principally induced me to undertake the reduction of these observations, coupled with the belief, founded on some research, that from the general inattention to the subject by more practised micrometrical observers I could not expect to find elsewhere any series offering more precision.

After rejecting about 80 diagrams in the first half of 1826, there remained suitable for my purpose about 80 others taken between August 3 and Dec. 31 of that year, and about 160 between January and July 31, 1827, after which, for a period of seven months, ending with Feb. 1828, no observations occurred. From the 1st of March, 1828, the record was resumed with increased care and with a little more precision in the recorded time of observation, and continued to December 31, 1829, each of the years 1828 and 1829 affording about 240 diagrams. Of these 720 disks in all, Professor Thilo's tract comprises the first third part only; and so far as I know no one previously to myself has instituted an examination of the last and more valuable portion, which happens to extend (as I have marked on the map) over a period of maximum frequency, as ascertained by Prof. Wolf of Zurich from the observations of Schwabe and Staudacher.

For the double reason that I decided on performing the reduction by graphical process, for which I had materials already prepared for disks of 12 inches, and that I desired to possess a duplicate of the records, I employed Mr. Criswick to copy the whole of the 720 drawings for me on a scale increased about 3·4 times from 90 millimeters to 12 English inches. This was done very expeditiously, in about three weeks, by the use of a pentagraph constructed to work in two different planes; and although the instrument employed, being of home-manufacture, did not possess perfect accuracy, its errors of indication were ascertained by trial, and so allowed for in transcription as not to introduce any material additional error of position. The copies thus made are bound in three volumes, which were exhibited at the evening meeting of the Society in January 1860.

For the reduction by graphical process of the positions of the spots at any one time, three elements have to be computed.

1st. The angle from the zenith and nadir points given on the record, at which the projection of the sun's axis cuts the circle.

2d. The angle of inclination to the plane of the sun's equator of the line joining the centres of the earth and sun, which is sensibly the same as the direction of vision.

3d. The meridian on the sun, which at the time of observation passed through the centre of the disk.

When these elements are known, and a set of projections of the principal meridians and parallels of latitude has been previously prepared for different convenient values of the second element, the position of any spot for any day may readily be seen on inspection by superposing on the record at the suitable angle (1) the suitable projection (2), and reading off the longitudes by their differences from (3). Angle (1) is the sum (with the proper signs) of three other angles, S the angle at sun's centre between the pole and zenith of Frankfort, β the angle by which the earth's axis would appear inclined to a vertical on the ecliptic to an observer in the sun, and γ the angle by which the sun's axis would appear inclined to a similar vertical to an observer at the earth's centre. Of these β is readily computed from the sun's longitude as argument, and γ from the sun's longitude less that of the node of the sun's equator (the elements employed for the sun were $N=74^\circ\ 8'$ and $I=7^\circ\ 10'$); but S depends on the hour-angle or time of observation, and is liable to considerable error in certain positions from insufficient precision in this datum. Angle (2) readily follows from the element I and the argument $\odot - N$; and angle (3) is found from the given time by the synodic period of rotation (taken as 27·275 days) from any meridian taken as the primary. In my reduction the values at G.M. noon on Jan. 1 are, for 1827, . . 241° 27′; for 1828, . . 103° 52′; and for 1829, . . 313° 3′: and the following extract will show the process followed and the degree of accuracy considered sufficient:—

1828.	F.M.T.	h	N.P.D.	$\odot$	$\odot-$N	β	γ	δ
	h m	h m	° ′	° ′	° ′	° ′	° ′	° ′
Mar. 1	8 0	4 13 E.	97 33	340 42	266 34	+22 16	−0 27	97 9
2	12 0	0 12 E.	97 6	341 52	267 44	+22 25	−0 17	97 9
3	8 45	3 27 E.	96 47	342 44	268 36	+22 31	−0 10	97 10
4	10 0	2 12 E.	96 22	343 47	269 39	+22 37	−0 3	97 10
5	8 45	3 27 E.	96 1	344 44	270 36	+22 43	+0 4	97 10
6	16 0	3 49 W.	95 31	346 2	271 54	+22 50	+0 14	97 10

1828.	Day.	S	Δ S	S+β+γ	Sheet.	M	Rotation.
		° ′	′	° ′	° ′	° ′	
Mar. 1	60·309	−35 38	+ 41	−13 49	7 20	34 27	22
2	61·475	− 2 21	+114	+19 47	”	19 4	22
3	62·340	−31 55	+ 59	− 9 34	”	7 38	22
4	63·392	−23 1	+ 84	− 0 27	”	353 45	23
5	64·340	−32 1	+ 58	− 9 14	”	341 14	23
6	65·652	+34 5	+ 51	+57 9	”	324 3	23

In the above, h is the hour-angle from apparent noon at Frankfort; the numbers in column "Day" are the days of the year to three decimals, in which Frankfort times have been converted into Greenwich mean civil times; and the quantities "Δ S" are the increments of "S" for 10 minutes' increment of the time of observation. Similar quantities to the above were computed by Mr. von Bose, under my direction, for the whole of the 720 disks, and the results subsequently checked throughout.

In the application of these elements, it was a question in what way the errors of observation, individually large, and the error of North Point, should be treated. The application of equations of condition, or any elaborate treatment, was out of the question altogether in results of this character; and it appeared to me a method of approximation based on the general principle of taking means would best serve my purpose. Accordingly I made trial of the following scheme, of first drawing in lightly the projected principal meridians and parallels resulting from the given time strictly; then assembling all the individual positions of the same spots, and adopting mean places of the first approximation; next by comparing the positions of each day with the previously taken means to ascertain by what allowable alteration of the angle S the discordances of the day could most be reduced; and so for each day in succession: finishing by finally drawing in the principal meridians and parallels with the corrected values of S, and taking concluded mean positions of the spots and groups. In my own hands this method presented no difficulty; but I perceived that the concluded means, for the most part, differed so little from those of the first approximation when a little judgment was used in the rejection of any manifestly bad subject, that the labour of repetition was hardly compensated. In the hands of a comparatively unpractised assistant I found it, on the other hand, difficult to lay down satisfactory rules for his guidance in my absence from home; and, finally, I fell back to the principle of simply taking mean results, rejecting manifest errors, and depending as little as possible on positions near the limb, in which the errors of S had full effect. It is desirable that I should give a single example, in order to show the degree of uncertainty attaching to the concluded places.

The following may be taken of a group:—

	Longitude.	Latitude.	Dimensions.		
	4°	17° S.	—° long	by	4° broad.
	9	16	2		4
	7	16	2		4
	0	12	3		3
	2	12	2		3
	355	18	3		2
	354	14	—		—
	2	19	2		2
	357	21	2		2
	0	20	3		2
Mean	1	16 S.	2		3

In this case no result was rejected as affected with extraordinary error, and though the process is rude I believe the final result for the group to be nearly as correct as any other that could be obtained by more refined means. The positions of groups thus obtained were as concluded laid down on a series of sheets (exhibited at the meeting) with a boundary line indicating the general limit of the observed area of disturbance, each sheet comprising half a rotation of the sun.

In the present state of the subject I do not think I should be justified in asking the Society to publish even the concluded results *in extenso*, and I limit myself to making known that the results are so far extracted from the observations, and are in my possession and open for use to any student of the subject to whose researches they may be of service. I propose only to give a graphical summary of the distribution in latitude for the four years, similar to that printed with my former paper of November 1858. The distribution in latitude about the time of the last minimum presented a remarkable and novel peculiarity; and it was with some curiosity that, in recovering four years of the sun's history a quarter of a century distant, comprising a period of maximum energy, I laid out a corresponding map to see if any peculiarity marked the maximum of 1829 The present map,* however, offers no such singular change of distribution; and the only remark it appears to give rise to is that during 1828 and 1829 the region of the equator seems to have been less respected than is usual, and that the size of the groups (although in some degree due to exaggeration in the original) as well as their number, was unusually great. The combined evidence of my two maps has made me very sceptica on the alleged observation at any time of spots in high latitudes I shall shortly give an instance of an extreme low south grou in latitude 44°, observed by myself on July 1, 2 and 4, in th year 1858; but with this exception I now have before m reduced records of nearly 10 years of the sun's history withou a similar or nearly a similar case; and I think I shall not be judged hasty in saying that I discredit the oft-quoted assertion that spots have been seen in a latitude of 70°, and that I believe the statement has arisen by error in reduction.

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* See map at end of volume.

It was stated above that I sought for the records of Sœmmerring with the view of obtaining the possible means of arriving at a more correct value of the period of the sun's rotation; and I have now to add that hitherto I have been baffled in every effort to recognise any indications of true recurrence which might enable me to arrive at this much-desired result, and that against my will I seem almost forced to the conclusion that every meridian of the sun within the bounding latitudes is indifferently visited by these singular phenomena, still wanting an adequate theory. In common with many others I have sought to identify the recurrence of individual spots and groups by peculiarities of size or form, and have gone about the inquiry prepared for a very different true period than that at present assumed. Failing in this way, I have grouped the outbreaks of successive batches of 4 or 5 rotations, with the object of recognising any preponderance of action which might exist in any quadrant, or any absence of outbreaks in corresponding regions, or again any tendency in outbreaks to reach higher latitudes in some longitudes rather than in others on the whole; but as yet the clue fails me. The problem is undoubtedly of high importance for the physical theory of the sun; and though the siege operations may be protracted, I have no doubt that a conclusion will in due time be arrived at. It need hardly be pointed out that whenever an indication of recurrence shall be ascertained, the existence of a body of observations such as the present will enable any computer in an hour to step at once to a period of rotation true to several places of decimals of a day.

On the Latitude and Longitude of the Sydney Observatory.
By W. Scott, Esq., Director of the Observatory.

"I send my determination of the latitude and longitude of the Sydney Observatory. The instrument employed is a meridian-circle by Jones, diameter of circle 42 inches, focal length of telescope 63 inches, aperture 3¼ inches.

"The circle has been recently repaired, considerably modified and redivided, by Messrs. Troughton and Simms. A comparison of the latitude results obtained from different stars will show how far the accuracy of its division can be relied on.

"A second report on the observatory is now in the press.

Latitude of Sydney Observatory.

Star.	No. of Obs.	Approx. Z.D. North.	Mean of Latitudes.	Greatest error from mean.
ρ Bootis	3	64° 51′	33° 51′ 41″·16	2″·4
α Bootis	7	53 46	42·07	2·6
η Bootis	9	52 58	42·45	2·8
α Herculis	7	48 25	39·25	3·9
α Serpentis	9	40 44	40·60	2·2
ζ Virginis	12	33 58	41·70	3·9
δ Ophiuchi	5	30 32	40·88	1·6
θ Virginis	8	29 4	40·82	1·5
β Libræ	4	24 59	40·82	1·5
α Virginis	9	23 26	42·15	2·3
$α_2$ Libræ	8	18 24	40·69	1·7
α Canis Major.	9	17 20	42·13	2·8
$β_1$ Scorpii	13	14 26	40·36	2·0
β Corvi	3	11 14	40·43	2·1
θ Ophiuchi	13	9° 0′	40″·13	2″·8
α Scorpii	9	7 44	41·52	3·1
		South.		
α Argûs	8	18 45	41·86	1·8
α Pavonis	7	23 19	41·05	1·1
β Centauri	8	25 50	42·48	3·0
β Chamæleon	4	44 40	40·85	2·6
β Hydri	4	44 10	43·22	0·8
α Triang. Aust.	13	34 54	41·29	2·5
σ Octantis	13	55 25	39·05	2·7
β Hydri S.P.	7	68 6	39·01	2·1

Mean of northern stars	33° 51′ 41″·07
Mean of southern	41·10
Adopted latitude	33 51 41·1

Longitude from Moon-Culminating Stars.

Limb I.

Assumed Approximate Longitude $10^h\ 4^m\ 50^s$

Date	1859, June 9	June 10	June 13
Stars	η Virginis q Virginis	α Virginis	δ Scorpii $β_1$ Scorpii
Observed R.A.....	12° 15′ 40″·07	13° 5′ 39″·07	15° 45′ 2″·71
Computed R.A. ..	12 15 40 ·47	13 5 39 ·40	15 45 3 ·04
Difference	+ 0 ·40	+ 0 ·33	+ 0 ·33
Correction to Long.	+ 11^s·98	+ 9^s·76	+ 8^s·79
Date	June 14	July 7	July 8
Stars	α Scorpii τ Scorpii	γ Virginis ψ Virginis	83 Virginis 89 Virginis
Observed R.A.....	16° 41′ 51″·84	12° 49′ 42″·18	13° 40′ 48″·33
Computed R.A. ..	16 41 52 ·49	12 49 42 ·50	13 40 48 ·56
Difference	+ 0 ·65	+ 0 ·32	+ 0 ·23
Correction to Long.	+ 16^s·98	+ 9^s·41	+ 6^s·64
Date	July 9	11	13
Stars	5 Libræ $α_2$ Libræ	σ Scorpii α Scorpii	θ Ophiuchi d Ophiuchi δ & λ Sagittarii
Observed R.A.....	14° 33′ 17″·20	16° 23′ 26″·21	18° 16′ 54″·06
Computed R.A. ..	14 33 17 ·50	16 23 36 ·29	18 16 53 ·96
Difference	+ 0 ·30	+ 0 ·08	− 0 ·10
Correction to Long.	+ 8^s·39	+ 2^s·11	− 2^s·68
Date	August 6	August 11	August 12
Stars	20 & ι Libræ α Scorpii	h_2 Sagittarii f Sagittarii	f Sagittarii $τ_2$ Capricorni
Observed R.A.....	15° 9′ 42″·75	19° 47′ 10″·62	20° 37′ 21″·99
Computed R.A. ..	15 9 43 ·27	19 47 10 ·49	20 37 21 ·81
Difference	+ 0 ·52	− 0 ·13	− 0 ·18
Correction to Long.	+ 14 ·05	− 3^s·76	− 5^s·49
Date	August 13	Sept. 7	Sept. 9
Stars	ι Capricorni γ Capricorni	σ Sagittarii $χ_3$ Sagittarii	θ Capricorni ι Capricorni
Observed R.A.....	21° 24′ 58″·44	19° 29′ 37″·17	21° 9′ 5″·84
Computed R.A. ..	21 24 58 ·25	19 29 37 ·06	21 9 5 ·69
Difference	− 0 ·19	− 0 ·11	− 0 ·15
Correction to Long.	− 6^s·08	− 3^s·13	− 4^s·72

Date	Oct. 5	Oct. 7	Oct. 8*
Stars	ω Sagittarii A Sagittarii	θ Capricorni γ Capricorni	θ Aquarii
Observed R.A.	20° 1′ 55″·89	21° 38′ 15″·61	22° 23′ 30″·79
Computed R.A.	20 1 55 ·85	21 38 15 ·49	22 23 30 ·31
Difference	− 0 ·04	− 0 ·12	− 0 ·48
Correction to Long.	−1^{s}·17	− 3^{s}·88	−15^{s}·95

Date	Oct. 9*	Oct. 10*	Oct. 11*
Stars	κ Piscium	ε Piscium 22 Piscium	ω Piscium
Observed R.A.	23° 7′ 59″·62	23° 52′ 42″·05	0° 38′ 41″·06
Computed R.A.	23 7 59 ·19	23 52 41 ·86	0 38 41 ·31
Difference	− 0 ·43	− 0 ·19	+ 0 ·25
Correction to Long.	−14^{s}·37	− 6^{s}·24	+ 7^{s}·90

Limb II.

Date	June 15	June 20	June 21
Stars	θ Ophiuchi d Ophiuchi	μ Capricorni	67 Aquarii λ Aquarii
Observed R.A.	17° 41′ 31″·57	21° 57′ 45″·54	22° 42′ 21″·41
Computed R.A.	17 41 32 ·26	21 57 45 ·59	22 42 21 ·46
Difference	+ 0 ·69	+ 0 ·05	+ 0 ·05
Correction to Long.	+18^{s}·63	+ 1^{s}·64	+ 1^{s}·67

Date	June 22	July 18	July 22
Stars	κ Piscium λ Piscium	σ Aquarii κ Aquarii	η Piscium
Observed R.A.	23° 26′ 31″·55	22° 26′ 58″·50	1° 28′ 2″·12
Computed R.A.	23 26 31 ·50	22 26 58 ·81	1 28 2 ·83
Difference	− 0 ·05	+ 0 ·31	+ 0 ·71
Correction to Long.	− 1^{s}·67	+10^{s}·39	+21^{s}·35

Date	July 23	Aug. 17	Aug. 19
Stars	α Arietis η Arietis	45 Piscium	α Arietis
Observed R.A.	2° 19′ 44″·61	0° 25′ 22″·79	2° 1′ 20″·50
Computed R.A.	2 19 45 ·17	0 25 23 ·15	2 1 21 ·22
Difference	+ 0 ·56	+ 0 ·36	+ 0 ·72
Correction to Long.	+14^{s}·92	+11^{s}·79	+20^{s}·89

Date	Aug. 21	Aug. 22	Sept. 13
Stars	η Tauri A_1 Tauri	β Tauri	45 Piscium
Observed R.A.	3° 53′ 16″·04	4° 56′ 16″·28	0° 11′ 6″·89
Computed R.A.	3 53 17 ·02	4 56 17 ·30	0 11 7 ·36
Difference	+ 0″·98	+ 1 ·02	+ 0 ·47
Correction to Long.	+24^{s}·23	+23^{s}·65	+15^{s}·43

Date	Sept. 17	Sept. 18	Sept. 19
Stars	17 Tauri	ν^1 Tauri	β Tauri ζ Tauri
Observed R.A.	3° 34′ 22″·61	4° 34′ 49″·00	5° 38′ 26″·76
Computed R.A.	3 34 23 ·37	4 34 49 ·99	5 38 27 ·73
Difference	+ 0 ·76	+ 0 ·99	+ 0 ·97
Correction to Long.	+19^{s}·51	+23^{s}·90	+22^{s}·58

Date	Sept. 20	Oct. 12*
Stars	μ Geminorum	ε Piscium η Piscium
Observed R.A.	6° 43′ 22″·66	1° 29′ 11″·10
Computed R.A.	6 43 23 ·64	1 29 11 ·71
Difference	+ 0 ·98	+ 0 ·61
Correction to Long.	+22^{s}·74	+18^{s}·20

* Assumed Longitude 10^{h} 5^{m} 0^{s}

Mean of results			
	Limb 1	+ 3^{s}·26	21 observations
	Limb 2	+ 16·46	17 "
	Mean	+ 9·86	

Resulting Longitude 10^{h} 4^{m} 59^{s}·86

Apparent Right Ascension of the Moon's Limb, and of Moon-culminating Stars, observed with the Transit Instrument of the Observatory, Washington.

(*Communicated by Lieut. Gilliss.*)

Date.	Name of Object.	No. of Wires.	Apparent R.A. Observed.	Apparent R.A. Computed.
1839.			h m s	s
Aug. 6	☽'s I. Limb	25	15 45 2·915	3·01
	δ Scorpii	15	52 3·338	
	β′ Scorpii	25	57 17·806	
	σ Scorpii	25	16 12 40·854	
	α Scorpii	25	20 49·610	
	τ Scorpii.	15	27 10·213	
8	A^1 Ophiuchi	15	17 6 44·566	
	θ Ophiuchi	25	13 24·977	
	d Ophiuchi	25	18 25·192	
	C^2 Ophiuchi	15	22 52·715	
	☽'s I. Limb	25	38 27·997	28·37
	4 Sagittarii	15	51 15·009	
	γ Sagittarii	15	56 49·283	
	μ′ Sagittarii	25	18 5 23·898	
10	φ Sagittarii	25	36 55·067	
	σ Sagittarii	25	46 35·535	
	τ Sagittarii	15	58 12·453	
	χ′ Sagittarii	15	19 16 45·835	
	☽'s I. Limb	25	28 7·479	7·74
	f Sagittarii	15	38 12·317	
16	ι Piscium	25	23 32 45·666	
	21 Piscium	25	42 18·107	
	☽'s II. Limb	25	0 8 53·401	53·37
	d Piscium	25	13 24·287	
	45 Piscium	25	18 29·507	
17	d Piscium	25	13 24·164	
	45 Piscium	25	18 29·423	
	δ Piscium	15	41 25·667	
	☽'s II. Limb	25	54 33·444	33·42
	e Piscium	15	1 1 9·840	
17	η Piscium	25	1 23 59·843	
	π Piscium	15	29 40·888	
18	η Piscium	25	23 59·921	
	π Piscium	25	29 40·936	
	☽'s II. Limb	25	42 48·392	48·40
	β Arietis	15	46 54·733	
	ι Arietis	15	49 42·405	
	α Arietis	25	59 17·147	
Sept. 2	☽'s I. Limb	25	15 23 47·935	48·10
	δ Scorpii	15	52 2·924	
	β′ Scorpii	25	57 17·471	
	α Scorpii	25	16 20 49·127	
3	δ Scorpii	15	15 52 2·889	
	☽'s I. Limb	25	16 21 13·021	13·09
	τ Scorpii	10	27 9·954	

Apparent Right Ascension of the Moon's Limb, and of Moon-culminating Stars, observed at Washington.

Date.	Name of Object.	No. of Wires.	Apparent R.A. Observed.	Apparent R.A. Computed.
1859.			h m s	
Sept. 6	ϕ Sagittarii	25	18 36 54·665	
	29 Sagittarii	15	41 21·803	
	σ Sagittarii	25	46 35·339	
	ζ Sagittarii	15	53 42·415	s
	☽'s I. Limb	25	19 10 14·053	14·51
	χ^3 Sagittarii	25	17 1·566	
	h^2 Sagittarii	25	28 11·573	
7	h^2 Sagittarii	25	28 11·646	
	f Sagittarii	15	38 12·137	
	c Sagittarii	15	54 3·410	
	☽'s I. Limb	25	20 2 18·750	19·14
	σ Capricorni	25	11 19·438	
	ρ Capricorni	15	20 53·020	
8	σ Capricorni	25	11 19·485	
	ρ Capricorni	25	20 53·106	
	ν Capricorni	25	32 5·453	
	☽'s I. Limb	25	51 39·785	40·01
	θ Capricorni	15	58 5·333	
	ν Aquarii	15	21 1 58·832	
	ι Capricorni	25	14 27·837	
9	θ Capricorni	15	58 5·291	
	ι Capricorni	25	14 27·818	
	γ Capricorni	15	32 20·846	
	☽'s I. Limb	25	38 37·681	37·92
	μ Capricorni	25	45 40·638	
	ι Aquarii	25	58 53·406	
	θ Aquarii	25	22 9 27·663	
10	δ Capricorni	25	21 39 19·668	
	ι Aquarii	15	58 53·439	
	θ Aquarii	25	22 9 27·603	
	☽'s I. Limb	25	23 51·791	52·00
	κ Aquarii	15	30 31·320	
	λ Aquarii	25	45 19·592	
12	γ Piscium	25	23 9 55·464	
	κ Piscium	25	19 46·392	
	ω Piscium	15	52 8·489	
	☽'s II. Limb	25	54 38·876	38·93
	d Piscium	25	0 13 24·645	
	45 Piscium	25	18 29·967	
	12 Ceti	25	22 54·626	
13	35 Piscium	15	7 47·232	
	d Piscium	25	13 24·672	
	45 Piscium	25	18 29·907	
	☽'s II. Limb	25	40 8·067	8·14
	ϵ Piscium	25	55 41·709	
18	υ' Tauri	25	4 17 56·268	
	τ Tauri	25	33 50·787	
	ι Aurigæ	25	47 52·899	
	ι Tauri	15	54 43·778	
	☽'s II. Limb*	25	5 14 53·466	52·83
	β Tauri	15	17 26·502	
	ζ Tauri	25	29 16·500	

The observations of September 2d were made by Professor ajor; the others by Professor Yarnall.

The computed place of the moon is taken from the American phemeris.

December 12, 1859.

* Moon's limb very jagged.

Results of Meridional Observations of Small Planets; Occultations of Stars by the Moon; and Phenomena of Jupiter's Satellites; observed at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

Hebe ⑥.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, Nov. 9	10 18 18·1	1 32 30·96	108 10 26·90
15	9 52 32·2	1 30 20·18	107 46 34·94
19	9 35 54·8	1 29 26·23	107 22 55·71
21	9 27 46·6	1 29 9·85	107 9 0·35
Dec. 1	8 48 55·5	1 29 37·89	105 40 39·90
5	8 34 13·6	1 30 39·85	104 58 1·34
7	8 27 2·8	1 31 20·97	104 35 23·33
8	8 23 30·4	1 31 44·53	104 23 48·37
13	8 6 11·6	1 34 5·73	103 23 0·12
21	7 39 50·3	1 39 12·52	101 38 17·89

Parthenope ⑪.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, Nov. 15	13 9 34·0	4 47 54·35	74 4 39·04
21	12 40 13·3	4 42 8·20	74 14 3·68
30	11 55 32·1	4 32 48·61	74 25 56·87
Dec. 5	11 30 39·6	4 27 34·80	74 30 51·13
7	11 20 45·0	4 25 31·72	74 32 25·27
8	11 15 48·8	4 24 31·28	74 33 6·89
19	10 22 24·6	4 14 20·45	74 35 42·96
28	9 40 35·4	4 7 53·41	74 30 24·14

Polyhymnia ㉝.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, Nov. 21	12 4 45·2	4 6 34·22	66 25 58·27
Dec. 5	18 55 37·4	3 52 26·86	67 2 6·62
28	9 10 18·1	3 37 31·17	67 54 49·40

Leda ㊳.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, Nov. 1	11 59 14·4	2 42 11·39	62 35 42·17

Alexandra (54).

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, Dec. 27	10 23 57·0	4 47 25·59	54 6 16·15

Mnemosyne (57).

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1859, Nov. 18	7 56 26·4	23 45 44·98	89 16 13·24

Eclipses, Occultations, and Transits of Jupiter's Satellites.

Day of Observation.	Satellite.	Phenomenon.	Mean Solar Time. h m s	Observer.
1859. Nov. 13	I	Ecl. disapp.	11 51 47·6	C.
15	I	Occ. reapp. (*a*)	9 46 50·7	C.
21	I	Ingress, 1st cont. (*b*)	12 1 2·4	D.
21	I	,, central bis. (*b*)	12 3 2·1	D.
21	I	Egress, central bis. (*b*)	14 21 39·3	D.
21	I	,, last cont. (*b*)	14 24 38·8	D.
24	III	Ecl. disapp. (*c*)	9 4 11·7	T.
24	II	Egress, last cont.	9 28 44·2	T.
Dec. 1	II	,, central bis. (*b*)	11 45 58·0	D.
1	II	., last cont. (*b*)	11 47 57·7	D.
7	IV	Ingress, central bis.	11 6 36·6	M. D.
7	I	Egress, central bis. (*d*)	12 23 24·0	M. D.
8	I	Occ. reapp. central bis. (*f*)	9 30 26·8	C.
8	I	,, last cont. (*f*)	9 31 56·7	C.
8	II	Ingress, total ingr.	11 19 39·0	C.
8	II	Egress, last cont.	14 4 12·1	C.
15	I	Occ. reapp. first cont.	11 13 46·3	M. D.
15	I	" last cont.	11 15 31·0	M. D.
15	II	Ingress, first cont.	13 28 54·1	M. D.
15	II	,, last cont.	13 34 23·2	M. D.
15	IV	Ecl. disapp.	13 38 58·5	M. D.
21	I	Ingress, first cont.	13 30 0·4	M. D.
21	I	,, central bis.	13 32 59·9	M. D.
21	I	,, last cont.	13 36 59·2	M. D.

(*a*), Planet very low and extremely tremulous; time uncertain to 2 or 3 minutes. (*b*), Unsatisfactory; planet very tremulous. (*c*), *Jupiter* very low; satellite exceedingly faint. (*d*), Cloudy; very unsatisfactory. (*f*), Image of *Jupiter* extremely bad.

The initials D., C., M. D., and T., are those of Mr. Dunkin, Mr. Criswick, Mr. Dolman, and Mr. Talmage.

Occultations of Stars by the Moon.

Day of Obs.	Phenomenon.	Moon's Limb.	Mean Solar Time. h m s	Observer.
1859, Nov. 8	101 Piscium disapp.	(*a*) Bright.	9 54 59·0	J. C.
15	δ Cancri disapp.	(*b*) Bright.	18 14 45·8	T.
15	δ Cancri reapp.	(*c*) Dark.	19 23 55·7	T.

(*a*) Not very accurate; star faint.
(*b*) Somewhat doubtful, owing to a passing cloud.
(*c*) Satisfactory.

The initials J. C. and T. are those of Mr. Carpenter and Mr. Talmage.

The observations of the occultation of the *Pleiades* by the moon, on December 8, have been already published in the preceding number of the *Monthly Notices*.

Comparison of the Probable Error of a Transit of a Star Observed with the Transit-circle by the "Eye and Ear" and Chronographic Methods. By Edwin Dunkin, Esq., of the Royal Observatory.

The probable error of a transit of a star has frequently been considered of sufficient interest to occupy the attention of astronomers, some of whom have discussed the subject in con-able detail, generally with the desire of testing the accuracy of meridional observations; but in the present communication it is not my intention to enter very fully into the question, or to offer any critical remarks on the Greenwich transits, but solely to confine myself to an investigation of the relative difference between the original method of observing transits and that which has superseded it since the introduction of galvanism as an aid in the registration of astronomical observations. The transits made at the Royal Observatory with the transit-circle furnish ample materials for such an investigation.

In these reductions I have, for the comparison, used two methods; that which gives the more direct result will be explained first. In this method a certain number of stars were selected without any partiality, whose total number of observations in each series amounted to about one hundred. The separate complete transits were then extracted from the *Greenwich Observations* for 1853, this being the last year of observation by "eye and ear," and from the volume for 1857 for the transits recorded by the chronographic method, taking care in each case to reject all when any wires had been omitted. Each transit was then treated as follows,—for the observations made in 1853, to the clock-time observed at each separate wire, has been applied the value of the interval, expressed in time, between that wire and the mean of the complete system of wires; and for those made in 1857, to each recorded clock-time has been applied the interval between the wire observed and the centre-wire, according to the values published in the *Introduction to the Greenwich Observations* for the respective years. Each clock-time is thus reduced to a mean, and ought, supposing no error to exist, to consist of a series of numbers absolutely identical. This theoretical perfection is never practically the case, therefore for our purpose it is necessary to find the difference between each individual observation and the mean of the whole series, thus obtaining a number of errors equal to the number of wires observed. The square of each error was then taken, and the remainder of the reduction treated for each observation according to the formulæ explained in considerable detail by M. Liagre in his *Calcul des Probabilités et Théorie des Erreurs*, Sections 105 to 113.

The following are the results obtained:

Transits observed by "Eye and Ear."

		No. of Obs.	Probable uncertainty of result.
Probable error of an observation at one wire	= ±0·074 s	707	±0·001
,, of a complete transit	= ±0·028	101	±0·001

Transits recorded by the Chronographic Method.

		No. of Obs.	Probable uncertainty of result.
Probable error of an observation at one wire	= ±0·051 s	909	±0·001
,, of a complete transit	= ±0·017	101	±0·001

It must be borne in mind that the probable error in this instance consists of a combination of the error of observation with the error of the determination of the intervals of the wires, though there is every reason to believe that the effect of the latter is nearly, if not altogether, insensible.

In the second investigation, I have taken the observation of every star whose N.P.D. is greater than 50°, which had been observed at least three times during the year over the whole number of wires. The separate results are given in the *Greenwich Observations* for 1853 and 1857 respectively. The limit of 50° was adopted in consequence of the scarcity in 1853 of complete observations of stars having less N.P.D.; and as the object of this paper is simply a comparison of the two observi methods in *ordinary* transits, it was considered prudent to commence from that point.

The process of reduction in this determination is slightly different from the preceding. In the section of the *Greenwich Observations* from which the numbers are extracted, the right ascensions are reduced to January 1 in each year. These results are affected by error of observation, errors of the position of the instrument, error of the correction applied to each observation on account of clock-error, &c. There are therefore so many errors affecting the result in this determination, which have no place in the preceding, that it would be impossible to discover what amount is due to the observer. But in an instrument of such stability as the transit-circle, it may be assumed that the instrumental errors in different years are determined with equal accuracy; consequently the difference obtained between the "eye and ear" and chronographic methods from this investigation may be considered as representing the superiority of one method over the other.

The method of reduction consists in first taking a mean of all the observations for each star, then the difference of each observation for the mean, giving a series of errors equal to the number of observations; these errors being squared were treated by the same formulæ as previously mentioned. The results are given in the following table:—

	Probable Error of each Observation.		Number of Observations.		Probable Uncertainty of each Result.	
N.P.D.	Eye and Ear.	Chronographic.	Eye and Ear.	Chronographic.	Eye and Ear.	Chronographic.
° °	s	s			s	s
50 to 60	0·044	0·037	46	61	±0·003	±0·002
60 to 70	·049	·035	237	165	± ·002	± ·001
70 to 80	·051	·029	313	210	± ·001	± ·001
80 to 90	·049	·032	312	278	± ·001	± ·001
90 to 100	·046	·032	169	122	± ·002	± ·001
100 to 110	·048	·035	106	167	± ·002	± ·001
110 +	·050	·043	76	189	± ·003	± ·001
Means	0·048	0·034	1259	1192	±0·0007	±0·0005

By inspection of the preceding table, the general agreement of the resulting errors at different N.P.D. is easily seen, and tends to give some degree of confidence to the mean result. The comparatively large error in the chronographic method obtained from observations of stars below 110°, is probably on account of an unusual number observed in 1857 near the south horizon for the determination of the constant of refraction. These objects are often seen with considerable difficulty, and their images under the best circumstances are generally very diffused.

The result of this investigation may be stated as follows:— For stars whose N.P.D. is greater than 50° in a transit over all the wires observed by "eye and ear," the probable error is ±0s·028, and by the chronographic method ±0s·017; and in a transit observed by "eye and ear" combined with the instrumental errors, &c., the probable error is ±0s·048, and by the chronographic ±0s·034, the differences of 0s·011 and 0s·014 respectively being in favour of the chronographic method.

Greenwich, 1860, *January* 4.

Suggestions connected with the Carrington-and-Hodgson Solar Phenomenon of 1st *Sept.* 1859. By Prof. C. Piazzi Smyth, Astronomer Royal for Scotland.

At the conclusion of the very interesting account, given at p. 24–26, in the *Monthly Notices* for November, of M. Le Verrier's late remarkable investigations, resulting in a high probability for the existence of one large planet, or a zone of small ones, revolving round the sun at rather less than half the mean distance of *Mercury*,—a hint is appended, that with the above may be usefully read, "Prof. W. Thomson's ideas respecting a zone of meteors in the neighbourhood of the sun, thrown out as a possible source of supply of the solar heat and energy, in a paper published in the *Philosophical Magazine*."

It is, however, proper to remark, where so valuable a thing as an original idea from the brain of man is concerned, that the first suggestion of the meteoro-dynamic origin of solar light and heat, was made by Mr. Waterston. His idea was afterwards taken hold of by Prof. W. Thomson, and worked out by him and extended in the most admirable manner, with invaluable numerical conclusions, in a paper first published to the world in the *Transactions of the Royal Society of Edinburgh*, which therefore should rather be quoted than the magazine in London which merely copied the printed article.

I trust that, without incurring a charge of needless criticism, I may express a doubt whether that solar-heat theory has any particular connexion with the results of Le Verrier's researches.

It is quite true, that upon the dynamical explanation of heat a planet or a planetoid falling into the sun, would, from the conversion of its previous mechanical energy, give out an awful blaze of light, such a blaze as, according to Prof. W. Thomson, would certainly scorch up in a moment all the inferior planets, and probably the earth also with everything upon it.

But until the planet does so fall, or approach so near the sun as to have its orbital motion mechanically interfered with by resistance of the solar atmosphere (supposed, by the way, to extend very little beyond the actual surface), no dynamic heat would be given out.

The question, therefore, of the connexion of Le Verrier's zone of asteroids with Waterston's and W. Thomson's meteoric solar fuel, resolves itself into as the first step,—how soon will the mean heliocentric distance of the former, at present some 18 millions of miles, be reduced to half a million only?

Now so far as the results of physical astronomy, founded on the theory of gravitation alone, may speak, supported too by observation on all the known planets, no such decrease will ever take place; for all the comparatively small periodic perturbations are secularly restored. And though from other branches of science we may conclude the existence of causes which must introduce a new element of change into the calculations; and in the end, even cause all the planets to fall successively to the sun,—yet, the interval of time that must elapse before the first of these casualties can occur, is so vast, as, for all practical purposes, to be infinity itself; and similar periods will separate the falls of all the other planets, as they come in, in their order. Hence it is surely obviously impossible for Le Verrier's supposed zone of asteroids (planetoids?) with a mean distance still of not less than half that of *Mercury*, to contribute towards solar heat, before countless ages shall have passed away. Not therefore from them, or any succession of *planet*-falls, as that word is usually understood, can come the nearly constant amount of light and heat which the sun has thrown out during the whole human period on earth, without a single moment's cessation, up to and including the present time. And it cannot even arise from a "zone" of meteors (*i.e.* meteoric stones) as long as their mean distances from the sun are anything at all approaching the planetary radii vectores.

What is wanted for keeping up, according to the dynamic theory, the observed amount of regular solar light and heat, is an infinity of meteors, with their orbits at all distances, and specially increasing in number with proximity to the sun, up to his very surface; as indeed we may almost fancy we can see them, on making an attentive examination of the steady increase in density of the zodiacal light, with every step of its apparent approach to the solar centre.

I am aware that some writers, even a few observers also, speak of the "ring" of the zodiacal light, but it has never so appeared to me; and I searched in vain when on the Peak of Teneriffe in 1856, for any symptom that could be so construed. Everything, on the contrary, tended so certainly to a continual increase in density, with every approach to the solar neighbourhood, (the opinion also, I believe, of so eminent an authority as Sir John Herschel,) that the frequent remarks met with in books, of the "ring" of the zodiacal light, appear to me to depend on some peculiar definition of the word quite different from the usual one, and therefore not imposing, as otherwise would be the case, a necessity for rejecting the idea of the zodiacal light being a visible representation of the present supply, and earnest of regular future supply, of meteoric heat-material to the sun.

Taking for granted, however, that the sun is fed by meteors, from wherever derived, partly in solid lumps of meteoric stone, as Mr. Waterston imagined, and partly by the same distilled away into smaller particles, as added by Prof. W. Thomson,—does not such an idea indicate a very possible interpretation of that strange solar phenomenon so fortunately witnessed, and well observed, by Mr. Carrington, and also by Mr. Hodgson, on Sept. 1st, 1859? Or, taking their observation (*Monthly Notices*, pp. 13–15) as our standing point, does not that seem greatly to confirm the Waterston and W. Thomson theory?

It appears, indeed, from what is printed in the *Notices* quite certain, that two bodies, elongated in the direction of their motion, moved rapidly along above the actual surface of the sun, suddenly bursting into view by means of their emitted light, and then dying gradually out after having described an arc nearly parallel with the ecliptic, and at the rate of about 7000 miles in a minute.

Now here we have a velocity such as nothing we know of, short of orbital motion in the neighbourhood of the sun, can give. But, as in the immediate neighbourhood of the sun, or close to his surface, where we may presume the observed bodies to have been, the true orbital velocity is about 16,000 miles in a minute, we have a destruction, from some mechanical resistance, of 9000 miles of motion in a minute; which multiplied into the mass of the bodies so retarded, would at once indicate the heat they ought to have given out; something so great, doubtless, as to be necessarily accompanied by the intense light which was testified to by either gentleman.

In all this we may see, ***mutatis mutandis***, precisely what has often been observed on the earth, when a meteorite, in the act of falling to it, shines out brilliantly in the clear blue sky at noon-day, like a speck of the sun itself; and describes thus luminously, in our upper atmosphere, the last part of its orbital motion.

Not the most ardent admirer of the theory had once apparently expected that human eye would ever be able to distinguish an individual meteor falling into the sun, and visibly adding to his fires; yet here does really seem to have been an instance. That it was so remarkably visible, evidently implies that its size must have been immensely greater than the average supply according to Waterston; or that, by its superior mass, it had successfully resisted Prof. W. Thomson's process of distillation of its material by heat, while revolving for ages in near proximity to the sun. This, however, presents no natural difficulty, when we judge of the meteorites that fall into the sun by those which fall to the earth; for numerous records show, that while the generality of the latter class of meteoric stones measure but a few inches, a particular one seen by many able observers in Scotland, England, and France, on August 18th, 1783, was estimated at more than half a mile in length.

There is still another point of view from which we may regard Messrs. Carrington and Hodgson's unique observation; for we have now, by its means, been made aware of a distinct class of solar phenomena, hitherto quite unexpected, and to the detection or record of which the usual methods of observing are accordingly not the most efficient that can be imagined; should not therefore something be done to follow up the opening thus made by an almost providential occurrence of circumstances?

A communication has been received from M. Chacornac, "Note sur l'intensité lumineuse du centre du Soleil comparée à celle des bords." Part of the note has been published in the *Comptes Rendus* of the 21st Nov. 1859, but the author's observations have been carried on to a later date, and the results of the entire series, extending from the 12th May to the 11th Dec. 1859, are as follows,—

Dist. from. Circum.	Azimuths.	Ratio of Intensities.	Primitive Intensities.
	° ′		
1·000	34 11	0·461	1·000
0·717	34 11	0·461	1·000
0·483	35 05	0·494	0·935

where the last column, containing the deduced values of the primitive intensities, shows that the central part of the sun's disk has a sensibly uniform brightness for a distance from the centre of about three tenths of the radius, and that the brightness of the centre surpasses by about one fifteenth part of itself, that of a point at a distance from the centre of fifty two hundredth parts of the radius. The author shows that these results as to the diminution of the brightness are consistent with those obtained by supposing that the sun (assumed to be of uniform brightness) is surrounded by an atmosphere, which treated as homogenous would be of a thickness $= 0·55$ of the radius, and such that the coefficient of extinction P is $= 0·511$, viz. brightness at centre: brightness at any other point $= P : P^{\epsilon}$ if 1, ϵ, denote respectively the thickness of atmosphere traversed by the ray at the centre and by the ray at the other point.

The President announced, at the December meeting, that Mr. Grant, having been appointed to the Chair of Astronomy in the University of Glasgow, would be compelled to resign the editorship of the *Monthly Notices*. He was happy to say that Mr. Arthur Cayley had consented to undertake the editorship; and whilst he congratulated the Society on their periodical being continued by so distinguished a man as Mr. Cayley, he thought that the Society would feel pleasure in acknowledging their high appreciation of Professor Grant's services for the last seven years, during which period the journal had increased in value and popularity. Mr. May followed, at the suggestion of the President, and proposed that the thanks of the meeting be given to Professor Grant. This motion was seconded by the Treasurer, and carried by acclamation.

The Approaching Solar Eclipse.

(*Letter from Dr. Lamont to the Astronomer Royal.*)

"Having just received the *Monthly Notices* of the Astronomical Society, I find that at the last meeting you called the attention of the Society to the eclipse of the sun on the 18th of July, adverting to the necessity of previously collecting information about the places that might be selected for observing the eclipse. Now, as I myself visited in 1857 nearly all the places within the band of total obscuration, I thought it might, perhaps, be useful to communicate the following particulars:—The place most easily arrived at is Santander, there being between that place and Nantes a regular communication by steam-vessels. At Santander there is only one good inn, kept by a Frenchman, and generally crowded with travellers during the hot season, when the people of Madrid and other principal towns in the interior are accustomed to resort to the sea-coast. From Santander there is a railroad to Reynosa (where, I was told, there is tolerable accommodation for travellers), and a small steam-boat goes every week to Portugalete and Bilboa. Both places would be proper stations for observing the eclipse; and the inn at Bilboa will be found sufficiently comfortable. From Bayonne there is a very good road to Vitoria and Burgos, and the diligence that goes to Madrid passes there every day; the only difficulty is to get a place in the diligence. A traveller may have to stay at Bayonne at least eight days in July, and three weeks in September, to get a good place inside. At Vitoria there is a good inn, and it will be possible at all times to find accommodation there. In Spain the climate of Vitoria is accounted cool,—which only means that it is less hot than at Madrid. From Bayonne there is another road to Madrid passing through Pamplona and Tudela, both in the band of total obscuration. This road is less frequented, and it will, generally, be found much easier to get to Pamplona than to Vitoria. A kind of omnibus goes daily from Pamplona to Logroño, and another from Logroño to Vitoria. Logroño must be very near the central line of the eclipse. The inn at Pamplona cannot be said to be comfortable, and that at Logroño still less.

"Those who intend to station themselves further to the south ought to take the way of Barcelona. There are French and Spanish steam-vessels to convey passengers from Marseilles to Barcelona, and a diligence passes from Perpignan across the Pyrenees. From Barcelona it is easy to get by the diligence (if the railroad that was constructing in 1857 is not yet finished) to Saragossa and Calatayud (very near the central line). At Saragossa there is excellent accommodation for travellers; the inn at Calatayud is a very inferior one. From Barcelona steam-vessels are going regularly to Valencia, and from thence there is a diligence to Castellon de la Plana and Oropesa. The two latter places are near the central line; Valencia (according to Mr. Encke's calculation) lies near the western extremity of the band of total obscuration. At Valencia travellers will find excellent accommodation; as for the two other places I can give no account of them, not having visited them myself.

"My own intention is to take up my station at Oropesa or Castellon de la Plana. The heat will be excessive, but there is no fear of the observation being frustrated by clouds. Now and then some clouds may be seen in the morning near the horizon, but they are always dissolved by the sun as soon as it attains a considerable altitude. If the cholera should break out in the south, as it did last year, I will probably go to Cataayud. I prefer a station in the south, because it is to be expected that the greater number of foreign astronomers will resort to the northern stations to avoid the heat: besides, I conceive that the climate of the south, near the sea-coast, will be especially favourable for deciding the points that I wish particularly to attend to. You may have seen the explanation of the red prominences that I have given in my 'Jahresbericht' for 1854 and 1858: I suppose them to be produced by thin clouds or masses of vapour floating in our atmosphere, and condensed by the depression of temperature in the shadow of the moon. All the phenomena that have been recorded seem to me to agree perfectly with this explanation, and I have no doubt that in a hot climate, where the air contains a great quantity of vapour, it will be possible, by using a telescope of small magnifying power and considerable aperture, to trace these clouds to a much greater distance than has been done yet, and to prove by their motion and changes that they do not belong to the sun."

The following account of the new Equatoreal at the Royal Observatory, Greenwich, is extracted from the Report of the Astronomer Royal read at the Annual Visitation, 1859, June 4:—

"The new south-east equatoreal is now far advanced, and though I am not yet able to exhibit it in a complete working state, I trust to be able, by referring to the parts or models of parts which are visible, and by description of the principles which have guided their introduction, to make it generally intelligible. It is known to the Visitors that on three occasions of erecting large equatoreals which were entirely intrusted to me, namely, at Cambridge, at Liverpool, and at Greenwich, I have thought it my duty carefully to examine the construction of the three principal classes of equatoreals (the old English or Shuckburgh's, the long polar axis with telescope on one side, and the German form). And I have never put out of sight the consideration that, in observatories like this, an equatoreal cannot be considered as a mere commodious support for a gazing-telescope, but must be regarded as a firm graduated instrument, with which right ascensions and polar distances can be determined with considerable accuracy. Remarking, then, the diameters of the graduated circles, but still more the diameters of the racked-circles, and the clamp-circles on which the steadiness of clock-movement or the firmness of clamps must depend; remarking also the engineering objections, as regards weakness and friction, to the support of an instrument which projects beyond its two bearings; I prefer the English form, and I have accordingly adopted it here, with no novelty of principle, except that the declination-axis is so advanced in front of the centre of the polar-axis, and the upper part of the polar frame is so cut away, that the telescope commands the meridian without interruption to and a little way beyond the pole. Each cheek of the polar-axis is a skeleton prism; and, as I have shown in the description of the Northumberland telescope, this form is braced against torsion if each of its slides is braced. The bracing here is effected by a series of diagonal tension-bars and transversal thrusting-bars (each being drawn or thrust by its appropriate screw-motion); the long pillars and the bars being of wrought-iron. The upper and lower ovals which carry these are of cast-iron. On the spindle of the lower oval turns freely the hour-wheel, 6 feet in diameter, on which the clock-movement acts: which hour-wheel can be clamped at pleasure to the oval, and can also (when transits are required with an unmoved telescope) be clamped to the foundation-plate. A fixed microscope views a graduation of the hour-wheel, which, in its ordinary motion, will always exhibit in the microscope the sidereal time: and a microscope carried by the lower oval views another graduation, which in the same circumstances will exhibit the right ascension of the celestial body under view of the telescope. These microscopes are not directed perpendicularly to the graduated limb, but at an angle of nearly 10°, the illuminator throwing its light at the same inclination on the opposite side of the normal to the limb, so that the light is specularly reflected up the microscope: a principle of illumination which I think important. The two microscopes of the declination-circle spread out nearly in the same way as those of the meridional transit-circle, so that though they view opposite graduations on a 5-feet circle, the two eye-pieces are only a few inches apart. The illumination of the divisions is effected as in the eye-piece for viewing the wires of the transit-circle by reflexion; the light enters a hole in the side of the eye-tube, where it is reflected downwards by diagonal plates of transparent glass; it falls upon the limb, whose surface is turned to a concave or dished conical form so that the axis of the microscope is perpendicular to the portion of the limb under view; and the light which has been thrown down that axis is therefore reflected specularly up the axis to the eye. For the observer's ascent to these microscopes, a swinging ladder is provided, of which the lower end rests on the floor, while the upper end is supported by a rope of adjustable length fastened to the top of the dome. There is a 5-foot clamp-circle attached to the opposite cheek of the polar-axis, whose clamp-screw and slow-motion are acted on by long handles from the eye-end of the telescope. For convenience of setting, and for easy reading of differences of polar distance, there is a radial bar at the side of the telescope-tube, turning on a pin near the centre of motion of the telescope, and having a graduation at its end near the eye-piece of the telescope; and this radial bar is bridled by an adjustable graduated sliding rod, of which the distant end is carried by a pin on one cheek of the polar-axis. The lights for all these purposes are gaslights; the upper pivot of the polar-axis is perforated, and a gas tube (connected with fixed gas tubes by a rotatory joint) is passed through the perforation, and distributes its branches where lights are required on the polar-axis; one branch passes by means of a rotatory joint through the declination-axis (which is perforated), and brings gas to a burner on the eye-end of the telescope. The communication of time is made by a small clock with 10-inch pendulum beating seconds, near the fixed microscope of the hour-circle, and by a galvanic chronometer carried by the eye-end of the telescope; but these clocks do not give independent times: the wire of a galvanic battery, whose circuit is completed at 59 seconds of every minute of the transit-clock, is carried through a coil into which swings a bar-magnet carried by the 10-inch pendulum, and is also carried through the coils of the galvanic chronometer, (being led from the fixed support to the polar frame by means of insulated springs on the fixed support, touching insulated rings on the polar frame, and being then led in a similar manner to the telescope,) and thus the 10-inch clock is regulated, and the chronometer is moved, in exact accordance with the transit-clock. The seconds' wheel of the chronometer has only 59 teeth, and the seconds' dial has only 59 divisions, one of which counts for the two successive seconds (28 and 29): this is necessary in consequence of the loss of one current per minute in the transit-clock, as before remarked. In the 10-inch clock, which is merely regulated by the current on the principle introduced by Mr. R. L. Jones, there is no such anomaly.

"Another pair of galvanic wires is brought, by insulated rings and springs, to a touch-piece at the eye-end of the telescope: these wires are connected with the pricker of the chronographic barrel, so that times may be registered on the same sheets as the times of observations with the transit-circle and the altazimuth.

"The clock-movement of the hour-circle is effected essentially by the same mechanism as that which I introduced at Liverpool. By transferring a water supply from a lower to a higher level, I have obtained a fall for driving a reaction machine (Barker's mill), revolving four times in a second. This, acting through two worms, drives the hour-circle. For its regulation, it gives motion to an axis revolving in two seconds, which acts upon a conical pendulum (carried by two pairs of springs that do the duty of a universal joint), by the remarkable contrivance called Sieman's chronometric governor, of which the effect is that, if there is a tendency to acceleration, the throttle-valve of the water-pipe is immediately contracted. It is necessary that the pendulum suffer a retarding force, which ought to be strictly tangential; to effect this, I have so connected a small spade with the rotatory apparatus, that the pendulum by enlarging its cone dips the spade into a trough of water. I anticipate from this a very smooth and uniform motion.

"Every part of the apparatus is provided with the proper means of gearing and ungearing, oiling, &c., according to the usages of the best modern engineering. The instrument may now be considered a splendid piece of mechanism; I hope that it will ultimately be judged to be a splendid equatoreal.

"The engineers' part of the work is executed by Messrs. Ransomes and Sims of Ipswich; the graduations and the optical parts generally (except the object-glass), including also the work of the eye-end, the radial bar and bridles, and the mounting of the object-glass, by Mr. Simms of Fleet Street; the object-glass by Messrs. Merz and Son of Munich. The pendulum of the clock movement, and the 10-inch clock, were furnished and mounted by Mr. Dent of the Strand. The galvanic chronometer was made by Mr. Shepherd of Leadenhall Street.

"The observing chair I propose to mount as a slider upon a vertical quadrant revolving round a point near the centre of the room, the part of the floor near the south pier being furnished with theatre seats, as for the Northumberland telescope. For enabling the observer to observe objects near the pole, without leaning upon the polar frame, I have surrounded the lower oval with a curb, for supporting a board that will pass between the cheeks of the frame, on which board the observer will rest.

"It is proper to mention that the focal length of the object-glass furnished by Messrs. Merz is considerably less than the length originally stated. The whole apparatus therefore is somewhat larger than is absolutely necessary. It might with some inconvenience be made to carry a telescope of above 20 feet focal length; and, if the regions near the pole were given up, it might carry one still larger."

MISCELLANEOUS INTELLIGENCE.

A supposed New Interior Planet.

The conclusions arrived at by the distinguished Director of the Imperial Observatory at Paris in his recent investigations of the corrections required in the theory of *Mercury*, and which he communicated in so striking a manner to the world by his letter addressed to M. Faye, appear to have already received signal confirmation. An abstract of this remarkable letter was printed in the first number of the present volume, and it may now be acceptable to the readers of the *Monthly Notices* to be made acquainted with the circumstances of the observation recently announced.

From various accounts received from Paris it appears that not long after the publication of M. Le Verrier's assertion of the probable existence of a planet or planets interior to *Mercury* it began to be rumoured in Paris that more than one person had already seen *the* planet, but that, as is usual, the rumour assumed different forms and was coupled with different names. Towards the close of the year, however, a communication was made of so definite a kind to M. Le Verrier that he considered his official position required that he should at once probe it to the bottom, and take such steps as should prevent the success of any attempt to palm off a fraud on the public. Noting the name and address of the asserted observer he proceeded on the last Saturday of the year to the nearest railway station to Orgères, in the department Eure et Loire, and accompanied by a friend whom he took as a desirable witness of his proceedings, made his way direct and unannounced to the house of M. Lescarbault, residing at Orgères and practising as a country physician. A very graphic account of what here passed is given by the Abbé Moigno in a recent number of his useful publication, *Cosmos*, and is stated to be given nearly as recounted by M. Le Verrier to an assembly of friends on his return. M. Lescarbault was subjected to a severe cross-examination by his unknown visitor, who pressed him hard from step to step till he had obtained such material and verbal evidence as no longer permitted him to doubt the reality of the observation or the good faith of the observer. Not content to leave the question of good faith, where so much rested upon it, dependent on the replies of the individual concerned, he was careful to obtain collateral evidence of the high character and worth of Dr. Lescarbault, from such other persons of station in the neighbourhood as should at once satisfy himself and others; and we believe that the precautions thus taken have been accepted as sufficient by all concerned.

At the first sitting of the Academy of Sciences after his return M. Le Verrier announced that on the afternoon of the 26th of March, 1859, a small dark body had been observed to transit a portion of the sun's disk by M. Lescarbault, and which bore all the appearance of being a new interior planet. And at the same time he stated that the observer had made such observations as led to the conclusion that the supposed new planet's distance from the sun was about 0·1427, its period less than 20 days, its ascending node situated at about 13° of longitude, and its inclination between 12° and 13°.

M. Lescarbault has since addressed a written account of his observation to M. Le Verrier which has been published, in which he states that having witnessed the transit of *Mercury* in 1845, he had himself at once inferred that the body was an interior planet, and that, living in retirement, he had kept his discovery to himself, in the hope of being able to come to some conclusion respecting its distance from the sun, by calculation, or by being so fortunate as again to observe its transit; but that being only a moderate geometer, and much pressed by professional engagements, the problem had baffled him, and he had at last been induced to break silence on reading an account of M. Le Verrier's theoretical conclusions, printed in the publication of Abbé Moigno, to which he was accustomed to subscribe.

M. Lescarbault first detected the body when a little way advanced on to the disk, and inferred the time of first contact by noting the interval which elapsed while it advanced over what he estimated to be an equal space. He then repeatedly measured the angle from the zenith or nadir of the point of the limb to which the body was nearest and its distance* from the limb, and watched its passage off the sun. Correcting his measured angles for the angle between the pole and zenith, it would appear that the first contact took place at $19^{\circ}\ 4'$ and the last contact at $52^{\circ}\ 42'$ from the sun's north point measured towards the west, the greatest distance from the limb being $41''\cdot 3$, and the times $4^{h}\ 0^{m}$ and $5^{h}\ 17^{m}\cdot 2$ Orgères local time, little more or less. The estimated diameter was about one-fourth that of *Mercury* when last on the sun; and as this was $11''\cdot 6$, the inference from the statement would be about $3''$ for the new body, if so small a quantity can be admitted as probable.

At the meeting of the Society on January 13, Mr. Carrington exhibited two diagrams in illustration of the original observation and of the inferences to be made from it, and pointed out in detail the very simple process required in such a case for arriving at the approximate elements of the orbit supposed circular. The concluded elements were not sensibly different from those given by M. Le Verrier, except in the case of the inclination, which was concluded to be about $11^{\circ}\ 51'$. The inclination to the orbit of *Mercury* was inferred to be about 7°.

The remark of principal immediate importance was that, inasmuch as the observation of March 26, 1859, was made about $7\frac{1}{2}$ days before the earth passed through the calculated line of nodes, and an equal space thereafter was similarly available, it followed that for the space of 15 days at each conjunction the orbit was projected on the sun's disk; and that as the revolution was performed in less than 20 days, it was more probable than not that, unless the original observation were more in fault than supposed, the body might be re-observed in the spring of this year. The attention of observers was particularly called to this circumstance. The days referred to are

	March 25	to	April 10	at descending node,
and	Sept. 27	to	Oct. 14	at ascending node.

The singular merit of M. Lescarbault's observation will be recognised by all who examine the attendant circumstances; and astronomers of all countries will unite in applauding this second triumphant conclusion to the theoretical inquiries of M. Le Verrier. The redetection of this new member of the solar system must next engage the co-operation of observers; and it is hoped that the astronomer at Madras will unite his efforts with those of European and American observers for the purpose.

In the 10th number of Professor Wolf's *Mittheilungen über die Sonnenflecken* several cases are quoted of the observation of planetary bodies in transit over the sun, some of which are evidently of another character, but the following deserving of attention:—

On January 18, 1798, Dangos saw a round sharply defined spot traverse the sun in about 2 hours.

On February 12, 1820, Stark saw about noon a singular and well-defined circular spot with indications of an atmosphere, which was not visible in the evening of the same day.

At the end of February, 1762, Staudacher saw a black round spot, which he missed the next day, and of which he made a note, "Might this have been a new planet?"

At the beginning of May, 1764, Hoffmann remarked a large round black spot which appeared to have independent motion.

On June 6, 1761, Scheuten at Crefeld believed that he saw traverse the sun a round and well-defined spot, one-fourth the size of *Venus* and as black (as that planet when in transit?).

On October 9, 1819, Stark saw a well-defined and truly circular spot about the size of *Mercury*, which he could not find again in the evening.

On October 10, 1802, Fritsch saw a small round spot pass over the sun.

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* There is a shadow of doubt on this point, which it is desirable to have removed. The distance (if measured) singularly accords with that inferred from the angles and radius.

On November 19, 1762, Lichtenberg soon after sunrise saw a large round black spot, which manifestly moved before the sun, and made its exit after traversing a chord of 70°.

The observations of Stark on Oct. 9, 1819, and of Fritsch on Oct. 10, 1802, may prove to be transits at the ascending node of Lescarbault's planet as the recorded dates fall within the days of visibility. Professor Wolf quotes also an observation of Julius Schmidt on Oct. 11, 1847, but Schmidt himself remarks that the *rapidly* moving point seen by him was probably an insect or distant bird, and there are no sufficient grounds for retaining his note as of further value.

[R. C. C.]

Solar Spots.

Hofrath Schwabe has published his usual annual census of the groups of solar spots observed by him at Dessau in the year 1859. Referring to the *Astron. Nach.* (No. 1230, p. 95) for the monthly numbers, it may be sufficient here, as an abstract, to state that the sun was viewed on 343 days, on none of which was the disk found to be blank, and that 205 groups were registered. In comparing this number with those of the years preceding, it will be perceived that the expected maximum is nearly, if not already, attained, and that the arrested progress of the curve seems to indicate that the present maximum of energy will fall far short of that in the year 1848.

M. Otto Struve writes from Pulkowa, under date of December 31st, 1859, to the effect that M. Oblomicosky has been appointed to be director of the new observatory ordered by the Emperor of Russia to be erected at Tiflis, with a special view to the department of "mountain-astronomy;" and that the Report on the Teneriffe astronomical experiment of 1856 will form the basis of his future proceedings, in the annual visits which he expects to make from his permanent station at Tiflis to the summit of Mount Ararat or Mount Elburz.

A letter was read from Major Tennant, B. E., announcing that he had assumed the charge of the Madras Observatory on the 13th of October, 1859, under his appointment by the Secretary of State for India.

Sir John Herschel communicated further delineations of the solar spots by the Rev. F. Howlett.

The *Compte Rendu* of the French Academy for 1860, January 2, contains letters by M. Rico y Sinobas and M. Verneuil, on the selection of stations in Spain for the observation of the total eclipse of July 18. It is stated that there are frequent clouds in the valleys of the mountains which separate the Asturias, Santander, and Biscay, from the rest of Spain, and that there is also a vaporous stratum in the upper regions of the atmosphere. The annual rain-fall in these provinces is about 140 inches.

Even in the valley of the Ebro and on the mountains of Pancorbo, though there are fewer clouds in the hot season, there is a dry fog after noon called the *caliña*.

The most favourable positions, in respect of clearness of atmosphere, appear to be the mountains of Gorbea near Vitoria, the plateau of Alaba, the peaks of Pancorbo near Burgos, the peaks of Urbion (Cameros), the mountains of Treviño, but above all the mountain of Moncayo near Soria, 2340 mètres high.

Aragon and the eastern counties of Spain appear less favourable, as there are always cirrus clouds after noon.

M. Verneuil describes the best approach to the Moncayo as to be made by taking the diligence from Bayonne to Madrid, *viâ* Soria, as far as Agredo, from which a five hours' walk leads to the hermitage of Virgen at 1600 mètres elevation, where lodging might be obtained.

The Peña Golosa in Valentia, and Oropesa, are also mentioned as advantageous stations.

Other stations, generally on mountains, are also described, as advantageously situated near the northern and southern limits of the eclipse.

RECENT PUBLICATIONS.

Untersuchungen über die Lichtstärke der Planeten Venus, Mars, Jupiter, und Saturn verglichen mit Sternen, und über die relative Weisse ihrer Oberflächen, Nebst einem Anhange enthaltend die Theorie der Lichterscheinung des Saturn. Von L. Seidel. 4to. Munich, 1859. (Forms a part of the *Monumenta Sæcularia*, published by the Royal Bavarian Academy of Sciences on the 100th anniversary of the foundation.)

The author refers to a paper in the *Memoirs of the Academy* for 1852, and to some results published in the *Gelehrte Anzeigen*, Sept. 1853, in relation to the planets *Venus*, *Mars*, and *Jupiter*. The observations were made by him and Prof. Leonhard by means of Steinheil's objective or prism photometer. The memoir contains formulæ for the reduction of the results to standard positions of the planets under consideration; and the author finds numerical values for the quantities of light of the four planets respectively, in comparison with the star *Wega*.

He infers from the observations that the changes (if any) in the quantity of light within the period of the observations (August 1845 to June 1858) are so small that they are concealed by the necessary uncertainty of the separate observations, and no change is indicated by a comparison of the results with two observations by Olbers as to *Mars* and *Saturn* in the years 1801 and 1803. The author defines the "Albedo" of a planet to be the proportion diffusedly reflected by an element of surface, of the solar light incident on such element; and he obtains from the observations numerical relative values (Alb. other planet: Alb. *Jupiter*) for the four planets, the result being, that the Albedo is nearly the same for *Venus*, *Mars*, and *Jupiter*, but only about one-fifth for *Saturn*. The addition contains a mathematical investigation as to the quantity of light reflected by *Saturn*, taking account of the ring.

Annales de l'Observatoire Impériale de Paris. Publiées par U. J. Leverrier, Directeur de l'Observatoire. Observations, t. II. Paris, 1859.

This volume contains the reductions of Arago's observations for the years 1837 and 1838. The processes of reduction are explained in an introduction, and the concluded right ascensions and polar distances of the centres of the Sun, Moon, and Planets are given at the end of the volume.

The *Connaissance des Temps* for 1861 has been published. The additions contain a memoir by M. Delaunay, "Nouvelle Methode pour l'intégration des équations differentielles du Mouvement de la Lune autour de la Terre": the most detailed exposition he has as yet given of the method employed in his researches in the lunar theory, some of the results of which, in relation to the acceleration of the mean motion, have been already published in the *Comptes Rendus.*

Johannis Kepleri Astronomi Opera Omnia. Ed. Ch. Fusch, Frankfort and Erlangen.

Volumes i. and ii. of this important publication have appeared.

Report on the Teneriffe Astronomical Experiment of 1856, *Addressed to the Lords Commissioners of the Admiralty.* By Professor C. Piazzi Smyth, F.R.SS.L. and E., &c., H.M. Astronomer for Scotland.

This paper has been printed at the expense of the Admiralty. Some of the principal results recorded in it have been already published in the Annual Report of the Council of the Society for the year 1857. The paper is divided into eight chapters.[*] The first chapter is devoted to an account of the circumstances connected with the origin of the expedition. In the second chapter the author describes the improvement of astronomical vision, depending on the increase of height above the level of the sea. The third chapter is occupied with an account of the astronomical observations made by the author during his residence on the Peak. The fourth chapter contains the physical observations. The fifth chapter is devoted to the meteorological observations; the sixth, to the geology of the mountain; the seventh, to the botany; the eighth, to the miscellaneous observations. Appended to the paper are a series of valuable drawings illustrative of the facts recorded in it.

By favour of General T. F. de Schubert, Honorary Member of the Imperial Academy of Sciences at St. Petersburg, copies of his "Essai d'une Détermination de la véritable Figure de la Terre" (printed in the *Mém. de l'Acad. Imp. des Sciences de St. Pétersbourg*, vii. Série, Tome i. No. 6) have lately been received in this country. The meridian arcs used by the author are the following, in which the longitudes are measured eastwardly from an imaginary meridian 20° west of Paris —

		°	′		°	′		°	
The Russian Arc	Longitude	45	20	to	70	40	Longitude	44	23
The Indian Arc	,,	8	10	to	29	31	,,	95	20
The French Arc	,,	38	40	to	51	2	,,	20	0
The Cape Arc	,,	−29	44	to	−34	21	,,	36	9
The Peruvian Arc	,,	0	3	to	− 3	5	,,	298	44
The Prussian Arc	,,	54	13	to	55	43	,,	38	10
The English Arc	,,	50	37	to	53	28	,,	17	40
		(in first calculations)							
		50	37	to	60	50			
		(in final calculations)							
The Pennsylvanian Arc	,,	38	27	to	39	56	,,	300	10

The author first exhibits the elliptic formulæ which he has used, and which appear well adapted to give results of great accuracy and with much convenience. He then applies them to the comparison of each of these eight arcs with every other, and thus obtains 28 systems of elements, presenting great discordances. For the earth's polar semiaxis, expressed in toises, the largest result is 3274069, and the smallest 3245754; for the earth's equatoreal semiaxis, the largest result is 3279418, and the smallest 3259832; for the denominator of the fraction expressing the oblateness, the largest number is 14501, and the smallest 116. These discrepancies appear to the author to show that a different method of treatment is required; and he proposes and applies the following:—

First, he remarks that we have assumed in the preceding calculations "that the terrestrial meridians are ellipses," and "that their minor axis is, for all, the same as the axis of rotation of the earth," which assumptions our observations do not enable us to disprove; but we have also assumed "that the earth is a solid of revolution, or that all the meridians are equal," an assumption which he conceives the discrepancies above mentioned show to be untenable, if the two preceding assumptions are maintained. As soon as this is given up, all comparisons of arcs of meridian in different longitudes become illusory. The next inquiry is, What element is common to all the ellipses? and this, it is evident, is the minor semiaxis. [We have been very much astonished to see that in several late discussions this important principle has been entirely forgotten.] It is clear that this minor axis cannot be determined from a combination of arcs in different meridians, but must be found from a comparison of arcs in the same meridian, or (which amounts to the same) by dividing a long arc into two segments, and comparing one segment with the other, or comparing each segment with the whole arc. Thus the author uses the three following arcs (the longest of the series):—

The Russian arc, divided into two parts at Dorpat, latitude 58° 23′, and giving for the minor semiaxis 3261429.

The Indian arc, divided into two parts at Damargida, latitude 18° 3′, and giving for minor semiaxis 3261547.

The French arc, divided into two parts at Carcassonne, latitude 43° 13′, and giving for minor semiaxis 3260365.

The author then expresses his unwillingness to combine this French result with the others, on account of its discordance; but remarks that this discordance would be removed by supposing an error of 2″ in the latitude of Carcassonne. [We conceive that this explanation brings to light what is really a weak point in the investigation. The curvature of the meridian changes more rapidly in the arc divided in latitude 43° than in any other arc; and if an error of 2″ is fatal to that determination, an error of 1″ might be fatal to either of the others.] Using the Russian arc with weight 2, and the Indian arc with weight 1, he adopts for polar semiaxis 3261468.

Using this polar semiaxis as a known quantity, in the formulæ applying to the three arcs of Peru, Russia, and India, (the selection of which is determined in part by their difference of longitude), the author finds the following values for the major semiaxes of the meridians of those arcs:—

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[*] The first five chapters form a paper which is printed at the expense of the Admiralty in Part II. of the *Philosophical Transactions* of the Royal Society, t. cxlviii. (1858).

Major Semiaxis of Peruvian Meridian,	Longitude 298° 44′,	3272383
Major Semiaxis of Russian Meridian,	Longitude 44 23,	3272650
Major Semiaxis of Indian Meridian,	Longitude 95 20,	3272581

The question now arises, What is the form of the terrestrial equator? The author assumes it to be an ellipse. The three numbers just found are the values of three radii of that ellipse; and are sufficient to determine its elements. They give,—

Major Semiaxis	3272671;	its Longitude	58° 44′	or 238° 44′
Minor Semiaxis	3272303;	its Longitude	148 44	or 328 44

(The combination of these with the polar semiaxis above gives for the oblateness of the two principal meridians $\frac{1}{292}$ and $\frac{1}{302}$.)

With these semiaxes he computes the radii of the equatoreal ellipse corresponding to the different arcs (which are, in fact, the major semiaxes of their meridians), and finds them as follows:—

Major Semiaxis of Peruvian Meridian ...	3272395
Pennsylvanian ...	3272388
English	3272513
French	3272528
Cape of Good Hope ...	3272618
Prussian	3272626
Russian	3272650
Indian	3272540
Also, in Meridian of Warsaw	3272628
Pulkowa	3272659

With the arcs computed from these elements and the astronomical differences of latitude, the geodetic measures are compared, and the residual discordances are as follows:—

Geodetic Amplitude — Astronomical Amplitude.

	″
Peruvian Arc	+0·077
Pennsylvanian	−6·687
English (entire arc)	+0·736
French	−1·607
Cape of Good Hope	−0·442
Prussian	+1·267
Russian	−1·289
Indian	+1·619

The Pennsylvanian arc, as is well known, deserves no *à priori* confidence, and has been introduced only as a matter of curiosity.

In the Indian arc no calculation is made of the effects of the attraction of mountains.

The author then applies his elements to compute the longitudes of Warsaw and Dorpat, as referred to Pulkowa. On comparison with the geodetic measures of parallel, it is found that the discordances are reduced as follows:—

Astronomical Diff. Long. — Geodetic Diff. Long.

For Warsaw, is reduced from −124·7 toises to +58·8 toises.
For Dorpat, „ from −57·9 „ to − 7·8

We think that this investigation merits the most careful attention of geodetists. In repeating or extending it, we would suggest for consideration whether the determination of the value of the polar semiaxis would not be more satisfactory if it were left to be decided, with the other semiaxes, by a general process combining the consideration of the lengths of all the arcs, introducing also the latitudes of middle stations.

Referring to the mechanical explanation of the existence of a fluid ellipsoid with three unequal axes (two nearly equal), it appears that it can be explained by supposing the existence of an ellipsoidal nucleus of three unequal axes. Should the existence of the former be established, the mode of original formation of the latter may be a problem for speculators in cosmogony. [G. B. A.]

VOL. XX. *February* 10, 1860. No. 4.

THE Annual General Meeting of the Society, Rev. R. MAIN, President, in the Chair.

George Frodsham, Esq., Change Alley;
John Pope Hennessey, Esq. M.P., of the Inner Temple; and
Geo. Johnstone Stoney, Esq. M.A., Secretary to the Queen's University, Ireland,

were balloted for and duly elected Fellows of the Society.

Report of the Council to the Fortieth Annual General Meeting of the Society.

The Council present their usual Annual Report.

The Report of the Auditors, subjoined, will show the state of the finances:—

RECEIPTS.

	£	s.	d.
Balance of last year's account	316	7	9
By dividend on £1700 Consols	24	19	5
By ditto on £109 8*s.* 9*d.*	1	12	9
By ditto on £3500 new 3 per Cents	51	8	2
Carried forward	£394	8	1

	£	s.	d.
Brought forward	394	8	1
By dividend on £1900 Consols	27	18	2
By ditto on £3500 new 3 per Cents	49	13	2
£109 8*s*. 9*d*. 3 per Cents reduced, sold at £94, commission, &c.	102	14	8
On account of arrears of contributions	58	12	0
102 contributions (1859–60)	214	4	0
7 compositions	144	18	0
25 admission-fees	52	10	0
20 first year's contributions	37	16	0
Sale of Publications	57	7	0
	£1140	1	1

EXPENDITURE.

	£	s.	d.	£	s.	d.
Salaries:—						
Mr. Grant, 1 year as Editor of the Society's Publications	60	0	0			
Mr. Williams, 1 year as Assistant-Secretary	100	0	0			
Ditto commission on collecting £420 9*s*.	21	0	6			
				181	0	6
Investments:—						
Investing £200 Consols				190	15	0
Taxes:—						
Income Tax, 1 year	1	0	10			
Assessed Taxes, ditto	5	7	6			
				6	8	4
Bills:—						
Geo. Barclay, printer	313	17	0			
J. Rumfit, bookbinder	10	4	0			
Malby and Son, lithographers	24	12	0			
W. Rutter, Esq. (living of Stone)	5	0	0			
Sundry payments out of Turnor Fund	9	14	6			
				363	7	6
Miscellaneous items:—						
Power of attorney to transfer stock	1	1	6			
Charges on books, and carriage of parcels	3	10	5			
Postage of *Monthly Notices*, letters, &c.	41	13	3			
Porter's and charwoman's work	22	15	8			
Tea, sugar, biscuits, &c. for evening meetings	15	12	0			
Waiters attending meetings	4	8	0			
Expenses of meeting, June 1858 (omitted 1858)	2	10	0			
Coals, &c.	12	0	0			
Sundry disbursements by the Treasurer	29	2	4			
Banker's clerks	1	1	0			
				133	14	2
Balance at Banker's				264	15	7
				£1140	1	1

Assets and present property of the Society, Feb. 8, 1860:—

	£	s.	d.	£	s.	d.
Balance at Banker's				264	15	7
13 contributions of 2 years' standing	54	12	0			
63 ——— of 1 ditto	132	6	0			
On account of arrears	3	18	0			
				190	16	0
Due for publications of the Society				0	14	6

£3500 new 3 per Cents.
£1900 Consols (including the Lee Fund).
Unsold publications of the Society.
Various astronomical instruments, books, prints, &c.

	£	s.	d.
Balance of Turnor Fund (included in Treasurer's account)	31	2	10

Stock of volumes of the *Memoirs*:—

Vol.	Total.	Vol.	Total.	Vol.	Total.
I. Part 1	28	IX.	195	XXI. Part 1 (separate).	315
I. Part 2	70	X.	207		
II. Part 1	87	XI.	218	XXI. Part 2 (separate).	100
II. Part 2	52	XII.	224	XXI. (together).	125
III. Part 1	112	XIII.	241		
III. Part 2	132	XIV.	425	XXII.	220
IV. Part 1	127	XV.	211	XXIII.	220
IV. Part 2	140	XVI.	233	XXIV.	235
V.	160	XVII.	211	XXV.	242
VI.	180	XVIII.	211	XXVI.	262
VII.	203	XIX.	224	XXVII.	541
VIII.	189	XX.	221		

Progress and present state of the Society:—

	Compounders.	Annual Contributors.	Non-residents.	Patroness, and Honorary.	Total Fellows.	Associates.	Grand Total.
February 1859 ...	153	186	39	5	383	52	435
Since elected	6	17	...	...	23	...	23
Deceased	−5	−4	−7	−1	−17	−1	−18
Resigned	...	−5	...	...	−5	...	−5
Removal	1	−1	...	...	...	...	...
February 1860 ...	155	193	32	4	384	51	435

The instruments belonging to the Society are as follows:—

The *Harrison* clock,
The *Owen* portable circle,
The *Beaufoy* circle,
The *Beaufoy* transit,
The *Herschelian* 7-foot telescope,
The *Greig* universal instrument,
The *Smeaton* equatoreal,
The *Cavendish* apparatus,
The 7-foot Gregorian telescope (late Mr. Shearman's),
The Variation transit (late Mr. Shearman's),
The Universal quadrant by Abraham Sharp,
The *Fuller* theodolite,
The Standard scale,
The *Beaufoy* clock,
The *Wollaston* telescope,
The *Lee* circle.

The *Sheepshanks*' collection of instruments, viz.,—

1. 30-inch transit, by Simms, with level and two iron stands.

2. 6-inch transit theodolite, with circles divided on silver; reading microscopes, both for altitude and azimuth; cross and siding levels; magnetic needle; plumbline; portable clamping foot and tripod stand.

3. $4\frac{6}{10}$ achromatic telescope, about 5 feet 6 inches focal length; finder, rack motion; double-image micrometer; object-

glass micrometer; two other micrometers; one terrestrial and ten astronomical eyepieces, applied by means of two adapters.

4. 3¼-inch achromatic telescope, with equatoreal stand; double-image micrometer; one terrestrial and three astronomical eyepieces.

5. 2¾-inch achromatic telescope, with stand; one terrestrial and three astronomical eyepieces.

6. 2¾ achromatic telescope, about 30 inches focus; one terrestrial and four astronomical eyepieces.

7. 2-foot navy telescope.

8. 45-inch transit instrument, with iron stand, and also Y's for fixing to stone piers; two axis levels.

9. Repeating theodolite, by Ertel, with folding tripod stand.

10. 8-inch pillar-sextant, divided on platinum, with counterpoise stand and horizon roof.

11. Portable zenith instrument, with detached micrometer and eyepiece.

12. 18-inch Borda's repeating circle, by Troughton.

13. 8-inch vertical repeating circle, with diagonal telescope, by Troughton and Simms.

14. A set of surveying instruments, consisting of a 12-inch theodolite for horizontal angles only, with extra pair of parallel plates; tripod staff; in which the telescope tube is packed; repeating table; level collimator, with micrometer eyepiece; and Troughton's levelling staff.

15. Level collimator, plain diaphragm.

16. 10-inch reflecting circle, by Troughton, with counterpoise stand; artificial horizon, with metallic roof; two tripod stands, one with table for artificial horizon.

17. Hassler's reflecting circle, by Troughton, with counterpoise stand.

18. 6-inch reflecting circle, by Troughton, with two counterpoise stands, one with artificial horizon.

19. 5-inch reflecting circle, by Lenoir.

20. Reflecting circle, by Jecker, of Paris.

21. Box sextant and 3-inch plane artificial horizon.

22. Prismatic compass.

23. Mountain barometer.

24. Prismatic compass.

25. 5-inch compass.

26. Dipping needle.

27. Intensity needle.

28. Ditto ditto.

29. Box of magnetic apparatus.

30. Hassler's reflecting circle, with artificial horizon roof.

31. Box sextant and 2¼-inch glass plane artificial horizon.

32. Plane speculum artificial horizon and stand.

33. 2½-inch circular level horizon, by Dollond.

34. Artificial horizon roof and trough.

35. Set of drawing instruments, consisting of 6-inch circular protractor; common ditto; 2-foot plotting scale; two beam compasses and small T square.

36. A pentagraph.

37. A noddy.

38. A small Galilean telescope, with the object lens of rock-crystal.

39. Six levels, various.

40. 18-inch celestial globe.

41. Varley stand for telescope.

42. Thermometer.

These are now in the apartments of the Society, with the exception of the following, which are lent, during the pleasure of the Council, to the several parties under mentioned, viz.:—

The *Wollaston* telescope, to Mr. Rees.
The *Lee* circle, to Mr. Burr.
The *Sheepshanks* instrument, No. 1, to Mr. Lassell.
Ditto ditto No. 2, to Mr. De La Rue.
Ditto ditto No. 3, to Rev. C. Pritchard.
Ditto ditto No. 4, to Mr. Carrington.
Ditto ditto No. 5, to Mr. Birt.
Ditto ditto No. 6, to Rev. J. Cape.
Ditto ditto No. 8, to Prof. Wheatstone.
Ditto ditto No. 16, to Admiral Bethune.
Ditto ditto No. 18, to ditto.
Ditto ditto No. 32, to ditto.
Ditto ditto No. 19, to Mr. Dayman.
Ditto ditto No. 25, to ditto.
Ditto ditto No. 41, to Rev. C. Pritchard.

Nothing definite has yet been ascertained respecting the other *Beaufoy* clock, the two invariable pendulums, and the quadrant (said to have been *Lacaille's*), reported for many years past as being in the possession of the Royal Society.

The Council have awarded the Gold Medal to Professor Hansen, for his Tables of the Moon, constructed on the principles of universal gravitation. The President will detail the grounds of this award in the usual way at the close of the Meeting.

The twenty-eighth volume of the *Memoirs* of the Society has passed through the press and will shortly be ready for publication. It contains several papers of great importance. The volume commences with a series of communications from Capt. Jacob, embodying the results of his recent labours at the Madras Observatory. Among these will be found a catalogue of 317 stars which were selected for observation on the ground of the large proper motions assigned to them in the British Association Catalogue. It appears, from the new places of the southern stars contained in this Catalogue, that the large proper motions which had been in many instances indicated by anterior observations are in a great measure unfounded. Another paper by Capt. Jacob, which is eminently deserving of attention, consists of micrometrical measures of 120 double or multiple stars. Appended to these results is a valuable disquisition on the orbits of several stars whose binary character has been conclusively established. The remaining papers by the same author relate chiefly to the Saturnian system. It would be out of place here to enter into any details, but the Council cannot refrain from directing the attention of the Fellows of the Society to a very elaborate discussion of the elements of the orbits of *Saturn's* satellites and to the author's researches on the mass of the planet, an element of so much importance in physical astronomy. The Council feel assured that the Society, while applauding the energy and ability which Capt. Jacob has exhibited in his various communications, will cordially unite with them in offering their sincere congratulations to him on the improved state of his health consequent upon his return to England.

The President, to whom astronomy is so deeply indebted for his researches on the subject of the proper motions of the stars, has communicated a valuable paper on the proper motions of the stars in the recently published Greenwich Catalogue of 1576 stars. The results are deduced by a comparison of the modern determinations with the corresponding places derivable from Bradley's observations as reduced by Bessel in the *Fundamenta.* This paper by Mr. Main combined with another by him on the same subject which is printed in the nineteenth volume of the *Memoirs* of the Society, and of which it may be regarded as a continuation, will be found to embody the most important results relative to the proper motions of the stars, which are deducible from the observations made at the Royal Observatory, Greenwich, during the period of eighteen years comprised between the years 1835 and 1853.

The Astronomer Royal has contributed two papers to the volume of the *Memoirs* for the past year. The first relates to the important problem of the motion of the solar system in space. The Council would direct the special attention of the Society to this paper, inasmuch as the method of investigation is totally different to that employed by the author's predecessors in the same field of inquiry. Instead of assuming the apex of solar motion as approximately known, and then deducing a more correct result by a combination of observations having reference solely to the angular motions of the sun and stars, Mr. Airy treats the problem independently of any previous results, basing his researches on linear motions, which he expresses in terms of rectangular co-ordinates referred to the sun as the origin. He then forms equations of condition which, besides the unknown quantities of the problem, involve also possible errors in the constants which enter into the formulæ for precession in right ascension and north polar distance, and also a quantity denoting the possible error of Bradley's observations in north polar distance. The equations are then solved on two extreme suppositions with respect to the irregularities of proper motion. According to one of these suppositions the irregularities are wholly due to chance observations of the stars; according to the other they arise entirely from the peculiar motions of the stars. Laying aside all consideration of the existence of possible errors in the constants of precession, and of a possible error of Bradley's observations, the results obtained for the co-ordinates of the apex of solar motion agree pretty nearly in right ascension, but differ a good deal in declination, from the mean of the results arrived at by previous inquirers.

The second paper contributed by the Astronomer Royal relates to the curious phenomenon of the apparent projection of a star on the moon's disk. The author first considers the hypothesis of the effect being due to the unequal refrangibility of the rays proceeding from the moon and star, but this view of the subject is found to be untenable. He then shows that the phenomenon admits of an intelligible explanation founded on the undulatory theory of light, in which case it must be regarded as an effect of irradiation. This solution of a very obscure point of inquiry is perhaps the most satisfactory that has hitherto been proposed.

Mr. Cayley, who is so well known for his powers as an analyst of the highest order, has contributed two papers. The former of them contains the first part of an investigation of the development of the disturbing function in the lunar and planetary theories. The author refers to a fundamental formula of Hansen's, which gives the expression of the coefficient of the general term of the reciprocal of the distance. A new investigation is given of this expression and its form is somewhat simplified. The object of the paper is to exhibit in as complete a form as possible the preliminary development in multiple cosines of the true anomalies; and to indicate the process of obtaining the ulterior development in multiple cosines of the mean anomalies. The latter of them, supplementary to the author's memoir on the Problem of Disturbed Elliptic Motion, shows how the formulæ contained therein are deduced by means of the general equations of the *Mécanique Analytique*, from the expression of the *Vis Viva* function in terms of the co-ordinates of the body in its instantaneous orbit, and completes the development which he was anxious to give of the dynamical problem.

The volume concludes with three short papers by Capt. Tennant, on subjects connected with the determination of the atitude by observations of transits of stars in the prime vertical r its vicinity. To astronomers in general, and more especially o persons engaged in geodetical researches, these communicaions will be found to offer considerable interest.

The nineteenth volume of the *Monthly Notices*, which has been published some time since in the usual octavo form, has been also printed in quarto sheets, and will be bound up with the *Memoirs* of the Society, conformably to a resolution of the Council submitted to the Society, and approved at the Annual Meeting of last year.

The Council have much pleasure in announcing the appointment of Mr. Grant to the Professorship of Astronomy in the University of Glasgow, vacant by the death of Dr. Nichol. The appointment is in the hands of the Home Secretary. While rejoicing at the acknowledgment which the author of the *History of Physical Astronomy* has thus received, the Council are fully sensible of the loss which their body has sustained in the retirement of Mr. Grant from the editorship of the *Memoirs* and *Monthly Notices* — a post the duties of which he has performed with singular assiduity and ability. But they are happy to state that Mr. Cayley has accepted the vacant office, and has relieved the Council of all feeling of anxiety about the performance of duties on which, more than any others, the utility and honour of the Society depend.

By the death of Miss Elizabeth Coxe Baily, the sister of Francis Baily, which took place on the 6th of last September, the house in Tavistock Place in which the pendulum experiments and the repetition of the Cavendish experiment were performed, falls by bequest into the possession of Sir John Herschel. It will be remembered that Miss Baily presented to the Society the marble bust of her brother which stands in our entrance-hall. For his sake she always felt a strong interest in the Society, and for his sake she desired that the house which he had so closely connected with the astronomical history of his time should still retain its old associations. It would have been impossible to point out a better mode of carrying the intention into effect.

The past year has been gladdened to many of the older Fellows of the Society by a visit to England of Mr. Maclear, who has been for twenty-five years pursuing his astronomical duties at the Cape, without any such vacation as he has at last allowed to himself. This visit was chiefly for recreation, and there is therefore not much to be said of it astronomically; though, from the attention which Mr. Maclear paid to what has been doing in England and on the Continent, the fruits of it will probably be apparent. He takes out a magnetic electrical train for the observatory, electrical triggers for the time-ball, and meteorological instruments, and copies of the standard weights and measures for the Colonial Government; and he hopes to get up a time-ball at Simon's Bay. His old friends were happy to see that his health and energy continue unbroken; and should he be continued in their possession, they feel confident that he has a successful career still before him.

The fellows are aware that a part of the munificent gift of 10,000*l.* to the University of Cambridge by the representative of the late Mr. Sheepshanks for astronomical purposes, has been devoted to the maintenance of an astronomical exhibition. The first appointment under this endowment was competed for in October last, and the examination papers were constructed so as to demand a very high union of theory and practice. Each candidate was also examined *vivâ voce* by the Astronomer Royal on the construction and use of some astronomical instruments, the other examiners being Professor Adams and Mr. Mathison. The number of candidates was six. The exhibition was gained by Mr. Stirling, of Trinity College, who has since that time gained the place of Senior Wrangler; thus giving the Sheepshanks' Exhibition the best possible introduction to public notice. A copy of the examination papers is placed on the table for the inspection of the Fellows.

About four-and-twenty years ago our frequent benefactor, Dr. Lee, gave 100*l.* to the Society as the commencement of a fund, the proceeds of which should be devoted to the succour of such widows and orphans of Fellows of the Society as might be in a condition to claim it. Though this little fund has as yet received no accession, the Council have had fit opportunities of devoting the yearly income to its intended object. Looking at the enormous sums which are bequeathed for such benevolent purposes in this country in every year which passes over our heads, the Council draw the attention of the Fellows to this *Lee Fund*, as it has always been called, in the hope that, by making its existence known, they may possibly be the means of procuring some accessions from those who delight to leave the memory of good deeds behind them.

The Council regret to have to report the loss by death of our valued Associate, Mr. W. C. Bond, of the Cambridge (U. S.) Observatory; Gen. Sir T. M. Brisbane, Bart.; I. K. Brunel, Esq.; Major-Gen. Blanshard; Samuel Cooper, Esq.; Earl De Grey; E. Hughes, Esq.; Rev. Dr. Inman; M. J. Johnson, Esq.; the Earl of Minto; Dr. Nichol; J. A. Nicholls, Esq.; Dr. Ogle; R. Stephenson, Esq., C. E.; Robert Stevenson, Esq., of Edinburgh; Admiral Trotter; and the Rev. W. W. Walton.

WILLIAM CRANCH BOND, M. A., Director of the Astronomical Observatory of Harvard College, was the youngest son of William Bond, who came from England in 1784, and settled as a merchant and shipowner at Falmouth (now Portland), Maine, where the subject of this notice was born in September, 1789. His father having removed with his family to Boston in 1790, at an early age he engaged himself in the occupation of a watchmaker. In 1806 his attention was especially attracted to astronomy by witnessing the sublime spectacle of the total eclipse of the sun, which took place in June of that year. From that time the contemplation of celestial phenomena became his highest pleasure.

He was the earliest discoverer, in America, of the celebrated comet of 1811, while it was yet barely visible to the unassisted eye, and several months before it was generally recognised. At this time, unprovided with a telescope, he was accustomed to make use of the expedient of shading the eyes from stray light, and gazing for several minutes into a dark well. By this means the vision was stimulated, so as to discern the fainter celestial objects which, without such precaution, would have been altogether invisible; the young astronomer little dreaming, in his utter destitution of the means of observation, that he would one day be in possession of unsurpassed advantages for prosecuting his favourite pursuits.

To supply immediate deficiencies, and especially to provide for an accurate determination of local time for the regulation of the ships' chronometers under his care, he constructed a quadrant, and an instrument for obtaining meridian and altitude passages of stars, and subsequently a chronometer — the latter the *first* of American manufacture.

In 1815, Mr. Bond visited England, with instructions from the authorities of Harvard College to examine and report upon the principal English observatories, with the view to the contemplated erection of one at Cambridge. The project was not, however, carried into execution until several years later.

By the addition of astronomical, magnetic, and meteorological instruments, and by the erection of a private observatory at Dorchester, his appliances for observation in these departments became the best in the country.

In the year 1838, he was appointed by the United States' Navy Department to prosecute a series of observations in connexion with the Exploring Expedition under the command of Captain Charles Wilkes. While thus engaged, he was invited by the corporation of Harvard College to remove with his apparatus to Cambridge. In compliance with this request, he continued his observations at Cambridge, under a provisional arrangement, until 1840, when he was appointed Astronomical Observer to the University.

The present Observatory of Harvard College, erected in 1843–44, liberally furnished and endowed by private munificence, has, under his charge, contributed in no inconsiderable degree to the scientific reputation of the country.

Mr. Bond was the first to point out a practicable and exact method by which the instant of time of an astronomical observation could be recorded, by an immediate connexion of the sidereal clock with the galvanic circuit. The idea thus conceived, and discussed between him and the late lamented Professor Walker, has since been perfected in its details at the observatory. Celestial photography was here also first recognised as a possible art, and conducted through its earliest stages. Each of these aids to observation, at the time of its introduction, was a new conception among astronomical methods. The superiority of the former to all other methods for the registration of time-intervals is now universally admitted. The latter, though in its infancy, is full of promise of future application to the most important purposes.

The observatory, under his conduct, has further advanced astronomy by the discovery of a new ring in the system of *Saturn*, and by adding the satellite *Hyperion* to its list of secondary planets. Ten or fifteen comets have also been independently discovered there, and much labour has been expended in the examination and delineation of nebulæ, and in the formation of star-catalogues, the greater part of which remain unpublished.

The accurate longitude of all points of the Continent relatively to the European meridians is wholly dependent upon that of Cambridge, which rests mainly upon the laborious accumulation of observations made for that purpose by Mr. Bond at Dorchester, and subsequently at Cambridge. In considering these, he has been in a position, by constant co-operation with the great national surveys of the coast and of the interior, to render important public services in establishing the geography of the country.

Mr. Bond was a member of various scientific bodies of the United States; a Corresponding Member of the Institute of France, of the Société Philomatique of Paris, of the Society of Natural Sciences of Cherbourg, and of the Academia de Nuovi Lincei at Rome. He died January 29, 1859.

The very recent date of the death of our most eminent Fellow, General Sir THOMAS BRISBANE, renders it desirable that an account of his astronomical career should be deferred until the next report. The Council therefore stop at the announcement of his death on the 28th of January last, at the age of eighty-five.

EDWARD HUGHES was born in Ireland in 1819, and received the rudiments of education in the Kildare Place Model School, Dublin, where his assiduity procured him the friendship of the master, Mr. Irvine, through whose recommendation he was accepted as the junior teacher of writing and arithmetic in the Clergy Sons' School at Edgeworth's Town in 1836. Through the interest of the same friend, Mr. Hughes removed, in 1839, to Aubyn's Pauper Model School at Norwood, which was then under the active patronage of the Committee of Council on Education.

Shortly after this, Mr. Irvine was appointed Head Master of the Lower School of Greenwich Hospital, and in 1841 procured the appointment of Mr. Hughes as his first assistant;

and in 1844, retiring in ill health, Mr. Hughes succeeded to his situation.

While at Greenwich, Mr. Hughes edited and compiled several elementary educational books, principally on geography and arithmetic, evincing considerable research and an appreciation of the requirements of modern elementary instruction.

In his last work—a reading-book, in four volumes—he secured the co-operation of many persons distinguished in their special departments of knowledge; but it may be conjectured that the expense which this entailed upon him, while it exhausted his means, harassed and distressed him, burthened as he was with a very numerous family. His health failed him for a considerable time before his death, which took place at Greenwich Hospital on July 30th, 1859, in his fortieth year.

James Inman, a distinguished member of our Society, who ably directed the application of astronomy to one of its most useful objects—the navigation of the seas—was the younger son of a Yorkshire *statesman*, whose ancestors had been landowners for centuries in the valleys which lie under the height of Penegant. He was educated at Sedbergh Grammar School, and proceeded from thence to St. John's College, Cambridge, where he graduated as Senior Wrangler in 1800. Some little time after his degree, he was sent by the Board of Longitude to assist Flinders in his astronomical observations in the South Seas. This duty was performed under some difficulties. The astronomer was diverted from his object, and was sent to take off the crew of the Porpoise from the reef on which she was wrecked. On that reef Dr. Inman first met Sir John Franklin, who was then a midshipman in the Porpoise, and commenced an intimacy which continued until the last fatal enterprise of that celebrated navigator. The observations made by Dr. Inman in Australia are to be found in the *Transactions* of the Royal Society, but were not brought home without peril. He found it necessary to return by way of China, and was in that well-known East India convoy which beat off the squadron of Admiral Linois. Among the college friends of the subject of this memoir was the celebrated Henry Martyn. On his lamented death in the East, Dr. Inman set out to take up the thread of his missionary labours, but learnt, while pursuing the study of Arabic at Malta for the purpose of his enterprise, that all hope of being useful in that quarter of the world was precluded by a declaration of war with Turkey. In his absence he had been nominated by the University of Cambridge to the post to which he was afterwards appointed by the Admiralty—the Professorship of the Royal Naval College at Portsmouth. It is very well known how assiduously he applied himself to the duties of this situation, how entirely he remodelled the education of our naval officers, how carefully he arranged and composed for their use many valuable works on all the subjects connected with their profession, and so entirely superintended their instruction from first to last that he has been considered by some of his more partial pupils to have fairly earned the title of the "Father of the Navy." Another consideration will give him a still greater claim to this designation.

About the year 1810, he was desired by the late Lord Melville to place the construction of ships of war upon a footing more in accordance with the principles of mechanical science. We have long lost all remembrance in how rude a state our naval construction was in those days. In a few words, our builders scarcely ventured to build a ship except upon a French model taken from the enemy. In many cases the ribs of the prizes were preserved and overlaid with new planking. Even the weights carried by ships of war were not established with any precision; and it was not supposed that the amount and nature of these weights should have any influence upon the form of the ship's body. So far did this ignorance prevail, that nearly to the last period of Dr. Inman's career no effort of his could procure the abolition of a fatal Admiralty rule—that a ship of a certain burden must carry a certain number of guns. Many of his favourite models were rejected, and afterwards reduced, to their great detriment, to support the requirement of this rule.

None of his nautical problems, we believe, were solved free from the operation of this unfavourable condition, so great were the difficulties and discouragements under which this patient son of science laboured.

Dr. Inman's first effort in naval architecture was the translation of Chapman's *Architectura Navalis*, with notes. This work was printed at the press of the University of Cambridge. It was followed by the formation of a superior class of shipwrights; and the object of the institution, which was afterwards termed the School of Naval Architecture, was not, as many have thought, to build ships on purely scientific principles, but to educate a class of men in so liberal and sound a way as to make them the fittest instruments for determining those great problems which might afterwards be brought before them. Few facts had been ascertained—fewer experiments had been made. The exertions of Dr. Inman and his school may, perhaps, be best described as a series of experiments, conducted under the guidance of scientific principle, towards the right establishment of truth. These exertions, when they began to throw a little light upon a difficult and unknown question, raised, as might be expected, a host of competitors, who contributed, both by their errors and success, to the elucidation of the common object. They multiplied experiments; they modified generally-admitted facts. How little way had been made toward the full construction of the science of naval architecture may be learnt from the *brochure* which its founder (for he deserves the title) issued toward the close of his life; how much had been practically done is best proved by the grand specimens of "ships of war" which now carry our flag.

The curtailment of the public expenditure which followed the Reform Bill closed both the Royal Naval College and the School of Naval Architecture, and Dr. Inman returned into private life, from which he was but occasionally summoned: once, to give evidence before a Committee of the House of Commons on the tonnage question as to how the method then in use of taking the tonnage of merchant-ships affected their build, and the best method of amending it; and a second time, when he presided over a Committee of Revision appointed to report on the best method of applying steam to men-of-war. It is said that his influence and determination mainly contributed to relieve our merchantmen from the old method of taking tonnage, and so contributed to the introduction of our modern clippers.

Dr. Inman did not receive, at the termination of his official career, any mark of approbation from the country which he had served so long and so usefully, and for which he had laboured at improvement in a matter affecting her very safety as a nation. His life was prolonged to the advanced age of eighty-three years.

Manuel John Johnson was born May, 1805. He was educated at Addiscombe for the service of the East India Company. In 1821, he was appointed to the St. Helena Artillery, and while in that island was aide-de-camp to General Walker, who encouraged his taste for astronomy. He twice visited Mr. Fallows at the Cape of Good Hope for instruction in matters connected with observing; and constructed the Catalogue of Southern Stars, which afterwards received the Society's medal, and was printed at the expense of the Court of Directors in our *Memoirs*. This labour was performed while he was in charge of a small observatory which the Court had ordered to be established, and in which operations were commenced in 1830. In 1832, the island was made over to the King's Government, and Lieut. Johnson retired on a pension. He entered Oxford as an undergraduate in 1833, and was appointed Radcliffe Observer in 1839. In this post he died, March 1, 1859. He was President of the Society in the years 1857 and 1858.

Mr. Johnson, as has been stated, was appointed by the Trustees of the Radcliffe Observatory at Oxford to the Directorship in May, 1839; but, on account of various circumstances, including the necessity of repairs to the dwelling-house, he did not come into residence till October of that year, and observations were commenced at the beginning of 1840. The instruments at this time consisted mainly of a transit instrument of 8 feet focal length and 4 inches aperture, and a 6-foot meridian-circle by Jones. There were, in addition, two achromatic telescopes, one of 10 feet focal length and 4½ inches aperture, and the other (equatoreally mounted) of 42 inches focal length and 3¾ inches aperture. The transit-instrument was for more than three years a source of great trouble to Mr. Johnson, on account of the bending of the axis; and, after bestowing upon its peculiarities a great amount of labour, and exhibiting considerable skill in the treatment of the observations made with it, he was induced, in the year 1843, to propose to the Trustees the establishment of a new one, such parts of the old one as were available being used in its construction. This transit-instrument and the original transit-circle are in use at the present time, the latter instrument being similar in its general construction to that used by Dr. Robinson at Armagh.

The personal establishment of the observatory for ten years consisted of the Director and one assistant; and with this small staff—and, as far as the writer of this memoir is aware, without any extraneous help—the observations with both instruments were made, and reduced, and passed through the press, and published, with a regularity which is believed to be without a parallel.

On taking the appointment of Director in 1839, and carefully considering to what class of observations the resources of the Radcliffe Observatory could be most usefully devoted in the existing state of astronomy at that time, he determined to apply himself to the observation of a catalogue of circumpolar stars.

In general, there are two great branches of the science to which, either wholly or in part, an observatory may be devoted, namely, to observations of the sun, moon, and planets, or to observations of the fixed stars; and the choice must depend, if it be made judiciously, both on the requirements of the science, and the kind of instruments at the disposal of the director. At that time the planetary observations had been amply provided for by the Astronomer Royal, and the plan laid down by him for the conduct of the Greenwich Observatory left little doubt that for a long series of years this department of the science might be absolutely and solely left to it.

It is true that at that time only four small planets were known to exist between the orbits of *Mars* and *Jupiter*, and that there was not the slightest tendency to that laborious and complete sweeping of the heavens which has resulted in the addition of 53 more of those minute bodies to our list of known planets, and has rendered it necessary to divide the labour of observing them amongst various European and American observatories. But this labour cannot be undertaken at all successfully without large instruments; and even at this present time the transit-instrument and meridian-circle of the Radcliffe Observatory could not with any great effect be devoted to them. Star-observations admit of being classed in two great divisions, namely, of meridional observations for determination of their absolute places and the formation of accurate catalogues, and of double-star observations. The latter, equally with the small planets, require a large equatoreal, and were therefore, in Johnson's case, quite out of the question. The only question left then was the principle by which he should be guided in the choice of his stars to be observed with his two meridian-instruments; and, without much hesitation, he decided on observing the stars within 50° of the North Pole, taking as his basis the Circumpolar Catalogue of Groombridge. The catalogue itself was of moderate size, consisting of rather more than 4000 stars, and was therefore fitted for an observatory with a small observing force, as giving hope of the completion of it within a reasonable time. All the stars were, at their upper transit, at a considerable elevation; and, finally, the interval of time from Groombridge's epoch (1810) gave facility for a moderately good determination of their proper motions. This catalogue, of which the observing portion was completed some years before Mr. Johnson's death, must be considered as his greatest and most important work; though we shall see in the sequel that, excellent and accurate as it is, it is far from forming the only claim to the high estimation in which his labours have been held by astronomers and scientific men generally.

The idea of the erection of a large heliometer was entertained long before it was introduced into the observatory; and, in fact, it had been ordered of Messrs. Repsold, of Hamburgh, almost at the commencement of Johnson's directorship, being mentioned in the Preface to his first volume, published in 1842, though, through delay of the makers, it was not received at the observatory till the year 1848. In connexion with this instrument, it is gratifying to learn from Mr. Johnson's own statement that the proposal of it originated with Sir Robert Peel, as did also that of the new transit-instrument, and of the purchase of the library of the former director, Mr. Rigaud.

The plan and subjects of observation having been settled, the records of the observatory for several years offer little subject for remark. The director and his single assistant, Mr. Lucas, worked with unremitting diligence by night and by day; and the results of the observations of each year were given to the world in a perfectly reduced shape with a punctuality which, under the circumstances, must be considered marvellous. In the adjustment of the instruments, and in the reduction and discussion of the observations, the most fastidious accuracy was used, and the best elements employed. The errors of division of the circle were determined and allowed for, the zenith-point for some years was determined by reflexion-observations of stars and afterwards by means of the reflected image of the wire, and the discordance between the direct and reflexion-observations of the same stars was determined from the combined results of all the star-observations with great accuracy.

But that which most deserves remark as characteristic of Mr. Johnson, was the constant aim at improvement in subjects and processes which did not seem to ordinary minds to admit of anything beyond routine treatment, and the originality of his ideas evinced in his discussion of such subjects. In proof of this it is sufficient to give as an instance his treatment of the magnitude of the stars of his great catalogue. To appreciate this fully it is necessary to read carefully and continuously the introductions to his volumes of observations. One can see how by successive steps the subject grew under his hands; how he soon recognised by comparison of the magnitudes of the same stars assigned by himself and his assistant, that these estimates were not of the vague character which might be supposed beforehand, but that the difference of estimation—the personal equation so to speak—was as well determined as that of any other class of phenomena subject to observation; how he found it necessary, as the eye became more sensitive to small differences in the light of different stars, to adopt a decimal notation to discriminate the gradations existing between one magnitude and the preceding and following; and how, finally, the result of his labours on this subject was a complete discussion of the subject in a perfectly original manner, which connects his name indelibly with that important branch of stellar astronomy.

Before we leave the subject of his meridional observations, it will be necessary to make a few remarks on his treatment of the proper motions of the stars, though his labours on this branch of astronomy were, alas! cut short by his untimely death, and his ideas on the subject may be considered to have been only in their infancy. It is not many years since an eminent astronomer stated that he considered our ignorance of the proper motion of the stars to be the opprobrium of modern astronomy; since that time a good deal has been done, but still it is but little compared with what is necessary to be done to give the history of the heavens for this particular epoch. This is not the fault of the astronomers of this age, that is, it does not arise from any want of care and labour on their part, but from the scantiness of the materials.

The so-called proper motion of a star is that residual error which remains when its place, determined at one epoch, is compared with that determined at another, by means of the application of its precessional motion in the interval between the two epochs; and, to determine this quantity with adequate accuracy, it is necessary that the interval of time separating the two epochs be considerable.

Now there are very few catalogues, or collections of reduced places of the stars, which are separated from our time by a sufficient interval, and which, at the same time, are sufficiently accurate to enable us to deduce the proper motions with any confidence. The chief of the catalogues which can be so employed for the northern heavens are, first, that of Bradley as given in Bessel's *Fundamenta;* secondly, that of Piazzi; and thirdly, that of Groombridge. The epochs of those cata-ogues are respectively the years 1755, 1800, and 1810; and to btain the proper motions of all the stars in any one of them it s necessary that they all be re-observed at a comparatively ecent epoch. For Bradley's catalogue this most useful labour as been performed by Dr. Robinson, and the results have been ecently given to the world in a volume of which honourable ention will be made in another part of this Annual Report; he comparisons with Bradley have been also made by him, and he proper motions of all the stars have been deduced.

The re-observation of Piazzi's Catalogue, as a whole, is ill a desideratum; but that of Groombridge has formed, as e have seen, one of the important portions of the labours of e Radcliffe Observatory under the direction of Mr. Johnson. s he was coming to the conclusion of this admirable work, d collecting results for the formation of his catalogue, his tention was naturally drawn to the subject of proper motion, d in his volume for 1856 he has given some excellent dis-ssions of his mode of treating the subject for particular stars. e does not depend upon *one* catalogue for comparison with s own observations, but carefully collects all the results tained by different astronomers at epochs antecedent to his n, and thus obtains ultimately a definitive proper motion assigning weights to the partial results depending on the ervals of the epochs and the number of observations on ich each result depends. This process is of course laborious, t to obtain the *best* results, or even to avoid great occasional ors arising from accidental mistakes in the results of two cata-ues (if such only be used) it *must* ultimately be undertaken; l Mr. Johnson's labours, cut short as they have been by his mature death, are an excellent specimen of the method which st be employed.

In addition to the catalogue of circumpolar stars, a considerable number of stars were observed in the years 1854, 1855, and 1856, selected as having special claims on the attention of astronomers. The chief of these stars are, first, those above the third magnitude; secondly, stars within 6° of the Pole; and thirdly, those having physical peculiarities or orbital proper motion. In the volume for 1856, all these stars, to the number of 1481, are collected in a provisional catalogue, which forms an excellent basis for the researches of other astronomers.

We now come to a new epoch in the annals of the observatory, namely, the establishment of the heliometer, which had been ordered of Messrs. Repsold at the commencement of Mr. Johnson's directorship, but which, from unexplained circumstances, was not given into his charge till the end of the year 1848. A new building was erected for it in 1849, and the instrument was mounted in the October of that year. The addition of this instrument made another assistant indispensable, and the situation of second assistant was in the summer of 1851 offered to Mr. Pogson and accepted by him, though he did not come into residence till the end of the year. Mr. Pogson was at that time engaged at the South Villa Observatory of Mr. Bishop, and had already distinguished himself by his zeal and activity in the making of double-star observations, and the choice of him was singularly fortunate for the Radcliffe Observatory. By the discovery of two of the small planets during his residence at Oxford, and by his researches on variable stars, he has established for himself a reputation amongst the most successful and zealous cultivators of the science, while his assistance to Mr. Johnson during the time of his engagement at the Radcliffe Observatory was of the greatest service at a very critical period of its existence.

Mr. Johnson proceeded with his usual vigour and skill to master the peculiarities of the magnificent instrument now put at his disposal. An elaborate description of the heliometer, with an account of the methods employed in making and reducing the observations, is contained in the introduction to the eleventh volume (for 1850). The difficulties attending the use of the instrument are increased by a defect in the relative positions of the two segments of the object-glass, which are too close to each other; and which has this bad consequence, that the moving image overlaps the fixed one, when there ought to be a coincidence of images, by a quantity at present amounting to at least 10″, and that therefore measures of very close double stars are impossible.

Mr. Johnson, who was particularly well read in German astronomy, had studied accurately the disquisition on the heliometer in Bessel's *Untersuchungen*, and for the most part uses his methods, though with due regard to the peculiarities of his own instrument.

Every refinement and correction thought necessary by that astronomer are introduced; the methods of determining the value of the scale, of flexures of various kinds, of refraction and aberration, all these matters are discussed with a rigour and an accuracy which can be judged of only by the actual perusal of the introduction to the heliometer observations in the fourteenth volume, and can only be hinted at in a brief memoir like the present.

In the year 1850, besides some measures of the diameters of *Mars* and *Jupiter*, a catalogue of twenty-six of the most important double stars was observed, but in the years 1852 and 1853 the observations were much more important and valuable, consisting, in addition to a re-observation of the chief stars in the *Pleiades*, of continuous and very numerous observations of stars which were known to have, or were suspected of having measurable parallaxes, the chief of which are 61 *Cygni*, Groombridge 1830, and α *Lyræ*. On each of these stars there is an exceedingly complete and instructive disquisition, which will repay the labour of perusal.

Mr. Johnson continued during the years 1854 and 1855 to observe with the heliometer, confining himself to the stars *Castor*, *Arcturus*, and *α Lyræ*, and the discussion of the results for these stars is given in the volume for 1855. After this time, whether from the failing health of the director or from other causes, the instrument appears not to have been used.

Thus far we have spoken only of the astronomical labours of Mr. Johnson, but there is another department of science, namely meteorology, which is in its degree equally indebted to him. Observations of the usual elements of the weather were taken during the whole of his directorship; but, during the latter years, an extension was given both to the observations and to the discussion of them, which has gained for the Radcliffe Observatory a reputation in this department of science equal to that which it has acquired in the purely astronomical department. It is interesting, also, to observe in this subordinate subject the same tendency to progression, extension, and improvement, which has been before remarked as characteristic of Mr. Johnson in the discussion of his astronomical labours. In the volume for 1847 we find the first discussion of the meteorological observations, and, for the next ten years, these observations and the exhibition of them in the best possible shape, appear to have occupied a steadily increasing portion of his time and attention. His predecessor, Mr. Rigaud, had commenced in 1828, and had continued without intermission, a series of observations. These, in the year 1853, Mr. Johnson connected with his own, and was thus enabled to exhibit the comparison of the results for the elements of each year, with the average results for the same elements deduced from the observations of a quarter of a century. From this time each element of the weather is subjected to a very severe discussion, according to the methods practised in the best meteorological observatories.

But, commencing with 1854, an organic change was made in this department by the introduction of photographic registration. The apparatus employed was designed by Mr. Ronalds, and is similar to that at the Kew Observatory, and the management of the photographic processes was for some time after their introduction conducted by Mr. W. Crookes, a professional photographer.

There may be a doubt about the propriety of introducing into the permanent work of any establishment subjects or methods of observation, which require the executive control of professional persons not on the ordinary staff of assistants, and over which the director can exercise very little more than administrative superintendence, and the inconveniences which may arise are sufficiently obvious, but every one must admire the energy of purpose which enabled Mr. Johnson in spite of difficulties to continue this class of observations, and to exhibit the results without any interruption, up to the very time when death snatched him from his labours.

Every important part of the labours of Mr. Johnson has now been discussed with sufficient detail,—a detail in fact which may prove wearisome to those who are not personally interested in him—to enable any one to understand the nature of the qualities which he brought to bear upon the administration of the Radcliffe Observatory. To a vigour of body which enabled him to endure for many years almost uninterrupted fatigue, and a powerful and well-trained intellect which enabled him to grapple successfully with difficulties, and to keep pace with the astronomical activity of his epoch,—he united an inflexible integrity and honesty of purpose, which forced him to give all his time and energy to the duties of his office. In looking at the observations of many consecutive years in which he took upon himself the duty of observing with one of the eridian instruments, it is difficult to find a period of even a fortnight in which he could have been absent from the observatory. It is now too late to deprecate this incessant labour in dull routine processes which could have been performed by an assistant, if his staff had been larger; but we cannot but admire the resolute perseverance and the patient confidence in the value of his work, which, without much present encouragement, animated him with hopes of the future, and induced him year after year to devote his days to the reduction of the observations which he had himself made by his nightly labour.

There is no doubt that to him labour was a necessity of his being, and that his application of it in the duties of his office was limited only by his physical capacity for endurance. But, for his friends at Oxford, who cannot so well appreciate all the excellencies of his professional character, there are other qualities to dwell upon; and the University of that city will long venerate his memory for those endowments of mind and heart that made him so popular while living, and, now that he is gone, regretted with so deep a sorrow. There was a largeness of heart about him which so well accorded with his largeness of view and depth of purpose, that the whole character was as well balanced as it was soundly composed. His social qualities were of the first order. Kindness of feeling towards others, combined with frank and agreeable manners and much love of jocularity, made him a pleasant companion in society and opponent in conversation. Though of very decided opinions and uncompromising maintenance of them, his mode of conducting himself did not leave behind it the sense of past difference. By what is a most excellent gift, not always possessed by men as worthy as himself, his thorough honesty and singleness of mind were made apparent by his conversation and demeanour. All these things put together added weight to his opinions and power to his representations. His memory will be cherished by all who knew him, as well in our Society as in the University of Oxford.

The Council cannot announce the name of Mr. Johnson's successor at the Radcliffe Observatory; no appointment having yet been made.

John Pringle Nichol was born in the year 1804 at Brechin in Forfarshire. He completed his education at King's College, Aberdeen, where he acquired the reputation of a first-class student. At an unusually early age he was appointed schoolmaster of the parish of Dun. From Dun he removed to the town of Hawick; from Hawick to Cupar; and from Cupar to Montrose. While engaged in teaching he studied with a view to the Church, but the tendency of his mind was rather towards scientific and literary pursuits. In 1836 he was appointed Professor of Practical Astronomy in the University of Glasgow. He wrote several works on popular astronomy of considerable merit, and was eminently successful as a lecturer on Astronomy to a popular audience. He died at Rothesay on the 19th of September, 1859.

Professor Nichol possessed a kind and genial disposition, which made him a favourite with all classes of society. He was twice married, and has left a son and daughter by his first wife.

John Ashton Nicholls, who died at the premature age o 36, was a native of Manchester; at the New College in which city the higher branches of his education were received. He was well versed in both the theory and practice of chemistry and, besides establishing a small laboratory for private investigation, he frequently gave chemical lectures to Mechanics and other similar institutions, illustrating the subject by popul experiments.

To the science of astronomy Mr. Nicholls devoted a larg share of his time and study. He possessed an observato furnished with a good refracting telescope, equatoreally mounted, a transit, and other instruments, mostly of the manufacture of Mr. Dancer of Manchester. Unfortunately, however, his health did not prove sufficiently robust to withstand repeated exposure to the nocturnal atmosphere; and he was reluctantly compelled to abandon the attempt at a systematic course of practical observation. He was well known to many astronomical amateurs in the district, and became a Fellow of our Society in June 1849.

But undoubtedly, the object which he had most dearly at heart, was the moral and social improvement of his fellow-men, and especially of the so-called working classes. Here he began *ab initio;* and, while educational efforts of every kind found in him an active friend and supporter, he was principally interested in the schools established under the Factory Act for the instruction of children employed, during half the day, in cotton-mills. And so convinced was he of the advantages of a *compulsory* system of education, that he contended for the extension to *all* sections of the operative community, of the enactment which provides, in factory labour, that no child shall obtain remunerative employment without having first secured a schoolmaster's certificate.

Among the numerous lectures which he delivered in various parts of the country, two well deserve preservation. One was a discourse upon Public Health, printed at the request of the Manchester and Salford Sanitary Association; the other was a lecture on Strikes (also published) delivered to a large audience of working men in Manchester, and the simple, though not superficial, exposition of facts bearing on *both* sides of the question, which he then laid before his hearers might have been circulated in London with infinite advantage during the recent unhappy dissensions between the master and operative builders.

For about two years before his death Mr. Nicholls held the position of Honorary Secretary to the Manchester Athenæum; a post which, though honorary, entailed very arduous duties upon its occupant. That excellent institution had not previously been prosperous; and, it is universally admitted that the beneficial change which it has undergone is largely owing to the indefatigable attention of the late honorary secretary.

He visited the principal countries of Europe, as well as Canada and the United States: but his object in travelling was rather to examine the condition and institutions of the people than the nature of their country. His observations, communicated from America in the form of letters to friends and to a local newspaper, upon the rapid development of border settlements in the States, with critiques on their systems of finance, were highly interesting.

On domestic policy his views were decidedly liberal, though by no means *ultra;* and, while consistent in his own religious belief, he would always extend to all others of different faith, that perfect freedom of action and expression of opinion which he claimed on his own behalf. He took a prominent and successful share in the elections for his native city, and was himself invited by an important section of the constituency of Nottingham to become a candidate for the representation of that place. This invitation, though offering the opportunity of embarking in a career for which he had both taste and qualification, he felt bound to decline; being unwilling to throw additional labour upon his father, with whom he was engaged in business.

The recollection of his private worth, of the kindness of his disposition, and the charm of his conversation, to which a fund of natural humour, aided by the resources of varied and extensive readings, gave a brilliancy not often equalled, will not soon be effaced from the memories of those who enjoyed the privilege of his personal friendship. His funeral was most numerously attended by mourners of every grade; and the monument, which is about to be erected to his memory *by working men*, will testify alike to the excellence of his brief career, and to the appreciation of his labours by those for whose benefit his earnest efforts were so disinterestedly bestowed.

James Adey Ogle was born in 1793, and was entered at Trinity College, Oxford, in 1810, where he devoted himself chiefly to mathematical studies, and took the degree of B.A. with a first class in mathematics in 1813. He graduated M.A. in 1816, and became soon after tutor of his college, taking chiefly the mathematical department; meanwhile he graduated in medicine (B.M. 1816, M.D. 1820). He married, but retained his tutorship; in 1822 he was elected F.R.A.S., and in 1826 F.R.S., he was appointed public examiner in mathematics 1825. In 1827 he was an unsuccessful candidate for the Savilian Professorship of Geometry; on which an eminent medical friend congratulated him, as success would have debarred him from all professional advancement, and he must have been contented to sit down for life on 300*l.* a-year.

He now entered upon medical practice; and, having been previously nominated Aldrichian Professor of Medicine in 1824, he succeeded to the Clinical Professorship in 1830, and lastly was appointed Regius Professor of Medicine in 1851. He had before this time been left a widower with a large family. One of his daughters became the wife of the late lamented Radcliffe Observer, Mr. Johnson. Dr. Ogle died in 1857.

Robert Stevenson, F.R.S.E., an eminent civil engineer, was born at Glasgow in 1772, and died at Edinburgh in 1850. By the death of his father, who was a West India merchant, he was left, in infancy, in very straitened circumstances; but, by the self-denying energy of his widowed mother, he received a good education, and even acquired some tincture of classical learning. He had, however, such a decided liking for the physical sciences, and early conceived so great an admiration of Smeaton, that he determined to adopt the profession of a civil engineer. Notwithstanding many disappointments, which he bore with fortitude, he was enabled to prosecute his studies first at the Andersonian Institution, then just opened at Glasgow, and afterwards at Edinburgh under Robison, Dugald Stewart, and Playfair. In 1797 he became engineer to the Board of Northern Lighthouses, and held that office for nearly fifty years. During that period he erected no fewer than 23 lighthouses, of which one was a Tower on the Bell Rock, off the east coast of Scotland, where the foundation is covered at spring-tides to the depth of twelve feet. This was a most arduous work, and made large calls, which were most manfully met, on the courage and perseverance of Mr. Stevenson, who also showed, in its design and execution, great mechanical skill and practical wisdom. Following the model of Eddystone he yet introduced some important improvements into his design; and, in particular, he turned the floors of the apartments, whose *outward thrust* Smeaton had counteracted by an ingenious device, into direct ties for the outer walls of the tower. He, moreover, greatly improved the optical and mechanical arrangements of lighthouses in general; and added some useful distinctions for enabling the mariner to recognise the lights on a line of coast. As a civil engineer he executed many important works in various parts of the United Kingdom, and he surveyed and traced the lines of many canals and railways which have since been made in accordance with his views. He also gave his advice in many cases of harbour and river navigations both in Britain and Ireland. We owe to his ingenious suggestion the form of suspension-bridges for small spans, by which tall piers are dispensed with. For timber bridges he also prepared a simple construction of arch, in which the *ring-course* consists of layers of thin planks bent to the circular form, and stiffened by means of the *king-posts* which support the roadway of the bridge. This form is now much used for railway bridges. He was, besides, the first person who pointed out the advantages of using malleable iron for the rails of our *iron ways*. His observations on the changes observed in the bed of the German Ocean and on the wasting effects of the sea on the shores of Great Britain show how much his mind was given to physical inquiries. He also, while engaged on the survey of the river Dee, in Aberdeenshire, designed an ingenious instrument for drawing water from the bed of the estuary and bringing it to the surface, unmixed with the upper waters, thus showing that the salt water flows under the fresh.

Mr. Stevenson was a man of great sagacity, fortitude, and perseverance, and was early remarkable for his scrupulous punctuality in all his engagements. When at work he allowed no interruption to take off his thoughts: and this habit he preserved throughout that portion of life when most men plead age as an excuse for indulgence and love of ease. His mind was ever guided by an unobtrusive and sincere piety. His love of independence and his gratitude for kindness were remarkably exhibited in the fact that he devoted the first humble rewards of his professional labours to the payment of what he considered a debt to a charitable institution, of which his mother was compelled, by her difficulties, to take advantage for his education in his childhood. The memory of his early struggles deeply impressed him, and produced unwearied efforts to forward the progress of young men through life, a duty from which no thought of trouble or expense ever made him shrink.

The Council do not attempt any detail of the lives and labours of the two celebrated engineers, Isambard Brunel, and Robert Stephenson, who died, the first on the 15th of September, aged 54, the second on the 12th of October, aged 56. When men of such note, of whom there is not much to say in direct connexion with astronomy, pass away from among us, their lives are given to the public through many channels, in a manner which would render detail superfluous. These two men lived at a fortunate time for the character of an engineer, the time in which the railroad system was brought into operation. How much both, but especially Stephenson, had to do with its first successes, is well known to the world. It more concerns us to note that their time was also that in which the peculiar knowledge of the engineer began to be of use to the astronomer; and it may be presumed that both Brunel and Stephenson saw the connexion when they joined our Society. The approximation of the physical sciences to one another, and to the arts which should always have depended upon them, is one of the characteristics of our day. The Society will always have reason to feel proud that it has enrolled such names as those of which we now speak. And though the Council do not feel competent to lay before the Society an account of their labours, worthy of their reputation, they may at least join the rest of the community in showing that they know how to appreciate their high intellectual and moral worth, and their unspeakable utility to mankind.

In the ordinary operations of the Royal Observatory, Greenwich, very little change has taken place since the preceding year. The observations are precisely of the same character as before, and the reductions have been made to keep pace with the observations. The volume of Observations for 1858 has been very nearly passed through the press, and will be ready for publication in a short time. The printing of the volume for 1859 has commenced.

The galvanic operations of the Observatory have been carried on without material alteration, but considerable difficulty has been experienced in keeping up the time-communication along the lines of railway and for the drop of the Deal ball, on account of the bad state of the wires leading from the Observatory to the Lewisham station of the North Kent Railway. These wires pass underground, and the difficulty of examining and repairing them is found to be so great, that preparations are being made for carrying another set above ground across the Park and through the town to the station of the Greenwich Railway.

It has been the custom at the Royal Observatory, since the year 1835, at the end of every sixth year to collect the results of the observations of stars and of the sun during the preceding period of six years, and thus to form a new and independent star-catalogue, with independent place of equinox. The fourth of these six-yearly periods expires with the termination of 1859; but, as there is reason to think that in the course of another year the observations necessary for forming a complete catalogue of stars of the 5th magnitude visible at Greenwich may be finished, the Astronomer Royal has decided to defer the preparation of the new or seven-year catalogue to the close of the year 1860.

The printing of the Reduction of the Greenwich Lunar Observations from 1831 to 1851 (in continuation of the Reductions from 1750 to 1830, and on the same general system of calculations and the same elements of tables, as far as circumstances permitted) is now advancing. The principal object in these Reductions was, to deliver the annual results of the Greenwich observations from the petty inconsistencies which had crept in by reason of the uncertainty on the elements and the theory of the moon's parallax (an uncertainty from which we are at last delivered by the researches of Professor Adams), and from the small uncertainty on the equinox; and to compare these with a lunar theory more legitimate in its form than that of Burckhardt's Tables. Every observation for a period of *one hundred and two years* is now computed by the same elements of reduction and compared with the same tabular elements. It will not be necessary to continue these reductions beyond 1851, as the parallax may be considered as established, and as Hansen's Tables are now applicable (and partly applied) to the years commencing with 1852. In the course of printing, some small errors of computation have been discovered; and these will effect, in an inappreciable degree, the numerical corrections of lunar elements given in the supplemental *Monthly Notice* for 1859.

During the last year several computers have been employed on the principal reductions of the magnetical observations made at the Royal Observatory from 1848 to 1857. They are based entirely upon the photographic registration of the positions of the three magnetometers; and will present, it is believed, the first example of reductions so made. The measures taken for every hour are not the ordinates of the photographic curves, but the ordinates of pencil curves traced by hand as representing the general sweep of the curves, suppressing inequalities of very short period. It is hoped that, in this way, results of considerable value may be obtained. The ordinates have been taken both for solar hours and for lunar hours; and are so classified as to present, on both systems, the monthly means for days and for hours. The calculations are now far advanced.

The Great Equatoreal at the Royal Observatory will, it is anticipated, be shortly ready for use. Much delay has arisen from the tardiness of experiments on the instrument and its driving clock, from the personal occupation of the Astronomer Royal on public business not connected with the Observatory, from the pressure of other business on the opticians, and finally from little errors of measure derived from the German opticians. It is expected, however, that the whole will soon be in a state fitted for active employment.

From the Royal Observatory, Edinburgh, we learn, that since the commencement of the present year, the restriction imposed in August 1858 on the printing and free distribution of a certain number of copies of the Observations, has been withdrawn; and the typographical work has accordingly been recommenced.

In December 1859, Sir W. Gibson Craig, President of the Edinburgh Board of Visitors, made a personal application at the Home Office, and effectually proved to the Secretary of State—that the additional expense of paper and press-work for printing off 250 free copies, in place of the authorised 50 only, was exceedingly small: so small, indeed, as actually to be covered by the value of the publications annually sent to the Observatory from its numerous correspondents, observatories, societies, and scientific individuals, at home and abroad; and sent in most cases, under honourable expectation of receiving in return a similar free gift of the Edinburgh Observations.

In consequence of this explanation, the full significance of which was appreciated by Sir G. Cornewall Lewis, the Comptroller of the Stationery Office has been instructed to allow the printing of the twelfth Edinburgh volume to proceed on the same plan as its eleven predecessors; and from the second week of last month it has been proceeding accordingly.

In the course of the last summer, while the settlement of the above question was still in abeyance, the Edinburgh Astronomer took the opportunity of a friendly offer from the owners of the Leith and Cronstadt screw-steamer Edinburgh, to make a voyage to Russia for the double purpose of testing his improved free-revolver apparatus in the observation of altitudes at sea, and of perfecting his acquaintance with practical astronomy by a visit to the already classic establishment of Pulkowa.

There, he was most hospitably entertained for many weeks; and had the peculiar pleasure of witnessing, one day on the banks of the Neva, and in company with the astronomers of Pulkowa, St. Petersburg, Siberia, and many other places, of assisting at the reception of the venerable father of astronomy in Russia, W. Struve, on the occasion of his return to his own country, after his long severe illness and convalescent tour. He had been absent nearly a year; or, in the terms of local illustration, had left on the 30th of August *old style* 1858, and had come back on the 30th of August, *new style* 1859. His health now happily re-established, he received affectionately and nobly the earnest protestations of regard from his many devoted pupils, who had come up from distant parts of the great empire; and his countenance—said to be more genuinely Anglo-Saxon than anything that England can show—was lighted up with a benignant smile, as he addressed a few words of excellent English to the British stranger in the crowd.

A residence for so long a time at Pulkowa, first under the direction of M. Otto Struve, and then of his father, could not but give an inquiring visitor many new ideas, and confirm some old ones if good. Among the latter the Edinburgh astronomer is happy to class the eminent qualifications of the official head of astronomy in Great Britain; for these appear to be as extensively and emphatically praised, and as continually heard of, n Russia as in England; what Mr. Airy has said, or what he has done, being always looked to with the greatest interest and respect. Among the former, besides various methods of eliminating from the instruments and observations, minute temperature effects and personal equations, there was a method f procuring equability of clock-rate under rapidly varying tmospheric circumstances, which recommended itself so powerfully from Mr. W. Struve's statement of the principle, as well s from its practical performance in the intensely changing limate of Russia, that a Pulkowa pendulum was asked for, nd brought away, for application to the Brisbane transit clock f the Edinburgh Observatory.

A letter from Pulkowa was found to be the best sort of introduction in travelling through Russia; for the astronomers, wherever they were stationed, were always at home; while Prof. Sawitch at St. Petersburgh and Prof. Schweizer at Moscow vied in their kindly attentions with the Struves themselves. Very interesting it was to find the former gentleman with Sir John Herschel's collected reviews before him, and fully appreciating the ring of their true eloquence and lofty views; and equally interesting, though in a perfectly different manner, was it to hear Prof. Schweizer describe his extensive operations for determining the local attraction about Moscow,—a comparatively flat country, yet with twelve seconds of deviation of the plumb-line existing over several square miles, and with the observatory accidentally pitched over just the focus of the disturbance.

Of the mountains in the South of Russia, important particulars were given, and are soon to be published by General Chodzko, who was encamped with 300 men on Mount Ararat between the two peaks the summer before last; and made geodesical observations on the summit, 16,000 feet above the sea. These experiences, combined with the result of the Teneriffe experiment, led M. Otto Struve, after reading both the popular book and the Admiralty paper on Teneriffe, to think so well of mountain astronomy, as to report to the Russian government on the advisability of establishing a new observatory at Tiflis, with a special view to the astronomer taking his instruments each summer up into the elevated and transparent atmosphere of the summit of the sacred mountain.

The minister having been favourably impressed with the project, transmitted it to the Emperor; who sent for the general, and after a favourable interview, empowered M. Otto Struve to carry out the whole arrangements. The consequence was, that within a few weeks after the first idea was submitted to the Russian Government, the realization of it was actually begun before the visitor's eyes; and on so efficient a scale, that 5000*l.* was given for the erection of the permanent observatory, and 2500*l.* for the purchase of the instruments.

To this new post in Russian astronomy, M. Oblomicosky has been appointed; and it may be confidently expected that he will reap, in his annual ascents from Tiflis, most of the results that a similar following up of the Teneriffe experiment would have earlier given.

The sympathetic appreciation of the Pulkowa astronomers was shown even more strikingly still with the Edinburgh astronomer's attempts to apply the principle of free revolution, to enable altitude observations of stars to be procured at sea. Some parts of his apparatus, which had given way on the voyage, were repaired in a first-rate manner by the intelligent mechanical artist of the central Observatory, M. Brauer; and it was then, with consent, shown to two Russian Admirals (Admiral Lütke and Admiral Zailyónee) of scientific eminence. These gentlemen were so much pleased with what they saw, as to call the attention of the Grand Duke Constantine to the subject; and he having had the instruments sent to his Palace of Strelna, and given them a full examination during two hours' trial, decided on endeavouring, with the assistance of the astronomers and mechanicians of Pulkowa, to adapt them to practice at sea. His Imperial Highness had already perceived with regret the contracted limits for star observation on shipboard; and having on sundry occasions attempted to improve the existing state of things, but without attaining all his wishes, he adopted with enthusiasm the free-revolver appliances, when brought before him on this occasion.

With eminent mechanical appreciation, the Grand Duke separated in the machine before him the effects of imperfect workmanship and deficient proportions from the scientific principle involved in it; and considering that he had in the latter what would enable him to realise his long-sought hope, of a naval instrument with sufficient optical power to show, even through thin haze, a bright image of a 2d–3d mag. star, and with mechanical arrangements that would enable the angular altitude to be ascertained with the solar precision of a sextant, he at once gave M. Otto Struve *carte blanche* for undertaking the manufacture of the new instrument, even to the extent of any number of preliminary experiments on materials and proportions.

At the Cambridge Observatory, during the year 1859, the observations of the small planets, both meridional and extra-meridional, have been discontinued for the purpose of gaining time for the reductions of the observations of the preceding year. The meridian instruments have been employed only in obtaining the places of stars employed in past years for comparison with planets and comets. Comets are still observed with the Northumberland equatoreal, and occultations of stars by the moon are not neglected.

A portion of the dividends of the Sheepshanks' Donation, which is mentioned in another part of this Report, having been appropriated for the payment of two computers to assist in the reduction of the observations of past years, this work is proceeding continuously with great vigour. The printing of the meridian observations for 1852 is in a great measure completed, except the comparison of observed places of planets with tabular places. Professor Challis is devoting considerable time and labour to the comparison of the observed places of the small planets with tabular places, calculating places from the elements where necessary.

The Council have to announce the retirement of Captain Jacob from the superintendence of the Madras Observatory. The place of this zealous astronomer has been supplied by the appointment of Major Tennant, of the Bengal Engineers.

Mr. Hartnup states, in his Annual Report to the Mersey Docks and Harbour Board, that the astronomical and meteorological observations taken at the Liverpool Observatory during the past year differ in no material respect from those mentioned in his Report for 1858. The observations taken with the self-registering anemometer and rain-guage have been tabulated for each hour of the day throughout the year. The Greenwich mean time has been regularly communicated to the port by the dropping of the time-ball and by the regulation of the electric clocks. The large clock in the Victoria Tower, which has six dials, each of which is eight feet in diameter, is now under the control of the normal clock at the observatory; and the first blow of the hammer at each hour of the day indicates Greenwich mean time with all the accuracy that can possibly be required for nautical purposes. On a calm day the striking of this clock can be heard at the observatory, and it is frequently compared with the normal clock. The time which sound takes to pass from the clock to the observatory is 2·6 seconds; and so perfectly does the small clock at the observatory control the large one at the tower, that this difference scarcely varies the tenth part of a second. The striking, Mr. Hartnup says, is not, however, the most convenient test of the accuracy of its performance. The large clock is made to send a galvanic current once every minute, and by so doing the needle of a delicate galvanometer is deflected at the observatory, and the deflection of this needle is found to be quite simultaneous with the beat of the normal clock. Mr. Hartnup is not aware that any clock so large as this was ever before made to perform with such marvellous accuracy. The new time-ball at the Victoria Tower, which is just completed, is 5 feet 6 inches in diameter, and when raised to the top of the mast it is 134 feet from the ground. The accurate dropping of this ball will not, as is the case with most time-balls, be affected by any interruption in the passage of the electric current at the instant of one o'clock each day. The clock at the tower, being controlled by electricity, is made to drop the ball by a mechanical arrangement; and in the event of an accident to the wire of such a nature that no current would pass, even for half-an-hour, the clock, being carefully regulated, would, Mr. Hartnup says, not vary sensibly from correct time. The number of new chronometers sent to the observatory to be tested previous to their being sold has become so great, that the charge for testing was increased, on the 1st of March last, from seven shillings and sixpence to twenty-one shillings. Notwithstanding this, Mr. Hartnup states that the number of chronometers at the observatory on the 1st of January, 1860, was considerably greater than on the 1st of January, 1859; and the subject of making extensive alterations, to facilitate and render more perfect the working of this department of the observatory, is now under the consideration of the Mersey Docks and Harbour Board.

Mr. Lassell has proceeded successfully with the equatoreal mounting of his 4-foot reflector, of which a drawing is now on the table. With the single exception of the figure of the great speculum not being as yet quite brought up to the standard of the 2-foot reflector, the instrument is entirely finished, and in all its mechanical arrangements and contrivances fully equals or surpasses Mr. Lassell's expectations. It requires, indeed, two assistants, one of whom is almost entirely occupied in driving the telescope in R.A.; his duty being simply to turn a handle once round with every beat of a neighbouring pendulum, beating approximately sidereal seconds. This arrangement, originally intended to be but temporary, answers so well that Mr. Lassell has not as yet attempted the contrivance of any machinery which shall do the thing better. A labourer of ordinary intelligence almost immediately acquires the art of carrying on the telescope so that a star remains, even with a high power, apparently immoveable in the field. The other assistant is chiefly occupied in moving on the turn-table which carries the tower, with occasionally a motion of the tower itself, which he has the means of doing so gradually and steadily as not to distract the observer. The four stories of the tower and the supplemental stages adapted (book-shelf fashion) to each, render it easy to adjust the observer's position to any required altitude of the telescope; and the amount of shelter from wind and dew, even at a high elevation, is such that Mr. Lassell could often fancy himself observing within the dome of the 2-foot equatoreal.

A moderate breeze has no sensible effect upon the stability of the telescope, nor does the moulding and whirling of the image seem to be proportionally so great in this skeleton tube as in the entire tube of the 2-foot telescope under its dome. On this latter point, however, more observations are required to speak positively.

The position of the axis of the tube has been so happily chosen that after all the furniture has been attached no counterpoise, properly so called, has been found necessary. The tube, when unlocked in polar distance, has just that slight tendency to the zenith which will insure the speculum-end always being the lowest.

Notwithstanding the size and weight of the telescope, the observer has the means, while his eye is at the eye-piece, of moving it himself small quantities of polar distance — a most essential condition, to enable him to place the object in any required part of the field of view.

The unusually violent gales of this winter have supplied a pretty severe test of the stability of the whole erection, especially of that of the tower, which has not suffered any injury whatever.

The speculum has a system of levers to prevent flexure at low altitudes, similar to those applied to the 2-foot, excepting that in these the short ends are introduced into cavities cast in the back of the speculum, combining the advantages of firmer attachment and a nearer approach to its centre of gravity. Their efficiency is shown by the penumbræ of stars near to focus appearing truly round.

The speculum was lately withdrawn from the tube, and in the laboratory, where Mr. Lassell was endeavouring to give to its surface the requisite amount of perfection; but, in a more recent communication, Mr. Lassell says that the speculum has recently come out of the laboratory with apparently a very fine figure; and, so far as the results of a somewhat interrupted starlight evening enable him to judge, he thinks that he may pronounce the figure perfectly satisfactory. At any rate, it has shown him some objects so as he never saw them before.

The observatory erected by the Rev. W. R. Dawes at Haddenham, Bucks, has recently been enriched by the acquisition of a larger and new Equatoreal of great excellence, the mounting of which abounds with ingenious mechanical contrivances to insure perfect steadiness and uniformity of clock-motion. The instrument has been amply described by Mr. Dawes, page 60 of the current volume of our *Monthly Notices;* and the whole reflects the highest credit upon the makers, Messrs. Alvan Clarke and Sons, of Boston, U.S.

Mr. Carrington's attention has been mainly directed to the subject of the solar spots throughout the year 1859. Though absent from home on five days out of each week, he succeeded in registering the state of the sun on seventy days; and his present assistant has nearly concluded the reduction of the recorded positions, which, at a suitable time, will be laid before the Society. Several weeks were spent on the records of Dr. von Sœmmerring, an account of which was given in the last number of the *Monthly Notices.* Mr. Carrington purposes, in the next place, to employ his assistant in laying down graphically the variations in declination of the magnet, as recorded in the Greenwich volumes, from the commencement of the year 1854, a piece of work in which a beginning has been already made, with the object of (if possible) tracing in some closer detail the relation which has hitherto been only generally established.

The very fine appearance presented by the sun's disk at the end of July last led to the insertion in the public newspapers of several letters from amateurs residing in different parts of the country, and, among others, one from Mr. R. S. Newall, of Gateshead, whose name is better known in connexion with submarine telegraphy. This gentleman, while finding some fault with what he deemed the insufficient success of more professed astronomers in the yet obscure though advancing subject of solar physics, mentioned the possession of a series of solar observations made by himself, and extending over a period of eight years. Mr. Carrington wrote to Mr. Newall a letter of inquiry respecting these alleged observations, but has not yet received a reply.

It is to be hoped that this Society will receive information of all records of solar phenomena which may possess value in themselves, and tend to elucidate the observations of others.

The Hartwell Observatory, under the management of Mr. Pogson, has been chiefly devoted to observations and researches upon the variable stars. An atlas comprising all the known or justly-suspected variables, together with the chief recorded temporary stars, is in a forward state of preparation. The want of such a work of reference has hitherto impeded the progress of this interesting branch of sidereal astronomy, and Mr. Pogson has long felt desirous of supplying the deficiency. For each telescope variable a map of 1° 20′ square will be provided, giving *all* stars down to the 12th magnitude; suitable comparison-stars being selected, and their magnitudes carefully determined and tabulated.

Nineteen of these maps are ready for engraving at the present time. For the naked-eye variables convenient maps will also be provided, containing all stars visible with an ordinary opera-glass. The atlas will be followed by a complete catalogue, and detailed history of its contents.

In addition to this as a chief work, twenty-two telescopic variables have been under continuous observation for the determination of their elements of change. Of these, although in some cases the light curves extend over a period of seven years, Mr. Pogson prefers to await the accumulation of a satisfactory number of maxima and minima to making premature announcements of approximate epochs and periods, soon to be superseded by later results.

The annual ephemeris of the variable stars, now in the third year of its existence, though useful to all persons interested in the subject, is especially intended to aid the meridian observer who desires to fix the positions of these objects with the precision requisite for investigating their proper motions, and yet to be spared the loss of time and labour occasioned by picking up wrong stars, the inevitable result of searching for them at any other time than near their maximum brilliancy.

The ephemeris of the magnitudes of the minor planets—also a work of great utility to the meridian observer, by forewarning him what sized object to expect—has now been adopted by the superintendent of the *Nautical Almanac* in the annual supplement of that work. The revision by observation of this ephemeris, and the determination of the constant M (mean opposition magnitude) for the later-found planets, is a further object continually borne in mind at this Observatory.

A careful examination of the space intervening between the Berlin academical charts and those published by Mr. Bishop is kept up by the aid of Mr. Pogson's manuscript charts, constructed in his leisure time while attached to the Radcliffe Observatory; and, numerous as the family of asteroids has already become, additions will, it is hoped, ere long result from the constant scrutiny maintained for this purpose.

In compliance with the Astronomer Royal's recent circular, the ephemeris-errors of such of the minor planets as are re-observed at Hartwell are communicated to Greenwich immediately on their recognition. Except for this purpose and the comparison of the magnitude-ephemeris, however, observations of the known asteroids form no part of the routine of the Hartwell Observatory. *Lætitia*, *Lutetia*, *Mnemosyne*, *Parthenope*, *Polyhymnia*, and *Proserpine*, have thus been observed, the ring-micrometer being usually employed in consequence of the minute size of the objects sought.

Measures of the interesting binary star γ *Virginis*, so long and ably followed up by Admiral Smyth, were obtained with the ocular crystal micrometer on seven nights during its apparition in the spring of 1859.

The Equatoreal, erected upwards of twenty years back by the late worthy member of our Society, Mr. G. Dollond, has during the last summer been overhauled by his able successor the present Mr. Dollond, and may now be considered in fair adjustment. The excellent Clock, made expressly for this observatory by the late eminent Mr. Vulliamy, has also undergone a thorough examination by the distinguished member of our Council, Mr. Charles Frodsham, and has since performed admirably. Until last October the observatory had the benefit of a marine chronometer, most courteously lent by the Royal Geographical Society, when from some unknown cause the mainspring snapped, and the chronometer has been since returned with thanks to the Society. The 5-foot transit instrument, constructed by the late Mr. T. Jones, F.R.A.S., has been employed exclusively for the requisite determination of local time.

The meteorological observations have been carried on as in former years by Mr. Samuel Horton, with his usual diligence and accuracy, and are regularly forwarded to the Registrar General for publication in his quarterly reports.

Mr. Pogson has now been a regular contributor to the proceedings of this Society for more than twelve years. For four years previous to his holding any official capacity as a professional astronomer, he laboured diligently at the calculations of the orbits and ephemerides of several comets and new planets under the guidance and kind encouragement of his friend and fellow-townsman Mr. Hind. In 1851 he was engaged as assistant at the South Villa Observatory, to aid in the zone observations for Mr. Bishop's ecliptical charts. At the close of that year Mr. Pogson entered the Radcliffe Observatory at Oxford as transit-observer, from which date all the determinations of right ascension up to the end of 1858 were made by him. Of his official duties in that capacity the seven successive volumes, xiii. to xix., are worthy and sufficient records, but it is of his voluntary and ex-official labours with the 10-foot equatoreal, that we have here to take especial notice. The limited leisure available after the hour for closing the meridian instruments was employed with such method and success as to achieve results of the highest importance. Numerous observations of the minor planets, published in the *Astronomische Nachrichten;* the first discovery of the planets *Isis*, *Ariadne*, and *Hestia*, the first of which was rewarded by the Lalande medal of the Académie des Sciences; the independent discovery of *Amphitrite*, and the records of five other undoubted planets, lost only in consequence of the disadvantages occasioned by the equatoreal work being subordinate to his meridian duties; the construction of seven charts subsidiary to Mr. Bishop's, but extending to the 12th magnitude, all unpublished, though long since ready for the engraver's hands; the discovery of ten new telescopic variable stars; an extensive series of observations of his own, and other variables, also unpublished; photometric investigations and other observations of occasional phenomena; all these were facts actually accomplished with a very roughly mounted instrument in the open air exposed to wind and weather, without the highly efficient and comfortable equipments now placed at his disposal in Dr. Lee's Observatory at Hartwell. Mr. Pogson entered upon his new duties as Director of this last-named Observatory on January 1st, 1859, and already a valuable work is announced as in fair progress for the promotion of a branch of our science too long neglected in this country.

Mr. De La Rue has continued to devote his own Observatory to the photography of celestial objects, and he has with the aid of Mr. Beckley, assistant at the Kew Observatory, brought the photoheliometer and the process employed to that degree of perfection that a photograph of the sun can always be obtained under favourable conditions. Mr. De La Rue read at the British Association a report on celestial photography, describing the methods employed, which will appear in full in the forthcoming volume of Reports.

Mr. De La Rue has proposed to the Royal Society to undertake experiments, having for their object to determine if it be possible to considerably increase the aperture of reflecting telescopes in proportion to their focal length with advantage in celestial photography.

Monsieur Goldschmidt of Paris, the indefatigable planet-hunter and discoverer of twelve of these bodies, was, we regret to state, compelled to retire from his labours temporarily for the benefit of his health, and to seek change of air by a residence in the country. He has, however, still pursued his astronomical recreations; and the most singular feature yet known in the history of the wonderful variable star U *Geminorum*, which has so unceremoniously cheated everybody by its premature return one-fourth of a period before its expected time, is entirely due to his vigilance and zeal. His interesting variable star ν *Virginis*, too, proves to be one of short period, being now visible and approaching another maximum.

Mr. Baxendell, of Manchester, in addition to a steady adherence to his selected objects of research amongst the variable stars, has recently added one more to the list, of his own discovery, in the constellation *Sagitta:* changing from the 8·4 to the 9·8 magnitude in about 74 days. His observations are chiefly made with the instruments of Mr. Worthington's Observatory; sometimes with the achromatic equatoreal, at others with the large Newtonian reflector, 13 inches in aperture, and judging from Mr. Baxendell's observations evidently of very superior quality and performance.

In regard to astronomical instruments, the principal novelty of the last two or three years is the introduction, by MM. Steinheil and Leon Foucault, of a method of constructing the mirrors of reflecting telescopes by grinding to the proper form a disk of glass, and depositing upon it a coating of metallic silver, by a chemical process which has been explained to the Society by our Secretary, Mr. De La Rue. The Astronomer Royal had the good fortune in the past summer to make a careful scrutiny of a large part of the moon's surface with one of M. Foucault's telescopes at the Imperial Observatory of Paris, and speaks in very high terms of its performance. The image was very brilliant, and many minute and delicate shadings, both on the volcanic and on the more level parts of the moon's surface, were brought out with a distinctness which he had never before witnessed. The reflector in question is made of a rather thin disk of glass, which bends by its weight, and a singular contrivance is used for keeping it in form. An air-cushion of caoutchouc is placed behind it, from which a flexible pipe with a stopcock is led to the observer's mouth, and the observer blows air in till the image is perfect. If he blows in too much the distinctness is lost, and a little air must then be allowed to escape. The success of this contrivance is complete.

There is strong reason to suspect that a new planet, interior to *Mercury*, was on the 26th March, 1859, observed to transit over the sun's disk by M. Lescarbault at Orgères in the department Eure et Loire. The circumstances attending this remarkable observation are referred to, and the conclusions deduced from the observations, as to the elements of the orbit, are mentioned in the last number of the *Monthly Notices;* it only remains to add, that by the collection of prior cases of suspicion which this observation has led inquirers to make, there appear to be not less than four cases recorded in which dark bodies have been seen on the disk of the sun, possessing abnormal motion, on dates which fall within the limits of visibility of Lescarbault's planet during transit at its ascending node. In addition to these four cases, which may be taken as of some value in confirmation of a unique observation, it cannot be overlooked that other accounts exist of considerable circumstantiality which indicate the transit of planetary bodies the orbits of which cannot be identical with that of the planet of Lescarbault. In an imperfect list of such cases, printed in the last *Monthly Notice*, one case is quoted of an observation by Canon Stark of Augsburg on Feb. 12, 1820. To the surprise no doubt of many will be found in the *Olbers-Bessel Correspondence* the following passage (vol. ii. p. 162) in a letter from Olbers to Bessel, "What do you say to *Steinhübel's* observation on the 12th of February this year (1820) of a dark round well-defined spot, which in five hours completed its transit across the sun's disk?" There is also the observation on Jan. 10, 1818, by Mr. Capel Lofft, the account of which has recently been republished by the Rev. T. W. Webb, in a little handbook, to which the Council refer with pleasure as useful and carefully prepared. Then there is the observation of Mr. Scott in the first fortnight of July 1847. Time will show, perhaps even the next two years, in spite of the known difficulty of proving a negative, whether any of the above are to be received as the first imperfect observations of actually existing interior planets, provided that disengaged observers will apply the telescope and acquaint themselves with ready means of recording the position of bodies they may see, so that such bodies may not again be confounded with ordinary solar phenomena. In this sense amateurs may be told that a new mine of observation has been opened.

On the 2d of April, 1859, a comet was discovered by M. Tempel at Venice. It continued visible till about the middle of May. The elements have been found to be sensibly parabolic.

On the 22d of September a new planet was discovered at the Observatory of Bilk by Dr. Luther. This forms the 57th of the group of minor planets revolving between *Mars* and *Jupiter*. The new planet has been called *Mnemosyne*.

Among the works which have appeared within the year the Council have particularly to notice the *Places of 5345 Stars*, observed at Armagh, by Dr. Robinson, a general account of which has already been given in the *Monthly Notices*. This work is a considerable contribution of materials of high class workmanship, available as data in the subject of proper motion in the solar system, and in stellar astronomy generally. The scheme followed has been the re-observation with modern precision of Bradley's Catalogue, and, as intervals became available by the exhaustion of the first working list, of Piazzi's Catalogue as a secondary one. Where exceptions occur to this plan they have been incidental. The arrangement of the published results is singularly lucid and economical of space, and on inspection will at once attract attention to a leading feature of the work, namely, that the instrument with which the results have been obtained has passed not only the maker's but the astronomer's hands. The division error applied is in the case of each star scrupulously stated, and the reports given in the Preface of the various precautions taken and considerations taken into account by the author, attest his high position as a practical astronomer.

A few weeks since, our indefatigable member of Council, Admiral Smyth, published, in a splendid volume,* printed at the expense of Dr. Lee, a continuation of his observations on double stars, with other observations, and much historical detail on many subjects. This volume, an essential supplement of the *Cycle* by the same author, is a remarkable union of utility and beauty of execution. As the details, so far as they consist in new observations, are in continuation of a well-known series, no further description is here necessary.

The approaching total solar eclipse of July 18, of the present year, has received from English and Continental astronomers that attention which its importance deserves. With regard to the facilities of travelling in Spain and the choice of stations for observing, M. Lamont has afforded excellent information in a letter to the Astronomer Royal, which is printed in the last *Monthly Notice*. The Astronomer Royal at our evenng meeting of December last entered very minutely into the subject, and detailed the various methods which had been proposed for observing the various physical phenomena which usually present themselves. Mr. Hind has published a circular with a map of the revised path of the shadow, drawn by the use of the best existing elements, Hansen's Lunar Tables and Le Verrier's Solar Tables being the chief authorities employed. In the Paris *Cosmos* of October 28, 1859, is a paper by M. Faye, on the methods of observing and the ends to be answered in the observations of the eclipse; and it has been proposed by M. Le Verrier at the Institute that he should have the direction of a commission to conduct the observations in Spain and Algeria.

At the suggestion of Mr. De La Rue the Royal Society has voted the sum of 150*l.* for the expenses of removing the Kew photoheliometer to Spain, for the purpose of photographing the eclipse, and Mr. De La Rue and Mr. Carrington intend to accompany the expedition, unless prevented by unforeseen circumstances.

Papers read before the Society from February 1859 to February 1860.

1859.	
Mar. 11.	On Determining the Latitude by Transits. Major Tennant.
	R.A. and N.P.D. of Planets, from Observations at Madras. Capt. Jacob.
	On Donati's Comet. Mr. Goddard.
	On the Planet *Saturn*. Rev. W. R. Dawes.
	On the Silvering of Glass Specula. Mr. De La Rue.
	On the Proper Motion of the Solar System in Space. Mr. Airy.
	On a new Process for Constructing large Reflecting Telescopes. M. Foucault.
	Second Paper on same subject. The same.
	On some Indications of Rotation in a Solar Spot. Mr. Birt.
April 8.	On the Disturbing Function in the Lunar Theory. Sir J. Lubbock.
	Extract of Letter on the Solar Spots. Dr. Peters.
	Observations of Small Planets at Greenwich. Mr. Airy.
	On a Cause of Irregularity in a Chronometer. Mr. Wackerbarth.
	Elements of *Europa*. M. Lepissier.
	On the Corrections of Transits on the Prime Vertical. Maj. Tennant.
	On a Phenomenon seen during the Solar Eclipse, March 1858. Lieut. Hardy.
	On a new Method of Clearing the Lunar Distance. Col. Shortrede.
	On the Secular Variation of the Eccentricity and Inclination of the Moon's Orbit. Mr. Adams.
	On the Apparent Projection of Stars on the Disk of the Moon. Mr. Airy.
May 13.	Observations of Small Planets at Greenwich. Mr. Airy.
	Comparison of Burckhardt's and Hansen's Lunar Tables with recent Greenwich Observations. Mr. Airy.
	Ephemeris of Variable Stars for 1859. Mr. Pogson.
	Letter to Admiral Smyth on Donati's Comet. Mr. Maclear.

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* *The Cycle of Celestial Objects continued at the Hartwell Observatory* to 1859, with a notice of recent discoveries, including details from the Ædes Hartwellianæ. By Vice-Admiral W. H. Smyth. London, 1860, 4to.

Occultation of *Saturn*, May 8.		Capt. Noble.
Ditto	ditto	Mr. Jeans.
Ditto	ditto	Rev. W. R. Dawes.
Ditto	ditto	Mr. W. Simms, Jun.
Ditto	ditto	Mr. Airy.
Ditto	ditto	Capt. Shea.
Ditto	ditto	Mr. Burr.
Ditto	ditto	Mr. Grove.
Ditto	dttto	Mr. Whitbread.
Ditto	ditto	Sir W. K. Murray.

June 10. On Traces of Eruptive Action in the Moon. Rev. T. W. Webb.

Suggestions as to the Structure of Tails of Comets. Rev. T. W. Webb.

Observations of Small Planets. Mr. Airy.

Occultation of *Saturn*. Mr. Lassell.

Sur la Variation du Moyen Mouvement de la Lune. M. de Pontécoulant.

Extract of a Letter to the Astronomer Royal. M. Hansen.

On the Polarisation of the Moon's Light. Prof. Secchi.

Occultation of *Saturn*. Mr. Pogson.

Observations of Transits of the Sun by means of his Projected Image. M. Ragonaatha Charey.

On the Successive Illumination of the Lunar Crater *Geminus*. Mr. Birt.

On the Star ν *Scorpii*. Capt. Noble.

Occultation of *Saturn*, May 8. Prof. Challis.

July 8. Elements of Comet I. 1859. Messrs. Safford and Hall.

On Abnormal Refractions during Eclipses of the Sun, and on the Determination of Longitude by Eclipses. M. E. Liais.

Note on the Occultation of *Saturn*. Mr. G. F. Pollock.

Corrections of the Elements of the Moon's Orbit deduced from the Lunar Observations made at the Royal Observatory, Greenwich, from 1750 to 1851. Mr. Airy.

On the Polar Distances of the Greenwich Transit Circle. Mr. Marth.

Nov. 11. Correction of an Expression occurring in a passage in a former Communication. Capt. Jacob.

List of Star-Magnitudes not included in Baily's Edition of Lalande's *Histoire Céleste*. M. Hencke.

Note on Mr. Pollock's Description of the Occultation of *Saturn* by the Moon. The Astronomer Royal.

On the Triple Star ν *Scorpii*. Rev. T. W. Webb.

On the Telescopic Appearances of the Lunar Craters *Geminus* and *Bernouilli*. Mr. Birt.

Description of an Equatoreal recently erected at Hopefield Observatory, Haddenham. Rev. W. R. Dawes.

Extract of Letter to Mr. Carrington. M. Schwabe.

On an Appearance seen in the Sun, Sept. 1, 1859. Mr. Carrington.

Observations of Comet VIII. 1858, and Small Planets. Mr. Pogson.

On Determining the Longitude by the Phase of the Moon. Mr. Brown.

On some lately discovered Double Stars. M. O. Von Struve.

On the Shape of the Moon. M. Gussew.

New Double Stars discovered by Mr. Alvan Clarke. Rev. W. R. Dawes.

On the Periodical Variations and Level and Azimuth of the Transit. Mr. Ellis.

On a curious Appearance seen in the Sun. Mr. Hodgson.

Dec. 9. Physical Observations of *Jupiter*. Sir W. K. Murray.

Observations of Small Planets and of *Jupiter's* Satellites. Mr. Airy.

On the Physical Constitution of Comets. Mr. O. G. Downes.

On a Remarkable Solar Spot. Rev. W. R. Dawes.

Observations of *Jupiter's* Satellites. Mr. Lassell.

On the Position of *Venus* during the Total Eclipse of July 18, 1860. Prof. Chevallier.

Occultations of Stars by the Moon. Capt. Noble.

On the Variability of α *Cassiopeiæ*. Mr. Birt.

Observations of Comets and Occultations observed at Sea. Capt. Callow and others.

Occultation of the *Pleiades*, 1859, Dec. 8. The Astronomer Royal.

Unexpected Appearance of U *Geminorum*. Mr. Pogson.

1860. On Variable Stars. Mr. Pogson.

Jan. 13. On the Longitude of the Sydney Observatory. Mr. Scott.

Suggestions connected with the Carrington-Hodgson Solar Phenomenon. Prof. C. P. Smyth.

On Sœmmerring's Observations of the Solar Spots. Mr. Carrington.

Apparent R.A. of the Moon's Limb and Moon-culminating Stars observed at Washington. Lieut. Gilliss.

Comparison of the Probable Error of a Star observed by the Eye and Ear, and Chronographic Methods. Mr. Dunkin.

On the Lunar Crater *Fracastorius*. Mr. Birt.

List of Public Institutions and of Persons who have contributed to the Society's Library, &c. since the last Anniversary.

Her Majesty's Government.
The Lords Commissioners of the Admiralty.
The Board of Trade.
The Secretary of State for India.
Royal Society of London.
Royal Asiatic Society.
Royal Geographical Society.
Royal Institution.
Geological Society.
Linnean Society.
Zoological Society.
Photographic Society.
Society of Arts.
British Association.
Art-Union of London.
Institute of Actuaries.
British Horological Institute.
University College, London.
The Radcliffe Trustees.
The Royal Irish Academy.
Royal Dublin Society.
Royal Society of Tasmania.
L'Académie Impériale des Sciences de l'Institut de France.
Le Dépôt Général de la Marine.
Le Bureau des Longitudes.
The Society at Cherbourg.
Imperial Observatory, Paris.
Imperial Academy of Vienna.
Royal Academy at Berlin.

The Royal Academy of Sciences, Munich.
Royal Observatory at Munich.
Royal Academy of Sciences, Brussels.
Royal Observatory, Brussels.
Royal Observatory, Königsberg.
The University at Göttingen.
The Academy of Sciences, Stockholm.
Society of Sciences, Batavia.
Philosophical Institute of Victoria.
The Department of State, Washington.
American Philosophical Society.
American Academy of Arts and Sciences.
Smithsonian Institution.
Observatory at George Town.
The Franklin Institute.
The Canadian Institution.
Council of the City of Manchester.
The Editor of the Athenæum.
The Editor of the Literary Gazette.
The Editor of the Critic.
The Editor of the Atalantis.

G. B. Airy, Esq.
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M. A. Bouché.
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Lieut. Gilliss.
Jas. Glaisher, Esq.
Dr. B. A. Gould.
Lt.-Col. J. D. Graham.
Prof. Hansen.
Prof. Heis.
Prof. Henry.
J. Herapath, Esq.
Mrs. Hannah Jackson.
M. Jeanjaquet.
Dr. Lawson.
Prof. Von Littrow.
Lieut. Maury.
J. C. Maxwell, Esq.
M. L'Abbé Moigno.
Sir W. K. Murray.
M. Pape.
H. Perigal, Jun., Esq.
Dr. Peters.
M. Quetelet.
M. Reslhuber.
M. E. Roche.
Sigr. Scarpellini.
M. Schubert.
Prof. A. Secchi.
Prof. C. P. Smyth.
Capt. Spratt.
Prof. Sylvester.
Hon. Mrs. Ward.
Rev. T. W. Webb.
Dr. Weisse.
Capt. Wilkes, U. S. N.
Dr. R. Wolf.
Lord Wrottesley.

Address delivered by the President, the Rev. Robert Main, on presenting the Gold Medal of the Society to Professor Hansen.

Gentlemen,—I wish it had fallen to the lot of one of the great physical astronomers of whom this country can boast, and whom this Society is proud to enrol amongst its members,—I wish, I repeat, that it became the duty of an Airy or an Adams, a Herschel or a Lubbock, to detail to you the grounds on which your Council have decided to award the Gold Medal of the Society to Professor Hansen for his Lunar Tables. The subject which it is my duty to expound to you in connexion with this award is so vast and so difficult, the analysis which must be treated of is so refined and complicated, that were I to confine myself solely to the task of setting before you in a popular shape only such a sketch of the subject as is necessary to give you an adequate idea of the merits of the great work under consideration, without reference to the successive steps by which the lunar theory has been brought to its present perfection, the task would be sufficiently formidable to men much more intimately acquainted with the subject than I can pretend to be. But when you consider that, in justice both to your Council and to the illustrious astronomer whose work is crowned in this award, it is necessary to give you some idea of the analytical processes of preceding astronomers, and of the merits of the Tables which have embodied their results,—and that these, too, must be taken in connexion with the successive advances of practical astronomy, which have gone hand in hand with the theoretical developments in producing the work before us,—I may well ask for your indulgence both as regards the manner in which my duty is performed, and for the rather wearying details which will occasionally task your patience and attention. Yet, Gentlemen, permit me, before commencing the serious portion of my subject, to congratulate you as members of this Society on the pleasing duty which we are met together to perform,—a duty pleasing at all times, and under all circumstances, as exhibiting at stated periods that no improvement either in the theory or the practice of astronomy escapes our notice, and that we are eager to recognise it by the bestowal of the only distinction which it is in our power to award, but particularly pleasing on an occasion like the present, which marks the accomplishment of an astronomical era rather than the bestowal of a reward, and reminds us that one of the greatest of the tasks which were imposed on the astronomers of this age has been successfully achieved.

To justify these remarks, Gentlemen, it is necessary that I take you back for nearly two centuries, to the infancy, in fact, of modern astronomy, and remind you of the single and expressed object which the cultivators of the science in that age had in view. The growing greatness of this nation and the rapid development of its commercial relations made a study of astronomy absolutely necessary, not as a matter of pure science or of abstract speculation, but with a view to the observations of the fixed stars and planets for the purposes of navigation.

And, if I should be addressing any who have made the astronomical methods in use at that day a subject of study,—any, in fact, who are acquainted with them in Jonas Moore's Mathematics, and who understand the use of one of the principal nautical instruments at that epoch, namely, the nocturnal,—he will appreciate the difficulties which mariners had to encounter in a voyage to the East or West Indies, and the urgent necessity which existed for the introduction of better instruments and better methods. And it fortunately happened that this age was an age of great men, of giants in intellect, who could lead the advancing intelligence of the time, and could alternately urge and assist the Government of the country by advancing schemes suitable for the scientific and social as well as the commercial interests of this country. It was the age of Newton, Boyle, Hooke, and Wren, not to mention many others who were quite prepared to take every advantage of the Continental scientific discoveries bequeathed to them by that immediately preceding, and above all, and as a natural consequence, it was the age which gave birth to the Royal Society. Well, Gentlemen, one of the first fruits of this intellectual activity was the establishment of the Greenwich Observatory by the Government, with the expressed object of advancing the art of navigation, or of enabling the mariner to navigate the ocean more safely, and to find by observation and calculation his position upon it at any given instant.

It required no great sagacity to point out the objects to be aimed at in the prosecution of this idea, and, accordingly, in the warrant of the first Astronomer Royal, his duty is defined to be,—"to rectify the tables of the motions of the heavens and the places of the fixed stars, in order to find out the so-much-desired longitude at sea, for perfecting the art of navigation." You all know this, Gentlemen, but I call your attention to it now in a special manner, because it is the groundwork of our discussion on the subject of Lunar Tables, and because, if the Government and its advisers of that period had not been wise enough to lay down these stipulations, and, if the series of Astronomers Royal, from that time to the present, had not, under occasional misrepresentation and under evil report and good report, acted upon it with an honesty and a fidelity that have no parallel, the Tables now before you would in all probability never have existed.

Then, Gentlemen, look what was to be done to carry out these intentions, and be careful not to measure the amount of work, the necessary inventions, and the necessary discoveries by the notions derived from the present state of science; but go back in thought to that epoch absolutely destitute of everything but the genius and the determination of the men entrusted with this precious charge.

The notion of the grand method of discovering longitudes at sea by what are called "Lunar Distances" was not new to the seventeenth century. Apian* and Gemma Frisius, in the early part of the sixteenth century, and Kepler, in his Rudolphine Tables, had separately proposed the method, and a French Commission was appointed in 1634 to examine into the method as proposed by Morin.† But there was not one of the necessary requisites for carrying it out, and in particular the Lunar Tables of that epoch were quite inappropriate. Even Halley's Tables, constructed on the Newtonian theory of gravitation, had frequent errors amounting to 7′ or 8′, and this on the equator would produce at times an error of 180 nautical miles in longitude.

The art of chronometry was also, equally with pure science, in its infancy. It was not till the impulse was given in the early part of the eighteenth century, by the large reward offered by the British Government for the discovery of the longitude at sea, that the series of chronometrical improvements was begun, which, commencing with Harrison, was taken up successively by Arnold, Earnshaw, and a multitude of artists, extending to the present day, who have made the chronometer to become an instrument of wonderful precision, and a worthy attendant on the noble science of Nautical Astronomy, which has been by similar well-directed efforts brought to its present state of perfection. To make the method of Lunar Distances applicable, there are required, in the first place, instruments capable of observing the distance and the time with great precision; and, in the second, tabulated places of the moon and stars, which shall represent their true places with all attainable accuracy. With regard to the first requirement, the chronometer and the sextant, with their successive improvements, have done all that is needed; but to satisfy the second, there have been required all the resources of our own National Observatory, continued for above a century — not to speak of others — and the intellectual energies of all the great European mathematicians and astronomers continued for the same period. To avoid unnecessary prolixity, I shall assume that you are quite familiar with the successive steps by which the places of all the bright fixed stars have been determined with such accuracy, that doubt or error to the amount of a second of space in the position of any one of them now becomes a matter of serious discussion. I need only remind you that Maskelyne's thirty-six stars formed the basis of all following star catalogues, and that the Greenwich Mural Circle by Troughton was the origin of the precision of modern observations. But, with regard to the moon, it is necessary that I beg your attention again to the fact that, from the time of Bradley, observations of that luminary were made at Greenwich unremittingly with scrupulous care, and with all the accuracy attainable by the instruments and methods in use at each particular epoch. Thus, when mathematicians began in earnest to study the movements of the moon, and required observations for the more accurate determination of the elliptic elements on which their calculations were to be founded, it was to Greenwich that they looked, and not in vain, for what they needed. But to estimate the labour which was to be undergone by any one who at that period engaged in lunar researches, it is to be remembered that all these observations were left just as they were made, that is totally unreduced. There *was* then the raw material, the rich ore which would, with due labour, give forth the precious metal, but which was absolutely useless till much severe labour had been bestowed upon it. Still the observations existed. Greenwich was the treasure-house where the Astronomer, provided with funds and skilful in the processes of the practical department of his science, could find the materials which he needed. But here lay the difficulty: — first, astronomers had not themselves imbibed that idea which seems intuitive to us, of the comparative worthlessness of unreduced observations, and funds were much less readily produced for such work than at the present time; and, secondly, it was extremely rare then, as it is at present, that the same individual should be a first-rate physical as well as a first-rate practical astronomer, and, till an individual possessing these rare qualities should be found, there was, in fact, no chance of obtaining Lunar Tables adequate for the finding of the longitude at sea.

With these preliminary remarks, I am now in a condition to point out to you the successive steps by which the theory of the moon has been brought to its present state of perfection. At the outset there is a curious fact to be noticed, namely, that half a century after the publication of Newton's *Principia*, there was no advance worth mentioning made in the analytical lunar theory. In England it is true the doctrine of universal gravitation was eagerly embraced; and the Cartesian philosophy was very soon expelled from Cambridge, and Newton's theory was expounded first by Dr. Samuel Clarke, and afterwards by Whiston, while at nearly the same epoch Keill taught it at Oxford. But, on the Continent, the hostility to it was for a considerable time universal, — Leibnitz and John Bernouilli, and, at first, Huygens, were unanimous in rejecting it, as well as the French astronomers Cassini, Maraldi, and Maupertuis; and it was not till the celebrated Voltaire gave to the world a popular exposition of it, that it began to gain ground and to conquer the delusions which the fascinations of the Cartesian philosophy had produced.

I need not dwell on the causes which led to this want of recognition of the truth; they are well known and obvious, and belong to human nature.

It is not impossible that there are men even now who would, through love of the study of Aristotle's ethical works, endeavour to maintain the exploded assumptions of his physical theories. The prejudices of education are with exceeding difficulty eradicated, and the Cartesian philosophy had been universally taught and universally admired from its apparent beauty and simplicity: the scientific men of the day gained their reputation by expounding its principles, and students flocked to every university to imbibe them. On the contrary, the Newtonian philosophy was severely mathematical, beyond the acquirements of that age. Many of the analytical processes were invented by the illustrious discoverer, and even those processes were concealed by an after-interpretation according to the old synthetic methods of geometry. Still this blindness could last only for a time, and the original tardiness of the Continental geometers was more than atoned for afterwards by a succession of brilliant analytical discoveries commencing with the middle of the eighteenth century, and continued without intermission to the present time; while, on the contrary, the English mathematicians, content with the fame of Newton and with the methods which he had barely sketched for them, continued even within our own remembrance to expound his doctrines without improvement, and almost to ignore in our seats of learning the analytical developments which had become rudimentary science to the astronomers of the Continent.

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* In his *Cosmographia*, published first in 1529.

† *Longitudinum Terrestrium necnon Cœlestium nova et hactenus optata Scientia.* Paris, 1634.

It will now be necessary, for the sake of exhibiting to you more clearly the progress of the Lunar Theory, to divide the time from the middle of the last century, when the analytical methods were first promulgated, into epochs. These will be four; namely, first, that of Clairaut, Euler, and D'Alembert; secondly, that of Mayer and Mason; thirdly, that of Burg, Burckhardt, Damoiseau, and Plana; and fourthly, the present, which I will designate as that of Hansen.

It is remarkable that three of the greatest geometers of the Continent employed themselves almost simultaneously on the analytical theory of the movements of the moon, namely, Clairaut, Euler, and D'Alembert. Of these, the most remarkable was Clairaut. In 1747 he read before the French Academy a paper on the subject, in which he announced his celebrated difficulty about the motion of the apogee, deducing from the Newtonian theory, as Newton himself as well as D'Alembert and Euler had done, only half the amount. But, his process being one which proceeded by successive approximations, he soon found the cause of his error, namely, that the motion of the apse required a higher order of terms to be taken into account than was necessary for the other large inequalities; and, in 1749, he read another paper before the Academy with the important announcement that he had succeeded in reconciling theory with observation. About this time the Imperial Academy of St. Petersburg had offered a prize for the best Essay on the Lunar Theory, the subject being defined thus:—"To determine whether all the inequalities which are observed in the motions of the moon are in accordance with the Newtonian theory; and what is the true theory of all these inequalities, by means of which the place of the moon can be most exactly defined at any given time."

Clairaut, who sent in his MS. in 1750, was the successful competitor, and his memoir was printed in 1752, a most important epoch in the history of the Lunar Theory. His nethod is essentially similar to that afterwards adopted by La Place, that is, he obtains an expression for the mean ongitude and radius vector in terms of the true anomalies of he sun and moon, and then, by reversion of series, he obtains he true in terms of mean quantities. He commences by reolving his forces in the direction of the radius vector and at ight angles to it, his plane of reference being the ecliptic, and nds the differential polar equation of the orbit by the elimiation of the time between them, according to methods which ave since been familiar.

A second edition of his work* appeared in 1765, in which terval of time he had taken the opportunity to revise it, and add Tables for the computation of the place of the moon, d to subjoin a comparison of two hundred places derived om French observations with the Tables. The maximum ror of the Tables is about $1\frac{1}{2}'$.

D'Alembert's "Théorie de la Lune" appeared in the first rt of the *Recherches sur Différens Points Importans du ystème du Monde*, published at Paris in 1754. It is accomnied by Tables, and offers little occasion for remark. Esntially the same method is followed as in the researches of airaut, including the reversion of series and the resolution the forces in the direction of and at right angles to the lius vector. He obtains the ordinary differential equation for the orbit of the moon in polar co-ordinates; and it may be remarked that neither he nor Clairaut makes any use of rectangular co-ordinates. I may remark that the Preliminary Discourse of D'Alembert is well worthy of perusal even at the present time.

Euler's Memoir on the Lunar Motions, entitled "*Theoria Motus Lunæ, exhibens omnes ejus Inequalitates,*" was published by the Imperial Academy of St. Petersburg in 1753, but it is not accompanied by tables. The Memoir appears to have been sent in to the Academy as an accompaniment to the judgment which he thought it his duty to deliver on the merits of Clairaut's paper, and as giving his own views of the method by which the Lunar Theory might be advantageously treated. He refers the place of the moon to rectangular co-ordinates, the ecliptic being the principal plane of reference. His method, which he had applied originally to the theory of *Jupiter* and *Saturn*, is totally different from that of his rivals, and is remarkable as containing the germ of the method of deducing the true or disturbed place of the moon by the method of the variation of the elements of the orbit. He arrives, however, at expressions involving the true anomalies of the sun and moon, and leaves them in that state without reverting the series. In his second treatise, however, published in 1772, he recognises the inconvenience of this form, and by a process totally different arrives at expressions in which the mean longitudes and anomalies of the sun and moon are involved. The tables which accompanied this work were inferior in accuracy to those of Mayer. Important as these papers were, we need not devote any more space to them, because the tables resulting from their theories were not of sufficient accuracy for any of the purposes of navigation. Yet, even at the present time, they would amply repay the perusal of any one who could afford time to do so, and are, considering the state of mathematical as well as physical science of that period, admirable productions.

We now come to the second epoch of the history of Lunar Tables, namely, that of Mayer, an epoch of extreme importance, because his tables were the first that could be applied effectively to the solution of the great nautical problem of finding the longitude at sea. Mayer was an excellent practical astronomer as well as mathematician, and, as such, was not only capable of taking up the analytical theory in the state to which Euler (whom he took for his model) had reduced it, but of correcting by means of observations the coefficients of the inequalities which were exhibited by theory. In 1751 he was appointed to the superintendence of the Observatory of Gottingen, and exhibited remarkable skill in the examination and mode of use of the six-foot quadrant which was committed to him, and in the treatment of the observations made with it. The observations, however, which he used for the correction of the lunar elements and inequalities, consisted chiefly of about two hundred eclipses of the sun and occultations of the fixed stars. His analytical method, though setting out with Euler's equations, as given in his first treatise, is sufficiently original to show his perfect mastery of the subject, and is, on the whole, very simple, so that it can be understood with considerable facility. After giving the general expression of his forces, and the formation of his differential equations, he proceeds to change the independent variable, using first the mean motion of the moon, and then another quantity depending upon it by a very simple equation, so that it can easily be reinstated at the end of the work; and, for the true longitude, &c. of the sun, he proceeds at once to substitute the mean values. He then proceeds to determine the values of the unknown quantities in his expansions by the method of indeterminate coefficients. The resulting true longitude of the moon contains fifteen inequalities, and of these he compares the values of the coefficients derived from theory with those deduced from observation and used in his Tables. Mayer died in the year

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* *Théorie de la Lune, deduite du Principe de l'Attraction*, par M. Clairaut. ond Edition, à laquelle on a joint des Tables de la Lune: à Paris, 1765.

1762, being then only 39 years of age, and his widow transmitted to the Board of Longitude a revised copy of his Tables. By the recommendation of the Board the sum of 3000*l.* was paid to her by the British Government, while, at the same time, Euler was rewarded by a grant of 300*l.* Mayer's Theory of the Moon was published in 1767 by order of the Board of Longitude; and in 1770 his Solar and Lunar Tables were published, under the auspices of the Board. Mason's Tables, published in 1782, may be considered as a new and improved edition of Mayer's. A few new equations are added, and the coefficients of some of them are slightly altered from the results of comparisons with 1200 Greenwich observations. By these means the tabular errors of longitude were on the average diminished in the ratio of 25 : 17, as is shown in the comparison of both sets with Greenwich observations, exhibited at the end of the Tables.

We now come to the third epoch, namely, that of Burg and Burckhardt. It must be remembered that I am not giving a history of Physical Lunar Astronomy, but only indicating a few of the principal features of the successive discoveries from the time of Clairaut to the present, in connexion with the Tables based on the theoretical developments. Time forbids me to do more than this. It must, however, be borne in mind that between the epochs of Mayer and Burg, La Place and Lagrange, not to mention Poisson and other mathematicians, had in generous rivalry excited the admiration of Europe by the continuity and profusion of their brilliant discoveries in physical astronomy, and that the commencement of the present century was characterised by the publication of the greater portion of La Place's immortal work, the *Mécanique Céleste*, in which all the researches of himself and his illustrious rivals were embodied. In this interval the analytical theory of the moon had been, by the labours of these illustrious men, rendered almost perfect. The cause of the secular acceleration of the mean motion had been discovered by La Place, and he had shown that the node and the apogee are subject to secular inequalities from the same cause, namely, the secular diminution of the excentricity of the earth's orbit. He had also discovered an inequality in longitude and latitude, arising from the spheroidal form of the earth, and had by his analysis found the value of that due to the solar parallax, or the parallactic equation. It became now necessary to make a practical use of all these brilliant discoveries, by the construction of tables based on theory, in which the coefficients of all the inequalities should be obtained without the aid of observation, by carrying the successive approximations so far that no sensible numerical error should remain in any of them.

An inducement to undertake the enormous labour requisite for carrying out these researches was offered by a series of prizes proposed by the French Academy, the funds for which were furnished through the bequest of a large sum of money by a French citizen, M. Rouillé de Meslay, in 1714.

The first prize, offered by the Academy in 1798, had a more limited object in view, and did not discard the employment of observations. In fact, the subject proposed was, "to determine, by at least 500 observations, the epochs of mean longitude of the moon, and of the apogee and nodes of her orbit." This prize was divided between Bouvard, since so well known for his Tables of *Jupiter* and *Saturn*, and Tobias Burg, an Assistant Astronomer at the Observatory of Vienna. Burg did far more than was required of him. He employed more than 3000 Greenwich observations, and obtained the required quantities for the beginning of the eighteenth as well as for the beginning of the nineteenth century, and thus gave occasion for the discovery of some inequalities in these epochs. A new prize of 150*l.* was now offered by the Consular Government at the instance of the Academy for the construction of Lunar Tables, based on these materials, and this, as might be expected, was won by Burg.

His Tables were sent in in 1802, and, on a favourable report being given of them by a commission appointed to examine them, of which La Place and Lagrange were members, the First Consul doubled the prize.

These Tables by Burg form an important epoch in the subject before us. They were far more elaborate and correct than any which had preceded them, and included in their construction all the physical discoveries which had been made up to that time. We may observe that, in the course of his calculations, Burg discovered a varying error of the epoch of mean longitude which La Place failed to account for, and which both he and his follower, Burckhardt, supplied empirically.

In December 1811, Burckhardt presented to the Institute his own Lunar Tables, in which he had employed more than 4000 observations to correct the elements and the coefficients of the inequalities employed by Burg. A commission was appointed to examine them, and, in an evil hour, they reported so favourably of their accuracy, as compared with those of Burg, as to bring them into general use, and in particular to cause their employment in the English *Nautical Almanac*. The commission executed their task with fidelity and care, and were certainly justified in their decision by the results of the comparison of 167 Greenwich observations, by which it appeared that the sum of the squares of the errors of longitude were less than those of Burg, in the proportion of two to three. Nevertheless, nothing could have proved more unfortunate to the lunar theory than their adoption. This has only become notorious within the last few years, but was well known to some great physical astronomers many years ago. Any one who will take the trouble of looking at the Greenwich observations since 1836, will easily find errors which in the present state of the lunar theory are intolerable, amounting, in fact, frequently to half a minute in longitude. It is well known that Mr. Adams,* in his lunar researches, a few years since, detected a very serious error in the parallax, which Burckhardt had calculated most elaborately from La Place's theory; but no one but himself can tell the labour which the tracing of this error and its correction must have cost him, on account of the artificial form of the arguments used in these tables. Burckhardt adopted this form after experimenting a great deal on the subject; but the result has been that it is very difficult to compare his tables with theory for the purpose of correcting the errors of the elements and coefficients of inequalities indicated by the observations, and that, in consequence, a vast amount of labour, which would have been otherwise unnecessary, has been thrown on the Astronomer Royal and the other astronomers who have contributed to bring the theory to that perfection, of which the result has been the grand work of Hansen, which we are now discussing.

Thus far observation had lent its aid, both for the correction of the elliptic elements of the orbit and for that of the coefficients of the inequalities. But the lunar theory had now been brought so near to perfection by the labours of La Place, Lagrange, and Poisson, that the French Academy in 1820 gave as the subject for the great mathematical prize the construction of Lunar Tables as exact as those hitherto furnished, but in which the coefficients, as well as the arguments of all the inequalities, should be determined solely by the theory of gravitation.

The successful candidates for the prize were, first, Damoiseau, a retired French artillery officer, and, secondly, the Italian astronomers Plana and Carlini, who shared between them the labour as well as the honour of the work which they sent in.

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* See "Appendix to Nautical Almanac" for 1856.

Damoiseau's method was, without any variation, that of La Place, which we have had occasion to mention before as consisting in the expression of the mean in terms of the true longitude, and in proceeding by reversion of series to obtain the true longitude in terms of the mean, increased by a series of inequalities depending only on the time. La Place was not blind to the advantage which would be gained by proceeding directly to form the equations of motion so as to have the time or the mean longitude as the independent variable; but he despaired of its success, on account of the comparatively slow convergence of the resulting series.*

There is no doubt that the method of La Place has the advantage of simplicity, and the differential equations of motion naturally present themselves under the form in which Clairaut, D'Alembert, and almost all geometers, had put them, that is, with the true longitude of the moon as the independent variable. But on account of the enormous labour, when the approximations are pushed to a high order, of the necessary operations which must be performed before the results are put into a shape fit for tabulation—part of which consists in expressing the true motion of the sun as function of the true motion of the moon, and part in afterwards expressing true longitude, true latitude, parallax, and all other required functions in terms of mean quantities—some geometers (including M. Pontécoulant and Sir John Lubbock) have tried to solve the problem by using either the time or the mean longitude as the independent variable. The former of these astronomers has been perfectly successful, and he has carried his approximations as far as Plana in his great work on the lunar theory, so as to afford a most valuable test of the accuracy of his coefficients.† Sir John Lubbock appeared to be equally successful up to the point to which he carried his researches; but, unfortunately, want of leisure prevented him from carrying his approximations far enough to afford an ultimate test of his theory. If time did not fail us, it would be necessary to speak with some detail of the great work of Plana, afterwards published in three large quarto volumes, which is an imperishable monument of his genius, as well as of his industry; but we must bear in mind that our immediate subject is the history and consideration of lunar tables, and not of the lunar theory generally, and this passing mention must suffice. It is, however, necessary to state that, independently of the extent to which the approximations are carried, Plana's investigations have tended in a remarkable degree to the improvement of the Lunar Theory by his method of expansion of the coefficients of the inequalities in converging series, as, by this means, security is given for the accuracy of successive approximations, and the operations are performed with much greater facility. We may also be allowed to express our regret and surprise that the work sent in to the French Academy by him and Carlini in competition for the prize, and which must have been accompanied by tables, was not printed; and we could scarcely believe the fact without eminent French authority for it that the memoir has not even been preserved.‡

The Tables of Damoiseau, you are well aware, Gentlemen, were printed by the Academy of Sciences, as well as the theory on which it was based—the latter in vol. i. of the *Mémoires des Savans Etrangers* (3d series). In the theory there is nothing to show but the enormous labour in pushing the approximations to a higher order than La Place, all the terms of the fourth order being taken in, and such of the fifth as are necessary for obtaining those of the fourth correctly, and the consequent value of his results, which, however, must be taken, when they are not verified by comparison with others, on his own authority, since there is scarcely any man living who would undertake to follow him through the complicated mazes of his analysis till the numerical results are arrived at. Still the importance of this work of Damoiseau was exceedingly great; and it is only to be lamented that, in the *Nautical Almanac* and other similar works, Damoiseau's Tables had not been chosen instead of those of Burckhardt.

To make this portion of our subject complete, so as not to omit the mention of any great name that tended to throw light by original research on the doctrines of the lunar theory, I will barely mention a remarkable paper by Poisson on the "Motion of the Moon round the Earth," printed in vol. xiii. of the *Mémoires de l'Institut;* in which the author deduces the lunar inequalities by the method of the variation of the elements, though the expansions are not completely effected. The memoir, therefore, becomes only an interesting subject for study, but does not present a complete solution of the problem, and thus only remotely concerns us at present.

There is still one more work of considerable importance to be mentioned previously to that of Hansen, namely, the American *Tables of the Moon*, published in 1853, under the superintendence of Lieutenant Davis, U.S.N., and by the authority of the Secretary of the Navy, according to the arrangement of the distinguished mathematician Professor Peirce. These Tables are based on Plana's theory, but the coefficients of the inequalities are corrected by the results given by the Astronomer Royal as arising from the discussion which closed his great work of the Reduction of the ancient *Greenwich Observations*, as well as by some corrections due to investigations of Professor Longstreeth. There are also included: Hansen's two long inequalities due to the action of *Venus*, as well as his value of the secular equation of the moon's mean motion, and of the motion of the perigee. If these Tables be considered merely with reference to their current value—that is, as giving tabular places of the moon worthy of considerable confidence—there can be no dispute with regard to the high estimation in which they should be held; but the want of uniformity in the chosen values of the coefficients of the inequalities must detract in some measure from their value, when considered in relation only to the advancement of the abstract theory of the lunar motions.

Having thus, Gentlemen, cleared the way for a consideration of the labours of Professor Hansen, by giving you an idea, however imperfect, of those of his predecessors, I am enabled to devote the remainder of this Address exclusively to the subject which more immediately concerns us; and it will perhaps induce to clearness if I first give you a sketch of what may be called the external history of the work, confining myself chiefly to those features which make it peculiarly interesting to us as Englishmen and English astronomers. I was not aware myself of the amount of anxiety which the bad representation of the lunar motions by the Tables of Burckhardt had caused, more than twenty years ago, in the minds of some of the greatest of our astronomers, till, when I was preparing for my present task, the Astronomer Royal, with his usual kindness, put into my hands his correspondence on the lunar theory, continued from the time when he was first called upon to occupy his present position, namely, the year 1835, to the present time. Even if I had not known how many great works he has brought to completion in the same period—how he has revolutionised the theory and the practice of the construction of astronomical instruments, as well as of the making and reducing of observations—how he has borne the chief labour in almost every Government commission for scientific purposes, of which we need only mention the Standard Commission—how he has been occasionally engaged in optical researches, or in the writing of profound memoirs in some branch of abstract or mixed science, some of which adorn our own *Transactions*—and all apart from and in addition to the direct duties of his office—I should have thought the amount of care and labour which this subject of the lunar theory brought upon him, even before he took upon himself the responsibility of the reduction of the *Ancient Greenwich Observations*, a very serious addition to the heavy duties which are necessarily imposed upon him by the ordinary administration of the observatory. And I assure you, Gentlemen, that till now I have been myself quite ignorant of the amount of his indirect labours in this department, though I have been for nearly five-and-twenty years so near his person, and generally cognizant of all his efforts for the advancement of lunar astronomy. I am not writing an *éloge* on the Astronomer Royal, which would ill become the place or the occasion on which we are met together, but simply stating a fact which will make what follows more intelligible. I wish, in particular, to point out to you that the imperfect state of the lunar theory (or, rather, of its embodiment in tables, for Plana's great work even then existed) had attracted attention. Sir John Lubbock was then engaged in the attempt, to which I have before alluded, of applying a totally new method to the lunar theory, by making the mean longitude the independent variable in the equations of motion; and in the course of his researches had frequent correspondence, involving many deep questions, with Mr. Airy; and the chief difficulty which they had to encounter, as I before hinted, was the disentangling of the artificial arguments devised by Burckhardt, and the finding out what values of the coefficients and what arguments as deduced from theory he really had used, so as to enable Sir John to compare his own results with them. A great deal of labour was fruitlessly bestowed on this part of the subject. Then, again, there was a dearth of reduced observations for comparison with theory: a few existed, but even of these the elements of reduction were questionable, and they were generally unfit for the purpose of putting the lunar theory upon a better basis; and it was interesting to me to observe how, after various expedients were devised and rejected, the idea gradually became clear that the *Greenwich Observations*, which were confessedly the only ones continuous and accurate enough for the purpose, must, some time or other, be reduced on a uniform system. The project of this great work, when viewed prospectively and barely hinted at by Mr. Airy in one of his letters, he calls, and justly so, "stupendous;" and indeed any one who should be called upon even now to reduce nearly 9000 observations of the moon, made with old instruments of which the theory had to be studied and by men who have passed away and of whom no question can be asked, would, I presume, think it a very formidable task, even without the amplification and partial reconstruction of tables as the first and indispensable step towards the laborious tabular computations necessary for the comparison with observations. But, Gentlemen, it was in this gradual way that the great work of the *Greenwich Lunar Reductions* became an accomplished fact; and in the history of science there are few, if any, that can rank as greater or more important. Yet, even while this great work was going on, the *Nautical Almanac* kept on the even tenor of its way, giving places of the moon and lunar distances so far at variance with the truth as to be an opprobrium to the astronomical science of the nineteenth century. This, even, was not without remonstrance; and it was urged to the late superintendent of that work that it would be comparatively easy to substitute better tables; but timid counsels prevailed for a time, and then the hope of the tables promised by Hansen made it prudent to defer any radical change till they had appeared. It was understood, for many years previously to the interference of the British Government with offers of help to complete Professor Hansen's Lunar Tables, that he was engaged on them and on researches connected with them. At intervals he gave some of the results of his researches to the world in the *Astronomische Nachrichten*, commencing with the year 1829; and it was generally understood that the Danish Government had undertaken to supply the funds necessary for the completion of the work. Unfortunately that Government became involved in the Schleswig-Holstein wars, and, in 1848, was obliged to withdraw the very moderate assistance which it had hitherto furnished.

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* See *Méc. Céleste*, lib. 7, Introduction.

† Pontécoulant, *Système du Monde*, vol. iv.

‡ See *Journal des Savans* for October and December 1857, and January 1858, containing "Review of Hansen's Tables," by M. Biot.

A year previously M. Hansen had shown his complete command of the analytical theory which he had employed for the calculation of the lunar inequalities by the discovery of two curious inequalities of long period in the epoch of mean longitude arising from the action of *Venus*. The *Ancient Greenwich Observations*, extending over eighty years, had brought this inequality to light in a very conspicuous way, and no existing theory gave any probable solution of it. The investigations of La Place and Damoiseau had tended to show that the action of the planets produced scarcely any appreciable effect on the lunar orbit, at least none which could by possibility prove sensible without taking into account higher powers of small quantities than had been till that time employed. This, however, Hansen proceeded to do, suspecting rightly that as the almost accurate commensurability of the mean motions of *Venus* and the Earth, produced a sensible perturbation in the motion of the latter dependent on terms of a high order, the same might take place with respect to the lunar orbit, even though it should be necessary to carry the approximation still farther. The attempt was quite successful, and the result was, that two large inequalities were brought to light, one depending on the direct and the other on the indirect action of *Venus* on the moon, having for their periods respectively 273 and 239 years. The coefficients assigned to these inequalities in the tables are 15″·34 and 21″·47. The determination of these coefficients, it ought to be explained, has been accomplished with very great difficulty, and M. Hansen, in attempting by a different method to verify the results obtained in his first calculations, did not perfectly succeed, so that probably the complete development of these two inequalities by theory is still one of the desiderata of the lunar theory. This digression is necessary, and historically valuable, because it was this brilliant discovery that increased the eagerness of the English astronomers, and especially of the Astronomer Royal, for the completion of Hansen's great work, and induced the latter gentleman to inquire of him directly, and also of Professor Schumacher, in the year 1850, concerning his progress. The answer was a melancholy one—as has been stated, the funds furnished by the Danish Government were suspended without much prospect of speedy renewal; the computers were discharged, and the work was comparatively at a stand. It was well, Gentlemen, that at this crisis the correspondence was in the hands of one who deservedly had the ear of the Government of this country—who could apply with confidence for funds for the completion or the performance of any great scientific work, without chance of refusal, because he had been never known to fail either in his estimation of the national importance, or in the successful prosecution of other great works which had been previously proposed by him. On learning the true state of affairs he lost no time in learning from M. Hansen that assistance from the British Government would be acceptable to him, and in making a direct application to the First Lord of the Admiralty for the estimated sum which was modestly demanded. And what was the sum, think you, Gentlemen, which was wanted and which stopped the progress of this great work? The whole cost of the remainder of the work, up to the time when the MS. of the Tables was fairly written out and ready for the printer, was 300*l*. The want of a sum which a private gentleman devotes to a yachting or a travelling expedition was sufficient to stop for a time the progress of perhaps the greatest astronomical work which has been produced in the present century—greatest, if not absolutely or intellectually, yet at least in its effects as giving the finishing stroke to that great problem of finding the longitude at sea in which the whole civilised world is vitally interested. Yet, Gentlemen, though the sum be small that was demanded of the Government, this does not detract in the slightest degree from their merit in readily granting it, and still more may we admire the spirit with which the Board of Admiralty took upon themselves the expense and responsibility of *printing* the Tables and providing for their distribution so liberally that there is scarcely an institution in the world where a copy could be advantageously lodged, which has not had a copy presented and forwarded to it. Time will not allow me to dwell longer on the history of the production of these Tables, because it still remains that I say a few words on their construction and on the theory on which they are based. It may be sufficient to state, that in November 1854, M. Hansen announced that the greater portion of his work was completed, and at the same time sent some comparisons with observations extending from 1824 to 1850, to test the accuracy of his results;* that in the year 1856 he came to Greenwich to superintend the printing of the Tables; and that the copies were distributed in the summer of 1857.

I might perhaps be excused from entering on the subject of the theory on which these Tables are based, because the Council, being well assured of the high character of the analytical investigations of M. Hansen, as exhibited in his elaborate work on the Lunar Theory, published in 1838;† in his work on *Jupiter* and *Saturn;* and in that entitled "*Determination of Absolute Perturbations in Ellipses of any Excentricity and any Inclinations whatever,*" and not having before them the full development of the theory (which however the scientific world is anxiously expecting), have been guided in their award of the Medal by, for the most part, practical considerations.

On the publication of the Tables, Mr. Hind, with that ready zeal which characterises all his efforts for the advancement of science, undertook, at the instance of the Astronomer Royal, the comparison with them of all the observations of the moon which have been made at Greenwich since the institution of the altazimuth in 1847. For this work of love, involving as it did so considerable an amount of personal anxiety and labour, Mr. Hind deserves the warm thanks of this Society in particular, and of the scientific world in general, because, in the absence of the complete theoretical investigation, neither would your Council have been able otherwise to make this award, nor would the exceeding great value of the Tables have become so readily apparent. In the *Monthly Notice* for April 1859, is given the result of the comparison for the year 1852, and you may see for yourselves that for this year the sum of the squares of the errors in comparison with Burckhardt are greater than those in the comparison with Hansen in the proportion of 8 : 1. The comparisons have been completed for almost all the remaining years including 1858; and, as they are not published, I beg you to accept my assurance that the errors are equally small during the whole space of time from 1847 to 1858, and that M. Hansen's own estimate of the accuracy of his results, as given in the letter before referred to, is not at all exaggerated. He estimates that the mean error in R.A. is rather more than $2\frac{1}{2}''$, and in declination rather more than $2''$, while that of Burckhardt is according to his own (Burckhardt's) estimation $5\frac{1}{2}''$. Thus, Gentlemen, without going into details, you may rest assured that the residual errors of the lunar theory are reduced now to most insignificant limits; and certainly within the errors incidental to meridional observation with first-rate instruments, and very far within the limits of accuracy of observations with the sextant;—that Hansen's Tables are practically perfect for all the purposes of navigation, and that the great nautical problem of finding the longitude at sea is now completely solved. That rebellious satellite, which has hitherto successfully resisted all the most complicated and refined methods of analysis, is now completely reduced to obedience to the laws of gravitation, and her predicted place corresponds to her true place in the heavens as nearly as in the cases of the other planets and the fixed stars; there remains indeed enough undetermined in the theory of her motions to excite the emulation and the genius of the most profound of our physical astronomers (for none but the *most* profound can meddle with the remaining questions), but there is nothing left unsettled which vitiates in the slightest degree the absolute accuracy of the tabulated places.

Still, though your Council have judged that they could not do wrong in resting with perfect confidence on these practical considerations, and that further delay in their award of the Medal would be as injurious to the reputation of this Society as it would be unjust to M. Hansen, yet there is sufficient knowledge of the theory to prevent any sensible doubt of the correctness of the principles applied to the formation of the Tables. You will have observed, in my historical sketch of the analytical processes employed by the predecessors of M. Hansen since the time of Clairaut, that though the developments of the disturbing function are prodigiously increased in the later researches, yet there is essentially the same method employed in all, and that the theory of the moon has always been treated as an exceptional case, and as requiring different treatment from that of any other case occurring in the general problem of the motions of three bodies mutually attracting; that, taking advantage of the fact of the ratio of the mean motions of the sun and moon being small, as well as the excentricity of the orbit and the inclination to the ecliptic, all the methods have been devised with the view of expanding the perturbation-function and all resulting functions for the deduction of the true longitude and latitude and parallax of the moon, in terms of powers of these quantities. Professor Hansen's object has been to do away with this exceptional character in the methods of dealing with the problem of the three bodies, and to arrange his calculations in such a shape that they shall be equally applicable to orbits of all degrees of excentricity and inclination, and with any ratio of the mean motions whatever; a method which should be equally applicable to the theory of *Jupiter* and *Saturn*, to that of the small planets between *Mars* and *Jupiter*, and to the moon.

I will devote a few words to the explanation of the most remarkable points in his method, which, as you may imagine by what has already been said, is totally different from that of any of his predecessors. The ordinary method, or that employed by La Place, and by almost all other geometers, consisted in obtaining the differential equation of the disturbed orbit, and solving this equation by successive approximations. The first part of the process for longitude consists in expressing the terms of the disturbing forces, or the disturbing function, which must be manifestly functions of the elements of the solar and lunar orbits, and of the position at any given instant of the sun, moon, and earth, in terms of the true longitudes of the sun and moon, and ultimately of the true longitude of the moon; and of correcting by successive approximations the solution of the equation which expresses the relation between the reciprocal of the radius vector and the true longitude of the moon; while at the same time the differential equation expressing the angular velocity of the moon gives means for expressing the time or the mean longitude in terms of the true longitude, and by reversion of series of expressing the true longitude in terms of the mean; and thus of ultimately expressing the parallax in terms of mean quantities. Professor Hansen's method is totally different from this from the very commencement. He does not attempt *immediately* to obtain the actual expressions for the perturbations of the true co-ordinates of the moon's place, but he does this ultimately by deducing these quantities from the disturbed mean anomaly, the greatest portion of the labour being devoted to the expansion of this function in terms of mean quantities. This disturbed mean anomaly he afterwards substitutes in the equation of the centre of the mean lunar orbit according to the ordinary elliptic theory, and thus obtains the true anomaly in the disturbed orbit, which can be reduced in the usual way to true longitude by the application of the tropical longitude of the perigee, and of the reduction to the ecliptic together with its inequalities. He has in fact convinced himself by his investigations that the perturbations of the mean anomaly when converted into series converge much more rapidly and more surely than those of any other element, and that none of those doubts and difficulties about the degree of convergency of the series occur which meet the computer at every step of the approximations by the ordinary theory.

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* See *Monthly Notice* for November 1854.

† *Fundamenta Nova Investigationis Orbitæ Veræ quam Luna perlustrat.* Gotha, 1838.

To effect his development M. Hansen of course proceeds as in other methods by successive approximations. Setting out from the mean or undisturbed elliptic elements, he calculates successively the terms of the development which are due to the first, second, and higher powers of small quantities, using, according to well-known methods, the expressions derived from each approximation as the basis of the succeeding. But there is an important peculiarity in his method which cannot be passed over, because it is one on which he himself very much insists, and because it has been the subject of some discussion between himself and M. Delaunay. Since the whole problem of the lunar theory consists in finding for any given instant the three true co-ordinates of the moon, namely, the longitude, the latitude, and the radius vector or the parallax, there are various ways in which this can be done; and it is perfectly legitimate to assume that three out of the six elliptic elements are absolutely constant (that is, to attribute to them throughout their mean values derived from observations), and to throw all the variation on the other three elements. The three which M. Hansen has chosen for his constants are the excentricity, the inclination, and the mean motion; and he has been able so to arrange his calculations, that these three primitive elements shall enter into the coefficients of all the inequalities which it is necessary to calculate; and he is thus enabled at each step of his process to replace the analytical values of his coefficients by numerical values, and to avoid that intolerable complication of terms which in the other method increases so rapidly, as higher degrees of approximation are aimed at, as in the end to conquer the most industrious and the most skilful of analysts. M. Delaunay, whose theory is not yet published, states that he has discovered a method by which he can retain the advantages (for some such must be allowed to be attached to it) of the preservation of the literal values of the coefficients throughout the whole of the investigation, together with the absence of this extreme complexity of operations, by dividing his whole process into several subordinate ones. As neither his complete theory nor that of M. Hansen is yet published, it is useless to speculate on the preference due to their respective methods, nor to attempt to judge them. We have hope that both will be speedily published, and the appearance of them will be hailed by the whole scientific world as the most precious additions to physical astronomy that have been made in the present century.

I have not time to enter more into detail concerning the theory of the construction of the Tables, and it only remains that I say a few words about their form. In the year 1854, when M. Hansen had made a provisional construction sufficient for his own purposes, so as to enable him to calculate a sufficient number of places, that astronomers might judge of their accuracy by their comparison with Greenwich observations of the moon, he used as the unit of his arguments the mean lunar day, that is, the mean of the intervals of time taken by the moon in departing from a given meridian and returning to it again; and he seemed inclined to use this unit in the final composition of his tables. Though this unit has some advantages, particularly for the construction of ephemerides, yet there are so many more cases in which it is absolutely necessary to compute for equal intervals of mean solar time, that he was induced to change his mind, and the unit of his argument in the tables is a mean solar day. It is known to many amongst you, Gentlemen, that this unit of argument in the construction of planetary tables was proposed first by Carlini, and employed in his Solar Tables; and the advantages are so obvious — especially for solar tables, from the facility with which the arguments are formed when their periods are known — that I need not dwell upon them. The chief part of the Tables is occupied with the formation of what M. Hansen calls the *Fundamental Argument* — that quantity, in fact, which, when substituted in the equation of the centre, produces the true anomaly. The tables necessary to form this *Fundamental Argument* are fifty-three in number, and this after the utmost possible skill has been shown in combining the terms expressing the perturbations of the mean anomaly, so as to produce as much compression as possible. Many of these are tables of double-entry, that is, having a vertical and a horizontal argument; and every one knows how troublesome and wearying these are in practice, even when they are expanded so as to reduce the intervals of the successive values of the arguments as much as possible. But M. Hansen has found an ingenious device by which one of the arguments depending on the mean anomaly of the moon may be considered absolutely constant for a whole period of the mean anomaly by adding to each of the various values of the other argument the movement of the first from its initial value to that which corresponds to the actual value of the other. These are the two chief peculiarities of the actual construction of the Tables which I find it necessary to point out to you, and the only ones which time will allow me to dwell upon. It may, however, be mentioned, in addition, that the Tables contain the means not only for computing the co-ordinates of the moon referred to the ecliptic — that is, the longitude and latitude — but also for deducing from them the right ascensions and declinations referred to the moveable equinox, which, of course, is the point to be arrived at in every computation of a place of the moon; and this appears to be effected with the greatest possible simplicity. The only remaining question which we have to ask ourselves is, whether these Tables, so faultless in theory and so skilfully elaborated, are, after all, convenient in practice, and, above all, whether ordinary computers would be able to deduce from them places of the moon with the requisite facility. The book is undoubtedly a large one, and, at first sight, the task of computing a place from it looks formidable; but this very size of the book is for the advantage of the computer, the tables being so expanded as to enable him to take out all his numbers at sight. This is not so with other Lunar Tables; and especially, I remember, that, in the use of Damoiseau's Tables, the first necessary step was to expand a great many of them very considerably to avoid second differences. But, without putting undue confidence in these reasonings, I was not content till I had satisfied myself of their convenience and facility of use by going through a computation of longitude, and had assured myself that for a great book it is a very easy book, and that ordinary computers might easily be trained to compute readily places of the moon from it. But I have a still better guide for my judgment in this matter in the expressed opinion of

Mr. Hind, who, at the present time, has had perhaps more experience of the use of the Tables than any other astronomer living, having, as I before stated, undertaken and very nearly completed the comparison of all the Greenwich observations from 1847 to 1858. At my request he, with his usual ready compliance, gave me the following opinion of them:—

"Hansen's Tables considered simply as machinery for the practical determination of the moon's places may, although the process is longer than that by Burckhardt's Tables, be considered a production of the first order, and their excellence is particularly evident in the construction of an ephemeris.

"The operation at first sight appears formidable even to the initiated in such matters; this arises principally from the rather serious array of double-entry tables; the use of these tables does not however involve the irksome process of cross proportion, as every thing required from them is adapted to tabular increments of the horizontal argument.

"It is no bad test of their excellence to say that they are popular in this office; and further, that the parties employed in the twelve-years calculations by them for the Astronomer Royal, although at all times eager to devise artifices for shortening labour, failed to discover much scope for the exercise of their ingenuity."

Thus, Gentlemen, have I completed, however imperfectly, the somewhat formidable task which I proposed to myself at the commencement of this address, by giving you as far as my limited time and my facilities for collecting information would allow me, an exposition of the various theories which have been used in the construction of tables of the moon since the time of Newton, as introductory to such an account of the Tables of M. Hansen, which are this day crowned by the award of your Council, as would enable you to understand the *grounds* of the award. I felt indeed, that if I performed my duty properly, I must take a somewhat different course from that usually pursued on similar occasions, because it appeared absolutely certain to me that the Council would require no justification whatever, either in your eyes or in those of any other scientific community. The general expression of eagerness and satisfaction with which the appearance of the Tables was greeted, on their publication,—the unanimous favourable verdicts of all competent authorities,—proved the universal conviction that a new epoch in physical astronomy had arrived, and that the difficulties in representing the moon's true place accurately by theory, which have, ever since the time of Newton, so occupied the minds of astronomers, and been such a hindrance to the perfection of nautical astronomy and navigation, belong solely to the history of the past. And your Council might have accepted this favourable judgment at the very first; and there is no doubt that the *immediate* award of the Medal to M. Hansen would not only have met with *your* approbation, but that their decision would have been amply ratified by all who are capable of judging accurately on this deep subject. But, with the caution which becomes a Society like this, that looks rather to the future than the present, and would rather be blamed for being tardy in its awards than liable by precipitation to have its judgment reversed, your Council chose rather to wait for such evidence as must be shortly forthcoming and which would prevent the possibility of error in their judgment. It was expected that the publication of the complete development of the theory on which the Tables are based would speedily follow them, and we were waiting impatiently for this formal justification, that we might crown the *entire work* with our award.

In defect of this publication your Council have felt that they have still the best possible guarantee for the justice of their decision in that most severe test of comparison with the Greenwich observations, to which the Tables have been subjected, and they feel sure that no additional knowledge of the fundamental theory can tend to invalidate this testimony to their unrivalled excellence. The work is indeed perfect in theory and practice, emanating from the profoundest conceptions of the analytical developments required, and elaborated by a union of skill and industry rarely found in the same individual.

I have previously said that it is exceedingly difficult to find in the same man all the attainments necessary to form a great theoretical and a great practical astronomer, but it is rarer still to find one who in his own person unites not only these qualities, but also the patient industry of the experienced computer, with a modesty which caused him unhesitatingly, when compelled by necessity, to accept the drudgery of performing the numerical computations of the simplest character. Indeed, I should have told you, Gentlemen, that when the funds furnished by the Danish Government were withdrawn, he was prepared to continue the necessary computations unaided, and only lamented the length of time which must on this account elapse before his great work could be completed.

Thus, whether we consider the moral or the intellectual character of the man before us; whether our attention be fixed on him who, *tenax propositi*, could look undaunted on difficulties apparently unsurmountable, and undertake for the present the meanest offices, that he might realise the brilliant future which he saw already within his grasp,—or whether we look to him who unites in his own person the qualifications of the profound analyst and the skilful observer,—the originator of new theories in physical, and of new methods for the treatment of instruments in practical astronomy,—he is equally the object of our respect and admiration, and one who confers honour on us by his acceptance of the award of our Medal.

(*The President, then delivering the Medal to Admiral Manners, addressed him in the following terms*):—

Admiral Manners,—I request that you will be pleased to transmit to M. Hansen this Medal, as a means of expressing in a formal manner the opinion of this Society of the excellence and the great importance of the work for which it is awarded. Request him to accept it as the tribute of our unqualified admiration of this great work, and of the brilliant talents, the rare sagacity, and the patient industry which have contributed to produce it; and convey to him at the same time our best wishes for his future health and happiness, and for the successful issue of any other works which he may be prompted to undertake.

The Meeting then proceeded to the election of the Officers and Council for the ensuing year, when the following Fellows were elected:—

President:

Rev. Robert Main, M.A.

Vice-Presidents:

Augustus De Morgan, Esq.
Rev. Baden Powell, M.A. F.R.S.
Rev. C. Pritchard, M.A. F.R.S.
Admiral W. H. Smyth, K.S.F. D.C.L. F.R.S.

Treasurer:

Samuel Charles Whitbread, Esq., F.R.S.

Secretaries:

Richard C. Carrington, Esq.
Warren De La Rue, Esq. F.R.S.

Foreign Secretary:

Admiral R. H. Manners.

Council:

J. C. Adams, Esq. M.A. F.R.S.
G. B. Airy, Esq. M.A. F.R.S. Astronomer Royal.
Arthur Cayley, Esq. F.R.S.
Captain Alexander Ross Clarke, R.E.
Richard Farley, Esq.
Rev. George Fisher, M.A. F.R.S.
Charles Frodsham, Esq.
Richard Hodgson, Esq.
William Lassell, Esq. F.R.S.
John Lee, Esq. LL.D. F.R.S.
Charles V. Walker, Esq. F.R.S.
Rev. Thos. Wm. Webb, A.M.

Vol. XX. *March* 9, 1860. No. 5.

The Rev. R. Main, President, in the Chair.

Capt. H. Ingold Strutt, Southampton;
Rev. Thomas Wiltshire, Rectory, Bread Street Hill;
W. Corbett Burder, Esq., Clifton, near Bristol; and
Richard Ford Heath, Esq., Totteridge Park, Herts,

were balloted for and duly elected Fellows of the Society.

The twenty-eighth volume of the *Memoirs* is now ready for delivery. It contains the longer papers read at the Meetings of the Society, and also the nineteenth volume of the *Monthly Notices* reprinted in a quarto form, thus giving a complete account of the proceedings of the Society during the year. The price is *2s. 6d.* to Fellows, and *7s. 6d.* to the public.

The Astronomer Royal addressed the Society in reference to the observation of the approaching Total Eclipse of the Sun, on 1860, July 18; which will be visible in no part of Europe except the north-easterly portion of Spain, its shadow entering on the northern coast (the centre being not far west of Santander) and crossing the peninsula nearly in a south-easterly direction.

The first point adverted to was the locality favourable for observation, due regard being had to the facility of approach. The remarks were, however, confined to the north-west portion of the shadow-path, as being that which is most likely to be visited by English astronomers. It was pointed out that the Pyrenees, (which, commencing from their Mediterranean extremity, range in a westerly direction to the longitude of Bayonne, and nearly so far serve as the boundary between France and Spain,) do not cease to exist as a mountain-chain at the longitude of Bayonne, but are continued in the same direction under the popular names of "the Cantabrian Pyrenees," "the mountains of Asturias," &c., ranging parallel to the north coast of Spain, at a distance of 40 or 50 miles from it. Their elevation can be judged of from the circumstance that the lowest gaps which could be found for the railways going southward from Bilbao and Santander are 2500 feet high. The tract between these mountains and the coast is reputed to be liable to fogs, but it appeared to be the opinion of persons locally acquainted with them, that excellent observing stations, offering every probability of clear sky, might be found on points of much greater elevation, perhaps not less than 5000 feet, in the neighbourhood of Orduña (on the road south of Bilbao), or in the neighbourhood of Reynosa (on the road south of Santander). From these mountains there proceed two chains in the south-east direction, bounding the valley of the Ebro. That on the north-east side of the valley is rather general high land with occasional valleys than a distinct mountain-chain; and, as it is far from the centre of the shadow, being generally near to its north-east boundary, it is less likely to be visited (although observations in this part of the track, south-west of Pamplona for instance, in conjunction with other observations in the centre and south-west part, will be exceedingly valuable). But the chain on the south-west side of the valley is nearly in the centre of the shadow, and is much more likely to be visited. This chain proceeds in the south-east direction for a considerable distance, till it is suddenly cut off by the valley of the Xalon (a large affluent of the Ebro); it then turns suddenly in the south-west direction, with a point of flexure almost as sharp as the apex of the letter V, and that point of flexure is the lofty mountain Moncayo. (In the hollow of this V rise the streams of the Douro.) This position must possess remarkable advantages for the view of the landscape under the successive phases of the eclipse; it is also precisely under the centre of the shadow; and for these reasons and for its elevation it has been selected by M. Faye with the French official expedition, as also (it is understood) by Don Antonio Aguilar with a party of Spanish astronomers, for their place of observation. The chain between Reynosa and the Moncayo presents undoubtedly many favourable positions; the mountains of Pancorbo, near the pass bearing the same name through which is led the royal road between Vitoria and Burgos, have been specially mentioned.

Many persons, however, may be unwilling to risk the possible inconveniences of mountain excursion in a country which has always had the reputation of presenting great difficulties to the traveller who leaves the highroads. To those who wish to confine themselves to towns, the following may be suggested: Vitoria, said to have good accommodations, on the royal road, north-east of the central shadow; Miranda, a smaller place, on the same road, nearer the centre; Burgos, on the same road, south-west of the centre, its accommodations are said to be not equal to those of Vitoria. It is probable that all these places will be very hot. Bilbao and Santander will offer good accommodations, but Santander, being a sort of watering-place for the inland inhabitants, will probably be as full of visitors in this hot season as our own Scarborough would be.

In regard to the access to these places:—a traveller by land reaches Bayonne without difficulty by the excellent French railways, and then if going in the direction of Vitoria and Burgos he will find numerous diligences. But he may have some difficulty in returning by the same route, as, during the watering-place season, every seat will probably be taken in the diligences going northward from Madrid. It is believed, however, that no difficulty will be found in procuring a voiturier at Bayonne, and the Astronomer Royal is authorised by Mr. Vignoles to say that he will be happy to aid in making arrangements for this purpose. Similar assistance has been offered by M. D'Abbadie, who dwells in the immediate neighbourhood of the French and Spanish boundary.

To those, however, who are willing to brave the inconvenience of a sea-voyage, it is evident that great facilities for the conveyance of instruments would be given by a direct voyage from England to Bilbao or to Santander. The Astronomer Royal distinctly expressed that he had no authority to make any promise or engagement in reference to this matter, but stated that he had no doubt that, if several persons joined in desiring a sea-conveyance to that coast, the Admiralty would appoint one of their steamers for this service. At the same time, he wished to point out two circumstances. First, For communication with the valley of the Ebro, and also for certain assistance to which he would shortly advert, the port of Bilbao would be the more favourable. But he had lately heard that its bar is in a bad state, and a large Government steamer might be unwilling to attempt the entrance. At Santander there is no difficulty. Secondly, It is obvious that a large steamer cannot be kept waiting long, but must be dismissed for her return as early as possible. Every astronomer, therefore, arriving by the steamer, must be held engaged to re-embark his instruments and himself if he wished to return with the same ship, with all possible despatch after completing the observations.

Supposing, however, that a footing is gained on Spanish ground, the arrangements for proceeding (in a country where above all others it is necessary to make such arrangements in good time) are next to be considered. And he had the gratification to announce that, by a most favourable combination of circumstances, facilities of no usual kind might be anticipated. A railway is now in progress from Bilbao, crossing the mountains in a southerly direction to Orduña, and turning down the valley of the Ebro to and beyond Miranda and Logroño. The whole of this district is occupied by English engineers, railway officers, and workmen, under the direction of Charles Vignoles, Esq., engineer-in-chief, Mr. Brassey, contractor, and Mr. Bartlett, sub-contractor; and the Astronomer Royal is authorised by Mr. Vignoles to state that every possible assistance, in the nature of lodgings, food, assistance of men, horses, and carriages, will be given by his party; and he believes that Mr. Brassey and Mr. Bartlett concur in this offer. Another railway is in progress from Santander, crossing the mountains in a southerly direction by Reynosa; and the Astronomer Royal is authorised by Philip Sewell, Esq., engineer of that railway, to offer the services of himself and his parties to astronomers.

It is proper to mention that these gentlemen, and others personally acquainted with the country, insist on the importance of arriving early in Spain. Movements of every kind are so slow that unless an unnaturally long period be allowed for every inquiry, reconnaissance, or journey, it will not be properly effected. Apparently all parties ought to have passed the frontier not later than July 8; and even an earlier time was recommended as preferable.

Pamplona, near the north-east boundary, will be easily reached by diligence from Bayonne. For the south-west boundary, it is said that there are coasting steamers to Gijon, whence Oviedo is reached; Torquemada and Aranda are on highroads from Burgos to Madrid.

A very serious difficulty had been anticipated by many persons, from the amount of the Spanish custom-duties on instruments imported (30 per cent *ad valorem*, without drawback on return), and from the severity of the custom-house examination. The Astronomer Royal had written officially on these points to Lord John Russell, Secretary of State for the Foreign Department, and had been assured by his lordship that the matter should be duly represented to the Spanish Government. But before such representation could have reached Madrid, a letter had been addressed to him by Don Antonio Aguilar, Director of the Observatory of Madrid, for a translation of which he is indebted to Mr. Main, President of the Society. The letter is to the following effect:

"*To the Director of the Observatory of Greenwich.*

"At the request of the Observatory of Madrid, the Spanish Government has made the following arrangements with the view of facilitating as much as possible the performance of its delicate responsibilities towards foreign astronomers who may come to the Peninsula in the approaching month of July to observe the eclipse of the sun which has then to be verified.

"1st. In the Spanish custom-houses will be admitted without payment of any duties the instruments which the astronomers may bring for observation in the study of the phases and peculiarities of the eclipse; without prevention, notwithstanding, that the aforementioned places take the necessary precautions for the purpose of hindering persons, strangers to science, from fraudulently introducing other effects resembling these, by taking advantage of a similar privilege.

"2d. In case of necessity, and at the request of the expeditionary *savans*, the local authorities shall afford effectual assistance, so that they may freely and securely be able to acquit themselves perfectly of the task intrusted to them.

"3d. The rectors of the universities nearest to the zone of the eclipse will take care also, with the utmost precision, to give information to the astronomers, from their practical acquaintance with the country and with the most habitual atmospheric phenomena; to accompany them in their scientific excursion; and to take, on the invitation of the same, a part more or less active in the operations which they will engage in.

"To the end that the good intentions of the Spanish Government may not be frustrated, it will be proper that the astronomers who intend to observe this eclipse should communicate with the Observatory of Madrid, indicating approximately the time of their coming, the point from which they intend to penetrate into Spain, the region in which they wish especially to station themselves, and the number and quality of the instruments which they will bring with them. In regard to such communications—which, on account of the nearness of the epoch, ought to be made without loss of time—the observatory will take care to give to the astronomers who will favour it with them, subjects of information which may be of interest, acquainting every person of the travelling arrangements made by the others; so that they may all, if they please, proceed systematically in avoiding excessive augmentation of observers at one point, and a complete neglect of other stations no less advantageous or interesting. Thus, admitting the convenience for the complete observation of the accidents and phases of the eclipse, of the adoption of four principal stations in the mountains of Oca, Moncayo, the neighbourhood of Calatayud, and Peña-Golosa—for example, the points of preliminary reunion might be at the cities of Burgos, Agreda, Calatayud, and Castellan de la Plana, of easy access, where the astronomers could have circumstantial information of the neighbouring territory, expert guides, and zealous authorities, to whom they might apply in case of need, and where every one could arrange his plan of operations with full knowledge of that adopted by the rest of his colleagues, and afterwards all might compare results with each other. On this point I may be excused the statement that, with decided preference, M. Le Verrier has already declared his fixed determination that the French expedition, of which M. Faye has the direction, shall be stationed upon the heights of Moncayo.

"The Observatory of Madrid will feel itself honoured if any will accept its services offered to the expeditionary astronomers, with this design, that an account of this short letter may be given in the academical and scientific publications; so that its aspirations and desires may speedily come to the knowledge of all those who have an interest in it.

"The Director of the Observatory,

"ANTONIO AGUILAR.

"*Observatory of Madrid, February* 25, 1860."

The Society would acknowledge gratefully this spontaneous act of the Spanish Government and the Spanish astronomers, expressing so fully and so carefully their readiness to remove every difficulty, and to render every assistance which under any possibility they might be able to give. At the same time, the Astronomer Royal invited particular attention to that part of the letter which requests the most exact preliminary information (at least as far as circumstances permit) of the names of the observers, their place of entrance, the number and nature of their instruments, and the place at which they propose to make observations. Careful compliance with this request will evidently enable the Spanish Government to make far better arrangements for the advantage of observers than could possibly be made without it. The Astronomer Royal, in his correspondence with Lord John Russell, had anticipated the necessity and advantage of such a course.

Remarking the number of points to be arranged with the British Government, the Spanish Government, the English friends who had offered their assistance, and others, it is evidently desirable that all correspondence should pass as far as possible through one centre of correspondence. The Astronomer Royal undertook that, if English observers should think that he could render the services to which he alluded, he would with pleasure place himself at their disposal; and he would beg them, as early as could conveniently be done, to address themselves to him in reference to all matters connected with the observation.

The Astronomer Royal stated that he hoped to have the society of M. Otto Struve, who, with M. Winnecke, proposed to accompany him wherever he should decide on going. He added, that in regard to the selection of a place, his thoughts turned towards Reynosa, or some position among the mountains not far south of Reynosa.

The Astronomer Royal then adverted to the observations and the mode of making them. And in the first place, he took the opportunity of correcting the impression which might be left by some remarks which he had made on a preceding evening. He had pointed out, as what he had certainly seen, that red prominences disappeared on one side of the moon, and new ones appeared on the other side, just as would happen if these red prominences were parts of the sun; but that he could not undertake to say that their elevation increased on one side and diminished on the other side by the proper quantity. He had now to say that this point had been entirely established by the measures made by M. Otto Struve at Lomja in observing the eclipse of 1851, which are detailed in his account printed in the *Mélanges Mathématiques et Astronomiques de l'Académie de St. Petersbourg*, tom. i. 3^e livraison. In two instances the change of elevation of the prominences was measured with considerable accuracy, and the results agreed well with the relative motion of the sun and moon in the same interval of time. This accordance greatly strengthened the presumption that the prominences distinctly belonged to the sun.

Still it might be conceived that the prominences were, in fact, produced in some unexplained way by the passage of the sun's light grazing some mountains or hollows of the moon, and that their apparent elevation was altered when the course of the sun's light became less deflected by the moon before it reached the observer's eye; and that nothing therefore was proved with certainty as to their origin, unless we could show that in some cases they shifted their apparent position upon the moon's limb. Very great value, therefore, would attach to observations which should define with accuracy the angular position of the red prominences, as referred either to the astronomical meridian, or to any other line whose angle with the astronomical meridian could be calculated. An agreement was already tacitly made between M. Otto Struve and himself, that while M. O. Struve should take care for the elevations of prominences on the front and rear of the moon, the Astronomer Royal should use his best endeavours to settle the angular positions of any which might be seen on those parts of the limb which are nearly parallel to the direction of the moon's apparent motion over the sun. His thoughts, therefore, had been directed mainly to the contrivances for measuring the angular position. Adopting the fundamental principle that no attempt is to be made to use a micrometer, but that measures must be confined to eye-estimations, he proposed the following wire apparatus. From the measured focal length of his telescope, and the known angular diameter of the moon, he would compute the linear diameter of the moon's image. Four wires (or lines etched on glass) would be placed in the field of view, in such a position as exactly to include this image; and by these, with ordinary rackwork motion, he conceived that he could at any time very quickly bring the moon's image to an almost perfectly definite position in the field. Each of the four wires should have an external parallel wire at the distance of 1′ for estimation of elevations. Twelve radial wires, at equal angles of 15°, should intersect in the centre, the wires which are parallel to the sides of the square being thicker than the others, for facility of reference. The great elevation of the sun above the horizon (about 59° at the beginning of the eclipse, 54° at the middle, for the station Reynosa) made it very desirable that the observations should be made with a diagonal reflector, turning the rays of light, however, not necessarily 90°; and, the reflector being supposed to be of glass, he thought it would be most convenient that the lines above mentioned should be etched on the surface nearest to the eye. In regard to the mounting of the telescope, he proposed to abandon all idea of equatoreal motion, knowing the difficulty of making the proper adjustments in a field-telescope, and intended to mount it with an ordinary altitude-and-azimuth motion, the verticality of the axis being ascertained by a small spirit-level. With this mounting it is easy to verify, by a terrestrial object, the horizontality of one wire; and this circumstance would determine the position of his system of wires. It appears, from the numbers given in Mr. Hind's ephemeris, that about the middle of the eclipse the moon will appear to be moving across the sun in a direction inclined about 16½° to the horizon. He would therefore use, as his principle of verification, that the line next to one of the strong lines of which he had spoken should be exactly parallel to the horizon; and then that strong line would show the direction, within 1½°, of the moon's apparent motion, and the observer would be fully prepared to decide on the portions of the limb to which his observations ought to be directed.

It is an unfavourable feature of this mode of observation that the power of the telescope must be rather low; but he thought it likely that the advantages accompanying the method would more than balance that disadvantage.

The peculiarities of the method which he had described attach only to the determinations of angular position. For the measure of elevations, even by estimation, each observer would devise his own method, of a different class from that above mentioned, and a high power might undoubtedly be used. Those observers who wished carefully to scrutinise the prominences would of course use high powers. But on those observations, and on those of colour, polarisation, &c., or on those of the corona, or of external appearances, he did not intend to speak.

The method of registering time had been a subject of repeated consideration, and he thought that it might be best to make no time-record whatever, except perhaps of disappearance and reappearance. He proposed to hold in his hand a book in which he could write without looking at the book, inscribing only a few symbols on each opening according to a system which he himself should understand. He proposed to divide the whole time into observations, estimated as of equal duration; any turning away from the telescope to look at the sky or the landscape being also an observation; and to appropriate to each of these one opening in his book (even though no entry should be made). The durations might be so nearly equal that no sensible error would be committed by assuming them to be equal; and then the precise epoch of any observation would be found with sufficient accuracy by simple interpolation between the times of disappearance and reappearance. G. B. A.

Extract of a Letter from M. D'Abbadie to one of the Secretaries.

"The *Monthly Notices* are not sufficiently explicit about the coaches which ply daily between Bayonne and Madrid. In July many rich Spaniards go to the north for coolness and sea-baths, and it is then necessary to bespeak places three weeks in advance. Not so in proceeding from Bayonne to Burgos; for the coaches, being generally empty in July, travellers get places easily, and at half price: those going to the north then pay double. I beg you to communicate this part of my letter to Professor Airy, and add that Pancorbo is an excellent station (separated from Miranda by a narrow mountain-pass) on the north end of an elevated plain, where the village Cubo is on the line of central darkness. There is no good map of those parts, and none shows Altotero, somewhere in the neighbourhood, and a geodetical station in the new Spanish survey."

Dated Paris, 1860, *Feb.* 24.

An Eye-piece for the Solar Eclipse.

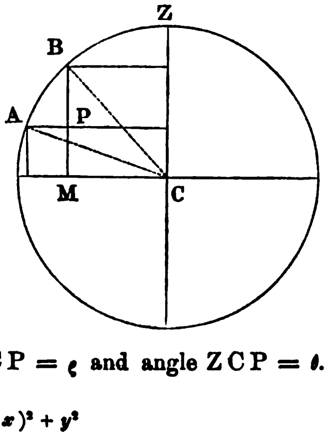
Mr. Carrington exhibited an eye-piece which he believed to be well adapted for reading off angles on the limb of the sun or moon during the ensuing eclipse, the main principle involved being that of Bessel (see *Ast. Nach.*, vol. xvi. p. 161), modified by Dr. Littrow (*Ast. Nach.*, vol. xxxiii. p. 129). The further modification proposed is the common diagonal reflexion, partly on account of the altitude of the sun in the coming eclipse, and partly because the light of the sun can in that construction be very conveniently reduced before totality by Mr. Hodgson's plane glass reflector, to be replaced by a silvered reflector or prism shortly before the totality. In the field is a glass plate, divided by fluoric acid lines into right-angled parallelograms, with sides in the proportion of 3 : 2 for ready distinction, and capable of rotation through 90° only. It has commonly been represented that the method of observing a position-angle on a limb by Bessel's wire consisted in placing the wire strictly as a tangent to the limb at the required point; but it is obvious that a chordal line parallel to the tangent equally gives the same angle, while the estimation of the equality of distance between the object on the limb and the two chordal points of intersection is more readily made than that of a true tangency. The observation proposed by Mr. Carrington would consist of noting the quadrant and the angle shown by the index which will be in all cases less than 90°; and he pointed out that in the construction exhibited, in which rotation through an angle of 90° was alone possible, confusion of adjoining quadrants was impossible, and the particular quadrant easily recorded. The quadrant would be recognised by any notation the observer might adopt to indicate that the preceding or following limb was placed on a long or a short side of one of the parallelograms. The size of any object seen would be readily estimated by comparison with a side of a parallelogram. Mr. Carrington further recommended the use of an altazimuth mounting, admitting of free reversion of the telescope, and that a small vertical circle be attached to the telescope, with a reading level for the purpose of adjustment of the vertical axis at the commencement of operations, and of its verification afterwards. Without further explanation the observer would see that by these means he would have his angles under perfect and ready command, and would have a convenient position for himself while observing. While making this proposal for the consideration of observers who prefer direct vision, Mr. Carrington still reserved a previously-expressed opinion about the desirability of others projecting the image on a properly shaded screen, and that photographic registration would be superior to either.

Proposed New Design for Vertically placed Divided Circles. By R. C. Carrington, Esq.

Mr. Carrington exhibited a small brass model of a circular plate for carrying the divisions in instruments of the larger dimensions, intended for the observation of zenith distances in or out of the meridian. It is well known that the theory of the flexure of vertical circles of large dimensions, provided with numerous radii, is complex, and that the best provision for equal temperature is not thereby attained; so that some have proposed to reduce the dimensions to little more than a foot, in order to replace a spoked wheel by a plane plate of not greater dead weight, and to compensate the smallness of the circle by increased power of the reading microscopes. Cast circles with spokes are also well known to be in a state of excessive molecular strain by the occasional occurrence of the rim parting company with the spokes while on the lathe. Mr. Carrington thinks the form exhibited worth consideration, namely, that of a thin corrugated circular plate, obtained by turning down in a lathe by opposed cutting tools an annealed cast plate till the skin of the original plate and unnecessary metal be entirely removed. By taking advantage of the strength afforded by the corrugated section, and by turning down from the rim inwards to the axis, the thickness and dead weight might be rendered very trifling, while the resistance to gravity flexure would be considerable and the same at all angles, and the greatest facility afforded for rapid equalisation of temperature. The cost of a pattern, which is a considerable item in the shop account, is avoided, and the engineer's finishing work reduced to lathe-work only. The finished plate might with advantage and at moderate cost be electro-gilt.

Formulæ for the Reduction of Pastorf's Observations of the Solar Spots. By R. C. Carrington, Esq.

Mr. Carrington communicated the following formulæ, in the hope that some gentleman, possessing more leisure than himself, might, by their use, extract some results from the three volumes lately presented to the Society by Sir J. Herschel. There is no probability of his doing so himself.



Z being the zenith point of the sun, Pastorf fixed the position of a spot P by measuring the lines PA $= a$ and PB $= b$. To render his measures available, it is required from these highly inconvenient data to find CM $= x$ and PM $= y$, or the distance CP $= \varrho$ and angle ZCP $= \theta$. Manifestly,

$$r^2 = (a + x)^2 + y^2$$
$$\text{and} \quad r^2 = (b + y)^2 + x^2,$$

By the solution of which we find

$$x = \frac{1}{2}(bm - a),$$

$$y = \frac{1}{2}(am - b), \qquad \text{where} \quad m^2 = \frac{4r^2}{a^2 + b^2} - 1,$$

whence also result

$$\varrho = \sqrt{r^2 - abm} \qquad \text{and} \quad \tan\theta = \frac{bm - a}{am - b}.$$

Example. 1832, May 5, 2^h P.M., *Mercury* on the sun (see *Ast. Nach.* No. 228, p. 199), $a = +427''$, $b = -707''$. By ephemeris, $r = 953''$. Whence result

$$x = +521''\cdot5, \quad y = -90''\cdot5, \quad \varrho = 529''\cdot5, \quad \theta = 99^\circ\ 51'.$$

On some previous Observations of supposed Planetary Bodies in Transit over the Sun. By R. C. Carrington, Esq.

In the *Notice* for January I gave a short list of cases extracted from a recent work of my friend Professor Wolf, after rejecting two or three, which a reference to the original will justify. The case of June 6, 1761, may also be rejected, as a reference to the *Ast. Jahrbuch* for 1778 showed that Scheuten was alleging the existence of a satellite to *Venus* seen by him to accompany that planet in its transit on that day. Whatever his observation may be worth, it does not bear on the present question of inferior planets. I find that, in hastily referring to Schmidt's observation of Oct. 11, 1847, I confused the German words *weder* and *entweder* in a way which materially affects his meaning; and the Rev. T. W. Webb has pointed out to me that Professor Wolf and I had overlooked a second similar case under date of Oct. 14, 1849. I do not myself attribute much importance to either of these cases; but as others may differ in opinion, and the dates are singularly concurrent with those of Stark in 1819 and Fritsch in 1802, I give the actual passages from Schmidt's *Resultate*, p. 26: —"1847 (Bonn), October 11. . . . There also (at 9 hours in the morning) rapidly passed across the sun a small black point. This was neither an insect nor a distant bird." Page 31. "1849 (Bonn), October 14. At 11 hours I saw a dark body, about 15″ in size, pass [*vorüberfliegen*] very rapidly from east to west before the sun. It was neither a bird nor an insect."

Schmidt has also a third case, at page 32 of his *Resultate*, under date Feb. 28, 1850, which again singularly concurs with the date of Staudacher's observation in 1762, at *the end* of February. Schmidt's note is:—"10^h 12^m A.M. A dark body, fully 30″, passed over the sun from west to east. I doubted at the time its being a distant bird:" [that is in an unfavourable sense. The original is: "*Ich zweifelte gleich, dass es ein ferner Vogel gewesen sei.*" As we should say, "My impression at the time was that it was not a bird."] But the direction of movement probably condemns the observation as one of a planetary body.

The account of Dangos's observation may be found in the *Jahrbuch* for 1804, pp. 185 to 187, as a "More accurate Notice of the Moving Spot seen on the Sun on Jan. 18, 1798, by M. Dangos, at Tarbes, in South France. Communicated by M. Mechain." The whole account is too long for translation, but the circumstantial part is as follows:—"About 2^h p.m. on the 15th and 17th of January, no spots visible; on the 18th, at about 1$\frac{1}{4}^h$, Dangos found a small spot in the west part of the sun, about half way between the centre and the limb, very dark, round, and sharp. At 1^h 58^m it was only one-fourth (of the radius?) distant from the limb, and the first and last contacts of the exit took place at 2^h 7^m 12$^s\cdot$5 and 2^h 8^m 48$^s\cdot$0 respectively." The reader must judge of the value of all this for himself.

In the case of the observation by Canon Stark, of Augsburg, on Feb. 12, 1820, so singular a confirmation exists that

I first give his entire account, rendered into English from his great *Meteorologisches Jahrbuch*, a work for many years (1813 to 1837) printed at his own expense. His words are:—"On the 12th, a singular spot of well-defined circular form, with circular atmosphere and orange-gold tint, was seen, the apparent size of which was nearly twice the apparent diameter of *Mercury*. At noon, 12^h, this spot was 11′ 20″ distant from the east limb, and 14′ 17″ from the south limb of the sun; and at $4^h\,23^m$ in the evening it was no longer to be seen. The appearance was rather indicative of the transit of a planetary heavenly body, having its path included within that of *Mercury*, than of a solar spot." A confirmation of this observation has been pointed out to me by Mr. Marth, in a passage of a letter of Olbers to Bessel, dated 20th June, 1820 (see *Corr.*, vol. ii. p. 162). Olbers writes:—"What do you say to Steinhübel's observation of a dark, round, well-defined spot, which, on the 12th of February of this year, completed its transit across the sun's disk in five hours? If the thing is a fact, it indicates a planet interior to the orbit of *Mercury*, and whose distance from the sun must be about 0·19, and period over 30 days. If Steinhübel, of whom I know no more than that he has published some sun-spot observations, is actually a good and true man, it may perhaps be worth the trouble for Littrow to endeavour to procure from him the more exact circumstances of this observation, and in particular . . . ." Thinking that the present Dr. Littrow might be able to add something to this information I wrote to him; but have received for answer that he only knew that Steinhübel was a private observer, who died about thirty years ago, and that in his opinion it was very improbable that he had any connexion with Stark, a canon of Augsburg. It seems impossible here to withhold the opinion that either a planet or comet in perihelion was observed by two independent private astronomers of Germany.

Mr. Capel Lofft's observation, on Jan. 6, 1818, is described very circumstantially in a letter to the *Monthly Magazine* of Jan 10, 1818, and the account republished by Mr. Webb in his *Celestial Objects for Common Telescopes*, p. 32. For the convenience of readers not in England the insertion of the account in this paper may be permitted: "I saw it about 11 A.M. (on Jan. 6) with my own reflector, with a power of about 80, with an excellent Cassigrain reflector made by Crickmore of this town (Ipswich) with about 260, and with a reflector of Mr. Acton's with about 170. It appeared when I first saw it somewhat about one-third from the eastern limb, sub-elliptic, small, uniformly opaque. About 2½ hours P.M. it appeared to Mr. Acton considerably advanced, and a little west of the sun's centre, and I think it appeared then 6 or 8 seconds in diameter. I had been able to see no spot on the 4th, nor again on the 8th, and even on the 6th Mr. Crickmore could not see it a little before sunset, though the telescope already mentioned gave him every advantage. . . . Its state of motion seemed inconsistent with that of the solar rotation, and both in figure, density, and regularity of path, it seemed utterly unlike floating scoria. In short, its progress over the sun's disk seems to have exceeded that of *Venus* in transit. . . ."

The observation by Mr. Benjamin Scott, in July 1847, of a body of the appearance of *Mercury* in transit, appears to be partly confirmed by an alleged observation at (as nearly as can now be ascertained) the same time and under similar circumstances by a Mr. W. Wray, optician, of 25 Torriano Terrace, Kentish Town, London, N.W. Mr. Scott has favoured me with copies of Mr. Wray's letters to himself in the present year and to Mr. James Breen, then of Cambridge, in 1848. This case may be referred to on a future occasion should either of the observers succeed in recovering the date with more certainty.

The above cases are merely given for as much as they may be worth in the reader's opinion, but are, at least, of unusual interest at the present moment.

Future Observations of the supposed New Planet.

By M. R. Radau.

(*Translated and communicated to one of the Secretaries by M. D'Abbadie.*)

The investigations of Professor Le Verrier on the theory of *Mercury*, and the procrastinated publication of Dr. Lescarbault's remarkable observation, have more than ever called the attention of the learned to the inframercurial planets. Since 1761, mention has been frequently made of one or several dark spots passing before the sun. Professor Wolf, of Zurich, has published, in 1859, a list of upwards of twenty such occurrences. The two observations of Lichtenberg (1762, Nov. 19), and of Hoffmann (beginning of May, 1764), seem both to point out one and the same planet; while those of Fritsch (1802, Oct. 10), and of Stark (1819, Oct. 9), have a very probable relation with the planet of 1859, March 26, for which the obvious name of *Vulcan* has been proposed.

I shall briefly state the results obtained by combining the incomplete results of Stark and Fritsch, with the much more available observation of Dr. Lescarbault. Neglecting small corrections, one may put the equation of the passage in the following form:—

$$c\,\mathrm{D}-(c-a)\,\mathrm{D}^2=a\sqrt{\frac{2\,\mathrm{D}\,\mathrm{D}_1}{\mathrm{D}+\mathrm{D}_1}},$$

D and D_1 being the distances from the sun to *Vulcan* observed in March and October (*i. e.*, near the descending and ascending nodes), c the chord travelled over the sun's disk, and a the angular displacement of the earth during the passage. This latter having lasted $1^h\,17^m\,8^s$, we have $a=3'\,11''$, and c was $9'\,17''$; $\therefore \frac{a}{c-a}=0{\cdot}521$, and

$$1{\cdot}521\,\mathrm{D}-\mathrm{D}^2=0{\cdot}521\sqrt{\frac{2\,\mathrm{D}\,\mathrm{D}_1}{\mathrm{D}+\mathrm{D}_1}}; \qquad (1)$$

Naming Δ the mean distance, ι the excentricity, putting $\mathrm{D}=x^2$, and $\Delta\,(1-\iota^2)=x^2\,(1+\lambda)^2$, we obtain

$$x+x^2=0{\cdot}521\left(1+\frac{\lambda}{1-x}\right),$$

and for a circular orbit

$$x+x^2=0{\cdot}521,$$

which would give $x=0{\cdot}381$, the distance $x^2=0{\cdot}143$, the time of revolution $t=x^3=0{\cdot}054$ year $=19{\cdot}7$ days. But this would be a particular case: generally speaking, we have only one relation between D and D_1 available to fix certain limits for the elements of the orbit. Taking at once 0·25 for the superior limit of *Vulcan's* distance from the sun, one finds by equation (1) that $\mathrm{D}<0{\cdot}176$ as long as $\mathrm{D}_1<0{\cdot}25$; and since $\frac{2}{\Delta\,(1-\iota^2)}=\frac{1}{\mathrm{D}}+\frac{1}{\mathrm{D}_1}$, we get $\Delta\,(1-\iota^2)<0{\cdot}206$, or $1-\iota<\frac{0{\cdot}206}{\Delta\,(1-\iota)}$. Now the aphelion distance $\Delta\,(1+\iota)<0{\cdot}25$; giving it different values between 0·206 and 0·25, an inferior limit is at the same time obtained for ι between 0 and 0·176, and for Δ a superior limit between 0·206 and 0·221. Therefore Δ will be in all cases $<0{\cdot}221$, and $t<38$ days. The inclination must be included between $11^\circ{\cdot}5$ and $12^\circ{\cdot}5$; therefore *Vulcan*, already 15′ 22″ distant from the centre of the sun's disk on the 26th March, would not be visible on the 25th of the same month. Computing the longitude of the ascending node with a supposed distance D, and the limit of visibility in October with the corresponding distance D, it follows that D_1 must be $<0{\cdot}19$ and $\mathrm{D}<0{\cdot}16$, in order that *Vulcan* may be visible beyond the 10th October. Therefore $\Delta\,(1-\iota^2)<0{\cdot}17$, the inferior limit of ι will remain between 0 and 0·3, and Δ will be $<0{\cdot}19$, $t<30$ days; the synodic revolution be $=\frac{365\,t}{365-t}<32^d$.

The intervals between the three observations are 6208 and $14413 = 2 \times 6208 + 1997$ days. We may suppose $6208 = ms + \mu$, $1997 = ns + \nu$, m and n being whole numbers, and μ and ν small uncertainties. The remainder of $\frac{6208}{1997}$ is 217, that of $\frac{1997}{217}$ is 44, &c., and $217 = (m - 3n)s + \mu - 3\nu$. Therefore 217 may still be considered as a multiple of s; but the ensuing remainders, 44, &c., are already too much altered by the uncertainties μ, ν, to show the traces of s. We already know that $s < 32^{\text{d}}$: we must, therefore, choose between the quotients $\frac{217}{7} = 31$, $\frac{217}{8} = 27$, $\frac{217}{9} = 24$, &c., a number which will be a common divisor to 6208 and 1997. Unfortunately these intervals are too large to prevent finding common divisors differing slightly from any one of the preceding quotients. Confining ourselves, therefore, to these two, 31 and 27, the others being smaller than the sun's rotation (which must not, according to Laplace, happen to any planet), we find nearly

$$6208 = 199 \times 31{\cdot}197 = 202 \times 30{\cdot}730 = 227 \times 27{\cdot}350 = 230 \times 26{\cdot}990,$$

$$1997 = 64 \times 31{\cdot}197 = 65 \times 30{\cdot}730 = 73 \times 27{\cdot}350 = 74 \times 26{\cdot}990.$$

The relation $s = t\left(1 + \frac{s}{365}\right)$ shows that $6208 = ms = 17 \times 365$ will contain $m + 17$ times t: consequently $t = \frac{6208}{216} = 28{\cdot}741$, and for the other cases likewise 28·305, 25·443, 25·113. The periods of return would be 1 Julian year +9, +4, +8, +13 days. One might conclude, therefore, that *Vulcan* will be seen perhaps, on April 4 next, at 2^{h} A.M. (in China), or March 29, about noon, or April 2, between noon and 5^{h}, or finally, April 7, between 10^{h} and 3^{h}, Paris civil time. Taking $t = 28{\cdot}74$ or $= 25{\cdot}44$, Δ would be $= 0{\cdot}18$ and $= 0{\cdot}17$. In the first case $\iota \geqq 0{\cdot}25$, $\Delta(1 + \iota) \geqq 0{\cdot}23$. On July 18, 1860, the places of *Vulcan*, excluding inequalities of motion, would be

Geocentric latitude	$+1^{\circ}{\cdot}1$,	$+0^{\circ}{\cdot}6$,	$+1^{\circ}{\cdot}2$,	$-1^{\circ}{\cdot}3$	in the 4 suppositions.
,, longitude	112·5,	122,	119,	125·5	

The sun's longitude will then be 116°, that of *Jupiter* 124°, and its latitude 0·5.

On that day, therefore, *Vulcan* must be looked for in a zone beginning at 2°, or 4 sun's diameters to the south of *Jupiter*, passing a little to the west of the latter on nearing the sun to within 1° or 2 diameters, and finishing 4° to the north-west of the sun.

Paris, February 17th, 1860.

On certain Inductions with respect to the Heat engendered by the possible Fall of a Meteor into the Sun, and on a mode of deducing the absolute Temperature of the Solar Surface from Thermometric Observation. By J. J. Waterston, Esq.

Mr. Carrington's observation of the sun on the 1st of September last having fortunately established the fact of an outburst of light above the solar surface, and thus favouring Newton's conjecture as to the sun receiving a supply of force from bodies descending upon it, it may be worth while, and perhaps assist in the formation of more exact ideas on the subject, if we compute, from Mr. Joules' value of the thermal unit, the quantity of heat and intensity of temperature that would accrue by a body falling into the sun or sun's atmosphere with the velocity due to a parabolic trajectory. The modern interpretation of Newton's conjecture is, that the comet, or by whatever name we call the body, does not supply fuel to a fire as oil to a flame, but that it supplies force to the central radiating energy,—force to be converted into heat and light.

It is true that no body belonging to the solar system can fall into the sun so long as the laws that keep them in their orbits are maintained. A body that has once rounded the sun in a parabola or elongated ellipse, will probably continue to do so, unless in the rare case of the perihelion distance being so close to the sun's surface that a slight perturbation from the other bodies of the system may bring it within the limits of the sun's radius. But when the orbit is a hyperbola it is the first and last appearance of the body, and if, as may chance to happen, the perihelion distance is less than the sun's radius the sun must inevitably absorb it. We are thus certain that the velocity of a body that impinges on the sun must exceed 419 miles per second, for such is the orbital velocity in the parabola that separates the lesser velocity in the ellipse from the greater velocity in the hyperbola.

With this velocity and Mr. Joules' unit it is easy to compute the quantity of heat due to the conversion of the force; and if the absolute zero of temperature coincide with −461° F., the zero of gaseous tension, the temperature attainable by a known substance under such conditions may also be exactly computed, independent of any theory or hypothesis whatever.

A pound of water falling through 772 feet acquires a velocity of 222 feet per second, and a force that if converted raises the temperature of the pound of water 1° F. If it falls through 4 times 772 the velocity acquired is twice 222 feet, and the rise of temperature 4°. Suppose the acquired velocity to be 1 mile per second, the corresponding rise of temperature is 564°, increasing as the square of the velocity, and 419 miles per second corresponds to $419 \times 419 \times 564 = 99{,}065{,}000^{\circ}$.

A pound of iron under the same circumstances would acquire the same force, but that force converted would raise its temperature nine times the above amount, because the same quantity of heat has nine times greater effect in raising the temperature of iron than it has on the same weight of water.

A pound of mercury would have its temperature affected about 30 times the above amount, and so on according to the specific heat of the substance.

Assuming that the specific heat of the body that impinges on the sun is the same as glass, the rise of temperature corresponding to the velocity 419 miles per second is 565 million degrees Fahrenheit.

Thus the intensity of the temperature engendered depends on the molecular constitution of the body; the *quantity* of the heat, however, is independent of everything but the velocity and the mass, and each pound of any body whatever that strikes the sun with the velocity 419 miles per second is endowed with force sufficient to raise a pound of water 100 million degrees of Fahrenheit's scale.

Has the mass of the sun been gradually collocated by matter thus descending? To estimate exactly the probability of this, two other data are required, viz., the temperature of the sun's surface, and the quantity of heat radiating from it in a given time. The first is unknown, but the second we know approximately from the observations of M. Pouillet, also from those of Herschel and Forbes. It is estimated that it suffices to melt a stratum of about two feet thickness of ice at the earth's mean distance, in 24 hours. This is equivalent to 1·04 feet of ice per second at the sun's surface. Now it is known that ice requires 142° F. to melt it, and each of those degrees is equivalent to the work expended in raising the weight of the ice 772 feet, against the force of gravity at the earth's surface. We thus deduce that a weight equal to that of the ice arriving at the sun with a velocity of 2649 feet, or half-a-mile per second (viz. the velocity acquired in falling through 142×772 feet) is equivalent to the force emanating from the sun in one second. If the same amount of force is brought to the sun by matter moving at the minimum velocity of 419 miles per second, the mass of this matter must be less than the mass of the ice, in the ratio of 1 to the square of twice 419. Such a mass is represented by a sphere of 11·068 miles in diameter, of the density of water.

Supposing the sun to have the same density and specific heat as water, and comparing its volume with that of the sphere of 11 miles diameter, and allowing for the time, we find that the sun's annual loss of temperature to be 6°·125. This is a simple arithmetical deduction from the fact, that the sun's heat can melt about 2 feet of ice daily. The observations of Herschel and Forbes fix it at 1·835 feet (*Phil. Trans.*, 1842); those of M. Pouillet at about one-third less (Taylor's *Scient. Memoirs*, vol. iv.). If we assume 1·5 feet as the correct thickness, the yearly reduction of temperature is 4°·59. If the specific heat of the matter of the sun corresponded with iron, this decrement of temperature would have to be multiplied by 9, and so on for other assumed values. But it is convenient to found our ideas of such quantitive relations on water as the standard most familiar.

Assuming the earth to have six times the density of water we may extend these calculations, and comparing volumes and time we deduce 69 as the number of *years* that the sun takes to throw out as much force as would accrue to it by the earth falling down upon its surface. The mechanical force thus supplied would be equivalent to the expenditure of heat-force for 69 years, and the rise of temperature of the whole mass of the sun, supposing the increment of heat uniformly diffused through its mass, would approach so much nearer to the maximum limit 317° (=69 × 4°·59) as the temperature of the planet after impact exceeded the temperature of the sun.

Again, if we suppose the planet after it has struck the surface of the sun to settle into a disk of 60,000 miles diameter, having the temperature of 100 million degrees, the temperature of the sun being 12 millions, we should have $\frac{1}{220}$ of its disk shining with eight-fold lustre, giving out probably eight times as much heat as an equal surface in the normal state. Thus $\frac{8-1}{220} = \frac{1}{31}$ represents the increment of solar heating power, and if we estimate the normal amount as what is required to keep the earth's surface at a mean temperature of 60° F. or 521° absolute, we see that $\frac{521}{31} = 17^\circ$ nearly, is the increment of mean temperature over all the earth that would arise from such a planet-fall.

M. Pouillet infers from his observations on solar radiation, that the temperature of the sun's surface is at least 2660° F. If it do not exceed this amount, the rise of temperature in the whole mass of the sun would be about 200°; but if, as before, we assume the planet to settle into a disk 2′ in diameter, we have $\frac{1}{220}$ of the sun's disk shining with 38,000 times the normal force, so that a planet-fall of this magnitude would increase the radiating power of the sun 171 times.

Such extravagant conclusions only show the insufficiency of our data, and demonstrate the uncertainty in which the subject is involved, so long as an approximation to the actual temperature of the solar surface is wanting.

Is it possible to ascertain the temperature of the radiating surface of the sun? Ordinary observations give us the temperature in the sun and in the shade. Suppose the temperature of the sun to be double its present amount, it is probable that the absolute temperature in the shade and in the sun would also be double the present amount; consequently, also, the difference between them. We might thus expect this difference, when due precautions are taken, to be a constant quantity and to be a function of the sun's absolute temperature.

Suppose a thermometer to be enclosed in a vacuum and surrounded on all sides by matter, having a uniform absolute temperature *t*: we may consider it to be the centre of a sphere, the interior surface of which radiates heat to it, and the balance of temperature to be thus maintained by reciprocal radiation: as much power *issuing from* the thermometer on all sides towards the concave surface of the enclosing sphere as *enters* into it by radiation from the concave surface. There is a dynamic interchange of force in constant operation. If the temperature of the sphere is augmented one degree, the thermometer rises until its radiating power increases to the same amount. If half the concave surface remains at *t*, while the other half increases from t to $t + 2^\circ$, the rise in the temperature will be the same as before, viz., 1°, because the supply to it is the same as if the whole surface were raised 1°. If $\frac{1}{1000}$ of the surface had the temperature $t + 1000^\circ$, the other parts remaining at *t*, we have

$$\frac{1 \times (t + 1000^\circ) + 999 \times t}{1000} = t + 1^\circ,$$

the resulting temperature as before. If $\frac{1}{1000}$th of the surface had the temperature of $t + 2000^\circ$, then

$$\frac{1 \times (t + 2000^\circ) + 999 \times t}{1000} = t \times 2^\circ$$

is the temperature of the thermometer, and generally, if $\frac{1}{n}$ of the surface of the sphere had the temperature $t + x^\circ$, we have

$$\frac{(t + x^\circ) + (n - 1)\,t}{n} = t + \tau^\circ,$$

the temperature of the thermometer. Hence $x^\circ = n\,\tau^\circ$, a simple relation, by which we can deduce the temperature of the sun's radiating surface, assuming for the present the non-absorption of rays in passing through the atmosphere. It will be remarked that τ° is the difference of reading between thermometers in the sun and in the shade taken with due precautions.

Some years ago, when in India, I tried this by enclosing a thermometer within three concentric boxes well protected from external influences, and capable of being equally heated all round to any temperature below 400°, by means of flues ascending from an Argand lamp. The sun's rays, when near the meridian (having an altitude of about 70° and with the atmosphere perfectly clear and calm), were admitted to fall, when required, on the bulb of the thermometer through a narrow triplet glass partition. I found that 50° was the rise that took place in consequence of exposure. There being glass partitions on both sides, the reading of the thermometer was very distinct by transmitted light. Beginning at 80°, without applying the lamp, the admission of the sun's rays caused the mercury to rise to 130°, where it remained steady. The lamp was then applied at low power, so as to maintain the inner box at this temperature while the sun was excluded. When perfectly steady, the sun's rays were readmitted, and the mercury again mounted with the same alacrity as before, until it reached 180°. This continued step by step up to 250°. No difference, either in the magnitude of the step or the time taken to effect it, could be detected.

Thus, for τ° we have the constant 50°, and n we obtain by comparing the disk of the sun with the surface of the sphere. At the earth's mean distance the sun's diameter is 32′ 3″·6, hence $n = 183960$ and $x = 9{,}180{,}000^\circ$.

If there is no fault in this mode of proceeding, we may with confidence estimate the solar temperature to be *above* 10 million degrees, probably 12 million, allowing a reduction of one-third from absorption in passing through the atmosphere and the three plates of glass.

A notable fact, in making these observations, is, that the step τ° seems wholly independent of the temperature of the medium in which the thermometer lies. Why this should be is apparent from the equation. Substituting $2t$ for t, and consequently $x - t$ in place of x, is tantamount to heating the box from 80° up to 620°. Let τ' represent (in degrees) the step at this higher temperature $2t$, we have

$$\frac{2t + x - t + (n-1)\,2t}{n} = 2t + \tau' \text{ and } x - t = n\tau' \text{ and } \frac{\tau'}{\tau} = \frac{x-t}{x}.$$

Thus the step diminishes only about the $\frac{1}{18000}$th part, or $\frac{1}{360}$th of a degree, during a change of upwards of 500° in the box.

From Mr. Carrington's observations it appears that the burst of light was of *much* greater intensity than the sun's normal surface. This is quite consistent with its assumed temperature, which is much less than any probable estimate we can make of the temperature of the conversion of the force of a body striking the sun's atmosphere with a velocity of from 400 to 500 miles per second. The existence of a transparent atmosphere seems also to be positively demonstrated by the blaze occurring above the spots.

Edinburgh, Feb. 15, 1860.

On the Variability of the Proper Motion of Sirius.
By the Rev. R. Main, President.

The object of this communication is not so much to discuss thoroughly the question of the variability of proper motion which has been recently attributed to *Sirius*, but to endeavour to disprove an assertion made repeatedly by Professor Calandrelli, director of the Pontifical Observatory at Rome, of the incorrectness of the N.P.D. of this star given for the epoch 1845 in the Greenwich Twelve-Year Catalogue. The papers in which this charge of incorrectness is made are the following:—

1. Pontificio Nuovo Osservatorio della Romana Università, Notizie Storiche del Prof. J. Calandrelli.
2. Sopra i Movimenti Propri delle Stelle. Memoria del Prof. J. Calandrelli.
3. Sul Movimento Proprio di *Sirio*, Ricerche Astronomiche, del Prof. Ignazio Calandrelli, &c.
4. Appendice alle Ricerche sopra i Movimenti Propri delle Stelle Fisse, del Prof. Ignazio Calandrelli, &c.

The first two of these memoirs are printed in the *Atti dell' Accademia Pontificia de' Nuovi Lincei* for 1853, April 3, and 1857, March 1. The last two are separate publications, printed in 1858.

The titles of the memoirs above will show that their principal object is the treatment of observations for the purpose of deducing the proper motions of the stars, and ultimately of resolving in particular the question of the variability of the proper motion of *Sirius;* and there is nothing in them which would require any special remark, if the assertion of the incorrectness of the value of the N.P.D. of *Sirius* given in the Greenwich Catalogue were not repeated in all of them so pertinaciously as to impress the reader with the belief that the fact is undisputed, and that it has arisen from some unexplained errors, either in the observations themselves, or the reduction of them.

The following investigations will, I trust, prove that the place given in the Greenwich Catalogue is correct, and that, in consequence, the greater portions of Calandrelli's reasonings with regard to the proper motion of *Sirius* in N.P.D. are defective.

It would be sufficient for my purpose to discuss only the north polar distances of this star; but I take the opportunity of collecting all the Greenwich observations, both of R.A. and N.P.D., which have been accessible to me in a reduced shape, as a small contribution towards the settlement of the disputed question of the variability of proper motion.

For R.A. the observations extend with tolerable continuity from the years 1765 to 1859; and I use,—

1. Maskelyne's "Ledgers of Stars," edited by myself, and appended to the *Greenwich Observations* of 1851. These extend from 1765 to 1807, with occasional interruptions.
2. Pond's Right Ascensions, given in his various catalogues from 1820 to 1835, with some interruptions.
3. Airy's *Greenwich Observations* from 1836 to 1858, both years inclusive.

A very few explanatory remarks are necessary, before giving the tabulated values of R.A.

As the result in N.P.D. for 1845 has been called in question, I have found it convenient to reduce all the results for individual years to this epoch, and I assume the following values:—

Peters' Annual Precession for 1845, in R.A. .. $= +2^{\mathrm{s}}{\cdot}681$
Main's Annual Proper Motion .... $= -0{\cdot}035$
Secular Variation of Precession............ $= -0{\cdot}0010$

I have corrected Maskelyne's "Right Ascensions," given in the catalogues contained in the before-mentioned work, by the quantities due to the assumed place of the equinox for each year, as given in the same papers.

As Pond's determination of the position of the equinox for each year was made with great care, I have not applied any correction to his results.

To Airy's "Greenwich Right Ascensions" I have applied the corrections assigned in the Twelve-Year Catalogue, and in the Six-Year Catalogue.

I now give the right ascensions deduced from the observations of each year, reduced to 1845:—

Year.	Mean R.A., 1845, Jan. 1.	Numb. of Obs.	Year.	Mean R.A., 1845, Jan. 1.	Numb. of Obs.
	h m s			h m s	
1765	6 38 19·14	44	1826	6 38 18·96	—
1766	19·16	5	1829	19·10	24
1767	19·08	9	1830	19·04	27
1768	(18·44)	1	1831	19·18	16
1769	(19·52)	1	1832	19·15	20
1779	(19·06)	1	1833	19·07	14
1780	18·93	9	1834	19·07	18
1781	19·18	5	1835	19·09	13
1782	19·01	15	1836	19·07	23
1783	18·83	13	1837	19·12	17
1784	18·69	15	1838	19·09	21
1785	19·05	6	1839	19·15	26
1796	(19·51)	7	1840	19·15	30
1797	(19·33)	5	1841	19·12	12
1798	19·33	12	1842	19·24	32
1799	19·21	2	1843	19·29	16
1800	19·34	8	1844	19·22	13
1801	19·22	3	1845	19·22	17
1804	18·85	6	1846	19·16	27
1805	18·97	17	1847	19·12	22
1806	19·12	3	1848	19·16	27
1807	18·95	11	1849	19·11	38
1820	19·08	—	1850	19·11	21
1823	19·06	—	1851	19·10	24

Year.	Mean R.A., 1845, Jan. 1.	Numb. of Obs.	Year.	Mean R.A., 1845, Jan. 1.	Numb. of Obs.
	h m s			h m s	
1852	6 38 19·09	29	1856	6 38 19·03	9
1853	19·05	7	1857	18·96	13
1854	19·00	13	1858	18·89	10
1855	19·04	15			

We next proceed to form the table of values of north polar distances for 1845 in a similar manner. The materials I have used are Bessel's *Fundamenta*, Piazzi's Catalogue, Pond's *Greenwich Observations* from 1813 to 1832, Airy's *Cambridge Observations* for 1833 and 1835, and Airy's *Greenwich Observations*, 1836 to 1859.

The values of constants are the following:—

Peters' Annual Precession in N.P.D. $= +3''.340$
Main's Annual Proper Motion...... $= +1''.24$
Secular Variation of Precession .... $= +0''.385$

The tabulated values of mean N.P.D. for 1845, Jan. 1, are as follows:—

Year.	Mean N.P.D., 1845, Jan. 1.	Numb. of Obs.	Year.	Mean N.P.D., 1845, Jan. 1.	Numb. of Obs.
	° ′ ″			° ′ ″	
1755	106 30 30·43	—	1842	106 30 28·49	11
1800	28·39	—	1843	27·30	13
1813	26·78	34	1844	27·07	9
1815	26·28	—	1845	26·34	10
1822	30·33	—	1846	26·47	10
1823	30·04	—	1847	26·47	5
1826	29·52	—	1848	26·36	11
1829	31·12	22	1849	26·49	23
1830	30·69	46	1850	27·28	25
1831	30·59	20	1851	25·86	27
1832	30·78	25	1852	26·39	26
1833	29·82	11	1853	26·58	8
1835	29·00	4	1854	26·67	15
1836	29·98	28	1855	26·62	16
1837	30·46	131	1856	26·85	9
1838	30·16	30	1857	27·76	12
1839	28·96	22	1858	28·25	9
1840	29·07	17	1859	27·68	11
1841	29·40	6			

Pond's north polar distances have been taken, without alteration, from his yearly catalogues; excepting that, as Bradley's refractions were used by him in the reduction of his observations, the correction $+1''.5$ has been applied for reduction to Bessel's refractions, or to make the results comparable with Airy's results. The result opposite the epoch 1755 is derived from the N.P.D. given in the *Fundamenta*, unaltered, and that opposite 1800 from Piazzi's Catalogue.

An inspection of the above table of north polar distances will show, first, that there is, on the whole, an admirable uniformity in the values for successive years, which proves the goodness of the observations on which the results are based; and, secondly, that there is a periodical fluctuation in the values of an interesting and well-marked character. This fluctuation I do not intend to speculate upon in the present paper, except by remarking that we might *à priori* have expected such a result to accompany the acknowledged and well-marked fluctuation of the corresponding values of R.A., which indicates a variability of proper motion.

If, now, we take the means of the six results from 1842 to 1847 inclusive, we shall have for the N.P.D., 106° 30′ 27″·02; and this is, as it ought to be, precisely the result given in the Greenwich Twelve-Year Catalogue for the year 1845, and which Professor Calandrelli assumes to be in error.

A little consideration of the grounds on which the Professor has arrived at this conclusion will show that he has not used sufficient caution in his assumption, especially as it involves the grave charge of an error amounting to more than 2″ in the place of a fundamental star in a work of so great authority as the *Greenwich Catalogue*.

In the first of his memoirs, namely, that in the *Nuovi Lincei* for April 3, 1853, p. 316, there is the following passage:—

"Nel recente Catalogo di Greenwich, pubblicato nel 1849, si riportano le distanze polari di molte stelle per le due epoche vicinissime fra loro 1840–1845. Accade spesso che, ridotte le posizioni alla stessa epoca, si trova une differenza di 2″, 3″. Per l' α del Cane Maj. si ha

$$1840,\ \Delta = 106^\circ\ 30'\ 6''.98$$
$$1845,\ \Delta = 106\ \ 30\ \ 27.02$$

Riducendo la prima all' epoca dell' altra si ottiene

$$1845,\ \Delta = 106^\circ\ 30'\ 30''.405$$
$$1845,\ \Delta = 106\ \ 30\ \ 27.020$$

e quindi pel 1° del 1855,

$$\Delta = 106^\circ\ 31'\ 17''.254$$
$$\Delta = 106\ \ 31\ \ 13.870$$

la prima è prossimamente quella che ho fissato dietro il confronto di altri cataloghi; la prima risulta da 234 osservazioni, l'altra da 58, dunque si conferma ciò che ho detto (87°) sugli errori inevitabili delle osservazioni."

In explanation of the above, it may be stated that the results for 1840 and 1845 are those given in the Greenwich Twelve-Year Catalogues for those epochs; the first being deduced by combinations of all the observations from 1836 to 1841 inclusive, and the second by all the observations from 1842 to 1847 inclusive; and Calandrelli's assertion is, that from mere error of observation the first result differs from the second by 3″·4, though the number of determining observations is 234 for the epoch 1840, and 58 for the epoch 1845. He also arbitrarily assumes, on account chiefly of the difference in the number of determining observations, that the result for 1840 is essentially correct, while that for 1845 is totally in error. Without wishing to be severe in our judgment, we may at least say that the charge of incorrectness is, in the instance before us, made without due examination, and on very insufficient grounds.

An inspection of the separate results for 1845, derived from the observations of the separate years from 1836 to 1841, and from 1842 to 1847, will show that the difference in the results of the two epochs noticed by Calandrelli is *real*, but that it has not its origin in any error either of observation or reduction: it depends upon a real fluctuation, most important and interesting, which will render the motions of this star a serious subject for study during the remainder of the present century.

In the other papers of Calandrelli to which reference has been made there are similar statements with respect to the incorrectness of the N.P.D. of *Sirius* given in the *Greenwich Catalogue*, but without any additional discussion. Thus, in the *Atti dell' Accad. Pontif. de' Nuovi Lincei* for March 1, 1857, he says, referring to the N.P.D. for 1845, which he has assumed: "In questa tavola la distanza polare di *Sirio* al 1° del 1845 non è quella che trovasi nel Cat. di Greenwich pubblicato in Londra nel 1849. Notai già in altro luogo, che in quella distanza polare essere incorso un qualche errore."

Again, in his memoir entitled "Sul Movimento Proprio di *Sirio*," published in 1858, p. 8, he makes a precisely similar statement: "Nel tomo secondo degli Annali dell' imperiale Osservatorio di Parigi, pubblicati da Le Verrier, dopo le correzioni date alle osservazioni di Bradley e di Greenwich si danno due posizioni fondamentali di *Sirio*, una pel 1° del 1755, e l' altra pel 1° del 1845. Nella declinazione o distanza polare di questa ultima è incorso un errore di circa 2″, come ho fatto notare in altri luoghi."

Lastly, in the Appendix to the paper quoted above, published also in 1858, pp. 26, 27, he repeats his statement, thus: "A me pare di aver dimonstrato evidentemente che le distanze polari di *Sirio* date pel 1° Gen. 1845 (Greenwich) e pel 1° Gen. 1850 (Madras) sono in errore di 2″ in 3″ colle altre date per le stesse epoche." And again: "Questo errore nella distanza polare di *Sirio* pel 1° Gen. 1845 fu già da me avvertito al num° 91 della Memoria pubblicata in Roma nel 1855, dove anche notai che la distanza polare di *Sirio* data nel Catalogo di Greenwich (1849) pel 1° Gen. 1840, risultava da 234 osservazioni, e che quella del 1845, dello stesso Catalogo risultava da 58 osservazioni."

It is sufficient at the present time to have drawn the attention of astronomers to this unfounded assertion of Calandrelli, without going into further details than have been found necessary for its refutation, because any astronomer who might find occasion to discuss the subject of the variability of the proper motion of *Sirius* would have found no greater difficulty in arriving at the truth than I have found myself. The tables which I have given in this paper of the values of R.A. and N.P.D. for Jan. 1, 1845, deduced from the observations of single years, will not only prove the general precision of the Greenwich results, but will, I think, establish the fact of periodicity of proper motion in both elements. In general, though this has been esteemed certain for right ascensions, it has been thought that there is not sufficient evidence of variation of proper motion in N.P.D. Yet an inspection of the preceding tables will show that it is pretty nearly as well marked in one element as the other, and will, I trust, give an additional motive for the study of the peculiarities of this star. If the subject were treated with the utmost rigour, a very nice scrutiny of Pond's observations, in comparison with the more modern Greenwich observations, would be requisite. Still I believe the remaining corrections, in addition to such as I have pointed out, would be very small, and that the results which I now give are entitled to considerable confidence.

Before concluding this paper, it may be interesting to give in a similar shape the results for the north polar distances of *Procyon*, in which variability of proper motion has been suspected. They are contained in the following table, from the year 1836 to the present time:—

Year of Obs.	Mean N.P.D., 1845, Jan. 1.	Numb. of Obs.	Year of Obs.	Mean N.P.D., 1845, Jan. 1.	Numb. of Obs.
	° ′ ″			° ′ ″	
1836	84 22 57·79	12	1848	84 22 54·99	13
1837	56·87	12	1849	54·85	12
1838	57·14	22	1850	55·59	18
1839	56·63	13	1851	55·78	24
1840	55·30	6	1852	56·08	33
1841	57·11	8	1853	55·81	1
1842	55·49	10	1854	56·40	21
1843	55·94	9	1855	56·71	19
1844	55·79	7	1856	56·52	14
1845	55·94	6	1857	56·60	16
1846	55·03	11	1858	57·22	13
1847	56·33	4			

For the reduction to the year 1845, I have used the following values of precession and proper motion:—

Peters' Annual Precession in N.P.D.	+7″·771
Main's Annual Proper Motion	+1·08
Secular Variation of Proper Motion.	+0·429

In the results there is a sensible trace of periodicity, the minimum values being about the epoch 1848–9; but as the number of observations in each year is not large, it would not be prudent to pronounce a definite opinion. By grouping the observations of every three years, a more distinct notion may be gained of the value of the evidence. Thus:—

Years of Obs.	Mean N.P.D., 1845, Jan. 1.	Numb. of Obs.	Years of Obs.	Mean N.P.D., 1845, Jan. 1.	Numb. of Obs.
	° ′ ″			° ′ ″	
1836–8	84 22 57·24	46	1848–50	84 22 55·20	43
1839–41	56·48	27	1851–3	55·95	58
1842–4	55·73	26	1854–6	56·54	54
1845–7	55·54	21	1857–8	56·88	29

Royal Observatory, Greenwich,
March 8th, 1860.

On a Method of getting rid of Personal Equation, by a Test for Micrometer Readings. By Lieut.-Col. R. Shortrede.

It is well known that different persons vary sensibly in their micrometer readings, the difference between the means of a number of readings being considered as the *personal equation* for the observers.

Many years ago I hit upon a method of reading by which I could (to the extent of the optical power of the micrometer) always myself get the same readings, and, after a few minutes' training, could get other people to read the same as myself. I conceive, therefore, that personal equation of this sort arises not from difference in the eyes of the observers, but from difference in the method of using them in judging of the bisection of the object in view.

In trying carefully to make the cross-wires bisect the divisions of the limb, I noticed that the angular points of the limb visible between the division and the cross-wires formed a quadrilateral; and that when the intersection of the wires was in the middle of the division, the quadrilateral was necessarily a parallelogram, the exactness of which could be instantly tested by comparing the short sides above and below the crossing of the wires.

If the intersection of the cross-wires be not in the middle of the division, the four points will not form a parallelogram.

If the division of the limb bisect the angle of the cross-wires, the four points will form a rectangle.

If the bisection of the wires be oblique to the division on the limb, they will form not a rectangle, but a parallelogram.

To apply this test, it is important that both the divisions and the wires should be clean and sharp, but it is not necessary that they should be very fine; they should be thick enough to be easily seen in the worst light likely to be used.

I may mention that this method of reading was not known at the head-quarters of the Great Trigonometrical Survey in India till 1840, nor at a principal observatory in this country till 1848.

By this method of reading, the intersection may always be brought over the middle of the division, even though the wires be unequally inclined. Unequal inclination of the wires, however, is an evil. In order to avoid it, the usual way of adjusting is to turn the body of the micrometer till the angle of the wires appears to be equally bisected by the divisions. This often leaves a considerable personal error, to avoid which I have found it a good method to bring the micrometer zero between the pair of equal lines in the middle of the quarter-degrees, and then move the micrometer-wires so as to be equidistant from the upper ends of the equal lines, when the lower ends also will be equidistant if the wires be equally inclined, or, in other words, if the divisions properly bisect the angle of intersection of the wires.

Of late years some observers profess to correct the runs of their micrometers for temperature. This seems to be a needless refinement; for the correction, though real, is in any particular case almost insensible, and vastly smaller in amount than another generally neglected, arising from the wear of the screw. As usually made, micrometer-screws are pressed up by a spring acting constantly on one side, in order to avoid slack: the consequence is, that the part of the screw commonly used gets worn away on one side; and after much use there may be a sudden change in the run, amounting, as I have found, to two or three seconds. I am not aware that this has received the attention it deserves.

February, 1860.

On the Shape of the Moon. By M. Gussew, Assistant at the Observatory of Wilna.

(*Communicated by W. De La Rue, Esq.*)

The author refers to a detailed account of an investigation on this subject, sent by him to the Academy of Sciences at St. Petersburg. He remarks that the sharpness of the lunar photographs obtained by means of the reflecting telescope of Mr. De La Rue is so well known to the Society that no justification is needed for making these lunar pictures the basis of scientific investigation. He mentions that he was led to the investigation by a peculiar egg-shaped appearance of the moon exhibited in the well-known stereographic lunar photographs of Mr. De La Rue. It appeared to him inexplicable how such an effect could be produced by two photographic pictures taken from the same sphere, and moreover entertaining a hope of finding a confirmation of the well-known theoretical results of M. Hansen, he commenced a series of micrometrical measurements, which were well suited to remove all doubts on the subject. The result obtained from the measurements is that the greatest mean deviation from the spherical form is about 0·05 of the radius, and the author concludes by remarking that his observations at least prove that the deviation of the form of the moon from that of a sphere is sufficiently great to be measurable by direct observation.

Morning Illumination of Hippalus (Lunar Crater). By W. R. Birt, Esq.

This crater (225 on Webb's index-map) is a fine specimen of a class but rarely met with on the lunar surface, viz., those partially inclosed by a mountainous chain that had *apparently* at some former period been so perfectly ring-formed as to have entirely inclosed the interior; but now a portion of the ring is gone, and that so completely as not to leave a vestige of it remaining in its immediate locality, which, in the instance before us, is towards the south-east.

The crater *Hippalus*, situated on the north-western border of an arm of the *Mare Humorum*, lies on a ridge of craters that may be traced as far as *Tycho*. The western and northern portions of the border of the crater are more or less complete; they consist of peaks varying very considerably in altitude. The appearance presented at the age and libration of the moon specified below is that of a nearly semicircular range of mountains of low altitude on the south, gradually increasing on the western side, and attaining the culminating point on the northern or rather north-eastern, where an *abrupt* termination of the chain occurs; the length of the shadows *without*, contrasting strongly with those on the opposite side *within* the crater. The western half of the floor, bounded on the east by a nearly straight line terminating at the southern extremity of the semicircular mountains, appears to be of a somewhat *higher level* than the eastern, and is *rugged* from the presence of a number of hillocks and two very minute craters; whilst the eastern part is *smooth*, strongly reminding the observer of the general surfaces of the *maria*. *Only one small hill is seen on this part of the floor, near the abrupt termination of the mountains.* Not the vestige of a shadow is to be seen between the two extremities of the mountains. The light passes unobstructedly from the boundary of the irregular western half over the smooth surface of the eastern, until it reaches the terminator at some distance to the east.

A most interesting question is suggested by the appearance of this crater. Has the ring at any earlier or remote period completely surrounded the interior? The smooth eastern and rugged western halves are remarkable; and it is worth inquiry if the ruggedness of the western has resulted from the *débris* of the destroyed portion of the wall on the east (?), or from a portion of the wall on the west having given way as indicated by Mr. Nasmyth, see *Monthly Notices*, vol. xiv. p. 158.

Epoch 1860, March $2^{d}\cdot44$ G.M.T. Moon's age, $10^{d}\cdot12$,
Libration N.W., $10^{d}\cdot11$, Epoch March $12^{d}\ 13^{h}\ 19^{m}$,
Perigee, $4^{d}\cdot14$, Epoch March $6^{d}\ 14^{h}$.

On a Phenomenon connected with a Transit of Jupiter's Third Satellite. By C. Leeson Prince, Esq.

I am induced to forward a brief report of a phenomenon connected with a transit of *Jupiter's* third satellite, which I had never previously observed.

The atmosphere during the evening of January 31st was remarkably clear and favourable for astronomical observations.

Upon examining *Jupiter's* disk about $10^{h}\,40^{m}$, and just before the shadow of the third satellite appeared upon the disk, I could not see the satellite shining upon the belt, but I observed what I at first thought was the shadow of another satellite which had passed the centre of the planet.

Upon reference to the *Nautical Almanac*, however, I found that the shadow of the first satellite had emerged at $8^{h}\,20^{m}$.

I then considered this dark object to be a spot upon the belt, and was not undeceived until I found that it travelled too fast, and became much fainter as it approached the limb of the planet.

About $11^{h}\,50^{m}$ I left the telescope, to allow a friend to look at the planet; and upon my return, at $11^{h}\,56^{m}$, I found to my surprise that during this short interval the spot had entirely disappeared, and that the satellite itself was shining *brightly* in its place.

I was not aware that the third satellite ever looked opaque during its transit.

When first observed, it appeared like the shadow of the first satellite when there is much tremor in the air.

The planet *Saturn* was also a fine object the same night.

From frequent observations during the last two years, I am convinced that the nebulous ring is becoming more and more illuminated.

The Observatory, Uckfield, February 20th.

From the particulars communicated in letters from Captain Boyd, R.N., commanding H.M.S. Ajax, and from a conversation with Lieut. Grant, R.N., of the same ship, the Astronomer Royal has no doubt that some of *Jupiter's* satellites were seen by those gentlemen and by other officers of the ship, in Kingstown Harbour, Dublin, on the evening of Sunday, Jan. 15, with the naked eye.

The evening was at first hazy, but became afterwards unusually clear.

The satellites and *Jupiter* were in the order

3, 2, 1, *Jupiter*, 4

(measuring from left to right).

The first satellite was very near to the body of *Jupiter*.

Capt. Boyd, and perhaps others of the officers, saw 3 and 2 with the naked eye; Lieut. Grant saw 3 only. It does not appear that any of the party saw 4 or 1. It will be borne in mind that 3 is much brighter than the others.

The satellites were identified by comparing their appearance with that of the complete system as viewed in the 2-feet navy telescopes.

Royal Observatory, Greenwich,
February 4th, 1860.

In a letter to the Assistant-Secretary, Mr. Pollock, after referring to his letter giving an account of the apparent projection of *Saturn* on the moon's disk, as seen by the naked eye on the occasion of the occultation of the 8th May last, and to the explanation by the Astronomer Royal of the observed phenomenon (see *Monthly Notices*, vol. xix. pp. 323 and 340), writes:—

"Had the moon been past the full at the time of the occultation, so as to make the planet disappear behind the *bright* limb, there might be no difficulty; but in this case the disappearance of *Saturn* took place behind the *dark* limb of the moon. And having regard to the comparative brightness of the planet and the dark limb of the moon, it is not easy to see how there could be such an enlargement of the diameter of the latter as to make the planet appear *entirely* on the moon's disk."

Results of the Observations of Small Planets made with the Transit-Circle at the Royal Observatory, Greenwich, during the months of January and February, and part of March, 1860.

(Communicated by the Astronomer Royal.)

Hebe ⑥.

Mean Solar Time of Observation.			Apparent R.A.	Apparent N.P.D.
		h m s	h m s	° ′ ″
1860, Jan.	2	7 7 2·7	1 49 41·73	98 48 55·46
	4	6 57 16·8	1 51 43·90	98 19 47·76
	6	6 51 31·5	1 53 50·72	97 50 30·48
	7	6 48 40·3	1 54 55·69	97 35 49·63

Iris ⑦.

Mean Solar Time of Observation.			Apparent R.A.	Apparent N.P.D.
		h m s	h m s	° ′ ″
1860, Jan.	17	14 4 11·2	9 51 3·73	84 50 58·43
	20	13 49 57·9	9 48 37·67	84 47 55·51
	26	13 20 56·1	9 43 10·49	84 36 51·88
	31	12 56 18·9	9 38 12·00	84 23 6·15
Feb.	6	12 26 29·4	9 31 56·93	84 2 6·87
	9	12 11 32·9	9 28 47·65	83 50 9·12
	10	12 6 34·5	9 27 44·96	83 45 59·29
	13	11 51 40·8	9 24 38·49	83 33 3·14
	23	11 2 46·5	9 15 1·74	82 47 14·29
	27	10 43 43·0	9 11 41·35	82 28 49·87
	29	10 34 19·9	9 10 9·76	82 19 47·03
Mar.	2	10 25 2·6	9 8 44·07	82 10 54·62
	5	10 11 18·7	9 6 47·55	81 57 10·35

Hygeia ⑩.

Mean Solar Time of Observation.			Apparent R.A.	Apparent N.P.D.
		h m s	h m s	° ′ ″
1860, Feb.	28	12 49 54·6	11 22 10·18	91 18 37·33
	29	12 45 15·2	11 21 26·65	91 15 3·63
Mar.	2	12 35 54·4	11 19 57·38	91 7 40·88
	5	12 21 51·1	11 17 41·44	90 55 58·40

Parthenope ⑪.

Mean Solar Time of Observation.			Apparent R.A.	Apparent N.P.D.
		h m s	h m s	° ′ ″
1860, Jan.	7	8 56 43·4	4 3 19·75	……
	12	8 35 53·9	4 2 9·67	74 5 17·87
	17	8 15 48·1	4 1 43·34	73 52 52·78
	21	8 0 15·4	4 1 54·34	73 41 31·54

Egeria ⑱.

Mean Solar Time of Observation.			Apparent R.A.	Apparent N.P.D.
		h m s	h m s	° ′ ″
1860, Jan.	16	14 2 18·2	9 45 13·77	47 35 41·60
	17	13 57 31·3	9 44 22·65	47 26 43·69
	20	13 42 59·6	9 41 38·31	47 1 6·36
Feb.	6	12 16 54·0	9 22 20·02	45 24 6·34
	27	10 31 38·3	8 59 34·65	45 48 14·59
	28	10 26 54·0	8 58 46·13	45 53 13·77
	29	10 22 12·2	8 58 0·13	45 58 31·25
Mar.	2	10 12 54·4	8 56 33·88	46 9 56·83
	5	9 59 14·0	8 54 40·94	46 29 6·61

Urania ㉚.

Mean Solar Time of Observation.			Apparent R.A.	Apparent N.P.D.
		h m s	h m s	° ′ ″
1860, Feb.	6	14 40 1·2	11 45 50·73	90 24 9·67
	28	12 57 36·4	11 29 53·26	89 5 37·58
Mar.	2	12 43 2·4	11 27 6·53	88 50 23·88
	5	12 28 24·6	11 24 15·99	88 34 34·84

Pomona ㉜.

Mean Solar Time of Observation.			Apparent R.A.	Apparent N.P.D.
		h m s	h m s	° ′ ″
1860, Jan.	16	12 55 26·4	8 38 11·00	80 39 44·46
	17	12 50 36·7	8 37 17·06	80 37 35·51
	20	12 36 5·0	8 34 32·67	80 30 39·75
Feb.	9	10 58 58·7	8 16 1·59	79 22 23·36
	10	10 54 13·2	8 15 11·84	79 18 23·13
	27	9 35 33·3	8 3 20·47	78 9 48·47
	29	9 27 55·3	8 3 34·25	78 2 12·97

Leda ㊳.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, Jan. 4	7 22 34·0	2 17 5·21	68 10 50·87

Lætitia ㊴.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, Jan. 16	11 56 10·7	7 38 45·60	79 15 36·49
20	11 36 55·2	7 35 13·11	78 55 16·25
24	11 17 45·9	7 31 46·90	78 33 56·75
Feb. 29	8 38 27·2	7 13 58·05	75 19 53·78

Ariadne ㊸.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, Feb. 27	12 20 32·2	10 48 46·43	88 47 29·48
29	12 10 36·0	10 46 41·69	88 36 30·44
Mar. 2	12 0 39·1	10 44 36·32	88 25 2·82
5	11 45 43·6	10 41 28·03	88 7 25·45

Nysa ㊹.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, Feb. 23	9 0 32·1	7 12 27·29	67 57 46·73
Mar. 2	8 31 16·8	7 14 39·57	67 42 31·21

Eugenia ㊺.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, Jan. 12	9 58 8·7	5 24 38·00	74 46 2·60

Doris ㊽.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, Feb. 13	13 21 12·7	10 54 22·11	87 41 4·50
29	12 7 4·2	10 43 9·33	86 8 35·74
Mar. 2	11 57 45·5	10 41 42·25	85 56 5·62

Pandora (55).

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, Jan. 20	13 5 47·5	9 4 20·02	63 2 28·39

All the observations of N.P.D. have been corrected for parallax.

Observed Places of the Small Planets Hygeia and Urania.

By M. Lépissier.

(*Communicated by the Astronomer Royal.*)

Hygeia ⑩.

1860.	Paris M.T.	App. R.A.		App. N.P.D.		Ref.	No. of Comp.
	h m s	h m s		° ′ ″			
Feb. 29	12 29 24·1	11 21 26·88	(8·419−)+Δ	91 15 0·3	(0·817−)+Δ	*a*	2 & 1
29	13 57 36·6	11 21 24·40	(9·070+)+Δ	91 14 57·0	(0·816−)+Δ	*a*	4 & 4
Mar. 1	8 44 26·9	11 20 49·46	(9·511−)+Δ	91 11 57·2	(0·812−)+Δ	*b*	3 & 3
1	12 21 27·1	11 20 42·98	(8·500−)+Δ	91 11 31·1	(0·816−)+Δ	*a*	7 & 5
2	11 3 46·3	11 20 0·48	(9·171−)+Δ	91 7 55·5	(0·815−)+Δ	*a*	4 & 3
3	10 18 11·8	11 19 16·85	(9·318−)+Δ	91 4 12·6	(0·814−)+Δ	*c*	5 & 5

Observer, Y. Villarceau, on March 1, second observation; on other occasions, Lépissier.

The observations are corrected for refraction.

Mean Adopted Positions of the Stars of Comparison, 1860·0.

h m s ° ′ ″
a = 353 Weisse 9th Mag. R.A. = 11 20 49·63 N.P.D. = 91 9 3·5
b = 312 Weisse 9th ,, 11 18 42·53 91 21 20·6
c = 3901 B.A.C. 6th ,, 11 20 44·63 90 55 48·3

Urania ㉚.

1860.	Paris M.T.	App. R.A.		App. N.P.D.		No. of Comp.	Ref.
	h m s	h m s		° ′ ″			
Mar. 8	11 34 36·3	11 21 24·98	(8·652−)+Δ	88 18 15·7	−(0·797)+Δ	3 & 3	*a*
	12 51 52·6	11 21 21·73	(8·797+)+Δ	88 17 59·9	−(0·797)+Δ	4 & 3	*a*

The observations are corrected for refraction.

Mean Adopted Position of the Star of Comparison, 1860·0.

h m s ° ′ ″
a = 21805 Lalande 7th Mag. R.A. = 11 20 23·20 N.P.D. = 88 15 58·0

The planet was a very faint object both from its smallness and the moonshine.

Occultations of Stars by the Moon, observed at Highbury.

(*Communicated by T. W. Burr, Esq.*)

Date.	Star.	Immersion. Dark Limb. h m s	Emersion. Bright Limb. h m s
1860, Feb. 28	19 Tauri	7 17 2·5	8 23 35·5
	16 Tauri	7 19 5·	7 53 35·
	20 Tauri	7 42 30·5	not observed.
Mar. 4	δ Cancri	6 6 11·8	ditto

The disappearances of 19 and 20 *Tauri* and δ *Cancri* were instantaneous. That of 16 *Tauri* rather sluggish. The reappearances are probably rather late, owing to the brightness of the moon's limb. The time recorded is local sidereal time. Telescope, 4 feet 4 inches focal length; $3\frac{3}{8}$ inches aperture. Power, 128; approximate longitude, $23^{s}{\cdot}8$ W.

Occultations of Stars by the Moon, observed at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

Day of Observation.	Phenomenon.	Moon's Limb.	Mean Solar T. h m s	Observer.
1860, Jan. 4	17 Tauri disap.	Dark	16 6 9·9	T
	19 Tauri disap.	Dark	16 13 6·8	T
	20 Tauri disap.	Dark	16 21 22·9	T

The initial T is that of Mr. Talmage.

Occultation of the Pleiades, 1860, *Feb.* 28.

Phenomenon.	Greenwich Mean Solar Time of Observation. Moon's Limb.	Hour and Minute.	Seconds observed by M.	E.	J. C.
19 Tauri disapp.	Dark	8 46	5·2	...	5·0
16 Tauri disapp.	Dark	8 48	18·7	18·9	19·2
21 Tauri disapp.	Dark	9 7	35·4	...	...
Bessel (8) disapp.	Dark	9 8	59·2	...	...
22 Tauri disapp.	Dark	9 11	29·1	...	...
20 Tauri disapp.	Dark	9 11	31·1	31·3	31·9
16 Tauri reapp.	Bright	9 21	9·5	21·4	13·7
19 Tauri reapp.	Bright	9 51	54·4	46·9	49·6
20 Tauri reapp.	Bright	10 0	29·0	27·5	27·7

Notes by Mr. Main [M.]. Observed with the Sheepshanks' equatoreal. The dark limb was distinctly visible; the disappearances were instantaneous. The times of reappearance of 16 *Tauri* and 19 *Tauri* were estimated, the stars being first seen at a little distance from the limb.

From the result given for the reappearance of 20 *Tauri*, as compared with the observations by E. and J. C., I suspect that the counting of the clock was 1 second in error: the wind was so high, that the beats could with great difficulty be heard.

Notes by Mr. Ellis [E.]. The reappearances of 16 *Tauri* and 19 *Tauri* were considered several seconds late, the star being in both cases a little separated from the limb when first seen. The reappearance of 20 *Tauri* considered good.

Notes by Mr. Carpenter [J. C.]. The disappearance of 20 *Tauri* uncertain; 16 *Tauri* and 19 *Tauri* were separated from the limb at the time noted for their reappearance. The reappearance of 20 *Tauri* considered good.

Star Occultations and Lunar Eclipse of Feb. 6, 1860, *observed at the Hartwell Observatory.* By Norman Pogson, Esq., Director of the Observatory.

(*Communicated by Dr. Lee.*)

			G.M.T. h m s
Disappearance of	π^2 Cancri ……………	Feb. 6	8 41 45·7
,,	18 Leonis ……………	March 5	9 12 28·2
,,	19 Leonis ……………	,,	9 52 45·7
,,	R Leonis (3345 B.A.C.)	,,	10 6 10·8

Lunar Eclipse of Feb. 6.

Shadow advancing.	G.M.T. h m s	Shadow retreating.	G.M.T. h m s
Grimaldi entered	13 7 1	Grimaldi entered	14 51 34
Aristarchus bisected	13 9 55	Grimaldi cleared	14 55 17
Grimaldi covered	13 14 17	Kepler bisected	15 9 37
Bessarian bisected	13 15 6	Bessarian bisected	15 13 14
Kepler ,,	13 17 5	Aristarchus bisected	15 14 14
Plato entered	13 20 50	Copernicus entered	15 16 27
,, covered	13 22 38	,, cleared	15 18 3
Copernicus entered	13 23 37	Pytheas bisected	15 22 15
,, covered	13 25 48	Manilius ,,	15 31 28
Manilius bisected	13 38 15	Menelaus ,,	15 34 38
Posidonius entered	13 40 16	Plato entered	15 37 6
Menelaus bisected	13 41 37	,, cleared	15 38 28
Posidonius covered	13 42 11	Proclus bisected	15 44 57
Proclus bisected	13 55 6	Posidonius ,,	15 45 59
Picard ,,	13 58 39	Picard ,,	15 47 44

Remarks.

Owing to the moon being at the full on the evening of February 6, the disappearance of π^2 *Cancri* was far less satisfactory than the other cases, in each of which the star vanished instantaneously. No projection or any other unusual phenomenon was seen, but the reappearances were too uncertain to be worth recording. The equatoreal was charged with a power of 118 for these observations, and the time carefully determined immediately afterwards by means of the 5-foot transit instrument.

The lunar eclipse was watched throughout with the equatoreal and power 84. The loss of light in the north-east quadrant was very decided about 12^{h} 33^{m}. The deep shade of the penumbra caused the actual shadow to appear so indefinite that neither its first nor last contacts could be observed. The only remarkable feature in the eclipse was the visibility—it might almost be termed the brilliancy—of *Aristarchus*. *Kepler* and other spots were comparatively lost, or at most barely discernible as soon as they became enveloped in the shadow, but not so *Aristarchus*, which evidently shone either by intrinsic or retained illumination. At the time of greatest obscuration the shadow extended southward within about two minutes of *Tycho*.

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The Rev. R. Main, President, in the Chair.

Robert Bellamy Clifton, Esq., St. John's College, Cambridge;
Horace Deighton, Esq., Principal of the Collegiate School, Trinidad;
Benjamin Scott, Esq., Chamberlain of London; and
Lieut. W. L. Searle, H.M. Indian Navy,

were balloted for and duly elected Fellows of the Society.

In a letter to the Foreign Secretary, dated Gotha, 1860, March 7th (written in German), M. Hansen, after expressing his warmest thanks to the Royal Astronomical Society for the Medal awarded to him, and requesting to be furnished with some copies of the President's address, continues as follows:

"The Lunar Tables are a work which has occupied me for several years, partly on account of the great number of the perturbation inequalities, which can only be accurately obtained by successive approximations, and partly because the comparisons with observations which are necessary for the amendment of the elliptic elements are very laborious and take much time. But as I had proposed to myself to work out as well the theory as the Lunar Tables themselves, as completely and accurately as was possible for me, it was necessary, besides the assistance which I was able to dispose of for the mechanical calculations, to occupy myself almost continuously therewith, and even myself to work out a part of the numerical calculations.

"It would have been impossible to accomplish this great work without pecuniary assistance, and I was fortunate enough at the commencement of the undertaking to obtain from the Danish Government an allowance, which, however, ceased long before the completion of the work. A sum for the same purpose was afterwards allowed me by the British Government, and by this means, and the application of what I could myself contribute, it is become possible for me to complete the Lunar Tables. I shall be always grateful to the above-mentioned governments for the assistance afforded me to the completion of the Lunar Tables, but I have not been able to finish them without the expenditure from my own resources of what is for me a considerable sum. It is therefore an especial pleasure for me to see that my Lunar Tables exhibit a very good agreement with observations, and that I may thus consider that I have accomplished not only an important scientific work, but one which will also be of utility for Geography and Navigation.

"I am now occupied in putting together the theoretical calculation of the coefficients of the Perturbations of the Moon, and shall soon have the first part thereof ready for the press."

Remarks on the Controversy respecting the Secular Acceleration of the Moon's Mean Motion. By the Rev. R. Main, President.

In a recent number of the *Astronomische Nachrichten*, namely, No. 1241, is a paper by Dr. Hartwig of Schwerin, entitled *Berechnung der Mondfinsternisse des Almagest mittelst der Hansen'schen Sonnen-und-Mondtafeln*, which deserves attention from its connexion with the disputed question of the amount of the lunar acceleration. The object of the author is to compare the observed times and magnitudes of nineteen ancient eclipses recorded by Ptolemy, with the times and magnitudes of the same derived by computation from Hansen's Tables, with the view of proving the superior accuracy of these tables, and the data which he employs are derived almost exclusively from a prize essay by Zech, published in 1851, entitled *Astronom. Untersuch. uber die Mondfinsternisse des Almagest.*

For the purpose of establishing the superiority of Hansen's tables he compares the data of the Almagest also with the results derived from Damoiseau's Tables, and finds, both for the magnitudes and the times of the eclipses, that the sum of the squares of the differences between theory and observation is considerably smaller for Hansen than for Damoiseau. There is, however, one remarkable circumstance connected with the tabular errors of the times of maxima, which the author has not adverted to, but which deserves considerable attention, namely, that out of thirteen recorded eclipses, the computed times of twelve are smaller than the observed times, or that, on the average, the eclipses happened later than the times given by the tables by about 26^{m}. The difference is rather greater by Damoiseau's Tables, but not considerably so; and we may fairly assume that, if Dr. Hartwig is correct in his deductions, the coefficient of the lunar acceleration employed by Hansen, namely, $12''\cdot12$, is too large by about $2''$.

In connexion with this subject of the lunar acceleration, I am desirous to say a few words with reference to the controversy which has within the last few weeks sprung up between MM. Le Verrier and Delaunay; but only with the view of preventing further misunderstanding with regard to my opinion as expressed in what I have occasionally written on that subject. In the Paris *Cosmos* for the 16th March last, and in that for the 23d March, the editor (the Abbé Moigno), who joins with M. Le Verrier in disputing the accuracy of M. Delaunay's results, makes reference to my opinion, or rather to the reserve with which I have avoided the expression of a definite opinion, on this difficult subject, in a way which, I think, does not fairly represent my meaning; and I feel that it would be unjust towards M. Delaunay not to set myself right in this matter. In the first instance the reference is made to my paper "On the Present State of the Controversy respecting the amount of the Acceleration of the Moon's Mean Motion," printed in the *Monthly Notice* for June 10, 1859 (vol. xix. No. 8). In that paper I endeavoured to sum up as impartially as was possible the difficulties which are met with in accepting the conclusions of either of the contending parties; and I explained that the results only could guide us in our judgment, partly because the theories of Delaunay and Hansen are not yet published, and partly because, if they were before us, the enormous labour which would be necessary in verifying the whole of the complicated analysis, would be probably a complete prohibition of the attempt to do so, even assuming that an analyst skilful enough to attempt such a task were to be found. But in that paper I have distinctly signified the high opinion which I entertain of the labours of M. Delaunay and my expectation that his resulting value of the lunar acceleration, or of that part of it which depends on the diminution of excentricity of the earth's orbit, will ultimately be found to be correct. We have in fact evidence so strong for the truth of this result that nothing but the fact of the discrepancy between it and the results of other eminent analysts, who deduce a value which better satisfies ancient observations, could induce any hesitation in the acceptance of it. The argument stands thus. Mr. Adams, in a paper published in 1853, points out an error which he thinks has been committed by Plana, in the deduction of the value of the acceleration by La Place's method, consisting of the omission of the effect of the variation of the areal velocity. He shows that this variation of areal velocity will produce a secular effect, and will modify the value of the acceleration obtained by La Place without consideration of it. The geometrical conception of the nature of the error committed is exceedingly simple, and I think that most persons competent to judge will on the *primâ facie* evidence agree with him, as to the necessity of accepting the correction which he proposes. Ultimately he carries his approximations to a very high order, and finds that another mathematician pursuing a process totally different not only from his own but from that of all other analysts, a process of the utmost generality, and which does not need at the commencement the special consideration of the variation of areal velocity,* arrives at precisely the same result as himself, and produces a numerical identity of which perhaps there is no similar example in a mass of computations of such complexity and magnitude. Now putting together these circumstances, 1st, the error in La Place's method, clearly enunciated by Mr. Adams, and admitting, as far as I can see, of no reply; 2d, the identity of the result for the acceleration corrected for this error with that deduced by another mathematician by a totally different process, and afterwards tested and verified by Poisson's method of the variation of elements, I contend that we have very great reason to accept the conclusion of these astronomers, and to believe, in spite of the difficulty which arises from the circumstance of Plana's value satisfying better the ancient observations of eclipses, that they have given the correct value of the acceleration depending on the diminution of excentricity of the earth's orbit. Still it is not to be denied that the difficulties that lie in the way of our acceptance of the result are not to be easily passed over, and that it is only as a *dernier ressort* that the necessity of admitting the action of a cause distinct from gravitation should be admitted. I do however protest against the unfavourable judgment which, before the publication of M. Delaunay's great work, it has been attempted to pass upon it on such a ground as this; and, whatever opinion we may entertain of the merits of the dispute between Plana, Hansen, and Pontécoulant on the one side, and Adams and Delaunay on the other, I consider that this prejudgment is in the highest degree to be deprecated as unphilosophical and injurious to science. At the best it is a small charge to make against an unpublished work, that, in a dispute on such a question as the lunar acceleration, the method employed leads to a result which agrees with that of one of the most accomplished of living analysts, and disagrees with the results of others, while at the same time the dispute is quite unnecessary, because we have every ground for hope that the theories both of Hansen and Delaunay will be soon given to the world.

An inference is also attempted to be drawn, unfavourable to Delaunay, from the silence respecting his labours which is observable in my address on presenting the Medal to Professor Hansen. With regard to this point I can only say that no such conclusion ought to have been arrived at; first, because I explained particularly that my object was not to discuss in any detailed way the various lunar theories which have been published, and, secondly, because it concerned me still less to give a prominent place to a disputed point of a speculative rather than a practical character. I have alluded with every possible respect to the labours of M. Delaunay, and I have alluded even to this subject of the lunar acceleration as one of those points which remain in the lunar theory still open for future investigators, but which can be profitably meddled with by none but the *most* skilful analysts. The actual value of the acceleration used does not affect the practical value of Hansen's Tables, so far as the interests of navigation are concerned; but it is, at least, a fortunate circumstance, even if we should be ultimately compelled to accept the value given by Delaunay, that the one employed in the tables as that deduced from the theory of gravitation satisfies ancient as well as modern observations, and therefore on practical grounds gives no cause for exception or dispute. I repeat, again, that nothing which I have said in praise of those tables was intended to detract in the slightest degree from the reputation of M. Delaunay; that the general silence which I maintained with regard to the disputed subject of the lunar acceleration arose from the necessity of reserve in speaking of an unpublished work; and that, finally, I look forward to the publication of his theory and that of M. Hansen with feelings of the most intense interest.

Reply to various Objections which have been brought against his Theory of the Secular Acceleration of the Moon's Mean Motion. By J. C. Adams, M.A., Lowndean Professor of Astronomy.*

If I have hitherto published no reply to the "Observations" of M. de Pontécoulant, contained in the Monthly Notices of July last, it is not because the task presented any difficulty, for the fallacies which pervade M. de Pontécoulant's communication were perfectly evident to me from the very first. I thought that any competent person who chose to look into my memoir "On the Secular Acceleration," and into these observations upon it, might be safely left to form his own judgment on the matter. Again, I had some hopes that M. de Pontécoulant might be led to see and acknowledge the errors into which he had fallen, and with that object in view I sent to him, on more than one occasion, through a friend, communications which appeared to me amply sufficient to expose the fallacies contained not only in his printed "Observations," but also in several private letters which he subsequently wrote upon the subject. I find, however, that M. de Pontécoulant, in a letter which he has lately caused to be circulated among the members of the French Institute, has ventured to ignore these communications of mine altogether, and to speak as if his observations had been admitted without dispute. Under these circumstances, as my further silence might be misconstrued, I beg leave to offer to the Society the following remarks.

In order to give a more complete view of the subject, however, and to obviate the necessity of my returning to it in a controversial manner, I shall not confine myself to the observations of M. de Pontécoulant, but shall likewise say a few words in reply to the objections of M. Plana and those of M. Hansen. I shall also take the opportunity of making some preliminary remarks which may tend to remove certain misapprehensions, which I have reason to believe exist in some minds with respect to the real nature of the matter in dispute.

First, then, I would call attention to the fact that the question is a purely mathematical one, with the decision of which observation has nothing whatever to do. It may be simply stated thus: if the excentricity of the earth's orbit be supposed to change at a given uniform rate and very slowly, what will be the corresponding rate of change, according to the theory of gravitation, in the mean motion of the moon? Now the solution of this question is effected by means of a purely algebraical process, the validity of each step of which admits of being placed beyond all possible doubt.

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* Mr. Adams also has explained that his method is perfectly general, and that the considerations respecting the variability of the areal velocity are introduced by him to show clearly, in a geometrical shape, the nature of the error which had been committed.

* Some additions and corrections by the Author are here made to the paper as printed in the 8vo. edition of the *Monthly Notices*.—Ed.

What conclusion must be drawn, then, supposing that ancient observations should show that the secular variation of the moon's mean motion is different from that which, according to theory, is due to the known change of the excentricity of the earth's orbit?

Why, simply this; that the mean motion of the moon is affected by some other cause or causes, besides the variation of excentricity which has been taken into account. This fact, if established, would be a most interesting one, and might put us on the traces of an important physical discovery. It is not difficult to imagine the existence of causes which may affect the mean motion of the moon, but whether it were so or not, any question respecting the validity of a mathematical process must be decided on mathematical grounds alone, quite independently of the agreement or disagreement of theory and observation.

In the case before us the mathematical question as stated above may be greatly simplified, without its ceasing to involve the point which is in dispute. The values of the secular acceleration given by M. Plana's theory and mine, differ in terms which are independent of the excentricity and inclination of the moon's orbit; consequently in deciding which of the theories is right, we may suppose the excentricity and inclination to vanish.

In the next place I would remark that the error which I attribute to M. Plana's theory on this point is not one of calculation which might require long and complicated numerical processes to be gone through for its correction, but that it is an error of principle, about which a mathematician ought not to have much difficulty in making up his mind. I am therefore inclined entirely to agree with M. de Pontécoulant's opinion, that the prolonged discussion of this subject would not be creditable to science, and indeed, considering the importance of the question, and the length of time which has passed since the publication of my memoir, I cannot but think it strange that any controversy respecting it should still exist at all.

Some persons appear to be under the impression that the contest lies between two values of the secular acceleration, that M. Delaunay and I agree in one value, and that MM. Plana, de Pontécoulant, and Hansen, agree in a larger value; but this is by no means the true state of the case. Between M. Delaunay's result and my own, indeed, there is a perfect agreement. He has carried the approximation much further than I have done, but all of the terms which I have calculated have been confirmed by him. Again, before publishing my memoir in 1853, I had obtained my result by two different methods, and I have since confirmed and extended it by means of a third. M. Delaunay arrived at his result by an independent method of his own, and he has lately found exactly the same result by following the method given by Poisson.

On the other hand, among our opponents there is far from being the same satisfactory agreement.

In his theory of the moon, M. Plana obtained one value of the secular acceleration. In 1856 he printed a paper in which he admitted that his theory was wrong on this point, and actually deduced my result from his own equations. Soon afterwards, however, M. Plana retracted his admission of the correctness of my result, and obtained a third result, differing both from his former one and from my own.

Again, M. de Pontécoulant, in the last communication which I received from him, gives two different values of the secular acceleration, one of which he has obtained by using the time, and the other by using the moon's longitude as the independent variable. Strange to say, however, he does not appear at all startled at obtaining two contradictory values, but seems fully inclined to defend both. Indeed, judging from the last paragraph of his letter in the *Monthly Notices*, he appears to have expected that the results of the two methods would differ from each other. One of the values which M. de Pontécoulant thus obtains agrees with that given in M. Plana's theory, as of course it must do, being found by means of the same principles. But he seems to be quite unaware that this value has been abandoned by M. Plana himself in his last paper above referred to, which is contained in the eighteenth volume of the Turin Memoirs.

M. Hansen's value of the secular acceleration is not given in an analytical form, like those of MM. Plana and de Pontécoulant, and therefore we can only compare the final numerical results. This comparison, which I shall presently give, shows that M. Hansen's value of the acceleration considerably exceeds either of those found by M. Plana.

Here then we find nothing to inspire confidence; certainly nothing like the cumulative testimony which there is in support of M. Delaunay's result and mine.

I may now be permitted to make some remarks on another point. In the introduction to my memoir of 1853, I gave some general reasoning to show that a change in the excentricity of the earth's orbit had a tendency to produce a change in the mean areal velocity of the moon, and that M. Plana was therefore wrong in assuming this velocity to be constant, as in his theory he does. Now this seems to have led some persons to imagine that my analysis in the following part of the memoir depended in some way or other on the validity of the general reasoning which had gone before, and therefore that my conclusions could not be regarded as established with mathematical strictness. But this is quite a mistaken view of the case. I make no assumption respecting the variability of the mean areal velocity. I prove mathematically that this velocity does vary by finding the amount of its variation, and the general reasoning given in the introduction is simply the translation, so to speak, of my analysis into ordinary language, in order to make the nature of my correction to M. Plana's theory, more generally intelligible. It may be remarked too that even if I had started with the assumption that the mean areal velocity was variable, no error could have been caused thereby, for if this velocity had been really constant I should have found its variation equal to zero. In mathematics the terms "constant" and "variable" are not looked upon as opposed to each other, but a constant is regarded as a particular case of a variable quantity.

It may be as well to guard against the idea that the extreme minuteness of the quantities which we have to deal with in this investigation, gives rise to any uncertainty in the result. The present rate of approach of the moon to the earth which accompanies the acceleration of its motion, is less than one inch per annum, but the theory can determine this minute quantity to within, say, a thousandth part of its true amount, just as easily and certainly as if the quantity to be found had been any number of times greater.

I will now proceed briefly to explain the principles which I employ in determining the secular acceleration, and to point out the errors which vitiate the several results of MM. Plana and de Pontécoulant which have been already referred to.

The principle of my method is simply this, viz., that the differential equations must be satisfied, and that quantities which really vary must be treated as variable in all the differentiations and integrations which occur throughout the investigation.

Now if e', the excentricity of the earth's orbit, be variable, the differentiation or integration of any term which involves e' in its coefficient will produce, in addition to the term which would result if e' were constant, another term involving $\frac{de'}{dt}$ in its coefficient, supposing t to be the independent variable.

In consequence of the existence of these supplementary terms, the ordinary expressions for the moon's coordinates when substituted in the differential equations will not satisfy them, but will leave terms multiplied by $\frac{de'}{dt}$ outstanding. In order to destroy these terms, it is necessary to add terms of the same form to the usual expressions for the moon's coordinates. The values of these new terms may, if we please, be easily found by the method of indeterminate coefficients, each of the coefficients being obtained by means of a simple equation.

If n, the moon's mean motion, be variable, the double differentiation of the moon's coordinates will produce in the differential equations, terms involving $\frac{dn}{dt}$ of the same form as those already mentioned which involve $\frac{de'}{dt}$.

Thus the same system of simultaneous simple equations that gives the values of the indeterminate coefficients, determines likewise the value of $\frac{dn}{dt}$, which is what we want to find.

If the moon's longitude v be taken as the independent variable, we must proceed according to the same principles, but there is one additional circumstance to be attended to.

In the former case, since e' is supposed to vary uniformly with the time, $\frac{de'}{dt}$ is considered constant, or $\frac{d^2e'}{dt^2} = 0$. In the latter case the terms which are introduced by the consideration of the variability of e' will involve $\frac{de'}{dv}$ instead of $\frac{de'}{dt}$ as before; and since the moon's motion in longitude is not uniform, the value of $\frac{de'}{dv}$ cannot be considered constant, or $\frac{d^2e'}{dv^2}$ cannot be neglected. To take this into account we must substitute for $\frac{de'}{dv}$ its value $\frac{de'}{dt}\frac{dt}{dv}$, in which $\frac{dt}{dv}$ is a known function of v, and then the remainder of the process will be exactly similar to that before described.

Let us now consider the method followed in M. Plana's theory, and also by M. de Pontécoulant.

In this method the terms above described involving $\frac{de'}{dt}$ are ignored, and consequently the differential equations as developed by these astronomers furnish no materials whatever for determining the value of $\frac{dn}{dt}$. Hence they are forced to supply the lack of data by means of an assumption, which is that one of the so-called constants introduced by integration is absolutely constant.

The value of any one of the constants so employed can be expressed in terms of n, e' and known quantities. If then this so-called constant were really so, we should be able by differentiating this relation to obtain $\frac{dn}{dt}$ in terms of $\frac{de'}{dt}$. But if on the other hand this supposed constant be really variable, we must take its variation into account, in order to obtain the true value of $\frac{dn}{dt}$ in terms of $\frac{de'}{dt}$.

In M. Plana's theory, in which v is taken as the independent variable, the constant so employed is h^2, which is added to complete the integral $2\int r^2 \frac{dR}{dv}\,dv$, in the equation

$$r^4\left(\frac{dv}{dt}\right)^2 = h^2 + 2\int r^2\frac{dR}{dv}\,dv,$$

in which $2\int r^2 \frac{dR}{dv}\,dv$ is supposed to consist of a series of cosines of multiples of v.

The quantity $r^2\frac{dv}{dt}$ is equal to twice the area described in a unit of time, or to twice the areal velocity, so that h^2 is the non-periodic part of the square of twice the areal velocity, the periodic part being supposed developed in cosines of multiples of v.

In M. de Pontécoulant's theory, the constant h is introduced to complete the integral $\int\frac{dR}{dv}\,dt$ in the equation

$$r^2\frac{dv}{dt} = h + \int\frac{dR}{dv}\,dt,$$

in which $\int\frac{dR}{dv}\,dt$ is supposed to consist of a series of cosines of multiples of t.

M. de Pontécoulant's h is not identical with M. Plana's h, but there is a simple relation between these quantities.

M. de Pontécoulant, however, does not employ the constant h in finding the value of the secular acceleration, but another constant $\frac{1}{a}$, which is introduced to complete the integral in the equation

$$\frac{1}{2}\frac{d^2(r^2)}{dt^2} - \frac{1}{r} + \frac{1}{a} = 2\int d'R + r\frac{dR}{dr},$$

all the periodic terms of which are supposed to consist of cosines of multiples of t.

If we neglect the excentricity and inclination of the moon's orbit, and also omit all powers of m above the fourth, the relations between these several constants and the mean motion n will be expressed as follows:

$$\mathrm{h} = n^{-\frac{1}{3}}\left\{1 - \frac{1}{3}m^2 + \frac{719}{576}m^4 + e'^2\left[-\frac{1}{2}m^2 + \frac{2635}{384}m^4\right]\right\},$$

$$h = n^{-\frac{1}{3}}\left\{1 - \frac{1}{3}m^2 + \frac{11}{144}m^4 + e'^2\left[-\frac{1}{2}m^2 - \frac{185}{96}m^4\right]\right\},$$

$$\frac{1}{a} = n^{\frac{2}{3}}\left\{1 + \frac{2}{3}m^2 - \frac{1253}{288}m^4 + e'^2\left[\quad m^2 - \frac{5593}{192}m^4\right]\right\},$$

the sum of the masses of the earth and moon being supposed to be unity.

From these relations we find by differentiation

$$\frac{dn}{n\,dt} = -3\frac{d\mathrm{h}}{\mathrm{h}\,dt} + \frac{d(e'^2)}{dt}\left\{-\frac{3}{2}m^2 + \frac{2187}{128}m^4\right\},$$

$$\frac{dn}{n\,dt} = -3\frac{dh}{h\,dt} + \frac{d(e'^2)}{dt}\left\{-\frac{3}{2}m^2 - \frac{297}{32}m^4\right\},$$

$$\frac{dn}{n\,dt} = -\frac{3}{2}\frac{da}{a\,dt} + \frac{d(e'^2)}{dt}\left\{-\frac{3}{2}m^2 + \frac{5337}{128}m^4\right\},$$

having taken care to observe that, since $m = \frac{n'}{n}$ and n' is constant, we have $\frac{dm}{m\,dt} = -\frac{dn}{n\,dt}$.

If $\frac{d\mathrm{h}}{\mathrm{h}\,dt}$ be neglected in the first of these expressions, we obtain the value of $\frac{dn}{n\,dt}$ found in M. Plana's theory, and one of those found by M. de Pontécoulant. If $\frac{dh}{h\,dt}$ be neglected in the second, the resulting value of $\frac{dn}{n\,dt}$ is what would have been found by M. de Pontécoulant, if he had taken his own h to be constant instead of M. Plana's h.

If in the third expression $\frac{d\,a}{a\,d\,t}$ be neglected, we obtain the value of $\frac{d\,n}{n\,d\,t}$ which M. de Pontécoulant communicated to me as the result which he had found by using t as the independent variable.

It is obvious that these several values of $\frac{d\,n}{n\,d\,t}$ contradict each other, and the reason is that the quantities h, h, and a are really variable, and that therefore $\frac{d\,\mathrm{h}}{\mathrm{h}\,d\,t}$, $\frac{d\,h}{h\,d\,t}$, and $\frac{d\,a}{a\,d\,t}$ have been wrongly neglected. In order to find the true value of $\frac{d\,n}{n\,d\,t}$ we must therefore determine the values of these last-mentioned differential coefficients, and substitute them in the several expressions for $\frac{d\,n}{n\,d\,t}$ given above.

Now the supplementary terms involving $\frac{d\,e'}{d\,t}$ which I have shown to exist in the expressions for the moon's coordinates, will introduce into the integral

$$2\int r^2 \frac{d\,\mathrm{R}}{d\,v}\,d\,v,$$

besides periodic terms, a non-periodic one of the form

$$\int \mathrm{H}\,\frac{d\,(e'^2)}{d\,t}\,d\,t, \quad \text{or } \mathrm{H}\,e'^2,$$

consequently, since in the equation

$$r^4\left(\frac{d\,v}{d\,t}\right)^2 = \mathrm{h}^2 + 2\int r^2 \frac{d\,\mathrm{R}}{d\,v}\,d\,v,$$

M. Plana considers h^2 to denote the whole of the non-periodic part of $r^4\left(\frac{d\,v}{d\,t}\right)^2$, h^2 must consist of an absolutely constant part together with the variable quantity $\mathrm{H}\,e'^2$ just mentioned

$$\text{and } \therefore \ \frac{d\,(\mathrm{h}^2)}{d\,t} \text{ must be equal to } \mathrm{H}\,\frac{d\,(e'^2)}{d\,t}.$$

Similarly $\frac{d\,h}{d\,t}$ may be found by determining the non-periodic term which is in the same way introduced into the integral

$$\int \frac{d\,\mathrm{R}}{d\,v}\,d\,t$$

in the equation

$$r^2\,\frac{d\,v}{d\,t} = h + \int \frac{d\,\mathrm{R}}{d\,v}\,d\,t;$$

and $\frac{d\left(\frac{1}{a}\right)}{d\,t}$ may be similarly found by means of the non-periodic terms introduced into the integral $\int d'\mathrm{R}$, in the equation

$$\frac{1}{2}\,\frac{d^2\,(r^2)}{d\,t^2} - \frac{1}{r} + \frac{1}{a} = 2\int d'\mathrm{R} + r\,\frac{d\,\mathrm{R}}{d\,r}.$$

When all this has been done, and the proper substitutions made, the three expressions for $\frac{d\,n}{n\,d\,t}$ are found to agree in giving

$$\frac{d\,n}{n\,d\,t} = \frac{d\,(e'^2)}{d\,t}\left\{-\frac{3}{2}\,m^2 + \frac{3771}{64}\,m^4\right\},$$

which is the result obtained by M. Delaunay and myself.

The supplementary terms in the moon's coordinates which involve $\frac{d\,e'}{d\,t}$ are of the order of the disturbing force, and therefore the terms which they introduce into the integrals.

$$\int r^2\frac{d\,\mathrm{R}}{d\,v}\,d\,v, \quad \int \frac{d\,\mathrm{R}}{d\,v}\,d\,t, \quad \text{and} \quad \int d'\mathrm{R},$$

will be the order of the square of the disturbing force

This is the reason why $\frac{d\,\mathrm{h}}{\mathrm{h}\,d\,t}$, $\frac{d\,h}{h\,d\,t}$ and $\frac{d\,a}{a\,d\,t}$ are all of the order m^4.

It may be well to mention, in order to prevent any misapprehension, that in my memoir of 1853, h has not the same signification as the h of M. Plana's theory.

It is proved in Art. 11 of the Memoir that

$$2\int r^2\frac{d\,\mathrm{R}}{d\,v}\,d\,v$$

contains the non-periodic terms

$$\mathrm{h}^2\left\{-\frac{285}{8}\,m^4\,e'^2 + \frac{495}{64}\,m^4\,e'^2\right\},$$

$$= \mathrm{h}^2\left\{-\frac{1785}{64}\,m^4\,e'^2\right\}$$

and the h^2 employed in the memoir is the absolutely constant quantity added to complete the integral, so that if for the sake of distinction $h_0{}^2$ be written for the h^2 of the memoir, we shall have

$$\mathrm{h}^2 = h_0{}^2 + \mathrm{h}^2\left\{-\frac{1785}{64}\,m^4\,e'^2\right\}$$

$$\text{or } \mathrm{h}^2 = h_0{}^2\left\{1 - \frac{1785}{64}\,m^4\,e'^2\right\}$$

The following relation exists between the h of M. Plana and the h of M. de Pontécoulant:—

$$\frac{\mathrm{h}}{h} = 1 + \frac{75}{64}\,m^4 + e'^2\left[\frac{1125}{128}\,m^4\right].$$

Now this relation at once shows that if e' be variable, h and h cannot both be constant; and since no à-priori reason can be given why one of these quantities should be constant rather than the other, we are not justified in assuming that either of them is so.

This argument, however, does not appear convincing to M. de Pontécoulant.

In the two methods which, as I mentioned before, I employed previously to the publication of my Memoir of 1853, the value of $\frac{d\,n}{n\,d\,t}$ was deduced from those of $\frac{d\,\mathrm{h}}{\mathrm{h}\,d\,t}$ and $\frac{d\,h}{h\,d\,t}$ respectively. In the method which I now employ, $\frac{d\,n}{n\,d\,t}$ is determined by direct substitution in the differential equations, without introducing either the quantity h or h, that is, without taking into consideration the mean areal velocity at all.

In M. Plana's Memoir, contained in the eighteenth volume of the Turin Memoirs, he no longer maintains the constancy of his quantity h, but he determines its variation incorrectly, only taking into account part of the terms which produce this variation. M. Plana here recognises the reality of the supplementary terms involving $\frac{de'}{dt}$, which I have proved to exist in the expressions for the moon's co-ordinates; and he finds values for δu and $\delta n t$ in pp. 14 and 20 of the Memoir, which coincide with mine, except in the terms with the argument $c' m v$, in which a mistake occurs in his coefficients, which, however, does not affect the coefficient of m^4 in the expression for the secular acceleration. It is very remarkable, however, that although he finds these values of δu and $\delta n t$, he does not substitute them in his equations, but puts $\delta u = 0$ and $\delta n t = 0$ instead of them. It is only by this strange process of suppressing part of the results which he himself has found, that M. Plana arrives at a different value of the secular acceleration from mine. Indeed, in the first form of this Memoir, as I have already mentioned, M. Plana did actually obtain a value coincident with mine.

M. Plana is led to make this suppression of his own results by a supposed à-priori proof that a certain integral which is equivalent to $2\int r^2 \frac{dR}{dv} dv$ can contain no such terms as those which would arise from the substitution in it of the true values of δu and $\delta n t$. Now, even if this proof had been ever so convincing, M. Plana was surely bound to show in what manner the terms thus arising from δu and $\delta n t$ were destroyed, as the different parts of his investigation would otherwise contradict each other.

In fact, however, this proof is entirely fallacious, for it rests on the assumption made at the top of p. 43 of the Memoir, that the terms multiplied by p, p^2, &c., in the equation given on the preceding page, may be neglected; and these are precisely the terms which are equivalent to those which M. Plana suppresses.

It may be as well to make another remark on this part of the investigation. In p. 42, M. Plana puts

$$e'^g \cos g\tau = \Sigma M \cos (pv + q),$$
$$e'^g \sin g\tau = \Sigma M \sin (pv + q),$$

and he assumes that all the coefficients p will be small quantities. But this will not be the case when $e'^g \cos g\tau$ and $e'^g \sin g\tau$ are thus expressed in terms of the moon's longitude. If these functions were similarly expressed in terms of the time, viz., if we were to put

$$e'^g \cos g\tau = \Sigma M \cos (pt + q),$$
$$e'^g \sin g\tau = \Sigma M \sin (pt + q),$$

all the coefficients p would be small.

The result which M. Plana obtains in this Memoir is

$$\frac{dn}{n\,dt} = \frac{d(e'^2)}{dt}\left\{-\frac{3}{2}m^2 + \frac{351}{64}\right\}m^4,$$

and the difference between this result and mine arises in the way I have explained, viz., from his having neglected to take into account the term

$$h^2\left\{-\frac{285}{8}m^4 e'^2\right\}$$

which is shown in Art. 11 of my Memoir to constitute part of the non-periodic term of $2\int r^2 \frac{dR}{dv} dv$.

M. Hansen's value of the secular acceleration is not exhibited in an analytical form, like those of MM. Plana and de Pontécoulant, and we can therefore only compare his numerical result with theirs. These differ considerably, and, in fact, much more than appears at first sight, on account of a reason which I will explain.

If we put

$$\frac{dn}{n\,dt} = K\frac{d(e'^2)}{dt},$$

where K is the coefficient found from theory, the secular equation to be applied to the mean longitude will be

$$K\int(e'^2 - E'^2)\,n\,dt,$$

E′ being the excentricity of the earth's orbit at the epoch from which t is reckoned.

Now I find that M. Hansen uses a smaller value of the integral $\int(e'^2 - E'^2)\,n\,dt$ than M. Plana does; that is, he supposes a slower change in the excentricity of the earth's orbit: and yet his resulting value of the secular equation is larger than those of M. Plana.

It may be inferred, either from the data in the Introduction to M. Hansen's Solar Tables, or from other data in the Introduction to his Lunar Tables, that the value of the integral

$$\int(e'^2 - E'^2)\,n\,dt$$

which he employs is $-1212''\cdot5\,t^2$, t being expressed, as usual, in centuries.

Now M. Plana, in his Theory of the Moon, supposes the value of the above integral to be $-1264''\cdot1\,t^2$, and in his Memoir in vol. xviii. of the Turin Memoirs he gives it the value $-1297''\cdot7\,t^2$.

If, then, we reduce the coefficients of the secular equation given by these authors, so as to make them correspond with the value $-1270''\,t^2$ of the above integral, which is that employed in my Memoir of 1853, they will become

Coefficient according to		M. Plana's theory	10″·60,
,,	,,	M. Plana's memoir (1856)	11·24,
,,	,,	M. Hansen's theory	12·76.

The difference between M. Hansen's coefficient and either of M. Plana's is much greater than could possibly have arisen if both values had been found on correct principles, and they had differed merely in consequence of the approximations not being carried far enough.

My value of the same coefficient, which was communicated to the French Institute in January, 1859, is 5″·70. And M. Delaunay, while perfectly agreeing with me in the terms which I have calculated, has added a great number of others depending on the excentricity and inclination of the moon's orbit, and thus increases the coefficient to 6″·11.

As M. Hansen's method of obtaining his coefficient has not yet appeared, it is, of course, impossible for me to point out the reason of the difference between it and my own, as I have done in reference to the results of MM. Plana and de Pontécoulant. I have very little doubt, however, that it arises from M. Hansen having tacitly assumed, like M. Plana, that one of his constants introduced by integration is an absolutely constant quantity.

M. Hansen has suggested that the difference between his result and that obtained by M. Delaunay and myself may arise from want of convergency in the series proceeding according to powers of m, by means of which we determine the coefficient denoted above by K.

If we confine our attention to the terms of K which are independent of the excentricity and inclination of the moon's orbit, and which are admitted by all to constitute by far the largest part of that quantity, we find that the terms involving the successive powers of *m* taken into account by me give rise to the following parts of the coefficient of the secular equation:—

m^2	$10''\cdot66,$
m^4	$-2\cdot34,$
m^5	$-1\cdot58,$
m^6	$-0\cdot71,$
m^7	$-0\cdot25.$

The sum of these is $5''\cdot78$. The convergence, although slow at starting, becomes more rapid in the later terms; and I inferred, in my communication to the French Institute above mentioned, that the remainder of the series would be very nearly equal to $-0''\cdot08$.

Now M. Delaunay has since calculated the next term of the series, and finds it $=-0''\cdot06$, which is in exact accordance with my anticipations.

Although I think that there can remain no doubt with respect to the convergency of the series, yet, in order to remove all possible objection, I have calculated the value of K by a method which does not require any expansion in powers of *m*, and the resulting coefficient of the secular equation is

$$5''\cdot70,$$

exactly agreeing with that found by means of the series of powers of *m*.

A very few words will now suffice in reply to the objections which M. de Pontécoulant brings forward in his observations in the Monthly Notices. In fact, almost all of them have been virtually answered in what I have said before.

At the outset of his paper, M. de Pontécoulant rightly describes the difference between my method of finding the secular acceleration and all preceding ones, as arising from the consideration of the variability of the excentricity of the earth's orbit in the differential equations of the moon's motion, in which this element had hitherto been considered as constant. He then refers to the statement in my Memoir, that when this consideration was introduced into the formulæ, I found exactly the same result whether the time or the moon's longitude was taken as the independent variable. But, adds M. de Pontécoulant, "il n'y a qu'une petite difficulté dans cette assertion, c'est quelle énonce un fait mathématiquement *inadmissible*."

Now I confess that I cannot see M. de Pontécoulant's "petite difficulté." I am far from looking upon the agreement between the results of different methods as a fact mathematically inadmissible. On the contrary, it appears to me a palpable absurdity to suppose that the result of a mathematical investigation can be different according as one independent variable or another is employed in obtaining it, or that two methods of solving the same problem may both be correct and yet lead to contradictory results.

In order, however, to show this mathematical inadmissibility, M. de Pontécoulant goes on to say, "En effet, M. Adams convient quelque part, je crois, et d'ailleurs je le démontrerais bientôt jusqu'à l'évidence, que la considération de la variabilité de l'orbe terrestre, n'exerce aucune influence sur la détermination de l'inégalité séculaire, lorsqu'on emploie pour l'obtenir les formules directes que j'ai adoptées dans ma théorie."

In thus stating that I admit that one of the methods of determining the secular acceleration is unaffected by the consideration of the variability of the excentricity of the earth's orbit, M. de Pontécoulant overlooks "une petite difficulté," viz., that instead of admitting this, I assert, in so many words, the exact contrary. In the concluding sentence of my memoir I say, "when both processes were corrected by taking into account the new terms whose existence I had already recognised, I had the satisfaction of finding a perfect agreement between the results."

For M. de Pontécoulant's demonstration "jusqu' à l'évidence," I am not responsible, and indeed, I think his paper tends to show that he has peculiar ideas as to what constitutes demonstration.

In the next place M. de Pontécoulant offers "une réflexion très simple," which he thinks ought to have struck me. "Qui est-ce après tout que le coefficient de l'équation séculaire?—une certaine fonction des élements des orbites de l'astre troublé et de l'astre perturbateur, qui se déduit des formules différentielles du mouvement; cette fonction est la même, selon M. Adams, par quelque méthode qu'on l'obtienne, dans le cas ou l'on considère comme variable l'excentricité de l'orbe terrestre; à plus forte raison elle doit l'être dans le cas où l'on régarde cette excentricité comme constante." I am at a loss to imagine what can be the meaning of this last clause, since the secular equation in question is entirely due to the variability of the excentricity of the earth's orbit, and would not exist at all if this excentricity were constant.

It must be admitted that my new determination of the secular acceleration has, as M. de Pontécoulant says, "l'inconvénient d'altérer profondément l'expression analytique admise jusqu'à présent, du coefficient de cette équation," but truth must not be sacrificed to convenience.

In the algebraical portion of his paper, M. de Pontécoulant is not happier than in his introductory remarks. Indeed, throughout the paper he expressly leaves out of consideration all the terms which give rise to the difference between M. Plana's result and mine.

Thus, at the bottom of p. 311, having found from an assumed term in $\frac{d\mathrm{R}}{d\nu}$, that

$$\int \frac{d\mathrm{R}}{d\nu}\,dt = -\frac{\mathrm{A}}{f}\,e'\cos(ft+l) + \frac{\mathrm{A}}{f}\,\frac{de'}{dt}\sin(ft+l),$$

he incorporates the term involving $\frac{de'}{dt}$ with the preceding under the form

$$-\frac{\mathrm{A}}{f}\,e'\cos\left(ft+l+\frac{de'}{e'\,dt}\right),$$

and then remarks:—

"On voit donc que la considération de la variation de l'excentricité de l'orbite terrestre ne fait qu' altérer d'une manière insensible la partie constante des angles des diverses inégalités lunaires multipliées par *e'*, elle ne change en rien la forme des séries qui déterminent les co-ordonnées du mouvement troublé . . ."

Now these alterations of the constant part of the angles on which the several lunar inequalities depend, which are neglected as insensible by M. de Pontécoulant, actually give rise to the terms in the moon's co-ordinates involving $\frac{de'}{dt}$, which I have been the first to take into account, and thus do change the form of the expressions for those co-ordinates.

The term $\frac{\mathrm{A}}{f}\,\frac{de'}{dt}\sin(ft+l)$ is not destroyed by being incorporated with the preceding term $-\frac{\mathrm{A}}{f}\,e'\cos(ft+l)$, as M. de Pontécoulant seems to suppose.

Again, in order to show that the integral $\int\frac{d\mathrm{R}}{d\nu}\,dt$ can contain no non-periodic term depending on *e'*, M. de Pontécoulant assumes, at the foot of p. 310, that $\frac{d\mathrm{R}}{d\nu}$ is made up of terms of the form

$$\mathrm{A}\,e'\sin(ft+l).$$

But $\frac{dR}{d\nu}$ is a function of r and ν; and since these quantities contain terms depending on the disturbing force and multiplied by $\frac{de'}{dt}$, $\frac{dR}{d\nu}$ will contain, in addition to the terms of the form considered by M. de Pontécoulant, other terms of the order of the square of the disturbing force, and of the form

$$B \frac{de'}{dt} \cos(ft + l);$$

among these there will be a term in which the angle $ft + l$ vanishes; viz., one of the form

$$C e' \frac{de'}{dt},$$

and consequently $\int \frac{dR}{d\nu} dt$ will contain the non-periodic term $\frac{1}{2} C e'^2$.

M. de Pontécoulant characterises the process which I have employed at the bottom of p. 402 in my Memoir, in order to find the non-periodic parts of certain integrals, as "une véritable *supercherie analytique.*" Now this "supercherie" only consists in taking account of the variability of $\frac{de'}{d\nu}$, by putting for it the identical quantity $\frac{de'}{dt} \cdot \frac{dt}{d\nu}$.

M. Plana, in equation [10], p. 12, of his Memoir, finds, for the terms thus objected to by M. de Pontécoulant, exactly the same values as I have done, though his process entirely differs from mine.

On this same point, in a note to p. 315, M. de Pontécoulant makes the objection that in the last step of the integrations referred to I make $d\nu = n\,dt$, contrary to the supposition I had previously employed. But my object was simply to find the non-periodic parts of the integrals concerned; and it is obvious that if I had put for $d\nu$ its complete value $n\,dt - \varphi(\nu)\,d\nu$, where $\varphi(\nu)$ is a periodic function of ν, this function would only introduce periodic terms into the integrals, and would cause no change whatever in the terms which I have found.

But one of the most remarkable objections in the whole course of M. de Pontécoulant's communication occurs in p. 316, where he says he is going to put his finger on the error I have committed. From an equation in my Memoir he deduces the following:—

$$e' = q + q'\left\{\nu - \frac{11}{8} m^2 \sin(2\nu - 2m\nu) - \frac{77}{16} m^2 e' \sin^2(2\nu - 2m\nu - c'm\nu) + \&c.\right\}$$

and then adds the remark,—

"C'est à dire, que l'excentricité de l'orbite terrestre, outre sa variation séculaire, serait soumise à toutes les inégalités du mouvement lunaire; c'est à dire, à des variations dont le période serait d'un mois, d'une année, &c. ce qui est contraire, quelque petitesse qu'on suppose au coefficient q', à tous les principes de la théorie."

Now it is astonishing that M. de Pontécoulant does not see that the quantity enclosed within brackets, in the above equation, is simply the expression of the moon's mean longitude nt in terms of the true longitude ν, so that the equation is equivalent to

$$e' = q + q'nt;$$

that is, the excentricity of the earth's orbit is made to vary uniformly with the time, which agrees with the supposition with which we started.

On the other hand, M. de Pontécoulant, by making

$$e' = q + q'\nu,$$

that is, by supposing the change in e' to be proportional to the moon's true motion in longitude, would evidently cause the excentricity of the earth's orbit to be affected by all the inequalities of the lunar motion.

All attempts to express e' in terms of ν, without introducing periodic terms, lead to this absurdity.

I have already alluded to the strange notion expressed at the end of M. de Pontécoulant's paper, that there may be two values of the secular acceleration, one applicable to the true longitude and the other to the mean longitude. The difference between the true and the mean longitudes consists wholly of periodic quantities, and cannot contain any term increasing continually with the time.

How M. de Pontécoulant could have so far deceived himself as to imagine that this paper settled the question of the secular acceleration, "sans contestation possible désormais," is, I confess, beyond my comprehension.

* * *

A short Note on the same subject has been received from M. Delaunay, which is as to the points therein adverted to in perfect agreement with the paper by Prof. Adams: this Note has already appeared in the *Comptes Rendus.* M. Delaunay remarks, first, that the argumentation of M. de Pontécoulant in a passage where he says that he neglects for greater simplicity the terms of an order superior to m^2, only applies to the quantities not affected by the correction indicated by Prof. Adams; secondly, that M. de Pontécoulant's expression for e' in terms of the true longitude of the moon (by which he points out with his finger the error fallen into by Prof. Adams) in that it contains periodic terms the arguments of which depend on the true longitude, is really an expression varying uniformly with the time.

* * *

M. de Pontécoulant, in a letter dated Paris, 19 April, 1860, addressed to the President, maintains the correctness of his results, which he states are verified by the calculation which he has made of the coefficient of the secular inequality directly from the differential equations with the time as the independent variable, and he refers to a Note (which, as appears by the postscript, is to be shortly sent) indicating the result of his researches, and to a Memoir which he is about to print containing the details of the calculation, and forming an appendix to Chap. 7 of his "Theory of the Moon."

* * *

Extract of a Letter, dated Feb. 14, from Lieut. Gilliss, of Washington, to Mr. Carrington, read at the Meeting of the Society.

"There seemed no probability of preparations to observe the July eclipse on this continent, until it was shown to Professor Bache how easily he could cause it to be done on our west coast. In consequence thereof, he has determined to despatch a party to the mouth of the Columbia river [129° W. 46° 20′ N.],* from whence its members will proceed to the central line of the shadow, some 10′ or 15′ further north. The duty will be assigned to Mr. Davidson, to whom my eldest son is assistant. The latter embarked for California (*viâ* Panama) on the 6th, having been duly instructed in his subordinate part.

"I hope to be able to get up a party for Cape Chudleigh [65° W., 60° N.], as the most appropriate place for us on this side the water to occupy, and have only been deterred from pressing it during the present month; firstly, from bad health; and secondly, because of doubt whether the land will be free enough of ice to permit a landing so early in the season. This doubt will be settled in a day or two; and if favourably, I trust the journey may effectually drive out the lingering remains of the Peruvian and Isthmus attacks in 1858. I have been almost unfitted for work ever since.

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* The longitudes and latitudes in brackets are not in the original, but are added for convenience.—R.C.C.

"Do you know whether there are any instruments at York Factory and persons capable of using them properly? It will be easy to ascertain at the office of the Hudson's Bay Company, and your countrymen are too liberally inclined towards science to deny the Astronomical Society any service its officers can render. Then, with Bache's party near Astoria, the Hudson's Bay force near York Factory [91° 8′ W., 56° 52′ N.], my own at Cape Chudleigh, yourselves near Santander, and the French on the N.E. coast of Spain and in Algeria, an amount of information may be obtained, far exceeding that derived from all previous eclipses. May the day prove auspicious for all!"

The above intelligence will be highly acceptable to this Society. and meet with its hearty wish for the success of so much astronomical enterprise.

The hint thrown out by Lieut. Gilliss has not been lost on Mr. Carrington, who put himself in immediate communication with the Secretary of the Hudson's Bay Company. He learns that, although probably no instruments are at York Factory fit for the observation desired, the British Commissioner will leave Montreal for his annual expedition early in April, and will, as far as may be in his power, endeavour to have carried into effect any simple instructions which may be supplied him for the direction of intelligent but not astronomical persons, in the Company's service. Mr. Hind has kindly supplied accurate data for the American continent, from which it appears that Fort York is a little within the north limit of totality, and Fort Severn very near the south boundary of the shadow. A landing at some intermediate point may probably be effected, and the wishes of Lieut. Gilliss more or less carried out. As the journey to Fort York is partly effected by boat and is tedious, it is very possible that Sir G. Simpson may select one of the trading stations on the Saskatschewan in preference. The selection of the best station can only be made by an officer of the Company acquainted with the relative capabilities of the subordinates at the different points.

The fifteenth volume, Part I. of the *Beobachtungen der Kaiserlichen Universitäts Sternwarte Dorpat*, which has recently appeared, contains an article by Dr. Mädler, on the Solar Eclipse of the 18th July next, with the circumstances of the eclipse calculated for 112 places in the Peninsula and the neighbouring countries, and illustrated by a map, and a plate of the position of the heavens during the eclipse, for the horizon of Saragossa. The Campvey of the Isle of Iviça (which is nearly on the central line) is indicated as an elevated situation likely to afford a magnificent spectacle of the passage of the shadow and the general phenomena of the eclipse.

The current volume of the *Atti della Accademia de' Nuovi Lincei* (January 1860) contains an article by P. Secchi, "Intorno all' Atmosfera Solare e ad alcune proprietà ottiche della Luce reflessa dalla Luna" (8 pages). After mentioning some experiments in support of the existence of a solar atmosphere, the author remarks that even if the existence of such atmosphere be proved, this will not completely account for the *corona* surrounding the sun during a total eclipse, and that it consequently occurred to him to inquire whether the moon had any part therein, and that he was thus led to study the modifications undergone by light reflected by the moon — the most important of these being the polarisation — which subject is discussed in the remainder of the paper. Some account of these researches is given in P. Secchi's letter in the January Number of the *Monthly Notices;* but the observations and reasonings are given in greater detail.

On the Appearance of Jupiter.

(Extract of a Letter by J. W. Long, Esq. to one of the Secretaries.)

"I send a copy of some views of *Jupiter*, lately taken, which I promised to forward to you when I last saw you in London.

"The point of most interest is the oblique marking, which you will notice has considerably altered in position and form since first seen. The colour of these markings is also interesting. The belts on the upper or south half of the planet are of a variety of yellowish red, while those in the lower or north half are of a bluish black, particularly the oblique marking."

In the annexed Plates, figures 1, 2, 3, which are the views referred to in Mr. Long's letter, are, by the permission of Messrs. Long and Baxendell, reproduced from the Memoirs of the Literary and Philosophical Society of Manchester. Figure 4 is taken from a drawing which accompanied Captain Jacob's notice of Mr. Fletcher's new equatoreal, (*post*, p. 247) and figure 5 from one of two drawings subsequently communicated by Mr. Baxendell, who, in a letter to the Editor, dated April 21, 1860, writes as follows: —

"With reference to these sketches, I may remark that since Mr. Long first observed the oblique streak on the 29th February, it has gradually extended itself in a preceding direction, or in the direction of the planet's rotation, at an average rate of 3640 miles per day, or 151 miles per hour; the two extremities of the belt remaining constantly on the same parallels of latitude. The belt has also gradually become darker and broader, some portions, however, being generally much darker than others. On the 14th of March I observed *Jupiter* with Mr. Worthington's 13-inch reflector, power 301, and noticed a faint, curved, dark mark, extending across the bright equatoreal belt from the upper end of the oblique belt to the small dark spot shown in the preceding part of the large belt in my sketch of March 5th. Perhaps I may here be allowed to remark that a transit of the third satellite occurred on the same night (March 14), when it appeared as a *dark* spot on the disk, and was estimated to be three or four times darker than the darkest part of the large belt; but as it approached the limb of the planet its darkness rapidly diminished, and at the moment of internal contact the satellite appeared as a *bright* spot. It was on the bright belt next above the large dark belt. Similar appearances were also presented during a transit of the same satellite on the 21st March. At the ingress the satellite appeared as a bright spot, and at least fully as bright as the brightest part of the planet's disk; but when it had advanced about one-third of its course across the disk it appeared as a large *dark* spot at least *three* times darker than the darkest part of the large belt. These observations had been made before I was aware of Mr. Prince's interesting observation of January 31st, reported at page 212 of the last number of the *Monthly Notices*.

"To return to the oblique belt. On March 21st the spot at the lower end of the belt had increased considerably in size and depth of shade, and appeared to consist of two spots in contact. The sketch of April 9 shows the belt, now considerably lengthened, extending completely across the planet's disk. The sketch of April 20 shows the preceding end of the belt in the middle of its path across the disk. This end is now very dark, and was estimated last night to be two or three times darker than the darkest part of the large belt. Its colour was also remarkable, being bluish-black when contrasted with the dull yellowish red colour of the large belt."

Remarks on the Appearance of Jupiter.

By the Astronomer Royal.

On viewing *Jupiter* with the S.E. equatoreal of the Royal Observatory, Greenwich, on 1860, March 26, about $9^h 30^m$, the following appearances were remarked:—

1. Near the apparent lower or north pole there were insignificant belts, as usual.
2. A little below the equator there was a broad discontinuous belt, consisting of about six unconnected light clouds, each of a rude circular form (with great irregularities); reminding me of one of Professor Piazzi Smyth's drawings of *Jupiter*, as seen on the Peak of Teneriffe.
3. In the region above the equator there was a double belt, inclined perhaps 15° to the equator and to the belt which I have just described. Its right hand or eastern end was nearly on the equator, but its left hand or west end was considerably above it or south of the equator. The left-hand ends were broader than the right-hand ends. The outlines irregular. The colour had a most decided reddish tinge; a pale brick-red.
4. Above this, three or four belts parallel to the equator, stronger than those of the lower part.

On the Appearance of Jupiter's Satellites while transiting the Disk of the Planet.

By the Rev. W. R. Dawes.

That some of the satellites of *Jupiter* while passing over the planet's disk appear as dark spots, is noticed by Sir John Herschel (see *Outlines of Astronomy*, Art. 540) as having been first observed by Schröter and Harding; probably, therefore, some thirty or forty years ago: but it is not stated where their observations of the phenomenon were published. For more than twenty years the fact has been, I believe, very generally known in this country; and it has been remarked that the third and fourth satellites sometimes appear so dark as at first sight to be mistaken for their shadows. From my own observations it seems that the shade of darkness in the same satellite varies considerably, being greatly influenced by the degree of *contrast* with the planet's surface, as the satellite may happen to transit along a bright zone or a dark belt; and also by the condition of the sky, which when at all hazy causes a diffusion of light over the dark spot of the satellite. I have noticed the *fourth* satellite to be always much the darkest, and though the dark spot is never quite round, yet that it is more nearly so than the third ever is. And this agrees with the appearance of the disk of the satellite when off the face of the planet, as examined under a fine sky and with high powers. I have occasionally noticed that portions of this satellite near its edge are scarcely visible and almost appear to be wanting, from their small reflective power; giving the disk of the satellite the form depicted in the accompanying sketch, copied from one made in my observatory journal at the time. (Fig. 1.)[*]

On 1843, Dec. 8, I observed the transit of the *third* satellite with Mr. Bishop's 7-inch refractor under fine circumstances, and in my journal have noted that it appeared as a *black* spot, which when approaching the western edge very rapidly changed to a bright one. The power used was only 108, and no irregularity of figure in the black spot was noted. But on 1849, Feb. 11, under a fine sky, with powers 323 and 460 on my 6⅓-inch Munich refractor, I observed the irregular form of the dark part of the satellite, and made a sketch of its appearance, of which an enlarged copy accompanies this paper. (Fig. 2.)

On the 31st of last January I observed, under favourable circumstances, the transit of the third satellite. My 8¼-inch refractor bore well a power of 530, and the shape of the dark portion of the satellite was satisfactorily shown as in the accompanying sketch. (Fig. 3.) A streak of light entered the dark part from the south side, and the north and north-east parts were decidedly the darkest. At $11^h 47^m$ G.M.T. the satellite was approaching the western limb, and had begun to change its appearance. The *southern edge* was then *rather brighter* than the part of the planet on which it was projected, and appeared as an *irregular bright crescent;* while the centre, and still more decidedly the northern part, remained *darker* than the planet. And when the satellite was so near the edge of the planet that the whole of it appeared brighter than that part of *Jupiter*, the form of the dusky region was still visible, and continued to be so after the satellite had moved off the disk. On the 24th of the same month, with Alvan Clark's 8-inch object-glass I obtained excellent observations of this satellite, both on and off the planet, with powers 300 and 400; and the sketch I then made of the dark portion agrees so nearly with the sketches on the 31st, and on 1849, Feb. 11, as to render it probable that the same aspect of the satellite was turned towards the earth on all those occasions.

The *first* satellite sometimes appears as an irregular gray spot, but never so dark by many shades as the third usually is. I have however been frequently unable to see it at all during the central half of its transit.

The *second* satellite has always seemed to me to be the brightest of the four *for its size;* and I have never been able to detect the slightest shading upon its disk during its transits over the planet, or at any other time.

It would seem to be quite beyond all reasonable expectation that so bright a satellite as the third looks when off the disk, and even when not far within it, should be so much less bright than the darkest parts of the darkest belts upon the planet; and also that when it reaches a certain part of the disk, its change from a dark spot to a bright one, or from a bright to a dark one, should be so rapid. Yet no more delicate photometer could be applied to the different parts of the planet's surface. If however the appearances are merely the effect of *contrast* between the brightness of the satellite and the less or greater brightness of different portions of the planet, it seems difficult to account for the fact, that if *two images* are produced by a double-image micrometer, and a satellite in one is brought upon the planet in the other, the satellite appears as a *bright* spot on whatever part of the planet it may be placed. And this is the case even when, by an unequal division of the cone of rays from the object-glass, the image to which the satellite belongs is made decidedly fainter than the other.

Observatory, Haddenham, Bucks, 12 April, 1860.

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[*] See figures of *Jupiter's* satellites in the second Plate.—Ed.

On the Appearance of Jupiter's Third Satellite on the Disk of the Planet. By Captain Noble.

At page 57 of the present volume of the *Monthly Notices*, is an account by Mr. Lassell of an observation of *Jupiter's* fourth satellite as a dark spot upon his disk. I wish to call the attention of observers to the fact that a similar phenomenon is presented by the *third* satellite. On the night of Wednesday, March 21, I watched the third satellite as it began its transit over the planet. At $9^h 50^m$ G.M.T., when it was fairly on *Jupiter's* limb, it presented the appearance which it theoretically should have done, that of a brilliant point of light. I left the telescope soon after this. On my return to it at $11^h 30^m$, I saw the satellite (now as nearly as possible half way across the disk of the planet) as a well-defined distinct dark spot, "almost as dark" (as I find from the entry in my observatory book) "as the shadow of the satellite itself on an indifferent evening." The contrast of tint was most marked between the satellite and the portion of the great southern belt upon which it was projected; and the bifurcations and markings of which were uncommonly well seen. I, unfortunately, could not observe the egress: a fact I lament, as it would have been a point of great interest to determine at what portion of its passage across *Jupiter's* disk the satellite exchanged its dark appearance for a brilliant one. I am informed by Mr. C. Leeson Prince that he and another Fellow of the Society, Mr. F. Brodie, witnessed a similar appearance of the third satellite during one of its transits some weeks since.

Forest Lodge, Maresfield, Sussex, March 24, 1860.

Account of the Performance of Mr. Fletcher's New Equatoreal. By Capt. Jacob.

Having lately had the opportunity of trying one of the largest telescopes yet made in England, it has occurred to me that some account of its performance may be interesting to the Society, and may serve to show that it is not now needful to go abroad in order to procure a first-rate instrument.

The telescope in question is the property of our worthy Fellow, J. Fletcher, Esq., of Tarnbank, where it has been but recently erected by him; the optical portion by Cooke, of York; but the mounting, a long polar axis of cast-iron in the form of a double cone, with 3 feet circles, has been executed under Mr. Fletcher's own direction at an engineering foundry at Whitehaven, belonging to one of his brothers.

The object-glass is of 9½ inches aperture and 12¼ feet focal length; the telescope is thus nearly identical in dimension with the great Dorpat telescope, but a trifle shorter.

The trial was made under some disadvantages, the adjustments being only approximate, the driving clock and tangent-screws not as yet fixed, and the glass not kept perfectly clean, the workmen being still employed about the Observatory, and making much dust. The weather also was not very favourable, for though the air was on two or three occasions pretty transparent for a short time, it was never quiet enough to admit of high powers being used with advantage. However, after several vain attempts, on Saturday, 10th March, a good view of *Saturn* was obtained with powers 190 and 250; the obscure ring, streaking of the south hemisphere, and faint line on the outer ring, were very well seen, and a small point of light was discerned just clear of the preceding *ansa*, which proved to be *Enceladus.*

On the 12th a good glimpse was also obtained just about dusk, and *Enceladus* was seen at its east elongation at $7^h 45^m$. *Jupiter* was also examined, and presented the unusual appearance shown in the accompanying sketch.* Nearly the same appearance was again seen on the 14th at 10^h; but the oblique belt had then extended itself and encroached on the equatoreal belt, forming a darkish spot at the point of junction. On the same night nearly as good a view of *Saturn* was obtained as on the 10th, and the whole of the seven old satellites were seen at once, as shown in the accompanying diagram A. *Enceladus* having come to his west elongation at $6^h 55^m$, while there was yet daylight enough to read the address of a letter! *Mimas* was judged to be at east elongation at about $10^h 48^m$ G.M.T. Though the features of the planet and ring were best brought out by a power of 250, it was found that a lower power answered better for the small satellites, and they could be steadily viewed only with an eye-piece of 134. The whole seven were again seen on the 15th, as in diagram B. The exhibition of so small a point as E, in such close proximity to the ring, is perhaps as good a test of the defining power of the object-glass as could be desired. *Mimas* appeared to be at his elongation about $9^h 25^m$.

The points marked *h* in the diagrams were at first suspected to be *Hyperion*, but as they are too far distant, and their place also does not agree with Lassell's observations, they must have been two distinct minute stars; they were about the minimum visible, certainly not half as bright as *Mimas.*

A few double stars were examined, but no extremely close or difficult ones, as the instrument was not in order for finding or following such objects, and the night would not bear power. *ζ Cancri* and *η Orionis* were about the closest tried; the former was, of course, easily separated, and the latter also appeared to be so by glimpses when the atmosphere was steady for an instant.

Procyon was examined for focussing and centering with satisfactory results, the image being round and concentric both in and out of focus on both sides, and the disks very small when in focus with powers up to 400.

The mounting seemed remarkably firm, and so smooth in its movement, that, with the telescope slightly under-counterpoised, a star could be well followed and kept pretty near the centre of the field by gentle pressure with the finger. On the whole, I should say it was creditable both to the designer and the maker, and that in such able hands it is likely to prove a most efficient instrument; but Mr. Fletcher will, doubtless, give a fuller description of it himself when he has got it into working order.

Malvern, April 5, 1860.

On the Satellite of Saturn, "Mimas." By the Rev. W. R. Dawes.

I got a capital view of *Mimas* on March 21, $12^h 0^m$ G.M.T. a little past his western elongation. I saw him well, though not without many interruptions from atmospheric disturbance, for an hour and twenty minutes, when his approach to the ring and the planet's diminished altitude rendered him no longer visible. I had only looked for him once before this apparition, and I now find that he was then on the face of the planet.

Alvan Clark has seen him frequently with his new 12-inch object-glass; and told me he was sure I could see him in a fine sky.

March 21 was very clear, though the planet was tremulous.

Hopefield Lodge, Haddenham, Thame,
April 12, 1860.

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* See second Plate, fig. 4. The other diagrams referred to are not printed. —ED.

Note on the Projection of Saturn on the Dark Limb of the Moon. By the Rev. Prof. Baden Powell.

In the last number of the *Notices* (March 1860, p. 213), Mr. Pollock has recurred to the phenomenon described by him in a former number, of the projection of *Saturn* on the *dark* limb of the moon on May 8th, 1859, and to the remarks of the Astronomer Royal, which do not seem to recognise the appearance as described.

As some doubt, however, still seems to attach to the case, perhaps I may be allowed to observe, that the subject of such projections was referred to by me long ago in two papers printed in the Society's *Memoirs*, vols. xvi. and xviii. For projections on the *bright* edge there may be little difficulty in theory; but practically, the point to be accounted for is, why they should be seen in some instances and not in others, under conditions apparently the same.

For the *dark* edge, in my first paper (p. 309), I pointed out generally a *peculiar optical principle* which might *possibly* afford an explanation. And in my second paper (pp. 84 and 90), I have referred to all the recorded instances of projections on the *dark* edge; yet how far *any* optical cause can here apply may be very doubtful; and it would seem more probably due to some atmospheric effect.

The appearance described by Mr. Pollock is, in my opinion, a very remarkable one, especially in regard to the magnitude of the effect; so striking as to be noticed by a person totally unprepared for it, and with the naked eye; while it was not visible in the telescope. All the recorded observations of this kind are telescopic, and this singular fact would seem rather to point to some *ocular* cause.

In reference to Mr. Pollock's note on the Occultation of *Saturn* in the last *Monthly Notice*, the Astronomer Royal observes:—

"Mr. Pollock cannot fail to remark that what we call the dark limb of the moon, though relatively dark, as compared with the bright limb, is absolutely bright, and brighter than the surrounding sky; and its image is, therefore, swelled beyond its geometrical limits upon the retina by any cause of indistinctness, whether ordinary irradiation, faulty adjustment of the eye, or any other cause; in the same manner as that of the bright limb, though probably not to the same visible extent. The proportion of its brightness to the brightness of *Saturn's* disk is unimportant, just as the proportion of the brightness of the moon's bright limb to the specific brightness of a star's spurious planetary disk is unimportant. The explanation of the phenomenon of projection is, therefore, exactly the same in the two cases."

Royal Observatory, Greenwich, 1860, *April* 7.

Suggestions for a true Estimate and Register of the Colours of the Stars. By S. M. Drach, Esq.

The author remarks, "The valuable chapters on this subject in Admiral Smyth's 'Hartwell Cycle,' and the conflicting opinions of simultaneous observers on the same single or double stars, induce me to suggest that the practical observer be furnished with a binocular eye-piece, whereof one shall have at its object-end a disk wherein the several gradations of one colour shall be marked as in Saussure's Cyanometer. By turning this disk till the tint be judged equal to the star-tint viewed by the other eye through the other eye-piece, an independent scale would be attainable. The disk might be made of transparent horn, so as to admit of illumination, and a number of silk thread patterns be disposed circularly round it, producing the effect known to inspectors of silk-fringe pattern-cards. These colours would probably be more permanent than oil or water paintings. We might introduce an interior smaller circle with tinted threads of another colour (*e. g.* various tints of red and green). A little daylight practice on this disk would be an excellent apprenticeship to judgment by the single eye-piece directed to the stars."

March 12, 1860.

Mr. Dawes, in a letter to the Editor relating to the employment, in a communication to the Society, of the expression "the Hodgson solar eye-piece," sends the following extract and remarks of his own on the subject:—

Extract from Sir John Herschel's "Results of Astronomical Observations at the Cape of Good Hope," p. 436.

"The image formed at the first surface" (of a transparent glass reflector) "being received by a *small plane glass reflector*, will, if the reflection from the second surface of that mirror be also got rid of)"—[viz. by leaving its back transparent and open to the air]—"be finally reduced before reaching the eye-glass to little more than $\frac{1}{900}$th part of the original intensity, &c."

Acting upon this suggestion soon after Sir John's work appeared, viz. in 1848, I constructed an eye-piece on this plan, and used it for some years with my 6⅓-inch Munich refractor. I afterwards got one made by Dollond upon the same plan, except that *both* surfaces of the glass reflector were *plane*, and the second surface was made rough in order to destroy any distinct reflection from it.

After the Meeting of the Royal Astronomical Society in November 1854, while examining the eye-piece which Mr. Hodgson had placed upon the table, and which was constructed upon the same principle, I adverted to Sir John's proposal, and expressed my opinion, that if any one's name were given to this form of diagonal solar eye-piece, it ought to be *Sir John Herschel's*.

Hopefield, Haddenham, March 1860.

Note respecting the Parallax of α Herculis. By Capt. Jacob.

I have lately been informed that objections have been made on the Continent to my discussions of the Parallax of *α Herculis*;* first, on the ground that it is absurd to expect a difference of parallax between two stars having a large and nearly identical proper motion; second, because there is a certain bias or personal equation to which the eye is liable in measuring double stars, depending partly on the distance from the meridian, which bias must be determined and corrected for, before any trustworthy results can be derived from the measures, as to parallax, or change of angle in any way.

Now, with regard to the first objection, it is simply inapplicable; *α Herculis* not having a large proper motion, the amount assigned in the British Association Catalogue being $0''{\cdot}05$; while, to assume the identity of proper motion between the pair, is simply begging the question, a factor for difference of proper motion having been introduced by me in both of my discussions.

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* In volume of *Madras Results* for 1848–52 (Appendix), and in vol. xxviii. of the *Mem. R. Ast. Society*.

As to the second objection, without pretending to deny the fact of such a bias generally existing, I may state that, in my own case, its influence has been found to be very small; in fact, nearly insensible, excepting when the object has been near the zenith, or otherwise so situated that it could not be observed comfortably, and such positions I have, therefore, habitually avoided when practicable.

But to obviate any possible objection on this score, in the case of *α Herculis*, I was careful to take all the measures as nearly as possible under similar conditions; viz., nearly all by daylight, and all those which would tell strongly on the parallax, on the same side and about the same distance from the meridian; considering that prevention was (in such a case emphatically) better than cure. This has been already stated in the first discussion in the Appendix to the Madras volume for 1848–52, at p. 21, but as the statement seems to have been overlooked, I have taken the liberty of calling the attention of astronomers to it.

The parallax being but small, had its amount depended on the application of any such uncertain corrections, I should, for my own part, have considered the determination as of very small value.

Malvern, April 5, 1860.

Note on one of the Minute Companions of σ Orionis.

By the Rev. T. W. Webb.

My attention was some time ago directed to the smallest star of the principal group connected with *σ Orionis*, designated as *a* in the Bedford Catalogue, by a communication received from Thomas Warner, Esq., of Brighton, in which he states that, while pointing his telescope, of 3¼ inches aperture and 4 feet 4 inches focal length, for a friend, on November 22, 1859, he was surprised to see, at the first glance, what he had not seen before; something which a moment's attention showed him to be a star, on the left of the chief star in the group. He was not thinking at the time that there was a small star to be looked for, and he certainly had not any knowledge where to look for it. He has since seen it several times, but never so clearly as on that day; however, in the beginning of the present year, a boy, who was staying with him, said he saw three stars in a row with the bright one, and that the smallest was to the left of the others. When Mr. Warner saw it, it was about as easy an object as the companion of *ζ Persei*, and certainly at that time easier than that of *κ Geminorum*. At the date of his communication (January 12) the latter was much less difficult than *a* in *σ Orionis:* hence, though atmospherical circumstances may have been the cause, he is rather disposed to infer variable light; and supposes that it may have been near its minimum when the elder Herschel, and subsequently Herschel junior and South, observed the others; that it may have been brighter when Struve said of it, "quanquam non est valde debilis," and classed it 10·3, like the companion of *λ Geminorum;* that it may have been brighter still when Smyth classed it 11, and the companion of *λ Geminorum* 12; and that it may have been near its maximum when it caught Mr. Warner's eye, November 22, 1859, as little, if at all, smaller than the companion of *ζ Persei*, rated 9·3 by Struve, and 10 by Smyth. He adds, however, that he had not compared the latter star with the suspected variable star under the same circumstances.

In consequence of this communication, I intended to have kept my eye upon this object, but the unusually cloudy and boisterous weather of the recent season, especially near a mountainous region where snow has been almost uninterruptedly visible for five months, has repeatedly frustrated my wish. I was able to see it, however, four times with my 5·5 inch object-glass, and once (at Gloucester) with an 8¼-inch speculum by Sir W. Herschel: but on no occasion did it appear to me sufficiently conspicuous to warrant the idea that it could have been perceived, at least without very great difficulty, in the telescope employed by Mr. Warner.

On two cases of Solar Spots in High Latitudes, and on the Surface Currents indicated by the Observations. By R. C. Carrington, Esq.

At pages 76 and 77 of the present volume I stated an opinion, based on pretty extensive research, that solar spots had not hitherto been observed in heliographical latitudes of anything like 70°, and that the assertion had probably originated in some numerical error. Lahire is the observer with whose name the observation is coupled; but I have not succeeded in finding any passage among his various notes on the subject, in which he himself states such a conclusion. It is important, however, to put on record, in a form not admitting of dispute, cases of the highest latitudes known to have been reached by veritable well-developed solar spots; and though the two following may already be surpassed by instances known to other observers, I offer them as satisfactory and indubitable in the evidence.

The first case is that of a large single spot, of a form generally round, with a nucleus of about 20 seconds diameter and average width of penumbra about 60 seconds, followed at a distance by one, and afterwards by three small detached spots, sufficiently removed not to affect the motion of the principal spot. This was observed at the Capo di Monte Observatory at Naples, in the year 1846, by my esteemed correspondent, Dr. Peters, now Professor of Astronomy in Hamilton College, New York, who has favoured me with exquisite copies of his drawings and the original numbers of his observations; so that I have been enabled to verify his concluded positions by reductions effected in my usual way, and with the elements of solar rotation which I have hitherto adopted.

The centre of the nucleus was observed by Dr. Peters in right ascension and declination, by differences in time and angle from the sun's limbs; from which I deduce the following angles and distances from centre in decimals of the sun's radius, and lastly the heliographical longitudes from node and latitudes:—

Naples, 1846.

	Naples M.T.	ρ	θ	l	λ
	h m		° ′	° ′	° ′
June 8	0 52	·9041	19 32	134 20	+50 2
13	0 26	·7799	339 32	199 42	+50 55

The spot was observed on these two days only, but the agreement and the check thus obtained are amply sufficient to establish its position beyond a doubt. If the longitudes be compared by deducting from the second 70° 40′ for the interval between the observations, on the supposition that the sun's rotation is effected in 25·380 mean solar days, we find the following differences in 5 days:—

−5° 18′ in longitude and +53′ in latitude,

or daily drifts at 50° N. by this one spot,

−64′ in longitude and +11′ in latitude;

conclusions which we will recur to below.

The second case is that of a smaller spot, elongated and at first with two small nuclei contained within one penumbral boundary, but which diverged and separated into two detached spots, both of which were sufficiently observed and sketched by myself in the month of July 1858. This group was not visible on the 30th of June. The motion of divergence, a kind of mutual perturbation, of which I have before given instances illustrated by drawings to scale, will here require to be allowed for, as will be seen. First of all, the result of observation is as follows:—

Redhill, 1858.

	G.M.T.		*ρ*	*θ*	*l*	λ
	h m			° ′	° ′	° ′
July 1	3 54	Q	·7540	187 47	215 32	−44 46
2	4 16	*q*	·7986	201 16	231 42	−44 15
		Q	·7910	198 23	228 27	−44 50
4	0 39	*q*	·8996	215 27	255 27	−43 59
		Q	·8836	212 30	250 59	−44 53

If we now correct the longitudes for rotation to noon on July 1, and with the same value of solar rotation 25·380, we shall have,—

		° ′	° ′	
For Noon on July 1	Q	212 59	−44 46	by July 1
	q	214 44	−44 15	by July 2
	Q	211 29	−44 50	,,
	q	212 16	−43 59	by July 4
	Q	207 48	−44 51	,,

In the next place it will be observed that the differences of longitude and latitude of *q* and Q are

	° ′		′
On July 2	3 15	and	35
On July 4	4 28	and	52

so that they diverged at the rates of 36′ and 8′ per diem. Without insisting on the utmost exactness for these values, yet taking them as generally quite in accordance with previous conclusions, and correcting the previous positions by dividing the divergence equally between the two nuclei, we shall lastly have the following positions, corrected for all but the mean surface drifts:—

		° ′	° ′
For Q	July 1	212 59	−44 46
	2	211 47	−44 46
	4	208 38	−44 43
and			
For *q*	July 2	214 26	−44 19
	4	211 26	−44 11

and from which the daily drift at 45° S. by the observations of this one group, is

− 85′ in longitude,

and uncertain (or slightly towards the equator) in latitude.

If, now, the reader will refer to page 83 of the preceding volume of the *Monthly Notices*, from which I bring forward the remainder of the following table, he will see the extreme value of collecting cases such as the present.

Mean Surface Drifts per Diem.

At 50°	N	−64	in long.	+11′	in lat.	by 1 spot
30	N	−25	...	+ 5	...	4 series
18	N	−14	...	+ 1	...	4 ,,
8	N	+ 8	...	− 5	...	14 ,,
11	S	+ 10	...	− 3	...	6 ,,
19	S	−10	...	+ 1	...	5 ,,
29	S	−21	...	+ 4	...	8 ,,
45	S	−85	...	(− 2)	...	by 1 group

The two lines now added will, I think, be allowed to add as much to the importance of the table, as they extend its range in latitude. The daily drifts in longitude above given are, however, relative and not absolute, being derived by the use of 25·380 days as the period of solar sidereal rotation. If a period of 25·000 days be used, the motions in longitude will all be more negative by 13′ per diem, and by graphical interpolation we should infer the following as a general sketch of the surface currents:—

With Period 25 Days.

At 50°	N	−94′	in Long.	+2	in Lat.	towards N. Pole
40	N	−64	...	+5	...	
30	N	−40	...	+5	...	
20	N	−22	...	+1	...	
10	N	−10	...	−4	..	towards Equator
0		0	...	0	...	
10	S	−10	...	−4	...	
20	S	−22	...	+1	...	towards S. Pole
30	S	−40	...	+5	...	
40	S	−64	...	+5	...	
50	S	−94	...	+2	...	

I am inclined, after some consideration, to think the above second table, in which all the daily drifts in longitude are retrograde, the more faithful representation of the actual motions, though the numerical values will for some time bear modification. And I cannot altogether refrain on this occasion from adding what I conceive are the inevitable conclusions to be derived from it. The matter which forms the photosphere, and which, undoubtedly, floats suspended in the sun's atmosphere, with as definite a lower surface as the upper visible one, appears by my observations to flow slowly towards the poles in each hemisphere, from the latitude of about 15°, this polar westerly drift becoming more and more westerly till at 50° or 60° the polar tendency is insensible. An accumulation must result, and as certainly find its vent. I conceive that this is probably effected by subsidence (any more definite expression might merely provoke objection,) into the second stratum, the surface of which we see as the penumbra of spots, and that as part of the second stratum, similarly floating above the sun's body in its atmosphere, and at its own lower level, the matter thus transferred passes again towards the equator, acquiring as it approaches it the velocity of rotation lost in its passage towards the pole in the upper stratum. On its arrival, or on the arrival of an equivalent portion of the lower stratum at the latitude of about 15 degrees, I conceive that a corresponding ascent takes place into the upper surface, the motion being still towards the equator, and in longitude again suffering retardation by the general cause (whatever for the moment that may be) which causes the marked retrogradation of the higher latitudes. At the equator I conceive that a second subsidence of the upper stratum into the lower takes place, and that the motion of the lower stratum is towards the pole from the neighbourhood of the equator to about 15 degrees of latitude, where an ascent again takes place to supply the loss of the upper current consequent on the polar drift we set out with. Those who are acquainted with Lieut. Maury's conclusions respecting the circulation of the under and upper currents of our own winds will, I think, see more analogy than difficulty in the above notions. He had to infer the upper currents from the observed lower currents: the case here is the exact converse. But whence the lost velocity of rotation which the increasing retrogradation in higher latitudes indicates? And how is the lost velocity in the upper current regained through the lower current? I imagine by the sensible contact of the upper stratum with a vast atmosphere or medium which does not wholly partake of the rotation of the body it surrounds; and that the compensation is by the action of the body beneath the second stratum rotating faster than that stratum. In other words, I conceive that if we arrive at the conclusion that 25 days is the least time in which any zone of the upper stratum rotates, it must be the greatest limit of the time of rotation of the body beneath.

According to the view here thrown out as a speculation for discussion, the system of under-currents of the second stratum will be represented by the same table as that last given for the upper currents, changing only the signs in latitude. Beyond 50° of latitude I imagine the motions of both strata to be the same and simply retrograde around the poles: the crossing of the two strata (consequent on the opposite signs in latitude) occurring only within the region frequented by spots.

Many will be reminded of the following words of Sir J. Herschel:—"The spots in this view of the subject would come to be assimilated to those regions on the earth's surface in which, for the moment, hurricanes and tornadoes prevail."

The immediate object of this paper was, however, the publication of two indisputable cases of spots in high latitudes, the highest known to me, and to challenge the production of cases equally authenticated which may have surpassed them.

Note.—Several drawings were exhibited at the meeting in illustration of the above paper.—ED.

Results of Meridional Observations of Small Planets; Occultations of Stars by the Moon; and Phenomena of Jupiter's Satellites; observed at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

Iris ⑦.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, Mar. 21	9 2 25·9	9 0 48·35	81 1 5·19
23	8 54 21·3	9 0 35·51	80 55 43·91
26	8 42 27·6	9 0 29·50	80 48 29·20
28	8 34 40·0	9 0 33·75	80 44 12·19

Hygeia ⑩.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, Mar. 21	11 6 57·4	11 5 40·24	89 46 46·28

Victoria ⑫.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, Mar. 21	13 8 28·1	13 7 30·93	107 18 26·24
26	12 44 42·1	13 3 23·83	106 49 0·29

Egeria ⑬.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, Mar. 28	8 25 35·7	8 51 27·95	49 55 11·51

Urania ㉚.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, Mar. 21	11 10 32·5	11 9 15·98	87 7 32·76

Doris (48).

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, Mar. 23	10 21 43·9	10 28 12·46	83 50 0·09

All the observations of N.P.D. have been corrected for parallax.

Occultations of Stars by the Moon.

Day of Obs.	Phenomenon.	Moon's Limb.	Mean Solar Time.	Observer.
			h m s	
1860, Mar. 4	δ Cancri disapp.	Dark	7 15 31·5	D.
4	δ Cancri reapp.	Bright	8 15 30·6	D.
5	18 Leonis disapp.	Dark	9 13 27·0	C.
5	B.A.C. 3345 disapp. (*a*)	Dark	10 7 22·3	C.
5	18 Leonis reapp.	Bright	10 18 59·4	C.

(*a*) Doubtful, the star very faint.

Eclipses, Occultations, and Transits of Jupiter's Satellites.

Day of Observation.	Satellite.	Phenomenon.	Mean Solar Time.	Observer.
1860.			h m s	
Jan. 1	IV	Ecl. disapp. (*a*)	7 42 56·3	J. C.
6	III	Ecl. disapp. (*b*)	8 56 4·9	D.
7	I	Occ. reapp. last cont.	10 54 47·0	E.
16	II	Ingress, first cont.	11 31 57·7	C.
16	II	,, central bis.	11 33 27·5	C.
16	II	,, last cont.	11 35 12·2	C.
16	II	Egress, first cont. (*c*)	14 18 30·4	C.
16	II	,, last cont. (*c*)	14 21 29·9	C.
24	I	Egress, last cont. (*d*)	6 6 20·3	C.
24	III	Egress, first cont.	8 44 26·4	T.
24	III	,, central bis. (*b*)	8 46 56·0	T.
24	III	,, last cont.	8 48 25·7	T.
31	I	Egress, first cont.	7 46 17·7	C.
31	I	,, central bis.	7 47 47·4	C.
31	I	,, last cont.	7 49 32·1	C.
31	III	Ingress, first cont.	8 38 39·1	C.
31	III	,, central bis.	8 41 8·6	C.
31	III	,, last cont.	8 44 8·1	C.
31	III	Shad. Ingr. last cont.	10 48 2·8	C.
31	III	Egress, last cont.	12 5 5·2	C.
Feb. 6	I	Ecl. reapp.	12 51 32·3	E.
7	III	Ingress, first cont.	11 59 10·8	D.
7	III	,, central bis.	12 3 10·1	D.
7	III	,, last cont.	12 7 9·5	D.
12	II	Ecl. reapp. (*e*)	6 51 14·6	C.
12	IV	Shad. Egr. first cont. (*f*)	7 18 50·1	C.
13	I	Occ. disapp. first cont. (*g*)	11 39 10·4	J. C.
13	I	,, central bis. (*g*)	11 42 24·9	J. C.
13	I	,, last cont. (*g*)	11 45 9·5	J. C.
13	I	Ecl. reapp. (*h*)	14 46 40·7	J. C.
14	I	Ingress, central bis.	9 4 13·2	E.
23	I	Egress, central bis.	7 37 12·2	E.
25	III	Occ. disapp. first cont. (*i*)	8 41 40·9	C.
25	III	,, central bis. (*i*)	8 45 40·2	C.
25	III	,, last cont. (*i*)	8 49 39·6	C.
28	IV	Ingress, first cont.	11 50 54·9	E.
28	IV	,, bisected	11 55 54·0	E.

Day of Observation.	Satellite.	Phenomenon.	Mean Solar Time.	Observer.
1860.			h m s	
Feb. 28	I	Ingress, central bis.	12 39 16·9	E.
29	I	Occ. disapp. last cont.	9 49 3·2	C.
29	I	Ecl. reapp.	13 5 21·0	C.
Mar. 2	I	Ecl. reapp.	7 34 25·1	D.
2	I	Ecl. reapp.	7 34 8·0	M. D.
2	II	Ingress, first cont. (*k*)	14 32 42·6	D.
2	II	„ central bis. (*k*)	14 35 42·1	D.

(*a*), Somewhat uncertain; clock beats nearly inaudible from high wind. (*b*), Disappearance very gradual. (*c*), Image much blurred and very tremulous. (*d*), Planet very tremulous. (*e*), Image of the planet much blurred. (*f*), Very cloudy; the planet disappeared before the shadow was bisected; the time noted was satisfactory. (*g*), Very tremulous. (*h*), Cloudy; planet very faint. (*i*), Cloudy. (*k*), Unsatisfactory; observed through thin cloud.

The initials D., E., C., J. C., M. D., and T., are those of Mr. Dunkin, Mr. Ellis, Mr. Criswick, Mr. Carpenter, Mr. Dolman, and Mr. Talmage.

Observations of Minor Planets. By Norman Pogson, Esq., Director of the Hartwell Observatory.

(*Communicated by Dr. Lee.*)

Polyhymnia ㉝.

Greenwich Mean Time.		Magnitude.	Apparent R.A.	Apparent N.P.D.	Log (Par. × Δ) R.A.	Log (Par. × Δ) N.P.D.	Comparisons.	Power.
1859.	h m s		h m s	° ′ ″				
Nov. 28	9 39 24	11·1	3 59 19·62	66 43 7·3	−9·266	−0·636	7 with *b*	52

Proserpine ㉖.

Greenwich Mean Time.		Magnitude.	Apparent R.A.	Apparent N.P.D.	Log (Par. × Δ) R.A.	Log (Par. × Δ) N.P.D.	Comparisons.	Power.
1859.								
Nov. 28	12 39 28	11·5	2 20 0·36	75 52 57·6	+9·383	−0·743	12 with *a*	52

Ariadne ㊸.

Greenwich Mean Time.		Magnitude.	Apparent R.A.	Apparent N.P.D.	Log (Par. × Δ) R.A.	Log (Par. × Δ) N.P.D.	Comparisons.	Power.
1860.								
Feb. 19	14 14 14	10·9	10 56 41·54	89 26 27·8	+9·034	−0·824	8 with *g*	84
	14 33 7	...	10 56 41·10	89 26 29·1	+9·133	−0·823	5 with *g*	52
23	13 39 43	10·8	10 52 48·70	89 8 2·9	+8·937	−0·822	6 with *f*	84
	13 57 21	...	10 52 47·64	89 8 0·7	+9·052	−0·822	6 with *f*	84
27	11 40 17	11·2	10 48 48·63	88 47 39·8	−8·828	−0·820	5 with *d*	52
	11 55 23	...	10 48 48·06	88 47 36·5	−8·640	−0·819	4 with *e*	84
28	11 27 31	10·4	10 47 46·57	88 42 19·1	−8·898	−0·819	7 with *d*	84
29	10 39 30	10·5	10 46 45·79	88 36 51·4	−9·154	−0·819	10 with *e*	84

Leucothea ㉟.

Greenwich Mean Time.		Magnitude.	Apparent R.A.	Apparent N.P.D.	Log (Par. × Δ) R.A.	Log (Par. × Δ) N.P.D.	Comparisons.	Power.
1860.								
Feb. 29	11 0 20	11·3	10 43 4·76	77 31 50·9	−8·897	−0·735	6 with *c*	84
Mar. 2	13 18 1	11·0	10 41 8·59	77 28 39·7	+9·082	−0·738	7 with *c*	52

Hygeia ⑩.

Greenwich Mean Time.		Magnitude.	Apparent R.A.	Apparent N.P.D.	Log (Par. × Δ) R.A.	Log (Par. × Δ) N.P.D.	Comparisons.	Power.
1860.								
Feb. 29	12 2 4	9·2	11 21 27·81	91 15 7·7	−8·856	−0·834	10 with *h*	84
Mar. 2	13 52 7	...	11 19 55·45	91 7 27·7	+9·046	−0·833	6 with *h*	52

Urania ㉚.

Greenwich Mean Time.		Magnitude.	Apparent R.A.	Apparent N.P.D.	Log (Par. × Δ) R.A.	Log (Par. × Δ) N.P.D.	Comparisons.	Power.
1860.			m s	′ ″				
Mar. 2	12 2 15	10·4	* −2 1·84	* + 12 50·9	−8·838	−0·820	10 with *i*	52

The observations were made with the Hartwell equatoreal and two ring-micrometers, and have been corrected for refraction and proper motion as usual. The adopted radii of that used with power 52 have been given on page 7 of this volume. The determinations for the other, used with power 84, fully bear out the remarks there made as to the accuracy attainable in such cases with ordinary care.

From stars in the *Pleiades*, as measured by Bessel, we have

Inner Edge of the Ring-Micrometer used with Power 84.

Determining Pair of Stars.	Radius.	Measures.	Probable Errors of One Measure.	Probable Errors of Mean Result.
	″		″	″
Anonyma 24 and 39	392·47	10	±0·16	±0·05
26 Tauri and Atlas	392·22	10	±0·28	±0·09
Adopted Mean	392·34	20	±0·24	±0·05

For the outer edge, we have in like manner,

Determining Pair of Stars.	Radius.	Measures.	Probable Errors of One Measure.	Probable Errors of Mean Result.
	″		″	″
Electra and Maia	489·91	5	±0·19	±0·08
Asterope and Anonyma 24	490·17	5	±0·21	±0·09
Adopted Mean	490·04	10	±0·19	±0·06

Besides the magnitudes recorded with the preceding observations, the following estimations were made, on nights when other duties prevented the use of the micrometer:—

Polyhymnia	1859, Nov. 21,	11·0 mag.;	Nov. 24,	11·2 mag.
Parthenope	Nov. 28,	9·5 mag.		
Lætitia	Dec. 8,	10·0 mag.;	Dec. 14,	9·8 mag.

The comparison star used with *Urania* on March 2 proved to be unobserved in any standard catalogue. The assumed mean places of the rest reduced to 1860 were as follows:—

Reference.	Authority.	Magnitude.	Mean R.A.	Mean N.P.D.
			h m s	° ′ ″
a	B. Z. 281, 9	9·0	2 17 50·02	76 5 22·0
b	B. Z. 393, 49 = 7481 Lalande	8·3	3 55 49·97	66 58 33·9
c	337 Greenwich Observations in 1854	7·3	10 43 46·96	77 40 45·2
d	55 Leonis, Gr. Obs. in 1856	6·0	10 48 30·21	88 31 2·9
e	57 Leonis { Maedler's Bradley, 2379 Armagh }	7·0	10 48 59·68	88 49 15·3
f	2394 Armagh	8·0	10 55 22·65	89 20 32·5
g	*p*² Leonis { Maedler's Bradley, 2400 Armagh }	6·5	10 56 26·59	89 14 53·0
h	Weisse xi. 290 = 21727 & 8 Lalande	6·5	11 17 49·74	91 26 28·6
i	(Approximate Place)	9·0	11 29 10	88 37 45·

Hartwell Observatory, March 31, 1860.

On the Figure of the Earth. By Capt. A. R. Clarke, R.E.*

The investigation was undertaken in consequence of the remarks of the Astronomer Royal in the *Monthly Notices* for January last, on General T. F. de Schubert's *Essai d'une Détermination de la Véritable Figure de la Terre.* The conclusion there arrived at is that the earth is an ellipsoid with three unequal axes, the dimensions of which are given: Captain Clarke remarks that these results are dependent on so small a number of observed latitudes that it becomes highly important to ascertain the probable errors before they can be received as representing the real figure of the earth, and that he had therefore repeated the calculation in a more precise manner, using a greater number of observed latitudes and bringing out the probable error of the elements. The arcs used are the same as those in General de Schubert's memoir, with the omission of the Prussian and Pennsylvanian arcs. It is noticed that whatever the real figure of the earth may be, if in the investigation we suppose it to be an ellipsoid, it is clear that the arithmetical process must bring out an ellipsoid of some kind or other, which ellipsoid will agree better with all the observed latitudes than any spheroid of revolution would do. Nevertheless it would scarcely, he conceives, be correct to say that we had proved the earth not to be a solid of revolution; to do this would require data which we are not in possession of at present, and several arcs of longitude would be necessary. But in the meantime it is interesting to ascertain what ellipsoid does actually best represent the existing measurements. The calculations are performed accordingly, and the conclusion is that from the magnitude of the probable errors which they exhibit, it appears that we are not yet in possession of data to bring out the excentricity of the equator with anything like certainty. The author mentions that the calculations were made in connexion with the geodetical operations of the Ordnance Survey, but that Colonel Sir Henry James had kindly given him permission to present them to the Royal Astronomical Society.

It will be recollected that the characteristic feature of General de Schubert's investigations is the employment, for the independent determination of the axes of any meridian, of the lengths of two or more sections of the meridian arc and latitudes of the extremities of such sections. In a paper, "Sur l'Influence des Attractions Locales dans les Opérations Géodésiques et particulièrement dans l'Arc Scandinavo-Russe," (*Ast. Nach.* Nos. 1245–1247), General de Schubert remarks that the publication of the Ordnance Trigonometrical Survey of Great Britain and Ireland has opened a new field to geodesy, by introducing into the measurements and calculations an element heretofore neglected, "the deviation of the plumb-line," which, if it does not entirely destroy the discrepancies between the different measurements of meridian arcs, promises at least to reduce them very materially; and, with a view to ascertaining *à posteriori* the extent to which this correction is required, he calculates by means of the various combinations of the observed latitudes of thirteen stations on the arc above referred to, and the distances, as deduced from the geodetical operations, the values of the semi-axes, which values, as obtained from the several different combinations, present discrepancies which afford ground for suspecting at several of the stations errors of latitude arising from a deviation of the plumb-line.—ED.

MISCELLANEOUS INTELLIGENCE.

The passage of a planet over the sun at the time of Dr. Lescarbault's observation of the 26th March, 1859, is positively denied by M. Liais, in an article in No. 1248 of the *Ast. Nachrichten.* M. Liais states that on the day in question he was observing the sun (with a view to the determination of the relative intensities of the different parts of the disk) at San Domingos, in the Bay of Rio Janeiro, from $11^h 4^m$ to $11^h 20^m$, and from $12^h 42^m$ to $1^h 17^m$, [S. Domingos mean solar time]. Taking account of the difference of three hours between the longitudes of Orgères and S. Domingos, the ingress of the spot, according to the observation of Dr. Lescarbault, should have been at $1^h 5^m$ and the egress at $2^h 23^m$, S. Domingos. At $1^h 17^m$ S. Domingos, the spot should have entered on the sun for 12 minutes, and from the rate of progress, according to Dr. Lescarbault, it should have travelled a space of 1′4 on the disk, and its smallest distance from the limb should have exceeded 20″, a quantity too great to be annihilated by the differences of the parallaxes for Orgères and S. Domingos; and M. Liais argues that if the spot had been visible at Orgères, he must have seen it at S. Domingos at the close of his observation, the more so that according to Dr. Lescarbault the spot entered at about 11° from the north pole of the disk, the region which M. Liais was exploring.

There is contained in the *Berichte* of the Royal Saxon Academy of Sciences for 1859 an account, by M. Hansen, of the establishment of the new Ducal Observatory at Gotha.

Situation.—The new observatory lies in the Erfurt suburb of the town of Gotha, on the east side of the Jägergasse, and on the north bank of the Leina Canal. It is found by trigonometrical measurements that the centre point of the tower of the new observatory is in the direction of the meridian 1022·6 meters to the north, and in the perpendicular direction 1342·5 meters to the west of the centre point of the transit-instrument on the Seeberg. The difference of latitude from the old observatory is therefore $33''{\cdot}10$, and the difference of longitude $4^{s}{\cdot}58$; giving provisionally for the latitude of the new observatory $50^{\circ}\,56'\,38''$, and for the longitude from Paris $33^{m}\,30^{s}{\cdot}2$. As the centre point of the transit-instrument on the Seeberg has been used in various measurements of degrees and triangulations, its position was, previous to the removal of the instrument, fixed by a cross on a block of stone buried three or four feet below the surface.

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* This paper will appear *in extenso* in the *Memoirs* of the Society.—ED.

Arrangement.—The observatory contains, as shown on the plan, first, a meridian-room, 18 feet from north to south, and 21½ feet from east to west, with two meridian-slits, each 2 feet wide; secondly, an observing-room, of the same size as the meridian-room, with a slit in the prime-vertical; thirdly, a tower with a revolving roof; the inner diameter of the observing-room in the tower is 15 feet, and the height of the summit from the floor 12¼ feet; fourthly, a corridor surrounding the tower, with entrances into the three observing-rooms, and containing a part of the library: it leads also into a second library-room, which forms part of the dwelling-house.

Construction.—The observatory and dwelling-house are built of stone—the observatory of *quaderstein* (sandstone), and the dwelling-house of *bruchstein* (a limestone). The foundations for the instruments are carried down to the ground, detached from the remaining wall-work. There is a commodious dwelling-house fronting the Jägergasse, and the whole is surrounded on the other sides by a small garden.

Instruments.—Under the western slit of the meridian-room is placed the Ertel's meridian-circle, which is so contrived that the requisite number of divisions is brought down to a very small number—72 on the circle, and 61 on each of the auxiliary graduations—and the complete determination of the graduation-errors is thus made practicable. The arrangement is different in detail, but the same in principle with that described by M. Hansen (*Ast. Nachr.*, tom. xvii.). The meridian-circle is also so arranged that the object-glass and eye-piece of the telescope can be interchanged—affording a simple method for determining the flexion of the telescope and circle. The object-glasses and eye-pieces of the microscope are by Schieck.

Under the eastern meridian-slit there is placed at present an astronomical theodolite by Reichenbach, and the slit serves in particular for observations in or near the meridian, by means of moveable instruments.

The clock in the meridian-room is by Tiede.

The same room contains a galvanic register-apparatus, for the graphical record of the observations with the meridian-circle: this consists of a telegraphic writing-apparatus by Siemens and Halske, provided with two electric magnets, one of which marks down on a continuously moving band of paper the seconds of clock-time, and the other the instants of observation, by means of points.

Under the slit in the prime-vertical is placed the Ramsden's 8-foot transit-instrument, the construction of which is essentially the same with that now made use of. The only fault of the instrument is, that the object-glass does not quite come up in goodness to the Munich object-glass.

In the tower is placed at present the Fraunhofer's heliometer, in which, however, the parallactic arrangement has been, by the Gotha mechanician Ausfeld, renewed, and provided with a larger and better-contrived circle; but the place is intended to be occupied by an equatoreal, with a 6-foot telescope and 2-foot circle given by the Duke, in preparation by the brothers Repsold. A detailed account is promised of this instrument, which, besides the improvements which the makers have introduced into their equatoreals, is to have others contrived by M. Hansen; and on this account, and from the skill and care of the makers, M. Hansen hopes that he will be able to employ the equatoreal for absolute observations.

The heliometer is being employed by Professor Habicht and M. Gussew, of Wilna (who has been staying at the observatory), in a series of observations of the distances and position-angles of the lunar spots, which are to be applied to the determination of the moon's figure, and of the libration.

As regards smaller instruments, the observatory possesses a number of telescopes, some old reflecting telescopes, an astronomical theodolite by Reichenbach, reflecting sextants, &c.

It is noticed, in conclusion, that the dwelling-house as well as the observatory is lighted by gas, and that the illumination of the clocks and of the fields of view in the principal instruments is effected in the same manner.

The following notice has been circulated by the Astronomer Royal among observers who had expressed their intention of observing the Eclipse of July 18 in the north-east part of Spain.

"I have written officially to the Board of Admiralty, on the subject of furnishing a steamship for conveyance of observers of the solar eclipse of July 18 to the north coast of Spain, and have also had an interview with one of the Lords of the Admiralty.

"There is every disposition in the Board to give all facilities in their power to observers.

"It appears that the accommodation on board the ships of the Royal Navy is so limited, that it would be almost impossible to find one which would convey the number of observers likely to desire a passage on this occasion. The Admiralty, therefore, contemplate as a probable course, the chartering of a merchant-steamer for the voyage; which would be in many respects more convenient. But nothing is yet officially determined.

"There will be no difficulty in landing a portion of the party at Bilbao, and proceeding with the rest to Santander (which port probably will be preferable for the vessel to lie at). And it is much to be wished that observers may be distributed between these two points.

"*Royal Observatory, Greenwich, 1860, May 5.*"

VOL. XX. *May* 11, 1860. NO. 7.

The Rev. R. MAIN, President, in the Chair.

Murray Gladstone, Esq., Broughton, near Manchester;
Norman Robert Pogson, Esq., Director of the Hartwell Observatory; and
James Buckingham, Esq., Westmoreland House, Walworth Common,

were balloted for and duly elected Fellows of the Society.

Observations sur les Formules employées jusqu'à présent par les Géomètres et les Astronomes pour déterminer par la Théorie le Coefficient de l'Equation Séculaire du Moyen Mouvement Lunaire, et sur la Nécessité de le calculer désormais par des Formules Directes. Par M. G. de Pontécoulant.

Dans une note insérée dans les *Monthly Notices* de la Société Royale Astronomique de Londres (No. 9, vol. xix. 8 Juillet, 1859), et qui avait pour titre "Observations sur les Nouveaux Termes que M. Adams a proposé d'introduire dans l'expression du Coefficient de l'équation séculaire de la Lune," après avoir montré la nécessité de reprendre à nouveau tous les calculs qui avaient été faits pour déterminer *l'équation séculaire* du moyen mouvement lunaire, parceque ces calculs, déduits des formules de Laplace où l'on a pris pour variable indépendante la *longitude vraie* de notre satellite, ne donnaient pas *directement l'équation séculaire* de la *longitude moyenne* dans l'expression de la *longitude vraie* (qui est la seule cependant qu'il soit nécessaire de connaître pour la formation des tables astronomiques, ou pour la comparaison des anciennes observations aux modernes), et qu'il était à craindre que la manière tout-à-fait arbitraire dont on déduit cette équation d'une formule inverse, ne laissât toujours échapper quelque partie (quelque minime qu'elle put être), de la quantité cherchée, comme il arrive, qu'on me permette cette comparaison grossière mais exacte, à un liquide qu'on transvase, d'un vase dans un autre, et qui perd toujours quelque chose dans cette opération, j'ai dit: "La détermination de *l'équation séculaire* par la formule directe sera encore plus laborieuse que par la méthode inverse, si l'on veut pousser l'approximation jusqu'au même ordre de quantités, mais l'importance de l'objet doit faire affronter toutes les difficultés, *et si personne ne se présente pour entreprendre cet utile travail, je ne balancerai pas à m'en charger moi-même.*"

J'ai depuis cette époque profité de quelque loisir pour remplir l'engagement que j'avais pris avec les lecteurs des *Monthly Notices;* j'ai déterminé *l'équation séculaire* du moyen mouvement lunaire par une formule directe, déduite des équations différentielles du mouvement troublé sous la forme où je les ai employées dans ma théorie de la Lune,* c'est à dire en supposant qu'on a pris le temps pour la variable indépendante du problème, j'ai compris dans mon approximation des termes même que M. Plana n'avait point considérés, et je suis arrivé à une nouvelle expression du coefficient de l'équation séculaire du moyen mouvement, *très différente* de la sienne, comme je devais m'y attendre d'après ce que j'avais dit à la fin de ma note, mais qui doit mériter désormais la confiance des géomètres et des astronomes, puisqu'elle est la seule *jusqu'ici* qui résulte directement des formules du mouvement troublé, sous la forme généralement adoptée pour la détermination de toutes les inégalités planétaires, et enfin des principes d'une saine théorie.

Je vais exposer ici, aussi succinctement que possible, les résultats auxquels je suis parvenu, je donnerai tous les détails de mes calculs dans un mémoire qui s'imprime en ce moment, de sorte que chacun pourra aisément les vérifier, et qu'on parviendra à donner ainsi à l'un des éléments les plus essentiels de la théorie du mouvement lunaire, toute l'exactitude qu'exige son importance.

Designons par n le moyen mouvement de la lune pendant l'unité de temps dans son orbite réelle, par $n_,$ la même quantité relative à l'orbite elliptique, c'est à dire à l'orbite que la lune décrirait autour de la terre sans l'action du soleil, nommons a et $a_,$ les demi-grands axes correspondants qui s'en déduisent par la loi de Kepler, en sorte qu'on ait, par exemple, en prenant pour unité la somme des masses de la terre et de la lune, $\mathrm{a}^3\,\mathrm{n}^2 = 1$ et $a^3_,\,n^2_, = 1$; la détermination du coefficient de *l'équation séculaire* exige que l'on connaisse le rapport de n à $n_,$, ou ce qui revient au même celui de a à $a_,$, c'est donc de la détermination analytique de ce rapport que nous devons d'abord nous occuper.

Pour faciliter cette recherche nous introduirons dans nos formules une nouvelle arbitraire a, que nous supposerons représenter la distance moyenne de la lune à la terre dans son orbite troublée, nous admettrons que cette quantité a été introduite à la place du demi-grand axe $a_,$, dans les formules même du mouvement elliptique, et que le développement de la fonction perturbatrice R, ou de la différence $\frac{d\mathrm{R}}{dv}$ a été effectué dans la même hypothèse, nous donnerons du reste le moyen de déduire ces nouveaux développements de ceux de ces mêmes fonctions donnés dans les chapitres ii et iii, du 7$^\text{e}$ livre de la *Théorie Analytique;* cela posé, il sera facile de déterminer le rapport de a à $a_,$, et par suite celui de n à $n_,$, en désignant par n le moyen mouvement qui repond à l'arbitraire a; en effet en représentant par r le rayon vecteur de la lune dans son orbite troublée et par R la fonction perturbatrice due à la seule action du soleil, on aura, par les formules ordinaires employées dans la théorie des perturbations planétaires*

$$\frac{d^2r^2}{2dt^2} - \frac{1}{r} + \frac{1}{a_,} = 2\int d'\mathrm{R} + r\frac{d\mathrm{R}}{dr}.$$

Si l'on fait abstraction dans cette équation des termes périodiques, en observant que, d'après la signification que nous avons donnée à l'arbitraire a, on a alors $\frac{1}{r} = \frac{1}{a}$, on trouve

$$\frac{1}{a} = \frac{1}{a_,} - r\frac{d\mathrm{R}}{dr},$$

cette équation fera connaître le rapport de a à $a_,$, et si pour plus de simplicité, en observant que $r\frac{d\mathrm{R}}{dr} = 2\,\mathrm{R}$, on fait

$$p = 2a\mathrm{R}, \qquad (1)$$

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* *Théorie Analytique du Système du Monde,* t. iv.

* *Théorie Analytique,* liv. vii. No. 1.

on aura

$$\frac{a}{a_,} = 1 + p. \qquad (2)$$

Il suffira de substituer dans cette formule à la place de p sa valeur qu'on déduira immédiatement du développement de R, effectué comme il a été dit plus haut, pour en conclure la relation cherchée entre a et $a_,$.

Ceci admis, on a pour déterminer la longitude vraie v de notre satellite, dans son orbite troublée, la formule

$$\frac{dv}{dt} = \frac{h}{r^2} + \frac{1}{r^2}\int\left(\frac{d\mathrm{R}}{dv}\right) dt.$$

Supposons le second membre de cette équation développé en une suite de *sinus* et de *cosinus* d'angles dépendants des longitudes moyennes nt et $n't$ de la lune et du soleil, ou de leurs multiples, et ne considérons que le premier terme, c'est à dire la partie constante de ce développement: admettons qu'on ait trouvé ainsi

$$\frac{dv}{dt} = \frac{h}{a^2}(1 + \mathrm{Q}).$$

Ou bien—en substituant pour h sa valeur que nous supposerons de cette forme $h = \sqrt{a_,}\,(1 + q)$, en representant par q une fonction déterminée des élémens e, e', γ, des orbites elliptiques de la lune et du soleil ordonnée par rapport aux puissances ascendantes de la très petite quantité m, qui représente le rapport des moyens mouvements du soleil et de la lune, et en observant qu'on a en faisant abstraction des inégalités périodiques $\frac{dv}{dt} = \mathrm{n}$,—on pourra supposer

$$\mathrm{n} = \frac{\sqrt{a_,}}{a^2}(1 + \Pi),$$

ou bien, en substituant dans cette formule pour a^2 sa valeur tirée de l'équation (2),

$$\mathrm{n} = a_,^{-\frac{3}{2}}\frac{(1 + \Pi)}{(1 + p)^2} \qquad (3)$$

formule dans laquelle il suffira de substituer pour p et Π leurs valeurs pour avoir le rapport cherché de n à $a_,^{-\frac{3}{2}}$, ou de n à $n_,$ élement essentiel pour la formation de *l'équation séculaire.*

En effectuant les opérations que nous ne pouvons qu'indiquer ici, mais dont le détail sera rapporté dans le mémoire que nous avons annoncé plus haut, et en portant les approximations jusqu'aux quantités du 8^{e} ordre par rapport à m, nous avons trouvé

$$\begin{aligned}
\mathrm{n} = n_,\Big\{ & 1 - m^2 + \frac{471}{64}m^4 + \frac{679}{32}m^5 + \frac{3407}{96}m^6 + \frac{15163}{288}m^7 \\
& - e^2\left(\frac{9}{8}m^2 + \frac{675}{32}m^3 + \frac{49293}{512}m^4 + \frac{700293}{2048}m^5\right) \\
& + e'^2\left(-\frac{3}{2}m^2 + \frac{5913}{128}m^4 + \frac{18333}{64}m^5 + \frac{1796123}{1536}m^6 + \frac{1715825}{384}m^7\right) \\
& - e^2e'^2\left(\frac{27}{16}m^2 + \frac{825}{32}m^3 + \frac{2838199}{4096}m^4 + \frac{90396389}{8192}m^5\right) \\
& + \gamma^2\left(\frac{9}{8}m^2 - \frac{27}{32}m^3 - \frac{3915}{512}m^4 - \frac{22965}{2048}m^5\right) \\
& + e'^2\gamma^2\left(\frac{27}{16}m^2 - \frac{153}{32}m^3 - \frac{71901}{1024}m^4 - \frac{9015061}{65536}m^5\right) \\
& + e^4\left(-\frac{3}{32}m^2 + \frac{1665}{256}m^3\right) \\
& + e'^4\left(-\frac{15}{8}m^2 + \frac{1075675}{6144}m^4\right) \\
& - \frac{9}{64}m^2e^4e'^2 + \frac{171}{64}m^2e^2e'^2\gamma^2 - \frac{99}{64}m^2e'^2\gamma^4 \\
& - \frac{a^2}{a'^2}\left(\frac{45}{64}m^2 + \frac{1035}{256}m^3 - \frac{1125}{512}m^2e'^2\right)\Big\}.
\end{aligned}$$

Si l'on compare l'expression précédente avec celle donnée par M. Plana dans la Théorie de la Lune, et qui résulte, comme nous l'avons dit, des formules inverses de celles que nous avons adoptées, c'est à dire des formules où la *longitude vraie* de la lune a été prise pour la variable indépendante, on voit que les valeurs de n déterminées par ces deux procédés diffèrent considérablement et même dans les parties les plus essentielles à la détermination exacte de *l'équation séculaire.* Ainsi, par exemple, le coefficient du terme multiplié par e'^2, dans l'expression du rapport $\frac{\mathrm{n}}{n_,}$, en négligeant les quantités de l'ordre m^5 et au-dessus, serait selon M. Plana

$$-e'^2\left(\frac{3}{2}m^2 - \frac{2763}{128}m^4\right)$$

au lieu de

$$-e'^2\left(\frac{3}{2}m^2 - \frac{5913}{128}m^4\right)$$

que nous avons trouvé pour son expression. La fraction qui multiplie le terme en m^4, n'aurait ainsi selon M. Plana que la moitié de sa valeur réelle, et *l'équation séculaire* qui en depend, éprouverait un accroissement de *plus d'une seconde* séxagésimale par suite de cette diminution. Cependant les calculs du savant Astronome de Turin sont d'une exactitude irréprochable, nous nous en sommes assuré en reprenant en entier la détermination de la valeur de n déduite des formules que l'on trouve dans le beau mémoire de Damoiseau,* et qui n'ont eu besoin que de très légères corrections et de quelques additions pour reproduire par leur développement cette quantité sous la forme même que M. Plana lui a attribuée. Il suit donc, du simple rapprochement que nous venons de faire, que les formules indirectes qu'avaient employées jusqu'ici les astronomes et les géomètres, qui ont traité la théorie de la lune et qui leur ont donné les expressions exactes de toutes les inégalités périodiques de cet astre, ainsi que celles des mouvements des nœuds et du périgée de son orbite, paraissent insuffisantes pour déterminer exactement le coefficient de *l'équation séculaire* ou du moins pour donner sa véritable expression analytique; soit que la méthode dont il s'agit ne soit pas en effet applicable à cette détermination, soit que l'hypothèse par laquelle on concède qu'on peut conclure le coefficient de *l'équation séculaire* de la *longitude moyenne* par le seul renversement de l'expression qui donne *l'équation séculaire* de la *longitude vraie* dans les formules où cette longitude a été prise pour la variable indépendante, soit trop arbitraire et contredite par les résultats d'une analyse rigoureuse.† Laplace qui ne s'était occupé que du terme de cette équation qui dépend de la première puissance de la force perturbatrice, ne s'était point aperçu de l'inconvénient que nous venons de signaler et que nous avions déjà rencontré dans la détermination de plusieurs inégalités à longues périodes; mais les géomètres et les astronomes, qui sont venus après lui, et qui ont poursuivi le développement de ses formules, ne se sont peut-être pas montrés assez scrupuleux en adoptant sans examen et comme hors de doute une supposition dont le principal mérite était de se plier sans peine aux besoins de leur analyse.[*]

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* *Mémoires de l'Institut, Savants Etrangers,* t. 1.

† Il y a dans la théorie du système du monde, disait Laplace, des choses qui échappent à toutes les prévisions du raisonnement et que le calcul seul peut nous apprendre.

Quant à M. Adams, dont les recherches sur *l'équation séculaire* ont vivement préoccupé les astronomes et les géomètres, dans ces derniers temps, par *l'étrangété* des résultats qu'il avait obtenus, comme il a suivi la même méthode que MM. Plana et Damoiseau, méthode dont nous venons de démontrer l'insuffisance, on peut être assuré d'avance que ses résultats ne sauraient s'accorder avec ceux que nous avons déduits des formules directes; mais les nouveaux termes, introduits par ce géomètre dans l'expression du coefficient de *l'équation séculaire*, par des considérations dont nous avons à plusieurs reprises signalé l'inexactitude, rendent la divergence bien plus sensible encore, elle ne va pas à moins de doubler et quelquefois de décupler la valeur de chacun des coefficients numériques qui entrent dans l'expression de n présentée plus haut. En sorte qu'on pourrait dire que la formule donnée par M. Adams n'offre avec la véritable formule d'où dépend *l'équation séculaire* aucun rapprochement possible; il suffira pour que chacun puisse s'en convaincre de déduire de la formule la valeur de n sous la forme dans laquelle nous l'avons présentée; en n'ayant égard qu'aux termes multipliés par e'^2, les seuls que M. Adams ait encore calculés, ou du moins publiés (*Monthly Notices*, No. 6, tome xix.), on trouve ainsi

$$\frac{n}{n_,} = e'^2\left(-\frac{3}{2}m^2 + \frac{4059}{64}m^4 + \frac{34047}{64}m^5 + \frac{280405}{96}m^6 + \frac{15161743}{1152}m^7\right),$$

valeur qu'il suffira de rapprocher de celle que nous avons présentée plus haut, pour en juger la dissidence.

Supposons maintenant qu'on différentie l'expression de n trouvée plus haut, par rapport à l'excentricité e' de l'orbite terrestre qu'elle renferme, et dont les variations sont, comme on sait, la véritable cause de l'accélération du moyen mouvement lunaire, remarquée par les astronomes longtemps avant que les géomètres fussent parvenus à en démêler le principe physique; en observant d'ailleurs que m est variable à raison de la variabilité du moyen mouvement n que cette quantité renferme, puisqu'on a en effet $m = \frac{n'}{n}$, ce qui donne

$$\frac{\delta n}{n} = -\frac{\delta m}{m},$$

on trouvera pour $\frac{\delta n}{n}$ une expression de cette forme

$$\frac{\delta n}{n} = K\,\delta\,.\,e'^2,$$

dans laquelle on fait pour abréger

$$\begin{aligned} K = {} & \frac{3}{2}m^2 - \frac{5337}{128}m^4 - \frac{18333}{64}m^5 - \frac{2073035}{1536}m^6 - \frac{2119151}{384}m^7 \\ & + e^2\left(\frac{27}{16}m^3 + \frac{825}{32}m^3 + \frac{2879671}{4096}m^4 + \frac{92066789}{8192}m^5\right) \\ & - \gamma^2\left(\frac{27}{16}m^2 - \frac{153}{32}m^3 - \frac{61533}{1024}m^4 - \frac{10286869}{65536}m^5\right) \\ & + \frac{27}{4}m^4e'^2 + \frac{9}{24}m^2e^4 - \frac{171}{64}m^2e^2\gamma^2 \\ & + \frac{99}{64}m^2\gamma^2 - \frac{1125}{512}m^2\left(\frac{a}{a'}\right)^2 \end{aligned}$$

En multipliant par $d\,t$ l'expression précédente et en l'intégrant ensuite, après avoir remplacé K par la valeur que cette lettre répresente, ou en a conclu *l'équation séculaire* $\int \delta n\,dt$ qui doit être ajoutée au moyen mouvement $\int n\,dt$ dans l'expression de la longitude vraie v, exprimée en fonction de la longitude moyenne $n\,t$. On a trouvé ainsi

$$\int \delta n\,dt = \left\{\begin{aligned} & \frac{3}{2}m^2 - \frac{5337}{128}m^4 - \frac{18333}{64}m^5 - \frac{2073035}{1536}m^6 - \frac{2119151}{384}m^7 \\ & + e^2\left(\frac{27}{16}m^2 + \frac{825}{32}m^3 + \frac{2879671}{4096}m^4 + \frac{92066789}{8192}m^5\right) \\ & - \gamma^2\left(\frac{27}{16}m^2 - \frac{153}{32}m^3 - \frac{61533}{1024}m^4 - \frac{10286869}{65536}m^5\right) \\ & + \frac{27}{4}m^4e'^2 + \frac{9}{64}m^2e^4 - \frac{171}{64}m^2e^2\gamma^2 \\ & + \frac{99}{64}m^2\gamma^4 - \frac{1125}{512}m^2\left(\frac{a}{a'}\right)^2 \end{aligned}\right\}\int (e'^2 - E'^2)\,n\,dt.$$

Dans cette formule E′ représente l'excentricité de l'orbite terrestre au bout d'un temps quelconque t, e' la valeur de cette excentricité au commencement de l'année que l'on a choisie pour époque.

Cette formule réduite en nombres, a donné:—

Equation Séculaire de la Longitude moyenne de la Lune.

$$\int \delta n\,dt = \left\{\begin{array}{l} \quad 0{,}008392850\ (2) - 0{,}001353514\ (4) \\ -\,0{,}000622120\ (5) - 0{,}000154749\ (6) \\ +\,0{,}000009844\ (7) = 0{,}006272311 \end{array}\right\}\int (e'^2 - E'^2)\,n\,dt.$$

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[*] Il est à rémarquer que Laplace dans son célèbre mémoire présenté le 19 Dec^bre^, 1747, à l'Académie des Sciences de Paris, et où il a pour la première fois annoncé qu'il avait trouvé la cause tant cherchée de l'accélération séculaire du moyen mouvement lunaire, s'était servi d'une *formule directe* pour en déterminer le coefficient, sans doute pour mettre sa grande découverte à l'abri de toute objection. Malheureusement sa formule ne s'appliquant qu'aux termes du premier ordre, dans la *Mécanique Céleste*, liv. 7^e^, il est revenu aux formules indirectes, mais sans pousser plus loin, comme nous l'avons dit, l'approximation.

En comparant ce résultat à celui que M. Plana a déduit de ses formules, on voit qu'il en diffère d'une manière notable, et que l'espèce de compensation qui devait s'établir selon ce géomètre, entre les quantités du 4ᵉ ordre et celles des ordres supérieurs, et qui semblait permettre de s'en tenir comme l'avait fait Laplace, aux termes résultant de la première approximation, n'existe pas réellement. La considération des puissances supérieures de la force perturbatrice altère sensiblement, au contraire, la valeur du coefficient qu'on obtient en faisant abstraction des quantités qui en dépendent, et comme tous les termes de la formule jusqu'aux termes du 7ᵉ ordre, sont affectés d'un signe *négatif*, la grandeur du coefficient qu'on s'était habitué à supposer à *l'équation séculaire* d'après les indications de Laplace, doit être considérablement *diminuée*. En effet, en substituant à la place de l'intégrale $\int(e'^2 - E'^2)\,n\,dt$ sa valeur $1274''{\cdot}185\,i^2$,[*] i représentant le nombre de siècles écoulés depuis l'année ou l'on compte le temps, *l'accélération séculaire* de la lune pendant le premier siècle, ou *l'équation séculaire* du moyen mouvement, se réduit à 7″·99218, c'est à dire que cette équation se trouvera diminuée de trois secondes à peu près de la valeur qu'on s'était habitué à lui attribuer.

Au reste il faut remarquer que les astronomes sont loin d'être d'accord entre eux sur l'évaluation exacte de cet élément si important du mouvement lunaire. Dunthorne, qui le premier a tenté de le déterminer par la comparaison des anciennes observations aux modernes, le faisait de 10″·0112; Mayer l'a réduit à 6″·998 dans ses premières tables; il l'a porté à 9″·0072 dans ses dernières. Lalande par une discussion nouvelle de cet objet a été conduit au résultat de Dunthorne. Enfin Laplace dit que Bouvard par une nouvelle discussion des éclipses anciennes déjà commes et de celles qu'il a extraites d'un manuscrit arabe d'Ibn Iunis, a trouvé un résultat qui diffère peu de celui que lui-même avait obtenu de la théorie, c'est-à-dire de 10″·18162 (nous ne connaissons pas exactement le résultat de Bouvard). On voit que le coefficient que nous avons déduit pour *la première fois* d'une analyse *directe* et *rigoureuse*, tient le milieu entre les deux déterminations de Mayer; il serait donc possible qu'une nouvelle discussion des anciennes éclipses, fit définitivement pencher la balance du coté de cet habile astronome et rétablit sur ce point important du système du monde, l'exacte concordance, toujours si désirable, entre la théorie et l'observation.

N. B.—Il résulte comme une conséquence désormais *incontestable* de notre analyse que la méthode jusqu'ici employée par les géomètres et les astronomes pour déterminer par la théorie le coefficient de l'équation séculaire du moyen mouvement lunaire, tendait à augmenter de 2″·50 à peu près le coefficient de cette inégalité; on doit donc accueillir avec une extrême réserve toutes les évaluations fondées sur l'emploi de ces formules défectueuses. C'est ainsi que le coefficient 12″·120 adopté par M. Hansen dans ses nouvelles tables lunaires, est évidemment trop fort de 3″ *au moins*, ce qui s' explique aisément, puisque ce coefficient est deduit, sans aucun doute, des anciennes formules de Laplace dont nous avons démontré l'insuffisance pour une détermination si délicate; mais je ferai d'ailleurs une remarque sur le procédé même par lequel M. Hansen dit avoir déduit ce coefficient numérique de ses formules analytiques. Il est évident par les résultats obtenus par M. Plana, et par le développement des formules de Damoiseau que j'ai complétement effectué, comme je l'ai dit dans un passage de ma note, que la correction que la considération des termes dépendants du carré et des puissances supérieures de la force perturbatrice, fait subir au coefficient de *l'équation séculaire*, est affectée d'un signe négatif; nous sommes parvenu par l'analyse précédente au même résultat; le coefficient de *l'équation séculaire* de quelque manière qu'on envisage la question et quelle que soit l'importance de l'altération qu'on lui fera subir, devra donc toujours tomber *audessous* de 10″·70, valeur qui lui serait attribuée par la première approximation où l'on n'a considéré que les termes dépendants de la première puissance de la force perturbatrice. D'après cela les trois valeurs donnés par M. Hansen dans le No. 7, vol. xix. des *Monthly Notices*, pour le coefficient de *l'équation séculaire* savoir 11″·93; 11″·47, et 12″·12, qu'il dit avoir obtenus par trois procédés différents, et dont la dernière est celle qu'il a employée dans les Tables de la Lune, sont donc également suspectes d'erreur, puisqu'elles tombent toutes trois audelà de la limite fixée. M. Damoiseau, dont j'ai développé la formule comme je l'ai dit plus haut, ce qui m'avait conduit comme M. Plana à une correction négative, a trouvé le coefficient déjà un peu trop fort 10″·7232 (tandisque d'après les données employées par cet astronome, ce coefficient devait tomber audessous de la limite 10″·196) il est évident que cette anomalie ne peut tenir qu' à la méthode des substitutions successives que ces deux astronomes ont employée pour résoudre leurs équations de condition, et en déterminer les arbitraires, méthode dont l'honorable M. Main a si judicieusement signalé naguère tous les inconvénients.[*] La méthode des développements en séries, plus pénible peut-être mais plus analytique, me semble offrir beaucoup plus de sécurité, et je ne partage nullement les idées de M. Hansen à cet égard[†], elle se prête à une révision continuelle des formules employées et l'exactitude d'ailleurs avec laquelle elle reproduit les coefficients de toutes les inégalités *périodiques*, à *longues périodes*, ou *séculaires* du mouvement lunaire, et donne des résultats parfaitement concordants avec ceux qu'on déduit des autres méthodes ou de l'observation, montre que l'objection qu'on avait voulu tirer contre elle de l'incertitude qui peut exister sur la convergence des séries est, du moins dans le cas général, sans fondement et sans valeur; c' était aussi à cette opinion qu' était revenu Laplace après une longue étude de toutes les méthodes employées pour résoudre les différents problêmes de la théorie de la lune, comme on peut le voir par ce qu'il dit dans un mémoire inséré dans la *Connaissance des Temps* pour 1823 (Voir page 229).

Postscript to the "Reply to various Objections which have been brought against his Theory of the Secular Acceleration of the Moon's Mean Motion."[‡] By J. C. Adams, M.A., Lowndean Professor of Astronomy and Geometry, Cambridge.

In the *Compte Rendu* of April 9, 1860, which has appeared since the foregoing paper was read, M. de Pontécoulant gives the value of the secular acceleration of the moon's mean motion, which he has obtained by taking the time as the independent variable, and which he considers to be "désormais à l'abri de toute objection."

This result, however, of M. de Pontécoulant's is the same as that which he formerly communicated to me, the error of which I have already pointed out.

M. de Pontécoulant thus describes his method, "En développant la formule qui donne l'expression de la longitude vraie en fonction de la longitude moyenne, et en n'ayant égard qu'au premier terme de ce développement, c'est-à-dire à sa

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[*] *Théorie Analytique*, Livre 7, No. 135.

[*] *Monthly Notices of the Royal Astronomical Society*, vol. xx. No. 4.

[†] Idem. vol. xix. No. 7.

[‡] For the "Reply, &c." see the preceding number of the *Monthly Notices*. The paper in the *Compte Rendu* of April 9, 1860, is to the same effect as the foregoing paper by M. de Pontécoulant, but the method is less fully explained, and the development is not carried quite so far; the letter H is there used in the place of the K of the foregoing paper.—ED.

partie non-périodique, j'en ai conclu le rapport du moyen mouvement de la lune dans son orbite troublée au moyen mouvement relatif à son orbite elliptique, c'est-à-dire à l'orbite que cet astre décrirait autour de la terre sans l'action du soleil. . . . En différentiant ensuite cette valeur par rapport à l'excentricité e' de l'orbite terrestre qu'elle renferme, . . . j'ai obtenu une expression de cette forme:

$$\frac{\delta n}{n} = \mathrm{H}\,\delta\,.\,e'^2."$$

The value of H thus obtained is

$$\mathrm{H} = -\frac{3}{2}m^2 + \frac{5337}{128}m^4$$

which, as I have shown in p. 232, is the result that would be found by differentiating the relation between n and a, and then neglecting the variation of a. The fallacy of M. de Pontécoulant's reasoning consists in his treating the moon's "orbite elliptique, c'est-à-dire, l'orbite que cet astre décrirait autour de la terre sans l'action du soleil," as if it were a real elliptic orbit with an unalterable semi-axis major, whereas the semi-axis major of the elliptic orbit spoken of by M. de Pontécoulant, which is the same quantity as that above denoted by the symbol a, is really variable, and its variation must be found by means of the differential equations in the way which I have before described.

The numerical value of the coefficient of the secular equation which M. de Pontécoulant obtains in this paper, when reduced so as to correspond with the value $-1270'' t^2$ of the integral $\int (e'^2 - \mathrm{E}'^2)\, n\, dt$ is

$$7''\cdot96$$

which, as we see, differs widely from the similarly reduced values of the coefficient according to the theories of M. Plana and M. Hansen, given in p. 236, as well as from the values obtained by M. Delaunay and myself.

After giving his formula for the secular equation, M. de Pontécoulant remarks, "En comparant ce résultat à celui que M. Plana a déduit de ses formules, on voit qu'il en diffère d'une manière notable, et que l'espèce de compensation qui devait s'établir, selon ce géomètre, entre les quantités du quatrième ordre et celles des ordres supérieurs, et qui semblait permettre de s'en tenir, comme l'avait fait Laplace, aux termes résultans de la première approximation, n'existe pas réellement. La considération des puissances supérieures de la force perturbatrice altère sensiblement, au contraire, la valeur du coefficient qu'on obtient en faisant abstraction des quantités qui en dépendent, et comme tous les termes de la formule, jusqu'aux termes du septième ordre, sont affectés d'un signe négatif, la grandeur du coefficient qu'on s'était habitué à supposer à l'équation séculaire d'après les indications de Laplace, doit être considérablement diminuée."

It is needless for me to point out how totally inconsistent these remarks of M. de Pontécoulant are with the conclusion at which he arrives in his paper in the *Monthly Notices*, "Il résulte, je pense, sans *contestation possible désormais*, de la discussion précédente, qui les formules employées jusqu'ici pour déterminer l'équation séculaire de la lune, ont toute la correction nécessaire à cet important objet."

On the Early Morning Illumination of a portion of the Mare Imbrium contiguous to Plato, and designated Newton by Schröter, but unnamed in Beer and Mädler's Map.

By W. R. Birt, Esq.

This portion of the *Mare Imbrium* of an elliptical form, just to the south of *Plato*, and about the same size as that crater, presents under the early morning illumination all the characteristics of a plain surrounded at some former period by a *ring*, some few portions of which still remain.

On the evening of April 28, 1860, I had an opportunity of observing the progression of the terminator over at least half the length of the plain, viz., from 0·25 reckoned from the western extremity at $7^{\mathrm{h}}\ 45^{\mathrm{m}}$ G.M.T. to 0·75, also reckoned from the same extremity at $9^{\mathrm{h}}\ 15^{\mathrm{m}}$ G.M.T, approximate in both cases, the major axis of the ellipse, viz., from west to east $= 1\cdot0$.

During the whole of the observations the northern part of the plain adjoining *Plato* was in shadow from a high mountain-peak and from smaller masses filling the angle between the rims of the plain and *Plato*.

Various points of light beyond or on the night side of the terminator were so disposed that the ring could be distinctly traced. On the north-east part of this imperfect ring a strong and somewhat large point of light, with a *double* summit, stood out boldly, a minute point existing between the mountain mass and *Plato*. On the south, and forming part of the ring, *Pico* was exceedingly brilliant, its shadow being scarcely discernible, its summit appeared to be double. A minute point of light eastward of *Pico* and situated on the apparent rim was seen.

On the south-west and west the rim appeared to be continuous but very low, and exhibited much of the character of the *soft ridges* so frequent on the *Maria*. Two ridges extend from this part of the rim, one towards the crater A, Beer and Mädler's map; the other towards the mountain mass B on the same map, this ridge passes between the mountain mass and *Pico*.

The floor of this plain has the appearance of being a portion of, and is evidently at the same level as the general surface of the *Mare Imbrium* to the south, it is however *under this illumination* slightly darker than the surface of the contiguous portion of the *Mare:* its colour appeared to be *greenish*, the general appearance being very similar to that of most of the *Maria* when very near the terminator. I have frequently observed the surfaces of the *Maria* to present a peculiar greenish tint when very obliquely illuminated, becoming *darker* the nearer they are to the edge of the shadow; this of course is the effect of the oblique incidence, it is however under such an illumination that the *green* tint is most perceptible. I have thus seen it in the *Mare Crisium, Mare Serenitatis, Mare Nectaris, Mare Humorum*, and others, while in a further advanced stage of the lunar day a more decidedly *greyish* tint creeps over the same surface, interspersed with numerous lighter patches giving the surfaces of the *Maria* a generally *unsettled appearance.*

Two minute points, apparently craters, were seen in the western area of the floor, they appeared to rise abruptly and nearly perpendicularly from the surface, as if they had been *surrounded* by the material forming the surface of the plain at a period subsequent to that of the formation of the craters. Several craters in the *Maria* present this feature. A very minute point of light on the floor near *Pico* may possibly be of a similar nature.

The fact that the *ring form* of this part of the *Mare Imbrium* can only be seen at early morning or just before sunset, combined with the circumstance that numerous formations of a somewhat similar character occur in various localities, mostly on the surfaces and near the borders of the *Maria*, invest all such formations with a high degree of interest. The marked difference existing between them as a class and the well-known craters *Plato, Tycho, Copernicus, Kepler* and others, seems to point, not so much to a difference of origin as to a remarkable modification which they may have undergone at some period of their history, which appears in some way to be connected with their proximity to the darker and mostly lower plains known as *Maria*.

The *greenish* tint above alluded to as being characteristic of the lunar plains is strikingly in contrast with the brilliantly *white* illumination of all mountain masses. *Pico* and the mountain on the north-east of the rim, which I propose designating by the letter S for future reference, show most brilliantly in the midst of the green tint of the plain.

Mr. Carrington exhibited a map on which were laid down the maxima and minima of frequency of the solar spots between the years 1740 and 1860. The ordinates were expressed throughout in the relative numbers of Prof. Wolf, the nature of which was briefly explained, and the principal data were taken from the *Mittheilungen* of the same astronomer. The series of numbers derived from the observations of Hofrath Schwabe and of Prof. Wolf was taken as the standard of the latter portion, and extended backwards to 1813 by the records of Stark, of Tevel, and of Mr. Adams, of Edmonton. For the latter half of the eighteenth century the series obtained from the observations of Staudacher, of Nuremberg, was taken as the standard; and it was explained how the shorter series of Oriani, Mallet, Horrebow, Zucchoni, and De Hagen, were employed as corroborating and further extending the conclusions. The principal gap was shown to lie between the years 1798 and 1813, being nearly coincident with the wars of the first French Republic and Empire,—a period during which no observations of any continuity have yet been found. Mr. Carrington explained the grounds on which Prof. Wolf was led to conclude that one maximum, about the years 1803 and 1804, preceded by a long-continued minimum, had alone occurred in this interval; and stated that, having generally examined the grounds for this result, he was inclined to concur in its correctness. The curve drawn through the summits of the ordinates showed in the clearest manner the variability of the period from minimum to minimum, while ocular inspection served to show that its mean duration was about eleven years. The height of the ordinates laid down indicated that, on the whole, the summits representing maxima were less elevated for the latter half of the eighteenth century than for the first half of the present; and it was remarked that this might be attributable to the use of less powerful telescopes during the former period, or to more careful enumeration in the latter, though no means apparently existed of deciding if the differences were real. Mr. Carrington named the two principal hypotheses which had been thrown out for the origin of the periodicity exhibited; but remarked that neither of them was, in his opinion, satisfactory, and that the curve laid before the meeting was still a grand riddle to be read, which deserved the consideration of all astronomers whose attention was directed to the subject of Solar Physics.

Notes on certain Variable Stars of Unknown or Doubtful Periods.

By Norman R. Pogson, Esq., Director of the Hartwell Observatory.

(*Communicated by Dr. Lee.*)

First approximations,—whether to a stellar parallax, a proper motion, the orbit of a comet, planet, or binary star, or to the period of a variable star,—are ever interesting; and I therefore take leave to forward a few of the latter class, arrived at from a cursory inspection of the unreduced records in my variable-star note-book; which, though liable to be slightly modified by future projection and discussion, are, nevertheless, sufficiently near for present purposes.

I will here remark, that, in compliance with a valuable suggestion of Sir John Herschel's, I now employ "*Julian dates,*" agreeably to the table on page 679 of the new edition of his *Outlines of Astronomy*, for a time co-ordinate, instead of "*Day of century.*" By this means the interval in days, between observations centuries apart, is found by a simple subtraction, while convenient tables are easily constructed, by which to change ordinary years, months, and days into seven-figure Julian dates. As the Julian date 2400000 corresponds to 1858, Nov. 16, by omitting the initial figures 24, five figures will suffice for a time record until the year 2131.

V *Virginis.*—This interesting star, discovered as a 7th magnitude by M. Hermann Goldschmidt, in May 1857, was stated in my *Ephemeris of the Variable Stars for* 1860 (page 42) to be due at a maximum, "*possibly in March and November.*" It attained its greatest brightness in the middle of February, a little above the 9th magnitude. In June 1859 it did not exceed the 10·5 magnitude. The greatest brilliancy is, therefore, unsteady, but for provisional elements we may employ the formula,

Maximum = 1860, Feb. 19 + *n* . 252 days.

R *Libræ.*—This star, announced as variable in the *Monthly Notices*, vol. xviii. p. 283, recently reappeared and passed a faint maximum (about 10·3 magnitude) towards the end of April. The interval elapsed since the previous maximum is 722 days, or nearly two years, and from frequent records of invisibility, this must be one entire period, and no multiple of a shorter time! The star is only visible for about a twelfth part of its period. In consequence of obtaining this first approximation to the elements of R *Libræ*, I have the pleasure to state, that not one of my ten variable stars now remains indeterminate in epoch or period!

R *and* S *Scorpii.*—These highly interesting adjacent variable stars, mapped by Admiral Smyth in 1837 and 1839, when engaged upon his *Celestial Cycle*, and found to be variable by M. Chacornac in 1853 and 1854 respectively, have each reappeared considerably earlier than was to have been expected from the scanty data available for future prediction. R has just passed its maximum; S was seen brightest at the end of February. Neither star exceeded the 11·5 magnitude, and S has now completely vanished. The positions of these objects have hitherto been so imperfectly known, and so unfortunately mystified by accidental errata, that I was glad to obtain some ring-micrometer comparisons of each with a neighbouring star on March 18. Both variables were then of the 12th magnitude; R increasing; S diminishing; and from their low altitudes anything but easy objects to observe, even with a dark field and the 84 power ring. Their places were,—

	Mean R.A. 1860.	Mean N.P.D. 1860.
	h m s	° ′ ″
R	16 9 19·13	112 35 11·6
S	16 9 20·22	112 32 59·5

The assumed position of the comparison star for the same epoch was deduced from Argelander's Zones, 213,19, 8·5 mag. = 382,12, 9 mag. = 386,6, 8 mag.; the mean of the three giving —

	h m s	° ′ ″
8·5	16 8 54·60	112 34 13·3

U *Geminorum*, after failing to appear on March 13 when due, has just shone forth nearly half a period behind time; but brightly and briefly as ever. It was well watched by Mr. Baxendell at Manchester, and also at Hartwell. Doubtless there will yet be found a compound periodicity even in these seeming irregularities.

It may save needless trouble and disappointment to remind observers that no complete maxima of R or S *Ophiuchi*, R *Sa-*

gittarii, or U *Capricorni*, are obtainable this year; their approximate or provisional elements being as follows:—

	Epoch of Maximum.	Julian Date.	Period in Days.
S Ophiuchi	1859, May 31	2400196	232
R Ophiuchi	1858, May 11	2399811	301
R Sagittarii	1859, Oct. 27	2400345	467
U Capricorni	1859, Dec. 22	2400401	420

In a future communication I shall request leave to propose a new and simple nomenclature for the variable stars, which is inexhaustible, and which has already received the unanimous approbation of several of the highest authorities in England.

Hartwell Observatory, May 10, 1860.

On the supposed Variability of the closest Companion of σ Orionis. By the Rev. W. R. Dawes.

When so acute an observer as Sir William Herschel overlooked an object which has since been seen with little difficulty, with the same or even less powerful means, we have at once some ground to suspect variability of light; and it becomes a matter of interest to decide the question. With reference, therefore, to the smallest and closest star of the quadruple group constituting *σ Orionis*, I beg to lay before the Society an extract or two from my astronomical note-books bearing on the point.

In the year 1829, while I was residing at Ormskirk, and previously to the erection of my 5-foot equatoreal, Mr. Lassell lent me a 7-foot Newtonian reflector of his own making, having an aperture of 7¼ inches, which I generally reduced to 6¼. With this instrument the observation is made which is thus recorded:

"1829, Dec. 15, *σ Orionis*, on meridian with a new negative eye-piece magnifying about 300 times, and aperture 6¼ inches, a *new star* was immediately recognised near *σ*. It is closer than the nearest of the bright triple set, and much fainter; but becomes far more positive with high powers. The negative power 360 showed it better still; and the highest, 600, rendered it very conspicuous. It was even steadily visible with *five inches aperture.* With 6¼ and power 120 it was barely visible —only by glimpses; with 180, just steady. But it was neat and steady with concave eye-glasses magnifying 140 and 250. If this new star had formerly been as bright, I conceive it would have been detected by myself in my former observations, if not by South with his 7-foot refractor [of 5 inches aperture]. At any rate Sir W. Herschel would have seen it."

It was not until Sir John Herschel presented me with Struve's large Dorpat Catalogue in 1833 that I was aware that this star had been seen by any other observer.

With my 5-foot refractor I obtained the following measures of position and distance, (see *Mem. R.A.S.*, vol. viii., p. 79),

1831·09, $P = 234^\circ{\cdot}84$; $D = 11''{\cdot}478$.

In the notes, I have remarked, "Bears a moderate illumination." . . . "Is seen better for a little illumination, but does not bear magnifying." . . . "Much better seen with 140 than with 226."

Subsequently I saw it well with a much smaller telescope, viz., a very fine 43-inch refractor of 2·7 inches aperture, by Peter Dollond, and have recorded as follows:—

"1843, Oct. 26. With 200 and 130 the smallest companion was occasionally discernible. I think that in fine circumstances and near the meridian it would be pretty steadily visible, perhaps more so with a lower power.

"Oct. 28. Very bad night. With power 200, the smallest companion occasionally visible. Power 77, it is sometimes steadily seen nearly throughout the field of view. I am surprised to see it so well on so indifferent a night."

With the larger telescopes I have since used it has always been visible; and any diminution in its apparent brightness I have attributed to the combined effect of its moderate altitude and varying atmospheric circumstances acting unfavourably on its delicately pale light. Yet the evidence taken altogether is perhaps sufficiently strong to warrant its being placed upon the list of *suspected* variables.

Observatory, Haddenham, Bucks,
1860, *May* 10.

A Catalogue of the Positions and Distances of 398 *Double Stars.* By Lord Wrottesley.

The following is extracted from the introductory observations which accompanied the Catalogue:—

"The positions and distances of the stars contained in the subjoined catalogue are the results of observations made by my assistants, Messrs. Simms, Philpott, and Frederick Morton, but the far greater number of the stars have been observed since the beginning of the year 1854 by Mr. Morton, to whose industry, ability, and skill I can bear ample testimony. He is a good observer and an accurate computer.

"The telescope employed is the equatoreal mentioned in my former communication, *Memoirs*, vol. xxiii., as forming part of the furniture of my observatory. It is 10 feet 9 inches in focal length, with a clear aperture of 7¼ inches; the mounting is in the usual English form; the declination and hour-circles are each 3 feet in diameter, and their verniers read to 10″ in space and to seconds of time. A wire micrometer only has been employed, the screw-heads of which are divided into 100 parts, the value of each of which is 0″·1564, and the position circle of 4 inches diameter is graduated in silver with a vernier reading to 6′ in space. The power generally employed was 450, but one of 320 was often used, and occasionally for close stars 600 or 820.

"The telescope is driven by clock-work. Notwithstanding some drawbacks it is quite capable of contending with others of its own class, and in a favourable and calm state of the atmosphere its power of showing faint companions leaves nothing to be desired. The working catalogue of stars for observation was selected from Struve's great Catalogue of 1837; and contained, 1st, stars confessedly binary; 2nd, others as to which there was strong presumptive evidence of their being so; 3d, stars of which the observations by different observers gave anomalous or discordant results. As these objects were worked out, more were added from time to time from among the finer and closer of the double stars, and from those which had been wholly observed at one epoch. Some of these latter exhibit orbital motion.

"The first of the catalogues contains the results of the observations of each star made on separate nights, the second the general mean of all the sets obtained in the same year. The catalogues are followed by notes, directing attention to any particulars worthy of remark, in reference to any star, but no notes have been made to such stars as *Castor*, *γ Virginis*, &c., which have received due attention from other observers, unless the fresh evidence seemed to require it."

Apparent Right Ascensions of the Moon's Limb and of Moon-Culminating Stars, observed with the Transit Instrument of the Observatory, Washington.

(*Communicated by Lieut. M. F. Maury, U. S. N.*)

Date.	Object.	No. of Threads.	Apparent Right Ascension. Observed.	Apparent Right Ascension. Computed.
1859.			h m s	s
Jan. 12	101 Piscium	10	1 28 14·892	
	Moon's 1st Limb	15	1 45 49·826	49·35
	β Arietis	10	1 46 52·040	
	B.A.C. 607	25	1 51 47·175	
13	ι Arietis	15	1 49 39·713	
	α Arietis	15	1 59 14·583	
	θ Arietis	15	2 10 18·057	
	Moon's 1st Limb	14	2 41 6·533	6·36
15	α Tauri	15	4 27 51·662	
	τ Tauri	13	4 33 48·903	
	Moon's 1st Limb	25	4 48 59·266	59·25
17	κ Aurigæ	5	6 6 25·668	
17	48 Aurigæ	15	6 19 32·550	
	49 Aurigæ	15	6 26 21·439	
	Moon's 1st Limb	25	7 10 39·788	40·37
	υ Geminorum	25	7 27 16·265	
	κ Geminorum	15	7 35 58·262	
18	υ Geminorum	15	7 27 16·171	
	κ Geminorum	15	7 35 58·273	
	λ Cancri	15	8 12 11·224	
	Moon's 2d Limb	15	8 21 4·219	4·09
	η Cancri	15	8 24 35·350	
	γ Cancri	15	8 35 9·650	
19	η Cancri	25	8 24 35·345	
	γ Cancri	15	8 35 9·623	
	δ Cancri	15	8 36 42·283	
	83 Cancri	25	9 11 8·642	
	Moon's 2d Limb	25	9 23 49·105	48·76
	ν Leonis	25	9 50 40·135	
	η Leonis	25	9 59 40·571	
Feb. 10	ζ Arietis	15	3 6 48·788	
	τ' Arietis	15	3 13 6·255	
	Moon's 1st Limb	15	3 17 41·175	41·07
	11 Tauri	15	3 32 22·348	
	η Tauri	15	3 39 7·529	
14	53 Geminorum	25	7 7 11·095	
	ι Geminorum	25	7 17 0·460	
	Moon's 1st Limb	25	7 43 14·466	14·53
	ψ² Cancri	5	8 1 59·905	
	χ Cancri	5	8 12 11·403	
	λ Cancri	15	8 35 9·777	
16	π² Cancri	25	9 7 29·047	
	83 Cancri	5	9 11 8·916	
	ξ Leonis	10	9 24 23·004	
	Moon's 1st Limb	25	9 47 59·392	59·74
	α Leonis	25	10 0 54·043	
Mar. 12	Moon's 1st Limb	15	6 9 47·027	47·02
	μ Geminorum	15	6 14 27·436	
	49 Aurigæ	15	6 26 20·932	
16	B.A.C. 3345	25	9 40 0·649	
	ν Leonis	25	9 51 40·571	
	α Leonis	25	10 0 54·071	
	Moon's 1st Limb	25	10 16 36·397	36·32
	45 Leonis	5	10 20 14·268	
	ρ Leonis	25	10 25 25·550	
April 9	Moon's 1st Limb	15	6 55 39·004	39·30
	ι Geminorum	15	7 16 59·644	
	υ Geminorum	15	7 27 15·542	
12	ν Leonis	15	9 50 40·339	
	Moon's 1st Limb	15	9 55 10·728	11·01
	α Leonis	15	10 0 53·860	
16	ψ Virginis	5	12 47 3·861	
	g Virginis	15	13 0 33·358	
	α Virginis	15	13 17 48·727	
	69 Virginis	15	13 19 58·843	
	Moon's 1st Limb	15	13 22 58·256	58·30
May 12	Moon's 1st Limb	15	12 12 48·099	48··4
	f Virginis	14	12 29 34·129	
	28 Virginis	13	12 34 42·768	
13	*f* Virginis	25	12 29 34·144	
	28 Virginis	25	12 34 42·735	
	Moon's 1st Limb	25	13 3 10·249	10·12
	58 Virginis	20	13 0 6·724	
	α Virginis	25	13 17 48·750	
June 9	η Virginis	15	12 12 43·725	
	Moon's 1st Limb	15	12 47 16·339	16·55
	58 Virginis	15	13 10 6·589	
	α Virginis	15	13 17 48·712	
11	86 Virginis	15	13 38 28·352	
	89 Virginis	15	13 42 15·539	
	Moon's 1st Limb	15	14 30 11·095	11·28
	20 Libræ	15	14 55 52·662	
	ι' Libræ	15	15 4 14·497	
	β' Scorpii	25	15 57 17·988	
17	*h*² Sagittarii	25	19 28 11·082	
	f Sagittarii	25	19 38 11·556	
	Moon's 2d Limb	25	20 4 50·706	50·59
	σ Capricorni	25	20 11 18·651	
	ρ Capricorni	25	20 20 52·188	
July 9	α² Libræ	25	14 43 7·889	
	20 Libræ	15	14 55 52·545	
	ι' Libræ	15	15 4 14·475	
	Moon's 1st Limb	25	15 7 30·455	30·59
	39 Libræ	25	15 28 31·729	
	42 Libræ	15	15 32 0·517	
	λ Libræ	15	15 45 12·597	
11	Moon's 1st Limb	25	16 59 37·512	37·52
	A' Ophiuchi	25	17 6 44·825	
	θ Ophiuchi	25	17 13 25·129	
	d Ophiuchi	25	17 18 25·379	
12	θ Ophiuchi	10	17 13 25·110	
	d Ophiuchi	5	17 18 25·330	
	Moon's 1st Limb	25	17 56 22·386	22·59
	μ' Sagittarii	25	18 5 23·944	
	δ Sagittarii	5	18 12 2·365	
	λ Sagittarii	20	18 29 20·365	

The clock-stars used are those of the *British Nautical Almanac* for the mean equinox of 1860·0.

The moon's computed place is taken from the *American Ephemeris*.

Results of Meridional Observations of Small Planets, and Phenomena of Jupiter's Satellites, observed at the Royal Observatory, Greenwich, in the month of April, 1860.

(*Communicated by the Astronomer Royal.*)

Iris ⑦.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, April 2	8 15 40·9	9 1 14·27	80 35 31·28
3	8 11 57·9	9 1 27·20	80 34 8·18
5	8 4 36·7	9 1 57·89	80 31 44·99
7	7 57 21·3	9 2 34·49	80 29 41·18

Hygeia ⑩.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, April 12	9 28 30·2	10 53 41·10	88 22 2·22
17	9 7 23·4	10 52 13·57	88 8 27·89
28	8 22 52·0	10 50 56·98	87 48 46·18

Victoria ⑫.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, April 2	12 10 53·0	12 57 5·04	105 58 27·87
9	11 36 47·7	12 50 30·00	104 58 47·74
16	11 2 51·2	12 44 3·77	103 52 56·13
17	10 58 2·2	12 43 10·57	103 43 11·49
27	10 10 50·2	12 35 16·37	102 5 33·61

Egeria ⑬.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, April 2	8 7 42·2	8 53 14·31	50 49 11·73
7	7 50 33·0	8 55 45·06	51 45 6·95

Thetis ⑰.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, April 16	12 53 14·5	14 34 45·23	94 36 36·72
21	12 29 26·9	14 30 36·54	94 9 38·73
28	11 55 47·9	14 24 27·89	93 35 31·25

Urania ㉚.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, April 12	9 29 35·2	10 54 46·30	85 36 53·02
16	9 12 27·5	10 53 21·99	85 27 13·57
17	9 8 14·4	10 53 4·77	85 25 8·13

Ariadne ㊸.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, April 7	9 10 44·9	10 16 10·11	85 7 40·39
10	8 58 5·9	10 15 18·74	84 57 12·59
17	8 29 45·2	10 14 29·24	84 38 25·08

Doris ㊽.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, April 7	9 17 9·2	10 22 35·43	82 41 13·33
17	8 36 35·4	10 21 20·56	82 10 31·38

No occultations of stars by the moon were observed during the month of April 1860.

Eclipses and Transits of Jupiter's Satellites.

Day of Observation.	Satellite.	Phenomenon.	Mean Solar Time.	Observer.
1860.			h m s	
April 1	III.	Ecl. disapp.	8 42 27·3	E.
1	I.	Ecl. reapp.	9 44 46·0	E.
2	IV. shadow	Ingress, bisected	9 45 31·8	J. C.
7	I.	Ingress, last cont.	11 1 14·7	E.
16	I.	Egress, last cont.	9 40 37·1	C.
17	I.	Ecl. reapp.	8 4 26·5	E.
30	II.	Ecl. reapp.	11 27 41·7	K.

The initials E., C., J. C., and K., are those of Mr. Ellis, Mr. Criswick, Mr. Carpenter, and Mr. Kerschner.

Observations of the Satellite of Saturn, Hyperion.

By W. Lassell, Esq.

I send herewith a few observations of *Hyperion*, the recently discovered satellite of *Saturn*, which I acknowledge I have of late somewhat neglected.

I do not find that any observations of this satellite have been published since the rather extensive series I obtained in Malta (*Monthly Notices*, vol. xiii. p. 181), nor have I learned that *Hyperion* has ever been seen, except in Mr. Bond's telescope and mine. Perhaps the possessors of some of the admirable refractors which have lately been constructed may now give it a passing notice. I find it faint and difficult to observe, except in a clear and dark sky, with which I am not here too frequently favoured.

The last observation (April 25th) was made with the 4-foot equatoreal; the haziness of the sky and the neighbourhood of the young moon rendering it unlikely that *Hyperion* would be measurable in the 2-foot equatoreal.

The period of revolution I deduce from the enclosed, compared with the Maltese observations, is 21·286 days.

Referring back to the measures of 1848, immediately upon the discovery of the satellite, and concluding its saturnicentric longitude at 1848, Sept. 19·76, to have been 82°, the period would be 21·284 days.

As somewhat connected with the subject, I may take the opportunity of remarking that I do not find that the generally received period of the closest satellite, *Mimas*, satisfies my observations. Comparing those made in Malta with some recently obtained, I deduce a period of 0·9427843, fraction of a day.

The measurements are referred to the apparent transverse axis of *Saturn's* ring produced, and to the centre of the planet.

		″
1860, April 14·406,	Satellite east of Saturn's centre	204·2
	and north of axis of ring (estimated)	35·0
15·385,	Satellite east of Saturn's centre	166·9
	and north of axis of ring	43·5
16·417,	Satellite east of Saturn's centre	84·9
	and north of axis of ring	50·1
20·458,	Satellite west of Saturn's centre	151·6
	and north of axis of ring	39·1
21·417,	Satellite west of Saturn's centre	192·2
	and north of axis of ring	33·5
23·396,	Satellite west of Saturn's centre	213·5
	and south of axis of ring	3·7
24·437,	Satellite west of Saturn's centre	198·5
	and south of axis of ring	16·0
25·427,	Satellite west of Saturn's centre	143·9
	and south of axis of ring	32·0

Bradstones, Sandfield Park, near Liverpool,
May 10, 1860.

On the Value of the Constant of Aberration as determined by Eight Years' Observations of γ Draconis, made with the Reflex Zenith Tube at the Royal Observatory, Greenwich. By the Rev. Robert Main, M.A., President.

This paper, the author remarks, must be taken in connexion with a paper by him printed in the twenty-fourth volume of the *Memoirs* of the Society, which gave the discussion of the observations from 1837 to 1848, made with the 25-foot zenith tube of the Royal Observatory, for the determination of the constants of aberration and nutation. The results then found were unsatisfactory, the value of the constant of aberration being considerably smaller than the received value, and the value of the annual parallax being a negative quantity amounting to nearly four-tenths of a second of space. It became, therefore, a matter of interest to determine what result for the aberration would be given by the eight years' observations which have been since that time made with the reflex zenith tube, since from the simplicity of construction of the instrument no errors are to be dreaded arising from mechanical or instrumental causes.

For the tables on which the computations are based reference is made to the preceding paper, as well as for a general explanation of the methods used in forming and solving the equations of condition. These are 106 in number, arising from groups formed from 486 observations, and contain three unknown quantities, namely, the correction to the assumed mean north zenith distance for the epoch chosen; the correction to the assumed value of the constant of aberration; and the constant of annual parallax. The equations are solved by the method of least squares, and the result for the constant of aberration is tolerably satisfactory, the resulting value being 20″·34. There results, however, a negative parallax amounting to 0″·24, which is larger than might have been expected, and would seem to denote some injurious effect, from atmospheric or other circumstances, on the observations.

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*Tables of the Developments of Functions in the Theory of Elliptic Motion.** By A. Cayley, Esq.

The notation made use of is as follows; viz., r, f, a, e, g denote

r, the radius vector,
f, the true anomaly,
a, the mean distance,
e, the excentricity,
g, the mean anomaly,

so that

$$\frac{r}{a}, = \text{elqr}\,(e, g), \text{ and}$$
$$f, = \text{elta}\,(e, g),$$

are known functions of e, g. Moreover, x denotes the periodic part of the quotient radius $\frac{r}{a}$, and y the equation of the centre, or periodic part of the true anomaly f; so that

$$\frac{r}{a} = 1 + x,$$
$$f = g + y,$$

and (x, y) are therefore also known functions of e, g.

Formulæ for the development in multiple cosines or sines, up to the terms in e^7, of the functions

$$(x^0, x^1, \ldots x^7) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, j\,y,$$

where j is an indeterminate symbol, are given by Leverrier in the *Annales de l'Observatoire de Paris*, tom. i. (1855), pp. 346–348. It occurred to me that it would be desirable to form tables such as are contained in the present memoir, of various functions of the forms

$$x^m \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, j\,f,\ \left(\frac{r}{a}\right)^{\pm m} \begin{smallmatrix}\cos\\ \sin\end{smallmatrix}\, j\,f,$$

obtainable from Leverrier's formulæ by assigning particular integer values to the symbol j. The calculations were performed under my superintendence by Messrs. Creedy and Davis; and I have to acknowledge my obligation for a grant to defray the expenses made to me out of the Donation Fund at the disposal of the Council of the Royal Society.

A cosine series is in general represented in the form

$$\Sigma\, [\cos]^i \cos i\,g$$

where $[\cos]^i$ is a symbol used to denote the coefficient of $\cos i\,g$ in such a series, i extends from $-\infty$ to $+\infty$, and the coefficients $[\cos]^i$ satisfy the condition

$$[\cos]^{-i} = [\cos]^i;$$

and in like manner a sine series is represented in the form

$$\Sigma\, [\sin]^i \sin i\,g$$

where $[\sin]^i$ is a symbol used to denote the coefficient of $\sin i\,g$ in such a series, i extends from $-\infty$ to $+\infty$, and the coefficients $[\sin]^i$ satisfy the condition

$$[\sin]^{-i} = -\,[\sin]^i, \text{ and in particular}$$
$$[\sin]^0 = 0.$$

In the case of a pair of corresponding functions $x^m \cos j\,f$ and $x^m \sin j\,f$, or $\left(\frac{r}{a}\right)^{\pm m} \cos j\,f$ and $\left(\frac{r}{a}\right)^{\pm m} \sin j\,f$, one of them expanded in the form

$$\Sigma\, [\cos]^i \cos i\,g,$$

and the other in the form

$$\Sigma\, [\sin]^i \sin i\,g,$$

the sums and differences of the corresponding coefficients $[\cos]^i$, $[\sin]^i$, are represented by the notation

$$[\cos \pm \sin]^i,$$

that is $[\cos + \sin]^i$ stands for $[\cos]^i + [\sin]^i$, and similarly $[\cos - \sin]^i$ for $[\cos]^i - [\sin]^i$; and it is clear that we have

$$[\cos + \sin]^{-i} = [\cos - \sin]^i,\ [\cos \pm \sin]^0 = [\cos]^0.$$

These coefficients $[\cos \pm \sin]^i$ are for many purposes equally useful with the coefficients $[\cos]^i$, $[\sin]^i$, and they are here tabulated accordingly.

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* The Tables will be printed *in extenso* in the *Memoirs* of the Society Ed.

The functions calculated are as follows :—

$$\left(x^1,\ x^2\ \ldots\ldots\ x^7\right) \qquad ,\ [\cos];$$

$$\left(x^0,\ x^1,\ x^2\ \ldots\ldots\ x^7\right)\begin{matrix}\cos\\ \sin\end{matrix} jf, \qquad j=1 \text{ to } j=7,\ [\cos],\ [\sin],\ [\cos\pm\sin];$$

$$\left(\left(\frac{r}{a}\right)^{+4},\ldots\left(\frac{r}{a}\right)^{+1},\ \log\frac{r}{a},\left(\frac{r}{a}\right)^{-1}\ldots\left(\frac{r}{a}\right)^{-5}\right) \qquad ,\ [\cos];$$

$$\left(\left(\frac{r}{a}\right)^{+4}\cdot\ \cdot\ \left(\frac{r}{a}\right)^{+1},\quad \left(\frac{r}{a}\right)^{-1}\ldots\left(\frac{r}{a}\right)^{-5}\right)\begin{matrix}\cos\\ \sin\end{matrix} jf, j=1 \text{ to } j=5,\ [\cos],\ [\sin],\ [\cos\pm\sin].$$

all the developments being carried up to e^7, the limit of the formulæ from which they are deduced.

Observation of the Occultation of Jupiter by the Moon, on May 24, 1860. By Capt. W. Noble.

The brilliancy of the sunlight, and the disturbance of the atmosphere, necessitated the employment of a low power, to obtain a sharp and distinct view of the planet; I accordingly observed with a power of 42. I cannot speak with any great decision as to the instant of first contact, but it was perceptible to me at $8^h\ 45^m\ 32^s$ L.S.T.; the final disappearance took place at $8^h\ 46^m\ 32^s\!\cdot\!3$ L.S.T.

The first indication of the emergence of the planet was visible

at $9^h\ 57^m\ 34^s$ L.S.T., and its complete emersion took place
at 9 58 33 L.S.T.

When *Jupiter* had reappeared to the extent of about two-thirds of his area I received the impression for several seconds of a dark shadow or line bounding the moon's limb where it crossed the planet, as described at pp. 70, 76, 77, and 80, of vol. xvii. of the *Monthly Notices*, by Messrs. Lassell, Mann, Dawes, Simms, &c. &c. It is a somewhat anomalous fact that I witnessed this same phenomenon during the occultation of *Mars* by the moon on 13th October, 1857; while, on the occasion of the occultation of *Saturn*, on the 8th of May last year, nothing whatever of the sort was visible (*Monthly Notices*, vol. xviii. p. 27, and vol. xix. p. 242). The moon and *Jupiter* were so apparently equal in brightness for a few seconds after the planet began to emerge, that he looked like a distorted portion of the lunar limb; as, however, the moon passed gradually off his disk he became sensibly paler; and when he was fairly clear of our satellite, his light was noticeably fainter than hers, his two principal belts were well seen throughout.

The telescope employed was my Ross equatoreal of 4·2 inches aperture and 61 inches focal length, with a power (as stated above) of 42, adjusted on *Jupiter*.

***Forest Lodge, Maresfield, Sussex, May* 29, 1860.**

On Projection in Lunar Occultations.
By the Rev. T. W. Webb.

The attention of astronomers having recently been directed to the phenomenon of projection on the lunar disk at the time of occultations, I beg to point out an instance, which, as far as I know, has not been alluded to; where this strange anomaly seems to have been perceived by the unassisted eye. The date of this occurrence was 1794, March 7, and it is described at length in the *Philosophical Transactions* for that year. I have not at present access to the volume, but from an abstract in my possession it appears that three persons at Norwich, and one in London, saw a star that evening in the dark part of the moon, which had not then attained the first quadrature; and from the two representations which are given, the star must have appeared very far advanced upon the disk. On the same evening there was an occultation of *Aldebaran*, which Dr. Maskelyne thought a singular coincidence, but which would now be acknowledged as the cause of the phenomenon.

In the *Cométographie* of Pingré, tom. i. p. 321, is the following passage. [A.D.] "547. Une étoile entra sur le disque de la lune. Quelques écrivains rapportent ce phénomène à l'an 553; d'autres le font arriver entre 547 et 549." The references are to Aimoinus, the chronicles of St. Denys, Gregory of Tours, and Toromachus. This looks like a similar occurrence to the one just mentioned; but in the uncertainty of the date it would be a fruitless task to endeavour to ascertain whether the occultation of any great star might possibly be thus recorded.

Without presuming to offer any opinion as to the cause of this anomaly, the inconsistencies of which seem to present a great obstacle to any mode of explanation, I may be permitted to mention that the effect of irradiation upon an object out of focus is greater than might be supposed by those whose vision is perfect; of this I have been made aware through my own near sight, in consequence of which luminous spaces are enlarged at the expense of adjacent dark ones, to an extent which might hardly have been anticipated.

***Hardwick Parsonage, May* 28, 1860.**

RECENT PUBLICATIONS.

An Account of the Total Eclipse of the Sun, on September 7, 1858, *as observed near Olmos, Peru.* By Lieut. J. M. Gilliss. (Smithsonian Contributions to Knowledge.)

The author having proposed to Professor Henry, the Secretary of the Smithsonian Institution, to proceed to the west coast of South America to observe the eclipse of September 7, 1858, and his offer having been accepted, no time was lost in making the necessary arrangements for carrying his project into effect.

Prof. A. D. Bache, Superintendent of the U.S. Coast-Survey, contributed the instruments for determination of time, and the geographical position of the station, and a tent for shelter at night; Commander M. F. Maury, Superintendent of the U.S. Naval Observatory, furnished two excellent pocket-chronometers; and Mr. Henry Fitz, of New York, who had made the object-glass of the equatoreal of the U.S. Naval Astronomical Expedition, with great liberality specially completed and lent a 4¼-inch achromatic telescope, mounted on an equatoreal stand suitable for all latitudes.

The author was accompanied by a young friend from New York, Mr. C. H. Raymond, who was desirous of observing the phenomenon. They left New York on the 5th of August, and traversing the Isthmus of Panama, reached Payta in Peru on the 21st of the same month.

On the day of the author's arrival at Payta he received, through Capt. D'Aignan of the French war-steamer Mégère, a message from the admiral of the French fleet in the Pacific, courteously offering to convey him in the Mégère to any point on the coast, or to facilitate in any other manner in his power the observations which were the object of his journey.

A residence of eight days at Payta having shown that the probabilities of a favourable morning near the coast were against them, the author resolved to proceed to the interior and select some favourable place for observing the phenomenon.

It was arranged that Capt. D'Aignan should proceed with the Mégère and observe the phenomenon at Punta de la Aguja, some 60 miles south from Payta, where the central line of the shadow would first touch the Continent. M. De La Pinelas, one of the officers of the Mégère, was selected by his commander to make the physical observations.

Having ascertained from a recently published map that Olmos, a town within the outer Andes range, would certainly be included by the moon's shadow, and possibly be very near to its central line, he directed his course thither. The party consisted of the author, Mr. Raymond, two muleteers, one of whom acted as guide, and eight animals, two of which carried provisions for their subsistence. They arrived at Olmos on the morning of the 4th of September. The station chosen for observing the phenomenon was situated on a small level spot, on the northern ridge of a spur from the Andes, the line of which from the main chain is a segment of a circle. From a mean of meridian and circum-meridian altitudes of *α Lyræ*, *α Cygni*, and *α Gruis*, made at camp on the nights of the 5th and 6th of September, with sextant and artificial horizon, its latitude was found to be S. 6° 0′ 2″. The longitude was obtained by the aid of the chronometers. Its definitive value, as subsequently computed by Mr. F. G. Hesse, Assistant U.S. Astronomical Expedition, was found to be 79° 42′ 53″·4 West of Greenwich.

The following account of the phenomena is given in the author's own words:—

"In anticipation of the difficulty which would attend micrometrical measures of either kind during the very brief period the total eclipse would continue, a cross of wires, at right angles to each other, was inserted in a perforated card disk, and fitted to the eye-piece. Then, to facilitate the location of, and approximate to the elevation of such solar protuberances as might become visible, a diagram was prepared. Three concentric circles were drawn upon a page of the note-book. The inner one, to represent the moon, was of 4 inches diameter, a black continuous line; the outer ones, respectively, one-eighth and one-quarter of an inch greater, but with broken lines. The inner circle was divided into equal segments of 22½° each, the vertical diameter on the page to correspond with the zenith and nadir of the station. The outer circles were to facilitate estimations in altitude of the protuberances, by comparison with the visible lunar diameter, all of which was in the field of the telescope.

"The night of the 6th and until daylight of the morning of the 7th September was bright and faultless; but just before sunrise, a few banks of heavy cumuli formed over and near the distant mountains to the eastward, the part of the sky least obscured being in the direction of the sun. Some of the intermediate peaks became enveloped in fog-banks at the same time, whilst thin cirri stretched to near the zenith. The wind was then from the westward, and the atmosphere humid, under the influence of which the lower strata of clouds increased in numbers and density, and the sun continued invisible until after the eclipse had made considerable progress. By $6^h\ 30^m$ all the fog-banks to the north and west were drifting towards the valley of Olmos, and at 7^h there was no portion of the sky entirely unclouded. Whenever the sun became momentarily visible during that period, the cusps could not be satisfactorily seen because of the masses of vapour continuously rolling across its disk. At about $7^h\ 15^m$ there was a striking change; the whole mass of cumuli to the eastward was lifted towards the zenith, leaving that portion of the sky clear, and it was only by close examination that a delicate film of vapour could be detected between us and the sun. At this period, the general light did not greatly differ in intensity from that of the moon when nearly full, though its colour resembled more that from burning spirit than that of the sun. Both intensity and colour changed under the feeble and gloomy illumination with perceptible rapidity, and for a minute before the first internal contact of the limbs, the hills near us appeared of a *black* hue.

"At from twelve to fifteen seconds before the beginning of totality the entire lune, then perhaps 35° in extent, broke up into masses of unequal length, showing detached portions wholly separated from the rest by dark lines. The occurrence was so sudden as to startle me, and although within the ten following seconds, the remaining visible fragments had become brilliant globules of nearly equal size, they differed from Baily's beads, as seen by me during the annular eclipse of September, 1838, by the darker spaces which separated them from one another.

"When the total obscuration took place at $7^h\ 30^m\ 42^s{\cdot}5$, four masses of luminous clouds became immediately visible at as many positions of the lunar circumference. Perhaps that to the left on the drawing was the first one seen, because my attention was directed there in watching the disappearance of Baily's beads, but I cannot say that they were not all simultaneously perceptible. The screen of glass was hurriedly removed, and in the brief instant of doing so I found, to my surprise, that all the phenomena were distinctly visible to the unassisted eye. Every effort has been made by the artist to represent in the accompanying lithograph, drawn from the sketch and notes, the scene then before me, and which still remains strongly impressed upon my memory.

"Of the four cloud-masses which had become visible, that on the upper border was much the most extended and serrated. It was more than 30° in extent, with an altitude, in the highest part, of about 30″. All of them were of as nearly the form and at the relative positions given in the plate, which the most careful estimations enabled me to assign to them, and sketch roughly in the note-book, whilst they were still in sight. Neither of them exceeded 1′ or 1′ 10″ in height. I could not discern any rose or pink colour on either of them, but they all resembled irregular masses of illuminated clouds of leaden hue, fringed with bright light, more especially at the edges furthest from the sun. Surprised at the absence of red—a colour I fully expected to witness—and for the moment supposing my physical condition might have some influence, Mr. Raymond was requested to note the colours carefully.*

"A corona-light flashed out at the instant of totality. It extended furthest from the sun, in lines drawn from the centre through the solar clouds, but was nowhere traceable more than 15′ or 16′ beyond the lunar disk. There were no radial streamers, or bundles of rays, but only a uniformly diminishing, and slightly orange-tinted light, whose brightness and extent were apparently influenced by the mist-film, as the colour of the clouds also may have been. Beyond the corona-light, the colour of the sky was of a grayish-black.

"It was a far more imposing sight without than with the telescope; and long as has been my experience in the observation of celestial phenomena, and calm and unimpassioned, at such times, as my temperament has become, the sublime majesty of the scene thrilled me with excitement and humble reverence. Nor was it less effective upon others. Two citizens of Olmos stood within a few feet of me, watching in silence, and with anxious countenances, the rapid and fearful decrease of light. They were wholly ignorant that any sudden effect would follow the total obscuration of the sun. At that instant, one exclaimed in terror, '*La Gloria!*' and both, I believe, fell to their knees, filled with awe. They appreciated the resemblance of the corona to the halos with which the old masters have encircled their ideals of the heads of our Saviour and the Madonna, and devoutly regarded this as a manifestation of the Divine presence.

"Every second of the time during which the total obscuration continued—*only sixty and a half*—was fully occupied in storing my memory with facts, and in tracing outlines of the solar clouds and corona. They did not permit me to turn away for the observation of collateral phenomena, before it was necessary to watch for the sun's reappearance. This took place at $7^h\ 31^m\ 43^s$. The emersion was sudden. But though the limb was then sharp, and without the least appearance of dislocation, during the next two or three seconds there were faint vertical striæ over the lune which gave to it a slight oscillating motion. Assuredly neither the illuminated clouds nor the corona disappeared at the actual instant of the return of light, but there was a brief period afterward when their images were still upon my retina.

"Though Mr. Raymond found a candle necessary to enable him to read satisfactorily the seconds-dial of the chronometer, and vernier-scale of the barometer, the darkness during totality could not have been very great, for my sketches were completed without the aid of artificial light. For some minutes previous all work in the valley below us had ceased, and even the strains of martial music, which the Governor of Olmos employed to cheer labourers digging for water, two or three miles from the town, were no longer audible. Superstition is still dominant here, and we could hear the solemn toll of the church-bell, whose sounds were intended to drive evil spirits from its vicinity. Neither at Olmos nor Piura did any *enceinte* woman leave her room during the eclipse, whilst some from curiosity, but more through fear, were in the streets, yet not daring to look upon the sun, lest malady should befall them. The sombre greenish light gave them the appearance of corpses, and they apprehended that a plague might be visited upon them. Afterwards the muleteers told us that their animals stopped eating, and huddled together in evident alarm."

The author has given a drawing of the phenomenon seen around the moon's limb during the totality of the eclipse. We certainly do not find in it such decided indications of complexity of structure as the Brazilian drawing presents. He has also appended to his own observations brief accounts of the eclipse as observed by Mr. Raymond and the French observers.

"*Observations of Mr. Raymond.*—Four solar protuberances were plainly visible to the naked eye, but they appeared to be of very nearly the same colour as the corona, making it look more intensely bright. There was no pink or red colour exhibited." Mr. Raymond looked at the sky to ascertain if any stars were visible, but clouds effectually concealed from view all those which might have been perceived.

"*Observations of the French Officers.*—Capt. D'Aignan states that although the sky was somewhat cloudy, several stars were seen during the totality of the eclipse. The obscurity was very sensible, and it seemed to pass off more rapidly than it came on. M. Pinelais had translated all the important portions of Mr. Carrington's pamphlet, containing 'Suggestions to Observers, &c.,' and made copies of its diagrams previous to the author's departure to the interior. In his brief account of the phenomenon he states that the spots appeared of a fine bright rose colour."

Précis de l'Histoire de l'Astronomie aux Etats-Unis d'Amérique, par Ed. Mailly, Aide à l'Observatoire Royal de Bruxelles (extracted from the *Annuaire de l'Observatoire Royal de Bruxelles pour l'Année* 1860), 12mo, Brussels, 1860, (88 pages). This little volume contains an interesting sketch of the history of astronomy in North America, from the foundation, in the year 1743, of the American Philosophical Society at Philadelphia, and the transit of *Venus* in 1769 (the occasion of the first recorded astronomical enterprise in the United States) up to the present time. The author mentions, as existing in 1856, inclusive of the two great observatories of Washington and Cambridge—twenty-five astronomical observatories (public and private)—in the United States; sixteen of them founded since the year 1843.

Annales de l'Observatoire Impérial de Paris. Publiées par U. J. Leverrier, Directeur de l'Observatoire, tom. v. 4to, Paris, 1859.

This volume contains the theory and tables of *Mercury* by M. Leverrier; the discrepancy as regards the secular motion of the perihelion which is found to exist between theory and observation, led, as is well known, to the suggestion by M. Leverrier of the existence of a planet or group of small planets interior to *Mercury*. The volume contains also a memoir by M. Foucault, on the "Construction of Telescopes with Silvered Glass Mirrors;" a memoir by M. Serret, "On the Theory of the Motion of the Earth about its Centre of Gravity;" a note by the same author, "On the Theory of the Elliptic Motion;" and "Researches on the Atmospheres of Comets," by M. Roche. The article of the *Comptes Rendus* which contains Dr. Lescarbault's letter in relation to his observation of a planet, and M. Leverrier's remarks on it, is reproduced in the appendix.

The Berlin *Astronomisches Jahrbuch* für 1862 has appeared. The Appendix contains a memoir by M. Encke on mechanical quadratures, and also a list of new elements and ephemerides for the year 1860, of fifty of the small planets.

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* The author had a severe attack of ague on the 4th of September, and on the 6th was labouring from excessive fever, accompanied by violent pains in the head and vertebræ. Before daylight of the 7th, however, he was up, and fortunately felt quite able to observe the phenomenon.

VOL. XX. *June* 8, 1860. No. 8.

The Rev. R. MAIN, President, in the Chair.

Sanford Gorton, Esq., Stamford Villas, Hackney;
Francis Abbott, Esq., Hobart Town, Tasmania; and
Louis P. Casella, Esq., 23 Hatton Garden,

were balloted for and duly elected Fellows of the Society.

Réponse aux Observations présentées par M. Adams sur différentes Objections élevées contre sa Théorie de l'Accélération du moyen Mouvement de la Lune. Par M. G. de Pontécoulant.

M. Adams, dans une note insérée aux *Monthly Notices* (tome xx. No. 6), a essayé de repousser à la fois les observations critiques qui avaient été soulevées contre son analyse par MM. Plana et Hansen, et par moi ; je ne repondrai ici qu'à ce qui m'est personnel ; les deux savants astronomes auxquels j'ai l'honneur de me trouver accolé en cette circonstance, défendront leur cause beaucoup mieux que je ne pourrais faire ; je dois d'ailleurs tâcher d'être aussi succinct que possible sur un sujet dejà plusieurs fois traité dans ce recueil, et je laisserai à l'intelligence du lecteur le soin de suppléer à ce que j'aurais pu omettre dans la seule crainte de le fatiguer par des répétitions fastidieuses.

M. Adams s'excuse d'abord de n'avoir pas répondu plutôt aux observations contenues dans ma note insérée dans le numéro des *Monthly Notices* de Juillet dernier, parceque, dit-il :—"I had some hope that M. de Pontécoulant might be led to see and acknowledge *the errors* into which he had fallen." Nous étions sur ce point encore une fois bien loin de nous entendre, car j'avais cru de mon coté que le silence de M. Adams provenait de ce qu'il avait reconnu lui-même les fautes que je lui avais signalées et je me reprochais dejà de les lui avoir indiquées peut-être avec trop peu de ménagements. Je vois que je m'étais trompé, je me trouve donc forcé de reprendre la question en entier, et de montrer que M. Adams, en prétendant redresser les *erreurs* contenues dans ma première note, en ajoute lui-même à celles de son mémoire de 1853 de nouvelles, qui montrent l'oubli de tous les principes suivis par les géomètres qui se sont occupés jusqu' ici de la théorie de la lune, et ce qui est plus grave encore, qu'il commet une faute essentielle contre les *premiers principes du calcul infinitésimal*, dans l'établissement de la formule qui sert pour ainsi dire de fondement à toute cette discussion. Comme il arrive toujours lorsqu' entré dans une mauvaise voie on se refuse à écouter les avertissements qui vous sont donnés pour en sortir.

1°. A l'argument sur lequel M. Adams s'appuie de nouveau, tiré de la concordance de ses résultats avec ceux qu'a trouvés de son coté M. Delaunay, par un procédé de calcul différent du sien, je repondrai ce que j'ai dejà en plusieurs fois l'occasion de dire à ce dernier, c'est que cet accord prouve seulement que ces messieurs sont partis, si ce n'est des mêmes formules, du moins des mêmes principes, et que si par conséquent nous prouvons que l'un des deux s'est trompé, il en résultera évidemment que l'autre est tombé dans la même erreur ; car on ne saurait supposer qu'une formule exacte et une formule fautive puissent conduire au même résultat, ce serait ébranler les bases de toutes les sciences fondées sur l'analyse mathématique.

2°. J'ai dit, ou plutôt j'ai fait préssentir, à la fin de ma note citée plus haut, que lorsqu'on calcule le coefficient de l'équation séculaire du moyen mouvement lunaire par les formules où la longitude vraie de la lune était prise pour la variable indépendante et par celles où l'on avait choisi le temps t pour cette variable, on obtenait des résultats analytiques très différents l'un de l'autre, bien que leurs valeurs numériques dussent sans doute présenter entre elles très peu d'écart ; cette assertion a paru beaucoup surprendre M. Adams, bien qu'il fut arrivé le premier à cette même conclusion, comme on le voit par le dernier paragraphe de son mémoire de 1853, la seule différence entre nous étant que M. Adams a cherché à faire disparaître cette discordance par des moyens qui lui sont propres, et que nous aurons à apprécier bientôt, tandis que moi je l'ai acceptée comme un résultat irréfutable du calcul. M. Adams va plus loin, dans un autre passage de son dernier écrit, il dit très nettement :—"I confess that it appears to me *absurd* to suppose that the result of an investigation ought to be different when a different independent variable is employed in obtaining it." Prenons un exemple bien simple et qui se rattache, comme on va le voir, à notre sujet; supposons qu'on veuille déterminer la partie non périodique de la fonction $\frac{1}{r}$, en désignant par r le rayon vecteur de la lune, et en faisant abstraction des forces perturbatrices; si l'on a pris pour variable indépendante la longitude vraie, on aura par les formules du mouvement elliptique $\frac{1}{r} = \frac{1}{a}(1+e^2+e^4+e^6+\text{etc.})$ et si l'on choisit pour variable le temps t on aura simplement $\frac{1}{r} = \frac{1}{a}$; ces deux valeurs sont très différentes, quoiqu'elles expriment la même chose dans les deux cas. Que l'on veuille bien réfléchir maintenant que le coefficient de l'équation séculaire du moyen mouvement lunaire, dépend précisement de la combinaison de la partie non périodique de la fonction $\frac{1}{r}$ dans l'orbite troublé, avec la partie correspondante de la fonction qui exprime la différentielle de la longitude vraie, prise par rapport au temps t, ou reciproquement, on verra que le fait que nous avons avancé, n'avait rien qui put exciter la surprise que M. Adams en a témoignée. Mais d'ailleurs il faut observer que ce qu'on appelle communement *l'équation séculaire* de la lune, n'est pas précisement la même chose dans les deux cas que nous venons de considérer, les formules directes en effet, donnent immédiatement et telle qu'elle doit être employée dans les tables, *l'équation séculaire* qui affecte le moyen mouvement de la lune, dans l'expression de sa longitude vraie en fonction de la longitude moyenne ou du temps ; les formules de Laplace, au contraire, donnent *l'équation séculaire* qui affecte la longitude vraie dans l'expression de la longitude moyenne en fonction de cette même longitude, on en déduit ensuite, par une inversion et des suppositions tout-à-fait arbitraires, l'expression de *l'équation séculaire* du moyen mouvement proprement dite, il ne serait donc pas bien surprenant que deux quantités qui ne sont pas absolument identiques, fussent exprimées analytiquement d'une manière différente ; et enfin, c'est là, comme l'a dit Laplace en une autre occasion, une de ces questions où les inductions les plus vraisemblables du raisonnement doivent céder à un guide plus sur et plus infaillible, les déductions du calcul.

3°. Voyons maintenant comment M. Adams a prétendu faire accorder les résultats que nous avons obtenus par les deux méthodes que nous avons successivement employées dans la recherche du coefficient de l'équation séculaire, c'est-à-dire la méthode suivie par MM. Plana et Damoiseau, et celle que nous proposons définitivement de lui substituer, comme plus directe et plus exacte. C'est ici qu'il est curieux d'observer à quel point la préoccupation d'une idée qu'on veut faire triompher, peut égarer un esprit qui s'est montré d'ailleurs éclairé et judicieux dans d'autres occasions.

M. Adams donne dans sa note (*Monthly Notices*, vol. xx. No. 6, p. 231) pour déterminer le moyen mouvement n de la lune dans son orbite troublée, les trois équations suivantes, dérivées, dit-il, de nos propres formules:—

$$\frac{dn}{n\,dt} = -3\frac{d\mathrm{h}}{\mathrm{h}\,dt} + \frac{d\,(e'^2)}{dt}\left(-\frac{3}{2}m^2 + \frac{2187}{128}m^4\right)$$

$$\frac{dn}{n\,dt} = -3\frac{dh}{h\,dt} + \frac{d\,(e'^2)}{dt}\left(-\frac{3}{2}m^2 - \frac{297}{31}m^4\right) \qquad (1)$$

$$\frac{dn}{n\,dt} = -\frac{3}{2}\frac{da_,}{a_,dt} + \frac{d\,(e'^2)}{dt}\left(-\frac{3}{2}m^2 + \frac{5337}{128}m^4\right)^*$$

Puis il ajoute:—"It is obvious that these several values of $\frac{dn}{n\,dt}$ *contradict* each other, and the reason is that the quantities h, h, and a are really variable, and that therefore $\frac{d\mathrm{h}}{\mathrm{h}\,dt}$, $\frac{dh}{h\,dt}$, and $\frac{da}{a\,dt}$ have been wrongly neglected. In order to find the true value of $\frac{dn}{n\,dt}$, we must therefore determine the values of the last-mentioned differential coefficients, and substitute them in the several expressions for $\frac{dn}{n\,dt}$ given above."

En conséquence dans une lettre qu'il m'avait fait l'honneur de m'adresser par l'entremise de M. Lubbock, notre ami commun (en date du 17 Octobre, 1859), M. Adams ajoutait:—"Now it is easily shown, without involving any point of dispute, that the following relation exists between h and h (ces deux quantités étant essentiellement différentes comme on le verra plus bas):—

$$\frac{\mathrm{h}}{h} = 1 + \frac{75}{64}m^4 + \frac{127}{32}m^5 + e'^2\left(\frac{1125}{128}m^4 + \frac{3429}{64}m^5\right) \qquad (2)$$

"Putting, however, for $\frac{d\mathrm{h}}{\mathrm{h}\,dt}$ et $\frac{dh}{h\,dt}$ their proper value, I find exactly the same value of $\frac{dn}{n\,dt}$ from the two expressions." Et c'est probablement à cette concordance *ainsi obtenue*, que M. Adams faisait allusion dans le dernier paragraphe de son mémoire de 1853, lorsqu'il disait:—"I have the satisfaction of finding a perfect agreement between the results."

Or, il n'y a à ce raisonnement qu'une petite difficulté, c'est que le rapport $\frac{\mathrm{h}}{h}$ donné par la formule (2), étant précisement tiré de la comparaison des deux valeurs de n rapportées par M. Adams dans la page 231 de sa note, et dont la différentiation a produit les deux premières formules (1), l'accord trouvé ne prouve rien du tout, si ce n'est qu'on a fait un *cercle vicieux*, et qu'on est revenu précisement au point d'où l'on était parti: M. Adams, il est vrai, n'a pas reproduit cet étrange argument dans sa dernière note insérée aux *Monthly Notices;* mais il ne parait pas cependant que l'observation que je lui en ai faite, ait produit tout le fruit que j'en attendais, car M. Adams après avoir reproduit l'équation (2), page 234, ajoute:—"Now this relation at once shows that if e' be variable, h and h cannot both be constant; and since no *à-priori* reason can be given why one of these quantities should be constant rather than the other, we are not justified in *assuming* that either of them is so.

"This argument, however, does not appear *convincing* to M. de Pontécoulant."

Oh! certes, M. Adams ne se trompe nullement dans cette dernière assertion. En effet les quatre lignes, que je viens de rapporter et que j'ai extraites exactement de sa Note, ne prouvent qu'une chose, et justifient complètement ce que j'ai avancé au commencement de cette dissertation, à savoir que M. Adams avait complètement mis en oubli, du moins pour le moment, tous les principes sur lesquels sont fondées les différentes solutions du problême des perturbations lunaires, données par les géomètres qui se sont occupés de cet objet. En effet ne semble-t-il pas croire que l'on est libre de donner dans la formule (2) à h et à h les valeurs arbitraires que l'on voudra choisir, de les regarder comme constantes ou comme variables, c'est-à-dire comme des fonctions indépendantes ou dépendantes de e', selon les convenances de la question qu'on traite; mais il ne faut pas oublier que la détermination de l'équation séculaire n'est qu'une question toute particulière de la grande théorie des perturbations lunaires, les valeurs des quantités h et h sont fixées d'avance par cette théorie, elles entrent dans les expressions de toutes les inégalités périodiques ou à longues périodes, et l'on voit que l'argument, que M. Adams s'étonne que je n'aie pas trouvé *convincing*, aboutissait, en définitive à la proposition d'une altération complète de toutes les inégalités déterminées théoriquement par le calcul des perturbations du mouvement de notre satellite.

Revenons aux vrais principes, et l'on verra aussitôt que la discordance, que M. Adams a cru remarquer entre les trois valeurs de $\frac{dn}{n\,dt}$ rapportées plus haut, et qui résultent en effet de mes recherches, en négligeant bien entendu les termes d'un ordre supérieur à m^4 et les termes dépendant de l'excentricité et de l'inclinaison à l'écliptique de l'orbite lunaire, n'a jamais existé que dans son imagination, ces valeurs ne se contredisent nullement, et viennent au contraire à l'appui de ce que j'ai avancé précedemment.

D'abord il faut remarquer, comme je l'ai dit plus haut, que les lettres h et h représentent des quantités essentiellement différentes, la première que j'ai désignée par h pour la distinguer de la seconde, est celle que Laplace a employée dans sa théorie, elle représente le double de l'aire décrite pendant l'unité de temps par la projection du rayon vecteur de la lune sur le plan de l'écliptique, le mouvement étant supposé elliptique; en sorte que si l'on néglige pour plus de simplicité l'inclinaison de l'orbite lunaire sur l'écliptique, on aura

$$\mathrm{h} = \sqrt{a_,(1-e^2)},$$

$a_,$ étant le demi-grand axe de l'ellipse que décrirait la lune autour de la terre sans l'action du soleil, h est donc une quantité indépendante de e', et par conséquent rigoureusement constante dans ce cas; et c'est ainsi que l'ont toujours supposé Damoiseau, M. Plana, et tous ceux qui ont employé en les étendant les formules données par Laplace dans le 7^e livre de la *Mécanique Céleste;* et si M. Adams insistait pour savoir les raisons de cette signification donnée à la constante h, nous repondrions que c'est qu'il a plu à Laplace de la choisir ainsi, mais que, ce choix une fois fait, toutes les arbitraires du problême ont été déterminées de manière à satisfaire aux équations différentielles du mouvement troublé sous la forme adoptée par Laplace, et qu'on ne saurait maintenant altérer en aucune manière la valeur de h, à moins de donner une théorie nouvelle des mouvements lunaires, sous peine d'altérer de la manière la plus grave toutes les expressions des coefficients des inégalités périodiques déterminées par Laplace, Damoiseau, ou M. Plana.

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* M. de Pontécoulant writes $a_,$ instead of the a of Mr. Adams' Note.—ED.

Cela posé, il est évident qu'on doit faire $\frac{d\mathrm{h}}{\mathrm{h}\,dt}$ égal à zéro dans la première des formules (1) et l'on a alors simplement

$$\frac{dn}{n\,dt}=\frac{d(e'^2)}{dt}\left(-\frac{3}{2}m^2+\frac{2187}{128}m^4\right),$$

valeur qui coincide, dans l'ordre de termes que nous considérons, avec celle donnée par M. Plana dans sa Théorie de la Lune, ce qui devait être, puisque c'est celle que nous avons obtenue en suivant, pour vérifier les résultats de M. Plana,[*] la méthode où l'on prend la longitude vraie de la lune, pour la variable indépendante.

Il ne nous reste donc qu'à examiner les deux dernières formules (1), qui dérivent des nouvelles recherches que j'ai faites sur la détermination du coefficient de l'équation séculaire; c'est-à-dire du développement des formules employées dans ma théorie analytique où le temps t a été pris pour la variable indépendante. Or, loin de se contredire, on va voir que ces deux formules coincident parfaitement ensemble, seulement il faut faire ici une observation, qui semble, *par une singulière inadvertence*, avoir échappé à M. Adams; la constante qui entre dans la troisième des formules (1) et qui sert à compléter l'équation différentielle du rayon vecteur, comme le dit M. Adams, page 231, n'est autre, comme on le voit par cette équation, que le demi-grand axe de l'orbite elliptique de la lune, c'est pour cela que nous l'avons désignée dans les formules (1) par $a_{,}$, cette quantité est donc nécessairement indépendante de l'excentricité e' de l'orbe terrestre. Quant à la constante h qui entre dans l'équation de la longitude

$$\frac{r^2\,dv}{dt}=h+\int\left(\frac{d\mathrm{R}}{dv}\right)dt,$$

l'inspection seule de cette formule montre ce qu'elle représente. En effet $\int\left(\frac{d\mathrm{R}}{dv}\right)dt$ ne contenant que des termes périodiques, si l'on néglige ces termes, on voit qu'on aura $h=\frac{r^2\,dv}{dt}$. C'est-à-dire que $2h$ représente, dans ce cas, la partie non périodique de l'aire décrite pendant l'unité de temps par le rayon vecteur de la lune dans son orbite troublée, de même que dans les formules de Laplace 2 h représentait la même aire dans l'orbite elliptique. On conçoit maintenant pourquoi dans nos formules la quantité h doit dépendre nécessairement des forces perturbatrices, et contient par conséquent dans son développement l'excentricité e' de l'orbite solaire; h dans ce cas est donc une quantité essentiellement variable, et il faut par conséquent avoir égard à sa variation dans la seconde des formules (1). Or la valeur de h n'est nullement arbitraire, elle est complétement déterminée par les approximations successives, par les conditions à remplir pour satisfaire aux équations différentielles du mouvement troublé; nous avons trouvé ainsi, en ne portant les approximations que jusqu'aux termes de l'ordre m^4 (*Théorie Analytique*, No. 50, livre 7),

$$h=\sqrt{\mathrm{a}}\left[1-\frac{m^2}{3}+\frac{11}{144}m^4-\left(\frac{m^2}{2}+\frac{185}{96}m^4\right)e'^2\right]$$

où a représente le demi-grand axe de l'orbite correspondant à n en sorte qu'on ait $\mathrm{a}^3 n^2=1$; ou trouve d'ailleurs en vertu de cette équation, et par la formule rapportée par M. Adams (page 231, ligne 12):

$$\sqrt{\frac{\mathrm{a}}{a_{,}}}=1+\frac{1}{3}m^2-\frac{1285}{576}m^4+\left(\frac{m^2}{2}-\frac{5657}{384}m^4\right)e'^2,$$

d'où l'on conclut:

$$h=\sqrt{a_{,}}\left(1-\frac{145}{64}m^4-\frac{2175}{128}m^4e'^2\right),$$

valeur remarquable où l'on voit que les termes en m^2 et m^3 ont totalement disparu.

Si l'on substitue cette valeur et sa différentielle dans la seconde des formules (1) on trouve

$$\frac{dn}{n\,dt}=\frac{d(e'^2)}{dt}\left(-\frac{3}{2}m^2+\frac{5337}{128}m^4\right),$$

c'est-à-dire une valeur de $\frac{dn}{n\,dt}$ identique avec celle que donne la troisième des formules (1) lorsqu'on y suppose, comme on doit le faire, $\frac{da_{,}}{a_{,}dt}=o$, puisqu'en effet $a_{,}$ est une quantité essentiellement constante. Ainsi donc nos trois formules (1) sont parfaitement exactes, seulement la première ne concorde pas avec les deux autres, à partir des termes de l'ordre m^4, ce qui n'a rien qui doive surprendre d'après ce que nous avons dit plus haut, puisqu'elles se dérivent de deux méthodes totalement différentes. On peut remarquer encore que lorsqu'on a remplacé $\frac{dh}{h\,dt}$ par sa valeur dans la seconde des formules (1), le second terme de la valeur de $\frac{dn}{n\,dt}$ ne diffère *plus de signe* du terme correspondant de la même valeur, donnée par la première de ces formules, comme l'avait remarqué M. Adams dans le dernier paragraphe de son mémoire de 1853, et qui n'était qu'une assertion tout-à-fait erronée, puisqu'il n'avait point tenu compte de tous les termes qui doivent concourir à former la quantité dont il s'agit. Ainsi disparait l'un des principaux appuis sur lequel reposait l'argumentation de M. Adams.

Nous voici arrivés maintenant au point capital de la discussion, puisqu'il s'agit de l'existence même des nouveaux termes que M. Adams a proposé d'introduire dans le coefficient de l'équation séculaire du moyen mouvement lunaire; nous avions cru avoir prouvé dans notre première note (*Monthly Notices*, Juillet, 1859) que de pareils termes étaient tout-à-fait *inadmissibles*, mais puisque la démonstration n'a pas paru suffisante à M. Adams, nous allons reprendre la question sous toutes ses faces et la couler tellement à fond cette fois, qu'il n'est pas probable que personne soit jamais tenté de la relever de ses ruines.

M. Adams pour retablir, dit-il, un accord, qu'il croit nécessaire, entre les trois formules (1), accord qui existait parfaitement, sans qu'il s'en doutât, du moins pour les deux dernières, prescrit d'introduire outre les termes périodiques, trois termes *non périodiques* de la forme H e'^2 dans les trois intégrales

$$\int r^2\frac{d\mathrm{R}}{dv}\,dt;\quad \int\frac{d\mathrm{R}}{dv}\,dt;\quad \int d'\,\mathrm{R}$$

qu'ils doivent servir à compléter. Voyons donc comment M. Adams prétend procéder à la formation de ces termes. Pour la première formule il nous renvoie à l'article 11 du mémoire de 1853, où il dit avoir prouvé que la fonction

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[*] Il n'est ici question que des résultats qui sont rapportés dans le grand ouvrage de M. Plana (*Théorie de la Lune*, 3 vols. in 4to.) et dont nous avons par la comparaison aux résultats de nos propres calculs vérifié la parfaite exactitude. M. Plana est depuis revenu plusieurs fois sur ce sujet, il a cru d'abord devoir rectifier ses premières valeurs pour se conformer aux idées de M. Adams, puis il est revenu sur cette décision, puis il a adopté une partie de ses premières corrections et en a rejeté une autre, nous n'avons point à nous occuper ici de ses variations qui montrent peu de fixité dans les idées de ce géomètre distingué, nous n'y voyons qu'une preuve de plus de la nécessité de détruire *sans ménagement* cette fois, toutes les vues *fausses* et *erronées* qui se trouvent dans le mémoire de M. Adams de 1853, puisqu'elles peuvent avoir une si funeste influence même sur des esprits très éclairés.

$2\int r^2 \frac{dR}{dv}\, dv$ contient les termes non périodiques

$$h^2\left(-\frac{285}{8} m^4 e'^2 + \frac{495}{64} m^4 e'^2\right) = h^2\left(-\frac{1785}{64} m^4 e'^2\right).$$

Or c'est précisement de l'introduction de ces termes que nous avons démontré le vice et *l'inadmissibilité* à la page 315 de la note insérée aux *Monthly Notices* (Juillet, 1859), reprenons donc la démonstration et rendons la, s'il se peut, plus péremptoire cette fois.

Admettons que la fonction $r^2 \frac{dR}{dv}\, dv$ contienne dans son développement un terme de cette forme $H e' \sin(lv + \epsilon)$, on aura, en ne considérant que ce terme, et en ayant égard en effectuant l'intégration à la variabilité de e':—

$$\int r^2 \frac{dR}{dv}\, dv = -\frac{H}{l} e' \cos(lv+\epsilon) + \frac{H}{l}\int \frac{de'}{dv} \cos(lv+\epsilon)\, dv.$$

Si l'on substitue dans le dernier terme, avant d'effectuer l'intégration indiquée, à la place de $\frac{de'}{dv}$ sa valeur tirée de l'équation $e' = f + f'v$, selon l'expression que l'on doit supposer à la variation de l'excentricité e' quand on fait usage des formules où la longitude vraie de la lune a été prise pour la variable indépendante,* il est évident qu'on aura

$$\int r^2 \frac{dR}{dv}\, dv = -\frac{H}{l} e' \cos(lv+\epsilon) + \frac{Hf'}{l^2} \sin(lv+\epsilon)$$

$$= -\frac{H}{l} e' \cos\left(lv + \epsilon + \frac{f'}{l}\right)$$

c'est-à-dire un terme périodique dans lequel l'angle $lv + \epsilon$ sera simplement augmenté de la quantité constante $\frac{f'}{l}$, ce qui n'altérera nullement la forme de la série, et l'on pourra, par conséquent, comme $\frac{f'}{l}$ est d'ailleurs une très petite fraction, continuer, sans erreur sensible, à ne pas tenir compte, dans le développement de la fonction $\int r^2 \frac{dR}{dv}\, dv$, ainsi qu'on l'a fait jusqu'à présent, de la variabilité de l'excentricité e' de l'orbite terrestre.

Tel est le résultat indiqué par une exacte analyse de la question, et ce résultat est conforme à ce qu'ont fait Laplace, Damoiseau, M. Plana, et tous les géomètres qui s'étaient occupés jusqu'ici de la théorie des perturbations de la lune. Voyons comment M. Adams a pu se trouver conduit à un résultat tout opposé. Pour cela, dans la valeur précédente de $\int r^2 \frac{dR}{dv}\, dv$, M. Adams introduit sous le signe intégral, au lieu de $\frac{de'}{dv}$ sa valeur $\frac{de'}{dt} \cdot \frac{dt}{dv}$, il obtient ainsi, en négligeant les quantités simplement périodiques, le terme suivant

$$\int r^2 \frac{dR}{dv}\, dv = \frac{H}{l}\int \frac{de'}{dt}\, \frac{dt}{dv} \cos(lv+\epsilon)\, dv.$$

Puis il remplace $\frac{dt}{dv}$ par sa valeur développée en série de cosinus d'angles proportionnels à la longitude vraie v de la lune, et qui est donnée par les approximations successives; parmi les termes de ce développement il s'en trouve nécessairement un qui a le même argument $lv + \epsilon$ que le terme $H e' \sin(lv + \epsilon)$ que l'on a considéré dans $r^2 \frac{dR}{dv}$, en représentant par $H' e' \cos(lv + \epsilon)$ ce terme, on aura, en négligeant les quantités simplement périodiques, et substituant $n\,dt$ à la place de dv:—

$$\int r^2 \frac{dR}{dv}\, dv = \frac{HH'n}{2l}\int \frac{e'\, de'}{dt} \cdot dt = \frac{HH'n}{4l} e'^2.$$

Il est donc évident que le terme non périodique que nous venons d'introduire dans l'expression de la fonction $\int r^2 \frac{dR}{dv}\, dv$ qui d'après l'analyse précédente ne devint renfermer, en vertu de la variabilité de e', que des quantités périodiques, ne doit son existence qu'à la supposition que M. Adams a faite, qu'on peut supposer $\frac{de'}{dv} = \frac{de'}{dt} \cdot \frac{dt}{dv}$; puis traiter $\frac{de'}{dt}$ comme une constante lorsqu'on substitue pour $\frac{dt}{dv}$ sa valeur développée en série de termes périodiques. Or, non seulement, cette supposition n'est pas justifiable, *elle est*, il faut le dire, *contraire aux premiers principes du calcul différentiel.* En effet pour qu'on put substituer à la place de $\frac{de'}{dv}$ sa valeur $\frac{de'}{dt} \cdot \frac{dt}{dv}$ il faudroit supposer que l'on a remplacé $\frac{de'}{dv}$ non pas par une constante, mais par sa valeur complète déduite de la théorie de la variation des constantes arbitraires, $\frac{de'}{dt}$ devient alors, après y avoir substitué à la place de t sa valeur en v, une fonction de cette dernière variable, et l'on a par conséquent $\frac{de'}{dv} = \varphi v \cdot \frac{dt}{dv}$; on conçoit donc que si l'on substitue pour $\frac{dt}{dv}$ sa valeur dans cette expression et qu'ensuite on développe la fonction résultante, on aura $\frac{de'}{dv} = f + P$, où P représente une suite de termes périodiques, termes qu'on doit négliger dans l'expression de $\frac{de'}{dv}$ puisqu'il ne faut avoir égard dans la valeur de e' qu'aux *termes séculaires*, et que tous les termes *périodiques* sont dejà supposés compris dans les trois co-ordonnées du soleil, conformement à la théorie ordinaire de la variation des constantes arbitraires. Il est donc évident d'après cette analyse qu'il suffira de substituer, à la place de $\frac{de'}{dt} \cdot \frac{dt}{dv}$ dans l'expression de $\int r^2 \frac{dR}{dv}\, dv$, la premier terme de sa valeur f, et alors on reconnaîtra, d'après ce qui a été dit plus haut, qu'il n'en saurait résulter dans la valeur de cette fonction que des termes périodiques.

Ce que nous venons de dire relativement à l'intégrale $\int r^2 \frac{dR}{dv}\, dv$ s'applique naturellement à l'intégrale $\int \frac{dR}{dv}\, dt$, qui est supposée développée en série de sinus d'angles proportionnels au temps t; il nous semblait même que *sur ce point* il ne pouvait y avoir aucune contestation possible, car comme M. Adams ne s'explique pas très clairement à cet égard, on ne conçoit pas même de quelle manière des termes non périodiques pourraient y être introduits par l'effet de la variabilité de e'. En effet supposons que la fonction $\frac{dR}{dv}$ contienne le terme suivant:—

$$\frac{dR}{dv} = A e' \sin(lt + \epsilon),$$

on aura en multipliant par dt et intégrant

$$\int \frac{dR}{dv}\, dt = -\frac{A}{l} e' \cos(lt+\epsilon) + \frac{A}{l}\int \frac{de'}{dt} \cos(lt+\epsilon).$$

Or il est évident que si l'on se borne à considérer dans l'expression de e' la partie séculaire, ce qui permet de supposer

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* Voir ce qui a été dit (*Monthly Notices*, Juillet, 1859, p. 313) et *Mécanique Céleste*, liv. 7ᵉ, No. 10.

$\frac{de'}{dt}$ égal à une constante, le second terme de l'expression précédente ne produira qu'un terme périodique qui s'ajoutera au premier (voir *Monthly Notices*, Juillet, 1859, p. 311) et qui n'altérera en aucune manière la forme de la série. On pourra donc satisfaire dans ce cas aux équations du mouvement troublé par les mêmes valeurs dans lesquelles on altérera seulement les constantes qui servent à compléter les arguments des différentes inégalités, les co-ordonnés de la lune conserveront les mêmes valeurs quelque loin que l'on pousse les approximations, et c'est par cette raison que nous avions supposé, que dans le cas au moins où l'on prend le temps t pour la variable indépendante, tout le monde était d'accord qu'on peut, dans l'intégration des formules différentielles du mouvement troublé, n'avoir aucun égard à la variation de l'excentricité de l'orbe terrestre. Je vois d'après ce que dit M. Adams dans sa note que je m'étais encore trompé sur ce point, mais comme il n'explique pas la méthode qu'il veut suivre pour calculer les nouveaux termes que la cause dont il s'agit introduirait, selon lui, dans les trois co-ordonnées de notre satellite, qu'il n'entre dans aucun détail qui puisse même nous rien faire préjuger à cet égard, nous attendrons qu'il se soit expliqué plus catégoriquement pour lui repondre si ce que nous venons de dire ne suffisait pas encore pour l'éclairer.

Il serait inutile de pousser plus loin cette discussion, du moment qu'il est prouvé, et je crois cette fois d'une manière incontestable, que l'introduction des nouveaux termes que l'on avait considérés dans les expressions des trois co-ordonnées lunaires, *tenait à l'oubli d'un des premiers principes du calcul différentiel* et que personne en insistant ne voudra se rendre complice d'une faute si grave. M. Adams levera lui-même toutes les difficultés qu'il a pu rencontrer dans ses recherches sur *l'équation séculaire* du moyen mouvement lunaire, et trouvera entre les formules, celles de M. Plana et les notres, et même avec celles de M. Hansen, s'il veut suivre la méthode des substitutions successives au lieu de celle des dévéloppements en série, les plus parfait accord, du moment qu'il consentira à supposer la valeur variable de l'excentricité de l'orbe terrestre exprimée par une série de cette forme $e'=f+f'v+f''v^2+$etc., dans le cas où l'on emploie les formules ou la longitude vraie de la lune a été prise pour la variable indépendante, ou par une série de cette forme $e'=g+g't+g''t^2+$etc., dans le cas où l'on emploiera comme je l'ai fait dans ma théorie de la lune, les formules directes ou le temps t a été choisi pour cette variable. C'est donc à cette seule proposition que doit aboutir définitivement toute cette discussion, et je ne puis sur ce sujet que renvoyer à ce que j'ai dit dans ma note précédente (*Monthly Notices*, Juillet, 1859) et en dernier lieu dans les *Comptes Rendus de l'Académie des Sciences*, tome l. No. 18.

Paris, 19 *Mai*, 1860.

On the Importance of making Observations on Thermal Radiation during the coming Eclipse of the Sun. By William Thomson, Prof. of Natural Philosophy at the University of Glasgow.

(*Extract from a Letter to the Editor.*)

"Prof. Grant thinks I ought to call the attention of the Astronomical Society to the importance of making observations on thermal radiation during the eclipse of the sun, by writing to you on the subject.

"All that I have to suggest is, that differential observations on the effects experienced by three or four thermometers with their bulbs placed, one in the centre of the image of the moon, in the focus of a lens or speculum, and the others in different positions round her disk, during the time of total obscuration of the sun, would probably give decided results as to the difference of radiation from the moon, and from the portion of space round the sun. The 'photosphere,' which I suppose is always seen round the moon's disk during a total eclipse of the sun, and which must be due to her light reflected from atmosphere or dust round the sun itself, must radiate heat, and would probably show a sensible effect, as compared with radiation from an equal angular area of the moon.

"The thermometers should have their bulbs blackened with soot or lamp-black, and be all of the same size. The bulb of each should be considerably smaller than the sun's image. This, if the focal length of the lens or mirror is 57 inches, would be a circle half an inch diameter, so that very ordinary thermometers would answer. I would recommend spirits-of-wine thermometers for superior sensitiveness; but, perhaps, mercury, with a rather fine bore, would be sensitive enough, and surer.

"A mirror would be better than a lens, because the hot dust round the sun must produce radiant heat of such colour as that of a hot stone or metal not at a bright red heat; and which, therefore, would be much absorbed by glass.

"There would need good arrangements to preserve the thermometers from disturbing influences. Until the totality, the radiation from the sun would need to be strictly excluded. At the instant of complete obscuration the cover would be removed, and the thermometers all observed. One observer could note them in succession. The sketch represents the positions such as I would have the thermometers in, the circle denoting the moon's unseen disk.

"There seems no doubt but that the thing is practicable, and would lead to results; but it is to be considered whether arrangements can be made, and an observer or observers secured, to execute it. Will you consult members of the Astronomical Society on the occasion of its meeting on Friday, if you find any who are disposed to consider the subject?

"From what Prof. Grant says, it appears that radiation experiments, as hitherto made or contemplated, have not attempted any definite concentration by means of a lens or otherwise. This appears to be as discriminating a way of investigating the phenomenon as it would be to attempt to record visible appearances by a photograph made by diffused light, with no lens to make a picture.

"*Birkenshaw Cottage, Thornliebank, Glasgow,*
June 5, 1860."

Mr. Carrington has received the following letter from the Superintendent of the U. S. Coast Survey:—

"It may interest the members of the Royal Astronomical Society to know what progress has been made in the United States for observing the solar eclipse of July 18th.

"1. The Coast Survey will have the services of Lieut. Gilliss, U. S. N., to observe near Olympia or Steilacoom, in Washington Territory, the central line of shadow passing near both towns. Two assistants have been detailed for him, and he has been fully equipped for the work, and has taken passage in the last California steamer.

"2. The Superintendent of the *Nautical Almanac* will send one or two assistants to Cumberland House, near Lake Winipeg, having received the cordial co-operation of the liberal Governor of the Hudson's Bay Company in the matter.

"3. A resolution has passed the Senate of the United States, and awaits the action of the House of Representatives and of the President, authorising the Superintendent of the

Coast Survey to send an expedition to the vicinity of Cape Chudleigh, where the central line of the shadow leaves the North American continent, to observe the eclipse. This will be composed of astronomers, and of a photographer volunteering for the expedition, and of the requisite assistants. The party is now making up, so as to be duly prepared if authority is given. Several institutions of learning and patrons of science have contributed the necessary funds for the incidental expenses of this expedition. (Signed) A. D. Bache.

"*Coast Survey Office, Washington, May* 22, 1860."

A letter from Lieut. Gilliss, of nearly the same date, states that he would leave New York on May 21, and expect to reach Puget Sound, *viâ* Panama and San Francisco, by June 22. He proposes to proceed from Steilacoom to an eminence of the Cascade Mountains, in order to command a good eastern horizon. The expedition to Cape Chudleigh would probably be under the command of Dr. Gould or Prof. Stephen Alexander.

M. Bulard, Director of the Observatory at Bou-zarea, near Algiers, has selected as his post of observation for July 18 the mountain Djebel Mhammel, in lat. 35° 31′, long. E. of Paris 3° 57′, nearly on the central line, and having an elevation of 7500 feet. The road lies from Phillipville through Constantine and Bathna.

On certain Babylonian Observations of the Planet Venus. By the Rev. Dr. Hincks.

(*Inclosed in a Letter addressed to the Secretaries.*)

There is a tablet of baked clay in the British Museum, the inscription on which, if I interpret it aright, contains a series of observations of the planet *Venus*, and a series of predictions grounded on the observations. The latter are of no value; but the former may in great measure, if not altogether, determine the law by which the Assyrio-Babylonian lunar year was regulated in respect to its intercalary months. The knowledge of this law, again, will either establish or disprove the view which I have long entertained, and repeatedly expressed, that the era of Nabonassar was an astronomical, and not a political one; and, I may add, it is not impossible that it may furnish a test of the genuineness of the works attributed to Quthami and other supposed ancient Babylonian writers.

For these reasons I am desirous that the observations which I suppose to be recorded should be submitted to astronomers. I now offer two, which will suffice to test the correctness of my interpretation of the records. If any astronomer will take the trouble to calculate whether what is here stated to have happened would have actually happened, and will communicate the result to me, "Rev. Doctor Hincks, Rectory, Killyleagh," I will, if he desire it, communicate to him other records of observations; as to the interpretation of some of which I feel less confidence than I do as to these.

I observe that the Babylonian months are expressed by monograms, for which I substitute Hebrew names of months. The Babylonian day began at noon; and that day in the evening of which the new moon was first seen was considered to be the first day of the month. I suppose, but am not very confident, that the year of the first observation was −685. The month of Thamuz would begin in the spring. The second observation was some years later.

"On the 25th of Thamuz, *Venus* ceased to appear in the west, was unseen for seven days, and on the 2d of Ab was seen in the east."

"On the 26th of Elul, *Venus* ceased to appear in the west, was unseen for eleven days, and on the 7th of the second Elul was seen in the east."

This being an embolismatic year, the day last mentioned was necessarily its 184th day, and was 200 days before the first day of the new year. If, then, this day can be determined from what is recorded of *Venus*, the commencement of two Babylonian years out of a cycle of eight will be determined.

Killyleagh, Co. Down, 6 *June*, 1860.

Mr. Hodgson's attention having been drawn to the remarks of Mr. Dawes, in the *Monthly Notices* for April, upon the solar eyepiece, he wrote to him upon the subject, and in correspondence Mr. Dawes says he did not intend the remarks to have been printed, but merely wished to draw the attention of the Editor to the quotations he enclosed.

Mr. Hodgson, however, still maintains his claim for the effective novelty of the arrangement, which has never before been questioned by Mr. Dawes, nor in his paper upon solar eyepieces, read before the Society in 1851, is this or any similar arrangement even hinted at.

Comparison of Burckhardt's and Hansen's Lunar Tables with the Greenwich Observations from 1847 *to* 1858. By G. B. Airy, Esq. Astronomer Royal.

More than a year ago I communicated to the Society a comparison, in detail, of the places given for the moon by Burckhardt's and Hansen's Lunar Tables, with those given by the observations made at the Royal Observatory, Greenwich, for the year 1852; and I expressed the anxiety with which I looked forward to the completion of the comparisons for the whole period, from 1847 to 1858.

Since that time, by Mr. Hind's continued attention, the computation of the places from Hansen's Tables has been finished, and the whole have been placed in my hands; and I have now prepared the complete comparison of both Hansen's and Burckhardt's Tables with the observations.

The places deduced from observation are in all cases computed with Professor Adams' parallax.

The entire series of comparisons forms a rather extensive table, which I propose to print in the *Greenwich Observations;* but I have thought that an epitome of the results might not be unacceptable to the Society.

A general idea of the relative merits of the two collections of tables may be derived from an exhibition of their apparent extreme errors in + and −, in the two co-ordinates of longitude and E.N.P.D., and for all the admitted observations made by both instruments, transit-circle and altazimuth; but in interpreting these extreme errors, it must be borne in mind that, in general, they are affected by extreme errors of observation with the sign opposite to that of the apparent error of tables. Suppose, for instance, that an ordinary extreme error of observation is 3″, then all the apparent maximum + errors of tables ought to be diminished numerically by 3″, and all the apparent maximum − errors ought to be diminished numerically by 3″; and these corrections will be the same for both Hansen's and Burckhardt's Tables. The effect of this application will be, to increase considerably the apparent inequality of merit of the tables.

A more accurate estimate will probably be formed by comparison of the sums of the squares of apparent errors. These, however, are on the whole increased by errors of observation, in such a manner as to make the apparent inequality of merit less than the true inequality.

In the tables which follow, the symbols B − O and H − O are used for the excess of the numbers derived from Burckhardt's and Hansen's Tables respectively above those derived from observation. The first table exhibits the extreme + and − values of B − O and H − O in each year, divided under the heads of Longitude and E.N.P.D., and also divided under the heads of Meridional and Altazimuth Observations. The second table exhibits the sums of squares of B − O and H − O for each year, and also their proportion, divided into similar groups; with the number of observations in each year.

It will be seen that this comparison for the twelve years entirely supports the opinion which I had formed on the relative merits of the tables from the comparison for the single year 1852.

Royal Observatory, Greenwich, 1860, *May* 30.

Maximum Positive and Negative Values of B − O *and* H − O, *in each Year, from* 1847 *to* 1858.

1847, *Meridional Observations.*

Longitude.				E.N.P.D.			
B − O.		H − O.		B − O.		H − O.	
+	−	+	−	+	−	+	−
July 29.	Feb. 28.	Oct. 29.	Aug. 20.	Sept. 26.	Nov. 30.	July 27.	July 29.
17″·4	2″·6	7″·2	3″·8	11″·5	11″·9	7″·6	5″·1

1847, *Altazimuth Observations.*

Nov. 10.	May 16.	June 30.	Dec. 23.	June 8.	May 24.	June 8.	Oct. 2.
24″·8	6″·3	11″·0	8″·9	16″·5	15″·1	7″·3	7″·6

1848, *Meridional.*

Sept. 20.	Sept. 6.	Sept. 20.	Feb. 12.	Sept 8.	Nov. 18.	June 11.	Oct 3.
19″·1	1″·9	8″·0	8″·1	9″·2	13″·4	8″·8	9″·0

1848, *Altazimuth.*

Sept. 12.	Mar. 31.	Sept. 12.	July 11.	Jan. 13.	Nov. 20.	Mar. 14.	May 24.
19″·5	8″·3	13″·1	9″·0	14″·1	14″·6	13″·2	9″·3

1849, *Meridional.*

Dec. 29.	Aug. 3.	July 8.	Mar. 2.	June 13.	Dec. 6.		Apr. 27.
16″·7	3″·3	7″·4	10″·7	15″·9	12″·7	″ ...	6″·7

1849, *Altazimuth.*

Sept. 19.	May 17.	Aug. 25.	Feb. 1.	June 13.	Dec. 5.	Feb. 28.	Nov. 4.
22″·8	9″·9	7″·8	12″·8	12″·6	17″·4	9″·4	7″·1

1850, *Meridional.*

Nov. 28.	Feb. 18.	Aug. 28.	Jan. 23 Feb. 21	June 2.	Nov. 24.	June 2.	Oct. 11.
19″·9	6″·1	5″·8	10″·4	17″·8	13″·2	10″·0	6″·9

1850, *Altazimuth.*

Oct. 8.	Nov. 27.	Oct. 11.	Feb. 19.	June 5.	Sept. 4.	Apr. 18.	Sept. 2.
36″·9	18″·6	9″·9	11″·4	10″·8	19″·8	10″·1	10″·9

1851, *Meridional.*

Longitude.				E.N.P.D.			
B − O.		H − O.		B − O.		H − O.	
+	−	+	−	+	−	+	−
Nov. 17.	Nov. 29.	Oct. 10.	Mar. 11.	Jan. 26.	May 10.	Aug. 3.	May 23.
21″·7	3″·6	7″·0	9″·6	9″·6	10″·7	6″·0	6″·4

1851, *Altazimuth.*

Oct. 26.	Apr. 25.	Oct. 26.	May 8.	Jan. 27.	Nov. 16.	Dec. 3.	Oct. 14.
43″·3	7″·3	11″·1	8″·6	9″·6	12″·9	9″·1	8″·7

1852, *Meridional.*

Sept. 29	Jan. 25 Feb. 25	Aug. 7.	Apr. 27.	July 2.	Oct. 6.	Feb. 2.	Aug. 7.
18″·2	1″·7	8″·6	9″·2	8″·3	10″·8	4″·6	4″·5

1852, *Altazimuth.*

Sept. 16.	Mar. 22.	Aug. 7.	Jan. 1 Nov. 30	Jan. 18.	Oct. 11.	July 22.	Oct. 15.
37″·0	7″·3	8″·9	10″·9	11″·5	14″·5	8″·0	11″·1

1853, *Meridional.*

Sept. 16.	Oct. 23.	Oct. 26.	July 15.	Apr. 1.	Jan. 21 Apr. 17	Jan. 31.	June 23.
22″·9	4″·1	6″·0	9″·6	14″·1	9″·5	5″·0	7″·9

1853, *Altazimuth.*

Sept. 5.	Mar. 11.	Dec. 16.	Nov. 26.	Sept. 16.	Sept. 5.	Sept. 16.	Feb. 1.
36″·4	5″·5	10″·3	12″·8	12″·0	12″·5	11″·0	10″·7

1854, *Meridional.*

Sept. 6.	Apr. 9.	July 12.	Sept. 28.	Apr. 18.	Jan. 8.	Apr. 1.	Sept. 28.
26″·5	4″·1	5″·6	13″·0	10″·2	8″·3	7″·9	4″·8

1854, *Altazimuth.*

Sept. 8.	May 9.	Oct. 8.	May 9.	Jan. 30.	Sept. 16.	Jan. 30.	July 14.
26″·3	11″·0	7″·2	18″·9	15″·5	12″·4	11″·0	10″·3

1855, *Meridional.*

Aug. 28.	Mar. 2.	Aug. 28.	Nov. 20.	July 25.	Dec. 20.	Dec. 21.	Dec. 29.
24″·8	5″·6	5″·8	7″·9	8″·9	10″·6	4″·5	5″·5

1855, *Altazimuth.*

Dec. 12.	Mar. 2.	Oct. 3.	July 19.	Dec. 12.	Dec. 20.	Dec. 12.	Oct. 4.
25″·1	6″·5	11″·1	9″·2	13″·5	13″·2	13″·4	8″·8

1856, *Meridional.*

Oct. 21.	May 18.	Oct. 21.	Apr. 16.	July 17.	June 26.	Apr. 9.	June 17.
19″·0	0″·2	7″·8	6″·7	8″·0	11″·5	2″·8	5″·6

1856, *Altazimuth.*

Jan. 10.	Mar. 30.	Sept. 22.	Aug. 10.	June 17.	June 28.	Jan. 10	Sep. 20. Dec. 16.
22″·4	7″·8	6″·5	10″·7	9″·2	15″·7	6″·6	9″·3

1857, *Meridional.*

Longitude.				E.N.P.D.			
B — O.		H — O.		B — O.		H — O.	
+	—	+	—	+	—	+	—
Oct. 9.	June 5.	Sept. 5.	Aug. 29.	Dec. 10.	June 15.	Feb. 15.	Mar. 16.
20″·8	1″·3	5″·4	7″·5	7″·1	9″·0	4″·4	3″·9

1857, *Altazimuth.*

July 23.	Apr. 20.	Oct. 7.	Mar. 28.	May 5.	June 14.	Oct. 4.	Aug. 12.
21″·5	8″·5	4″·8	10″·7	8″·7	12″·2	6″·0	6″·8

1858, *Meridional.*

Mar. 25.	Sept. 16.	Aug. 28.	Feb. 22.	Nov. 26.	Aug. 20.	Jan. 1	June 1. Aug. 4.
21″·7	3″·9	4″·8	9″·0	8″·1	9″·5	4″·5	7″·9

1858, *Altazimuth.*

June 13.	June 7.	June 8 25	June 7.	Dec. 1.	June 7.	Feb. 19.	Dec. 24.
23″·8	10″·8	3″·7	10″·6	7″·8	15″·9	6″·7	7″·8

Sums of Squares of B — O *and* H — O, *in each Year, from 1847 to 1858, and their Proportions; with the Number of Observations in each Year.*

1847, *Meridional Observations.*

Number of Observations.	Longitude. B — O.		Longitude. H — O.	E.N.P.D. B — O.		E.N.P.D. H — O.
107	7426·94		708·42	3174·96		564·05
		10·5			5·6	

1847, *Altazimuth Observations.*

126	11489·62		1616·07	4353·99		1130·72
		7·1			3·9	

1848, *Meridional.*

113	7662·82		1132·64	2366·69		810·21
		6·8			2·9	

1848, *Altazimuth.*

199	18880·81		2567·76	6497·61		1989·89
		7·3			3·3	

1849, *Meridional.*

109	6724·43		1083·60	3577·12		1006·45
		6·2			3·6	

1849, *Altazimuth.*

217	15170·75		2428·15	8154·33		1678·73
		6·2			4·9	

1850, *Meridional.*

115	8384·50		1156·54	4337·91		673·75
		7·2			6·4	

1850, *Altazimuth.*

Number of Observations.	Longitude. B — O.		Longitude. H — O.	E.N.P.D. B — O.		E.N.P.D. H — O.
215	24451·41		2903·65	7896·17		2543·45
		8·4			3·1	

1851, *Meridional.*

110	11005·70		1383·80	2375·05		452·16
		8·0			5·3	

1851, *Altazimuth.*

209	31952·11		3052·09	6576·60		1812·79
		10·3			3·6	

1852, *Meridional.*

109	9552·94		1221·76	2417·09		276·22
		7·8			8·7	

1852, *Altazimuth.*

210	28366·06		2876·17	7116·04		1737·40
		9·9			4·1	

1853, *Meridional.*

107	9603·01		1312·16	2768·48		336·75
		7·3			8·2	

1853, *Altazimuth.*

194	23630·65		2856·70	6843·10		1748·47
		8·3			3·9	

1854, *Meridional.*

121	13072·49		1672·84	2254·02		388·59
		7·9			5·8	

1854, *Altazimuth.*

204	23141·97		3431·22	5885·34		2405·24
		6·8			2·4	

1855, *Meridional.*

94	11395·78		723·55	1629·55		286·09
		15·5			5·7	

1855, *Altazimuth.*

187	20173·67		2304·23	5747·57		1859·22
		8·7			3·1	

1856, *Meridional.*

94	8863·97		881·59	1828·90		179·68
		10·1			10·2	

1856, *Altazimuth.*

182	16115·46		2488·05	6537·15		1718·45
		6·5			3·8	

1857, *Meridional.*

115	11994·96		1458·47	1295·12		229·29
		8·4			5·7	

1857, *Altazimuth.*

Number of Observations.	Longitude. B − O.	Longitude. H − O.	E.N.P.D. B − O.	E.N.P.D. H − O.
217	18940·23	4438·30	4702·54	1422·40
	4·3		3·3	

1858, *Meridional.*

Number of Observations.	Longitude. B − O.	Longitude. H − O.	E.N.P.D. B − O.	E.N.P.D. H − O.
130	16844·30	1795·76	2634·72	524·08
	9·4		5·0	

1858, *Altazimuth.*

Number of Observations.	Longitude. B − O.	Longitude. H − O.	E.N.P.D. B − O.	E.N.P.D. H − O.
201	21369·01	3440·31	5054·45	1735·24
	6·2		2·9	

Right Ascensions of the Moon and Moon-culminating Stars, observed at the Astronomical Observatory, Victoria. By R. L. J. Ellery, Esq., Director of the Observatory.

(*Communicated by the Astronomer Royal.*)

Date.	Object Observed.	App. R.A.	Date.	Object Observed.	App. R.A.
1859.		h m s	1860.		h m s
Nov. 7	☽'s 1st Limb	0 19 57·07	Mar. 7	φ Leonis	11 9 35·35
	ε Piscium	0 55 41·94		☽'s 2d Limb	11 13 17·10
	ζ Piscium	1 6 26·58	31	κ Geminorum	7 36 1·83
9	β Arietis	1 46 56·03		φ Geminorum	7 44 57·78
	☽'s 1st Limb	1 58 19·60		☽'s 1st Limb	7 48 7·52
1860. Feb. 2	υ^1 Tauri	4 17 57·85		η Cancri	8 24 38·91
	☽'s 1st Limb	4 26 39·16	Apr. 1	ο Leonis	9 37 56·63
	τ Tauri	4 33 52·46		☽'s 1st Limb	8 47 55·73
	χ Orionis	5 46 7·90		π Leonis	9 52 51·29
6	☽'s 1st Limb	8 47 29·29		α Leonis	10 0 57·41
	83 Cancri	9 11 12·40	2	☽'s 1st Limb	9 45 41·68
Mar. 3	☽'s 1st Limb	7 10 53·66		π Leonis	9 52 51·29
	δ Geminorum	7 11 48·18		α Leonis	10 0 57·30
	υ Geminorum	7 27 20·23	5	χ Virginis	12 32 4·00
4	☽'s 1st Limb	8 14 10·18		☽'s 1st Limb	12 33 23·01
	η Cancri	8 24 39·20		ψ Virginis	12 47 7·28
	79 Cancri	9 2 20·92	7	☽'s 2d Limb	14 32 34·39
5	79 Cancri	9 2 20·84		56 Hydræ	14 39 37·89
	83 Cancri	9 11 12·48		α^2 Libræ	14 43 11·11
	☽'s 1st Limb	9 15 16·78			
6	π Leonis	9 52 51·50			
	α Leonis	10 0 57·57			
	☽'s 1st Limb	10 14 0·83			

Corrected for all errors and clock-rate.

Results of Meridional Observations of Small Planets; Occultation of a Star by the Moon; and Phenomena of Jupiter's Satellites: observed at the Royal Observatory, Greenwich, in the month of May 1860.

(*Communicated by the Astronomer Royal.*)

Calliope ㉒.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, May 21	12 51 24·9	16 50 54·76	113 45 15·45
23	12 41 45·4	16 49 6·79	113 49 24·29

Thetis ⑰.

Mean Solar Time of Observation.		Apparent R.A.	Apparent N.P.D.
	h m s	h m s	° ′ ″
1860, May 1	11 41 21·2	14 21 48·46	93 22 45·37
2	11 36 32·8	14 20 55·80	93 18 48·63
3	11 31 44·6	14 20 3·36	93 14 59·82
9	11 3 5·3	14 14 58·72	92 55 46·12
21	10 7 25·2	14 6 28·13	92 38 22·53
22	10 2 55·2	14 5 53·92	92 38 41·05
23	9 58 26·5	14 5 21·08	92 38 27·27
(*a*) 26	9 45 8·4	14 3 50·21	92 40 17·24
(*b*) 26	9 45 10·3	14 3 52·32	92 40 11·98

Mr. Dunkin observed two objects for *Thetis* on May 26, which were so close to each other, that it was impossible to decide which was the planet; and the weather, as well as the increasing daylight, prevented re-observation of the star on the meridian. Mr. Dunkin, however, thinks that the object marked (*b*) was the planet, as it agreed more nearly in magnitude with *Thetis* than the other.

All the observations of N.P.D. have, as usual, been corrected for parallax.

Occultation of a Star by the Moon.

Day of Obs.	Phenomenon.	Moon's Limb.	Mean Solar Time.	Observer.
			h m s	
1860, May 2	*q* Virginis, reapp.	Bright	7 49 32·2	C.

The star was invisible at the disappearance. It was somewhat separated from the moon's limb when first seen at the reappearance. The initial C. is that of Mr. Criswick.

Occultation and Transits of Jupiter's Satellites.

Day of Observation.	Satellite.	Phenomenon.	Mean Solar Time.	Observer.
1860.			h m s	
May 1	I	Occ. disapp. first cont. (*a*)	8 22 18·1	J. C.
1	I	,, central bis. (*a*)	8 23 47·9	J. C.
3	III	Ingress, bisected	9 55 11·1	E.
9	I	Egress, last cont.	10 2 39·0	C.

(*a*), The planet very tremulous and diffused.

The initials E., C., J. C., are those of Mr. Ellis, Mr. Criswick, and Mr. Carpenter.

Observations of the Occultation of Jupiter by the Moon, made at the Royal Observatory, Greenwich, 1860, *May* 24.

(*Communicated by the Astronomer Royal.*)

Phenomenon.	Greenwich Mean Solar Time of Observation. Hour and Minute.	Seconds observed by M.	Seconds observed by D.
	h m	s	s
First Contact	4 34	49·0	...
Disappearance	4 35	53·4	54·2
Reappearance	5 46	17·6	12·6
Last Contact	5 47	20·4	20·4

Notes by Mr. Main [M.]. The observations were made with the Sheepshanks' equatoreal. At the reappearance of the first limb of *Jupiter*, the time recorded is probably a second or two late, the planet having emerged when first seen; all the other observations are perfectly satisfactory.

By Mr. Dunkin [D.]. The instrument used was the new S.E. equatoreal. At the first contact the occultation of the first limb of the planet evidently took place before I noticed any alteration in the appearance of *Jupiter* at $4^h\ 34^m\ 50^s$. The remaining observations are considered very satisfactory.

Notwithstanding the daylight and the excessive faintness of *Jupiter* at the reappearance at $5^h\ 47^m$, I was surprised to notice the visibility of the belts, which appeared as several exceedingly thin parallel lines.

The satellites of *Jupiter* were not visible.

Observations of the Occultation of Jupiter by the Moon on the 24th May, 1860. By Thomas Gaunt, Esq.

(*Extract from a Letter to one of the Secretaries.*)

"I send you the following account of the occultation of *Jupiter* by the moon on the 24th ult., as seen by me at Stoke Newington. The observation was made with an achromatic of 3·3 inches aperture, 50 inches focus, the immersion with a power of 50, and the emersion with a power of 70.

"At the immersion I could not see the dark limb of the moon until the planet appeared to touch it, and then only to the extent of the diameter of the planet; but what I was most struck with was the appearance on the moon as it passed over the planet. It appeared as though the planet was a dark object, and glided on to the moon instead of behind it, and the appearance continued until the planet was hid, when I suddenly lost the dark limb of the moon altogether. The equatoreal belts were visible up to the edge of the moon.

"At emersion the planet was but a little paler and greyer than the moon, and when about one-third out there appeared a dark broad line down the bright limb of the moon over the planet, and it remained visible until the whole of the planet was seen, it was about double the width of the equatoreal belts, but much darker. The belts were very distinct up to the dark band down the moon's bright limb.

"A lady observed the occultation at the same place with an achromatic of 3·25 inches aperture, 42 inches focus, and a power of 45, but she did not see the appearance I did at immersion, when the planet touched the moon the dark limb became visible to the extent of about three diameters of *Jupiter*, as though it was enlightened. At emersion she also saw the dark band down the bright limb of the moon. No distortion of the planet was noticed by either of us. The sun was very brilliant both at immersion and emersion.

"I enclose you a sketch of the appearances as I saw them.

"*June* 6, 1860."

Observations of the Occultation of Jupiter by the Moon, made at Highbury, 1860, *May* 24. By T. W. Burr, Esq.

	Local Sid. Time. h	m	s	Moon's Limb.
The Planet's 1st Limb disappeared at	8	44	6·7	Dark
,, 2d ,, ,,	8	45	4·9	
,, 1st ,, reappeared at	9	55	48	Bright
,, 2d ,, ,,	9	56	44·7	

The times of first contact and first reappearance are uncertain, and probably one or two seconds late. The observations of last contact in each case are very good. The clock-error was checked by a transit of *Regulus* immediately after the emersion. The planet was well seen, notwithstanding the strong sunlight ($4^h\ 34^m$ G. M. T.), but, of course, without any belts. The moon's dark limb could not be detected until it touched the planet, when it was seen very sharply defined and black; and as it passed, the disk of *Jupiter* in front appeared to brighten, so that the moon's limb was preceded by a bright band of light; doubtless an effect of contrast. At the reappearance nothing unusual was noticed, but the difference of hue and brilliancy between the planet and our satellite was very marked, *Jupiter* being, as noted in prior occultations, of a faint indigo tint.

The telescope used was by Ross, of 3¾ inches aperture; focal length, 52 inches; power, 58. Longitude, $23^s{\cdot}8$ W.

Highbury, June 4, 1860.

Observations of Jupiter. By Sir William Keith Murray.

(*Extract from a Letter to one of the Secretaries.*)

"I enclose four diagrams of *Jupiter*,* selected from my notebook, showing some of the changes displayed in the belts.

"As really good definition is rarely to be expected, except for about half an hour before dark, the appearance of the belts varies exceedingly, according to the time of night they are observed.

"I find it generally hopeless to see the white spots or the minute detail of the belts at any other hour of the night, at this season and position of the planet. On April 26th, the third satellite appeared so dark that had I not looked in the *Nautical Almanac*, I should have had no doubt of its being the shadow, instead of the satellite.

"On May 1st I saw the spots and markings on the belts in great perfection. I never happened to observe so many on the northern belt; but I find it very difficult to make a perfectly faithful sketch of the position and dimensions of the spots; therefore the sketch must rather be considered to give a general idea of the appearance of the planet than an absolutely true representation. Even in the best states of the atmosphere there are flickerings which make it almost impossible to produce a perfect representation of the minute detail of the belts, as seen with the 9-inch object-glass, and powers of from 200 to 360.

"The weather, generally, this winter, in this quarter, has been most unfavourable for delicate observation for the last five months; there has, till lately, hardly been a night without frost; the waters have been covered for nearly four months with 7 or 8 inches of ice; and there are still deep wreaths of snow on the hills within two miles of the observatory.

Ochtertyre, Crieff, 11*th May*, 1860."

New Variable Star in Ophiuchus. By Norman Pogson, Esq., Director of the Hartwell Observatory.

(*Communicated by Dr. Lee.*)

A small star of the eleventh magnitude was detected with the Hartwell equatoreal, on April 16th, in a position in which most certainly no such object had been previously seen, although in a portion of the heavens regularly examined each spring since 1853.

It was at first supposed to be a new planet; but ten micrometric comparisons with a star north preceding established its fixity, and gave for its mean place, reduced to 1860,—

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* These diagrams were exhibited at the Meeting.—Ed.

$$\text{R.A.} = 16^{h}\,25^{m}\,43^{s}{\cdot}85, \qquad \text{N.P.D.} = 105^{\circ}\,49'\,49''{\cdot}8.$$

The comparison star employed was

A.Z. 205,89 = A.Z. 297,91 = 30055 Lalande,

and its assumed place for the same epoch,—

$$\text{Mag. } 7{\cdot}7 \qquad \text{R.A.} = 16^{h}\,24^{m}\,39^{s}{\cdot}35, \qquad \text{N.P.D.} = 105^{\circ}\,40'\,43''{\cdot}9.$$

Moonlight and cloudy weather prevented its changes from being closely watched; but the following records suffice to establish its variability, and to show that the maximum is of brief duration; a circumstance also corroborated by the numerous occasions on which, for eight successive years, I have swept this part for new planets, without having entered it upon my chart, although much smaller stars in the immediate vicinity are all duly inserted:—

1860, April 16,	11·0 mag.	May 26,	Quite gone, and under 12·5 mag.
17,	11·0	28,	„ „ 13·0
20,	10·8		Carefully sought for on this last occasion,
21,	10·5		under very favourable circumstances.

The occultation of *Jupiter*, on May 24, was a complete failure here, owing to passing clouds, although two hours before the phenomenon the sky was so clear that the planet was well seen by Mrs. Pogson and a visitor, in full sunshine, with the 5-foot Smythian telescope, when the aperture of the object-glass was reduced to 1½ inch. Had it been as distinct with the equatoreal at the requisite moments, good observations might have been secured, instead of the two following dubious records, which are scarcely worth publishing, but all I could obtain with power 93:—

	h m s	
Disappearance of Jupiter's western limb	4 33 15	Greenwich M.T.
Reappearance „ „	5 43 49	„

Hartwell Observatory, June 6, 1860.

Note on a Grey Stripe extending from Burckhardt northward, and coincident with the Eastern Flank of Geminus, both Lunar Craters. By W. R. Birt, Esq.

The various tints reflected from the surface of the moon, considered apart from shadows, are dependent on three conditions: some may result from the colour of the lunar surface, properly so called, arising from the material of which that surface may be composed, and the different tints thus reflected will be as numerous as the materials forming the surface; others may be dependent not so much on the surface itself as on various superficial coverings by which many parts of the surface may be obscured: if these coverings possess much continuity, and are sufficiently thick to obstruct or prevent reflection from the surface beneath, the colour of the superficial covering only will be reflected; while, on the other hand, if the superficial covering be scanty, leaving portions of the surface here and there bare, a mixture of colour will be produced, giving rise to a tint, the result of the blending of the two.

The mid-day illumination of *Geminus* reveals a stripe of a greyish hue which skirts the eastern border of the crater; it extends from *Burckhardt* to a small crater marked a* on the diagram illustrating my paper on the mid-day illumination of *Geminus* (*Report of the British Association for the Advancement of Science*, 1859, p. 33 of the Transactions of the Sections). It is the topography and general features of this stripe that I am at present desirous of settling, so far as my observations during the past summer and present spring will enable me to do. My observations are not yet sufficiently extensive to enable me to say if this stripe extends further, either in a northerly or a southerly direction.

Under the morning illumination of *Geminus*, the external shadow of the eastern wall of the crater is not so strong nor so well defined as the internal shadow of the western. About the middle portion of the eastern flank (externally) there are strong indications of a small crater: they are not however constantly visible at the same ages of the moon. One of the most important features connected with this stripe is the existence of a mountain chain on which the crater a* is situated: this is seen in bold relief, both under the morning and evening illuminations, and is probably equal in length to the diameter of *Geminus*. This mountain range, as such, disappears entirely under the mid-day illumination, the crater presenting a lucid ring with a *grey* interior. So far as my observations have yet proceeded, I am disposed to come to the conclusion that the grey stripe passes clear, or very nearly so, of the mountain range, just skirting its western edge: it is not visible under the *early* morning illumination.

Under a suitable angle of illumination, the mountain range presents a very interesting feature: it is a very *narrow* shadow bounding the western base, as if the entire mass rose from a depressed but very shallow basin, and by which it is isolated from the surrounding country.

There is a peculiarity of the grey stripe which merits attention: the external shadow of the eastern wall of *Geminus*, in the neighbourhood of the indicated crater, is replaced by the darkest portion of the stripe; and long before the shadows are obliterated by the advance of the lunar day, the general direction of the stripe is seen; indeed the stripe itself appears much earlier than the features that characterise the mid-day illumination.

How far this stripe may be dependent on any of the above-named conditions future observations alone can decide. Next to the separation of *Geminus* into a dusky and lucid portion, and the lighter area with which the lucid portion is connected, it is the most interesting feature in the locality. If it derive its colour from the material forming the surface hereabout, it is likely to be persistent; but should it result from a *superficial* covering, there may be a probability of its undergoing some change in the lapse of time. The character of the markings eastward of it deserves attention.

Many portions of the moon's surface deepen in tint as the lunar day advances, particularly some of the large grey plains: the *Mare Serenitatis* is an instance; also the dark floor of the crater *Plato*, which, from having a dash of green under the early morning illumination, passes from a greenish through a blackish grey; and when the sun has just passed its meridian, the prevailing tint is a diluted inky black, with a branching oblique streak at the present epoch.

Observations of Donati's Comet, made at the Royal Observatory, Cape of Good Hope, between October 11th, 1858, and March 4th, 1859. By Sir Thomas Maclear, Director of the Royal Observatory, Cape of Good Hope.*

(*Communicated by the Astronomer Royal.*)

The clouded state of the sky prevented the detection of this splendid comet at the Cape Observatory before the 10th of October. On the evening of that day there was a complete view of it for a few minutes, afterwards the tail only could be seen through occasional breaks in the clouds. Rain fell on the forenoon of the 11th, after which the clouds dispersed, and as the evening twilight closed in, there is no exaggeration in the statement that the westward view from the observatory was magnificent beyond description. *Venus* in full splendour, the young moon, and the comet, presented an imposing display of celestial objects rarely witnessed, while the feelings were rendered more sensitive to the sublime and the beautiful by one of the serene evenings so delicious in the Cape climate when the atmosphere is calm and moist.

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* Ordered to be printed *in extenso* in the *Memoirs* of the Society.—Ed.

Observations for orbit being the primary consideration, and the time short for making them during the first few evenings of the appearance, the 8½-feet equatoreal could not be spared for elaborate physical examination. This, however, is of little importance, since more powerful instruments were available in Europe and America; and the decision has been verified by the published observations, up to the 20th of October, which have reached the Cape, and which are in accordance with our own so far as the latter were carried. After that date there was little to record beyond the gradual fading away of the image.

Two series of observations for orbit were made, viz., one by Mr. Mann, with the 8½-feet equatoreal, which comprehends the whole interval of visibility, commencing on the 11th of October and ending on the 4th of March following. The other with the 46-inch telescope by myself and G. W. H. Maclear conjointly, commencing on the 16th of October and ending on the 15th of November, when the comet became too faint to be observed with that instrument armed with a spider-line micrometer.

From the commencement of the observations with the 8½-feet equatoreal to Nov. 16th, the measures were taken with a position micrometer fitted with flat wires 10″ in breadth—the spider-line micrometer belonging to the instrument being then in Mr. Simms' hands for the execution of certain alterations. On November 17th it was received from England, and employed in observation up to December 29th, when the flat wire was again substituted, as the comet had then become too faint to bear the slightest illumination of the wires.

The powers generally employed were 126 with the flat wire micrometer, and 161 with the spider-line micrometer.

The differences in R.A. were obtained by direct micrometer measurement on the parallel, using the "driving clock;" except on November 18th and 22d, and December 11th and 14th, when the transits of both objects were observed with the system of fixed lines in the spider-line micrometer. The observations with the 46-inch telescope were made by means of a spider-line micrometer, having one line fixed and two moveable; and the differences in R.A. were measured in time by transits over the fixed wire—the differences in polar distance, by the micrometer screws at the instants of transiting the fixed wire.

The observations are of inferior value compared with those made with the more powerful instrument and the facilities afforded by its superior mounting; however, they are useful as vouchers for the interval they embrace, as the same stars were seldom used with both instruments.

Of the 114 stars of comparison, several are of very small magnitude—a matter of no moment with respect to relative accuracy of position, as the whole list has been observed with the large transit-circle, each star at least three times.

As the headings of the several columns are sufficiently explicit, and the observations are given in the usual form, no further explanation is needed, other than that the corrections for refraction have been applied, and that the columns headed log $\frac{p}{P}$, log $\frac{q}{P}$, contain the logarithms of the parallax factors in right ascension and polar distance respectively.

Royal Observatory, Cape of Good Hope,
1860, *April 20th.*

Eléments de la Comète découverte à Olinda le 26 Février, 1860, *d'après les Observations faites à l'Observatoire de la Commission Scientifique, du 26 Février au 13 Mars.* Par M. Emm. Liais, Astronome à l'Observatoire Impérial de Paris en Mission Scientifique du Gouvernement Français, et Président de la Commission Scientifique chargée par le Gouvernement Brésilien de réviser la Carte de la Côte du Brésil.

(*Inclosed in a Letter addressed to the President.*)

Distance Périhélie = 1·197342
Inclinaison = 79° 35′ 54″·5
Longitude du Nœud = 324 3 25·4 } Equinox Moyen,
Longitude du Périhélie = 173 45 21·1 } du 1 Janvier, 1860.

Passage au Périhélie le 16 Février, à 13ʰ 50ᵐ 6ˢ·5 Temps Moyen d'Olinda.
Sens du Mouvement, *Direct*.

Ces éléments s'appliquent à la grande nébulosité de la Comète, qui offrait le singulier aspect de deux nébulosités visibles à la fois dans le même champ. Ils représentent les observations avec une assez grande approximation. La plus grande différence sur l'A.R., est de 1ˢ·34, le 10 Mars, et sur la déclinaison de 21″·2, le 12 Mars, 1ᵉʳᵉ comparaison.

La seconde nébulosité suivait la première,—

Le 27 Février, à 10ʰ·25 de 25ˢ en A.R.
Le 3 Mars, à 11ʰ·16 de 23ˢ
Le 10 Mars, à 8ʰ·25 de 21ˢ

Sa Déclinaison S. était moindre les mêmes jours respectivement de 1′ 8″, 46″, et 21″.

Sa lumière était très foible, et a présenté des variations assez notables. Le 13 Mars, il a été impossible de l'apercevoir. Elle était circulaire, tandis que la grande nébulosité était allongée dans le sens du rayon vecteur au soleil.

Olinda, 12 *Mai*, 1860.

Notice of some Errors in Baily's Tables and Formulæ.
By J. O. Farrell, Esq.

The late Mr. F. Baily's *Astronomical Tables and Formulæ* (London, 1827) being a work of considerable reputation and of frequent reference by the practical and theoretical astronomers of this country; perhaps it may be useful to point out an error in one of the formulæ, as it has led the author to insert, in the *Elements of the Solar System*, an erroneous value of the excentricity of the earth's orbit. I refer to formulæ xlvi. (p. 119), the first two of which give the relations between the excentricity (e) and the greatest equation of the centre (E), viz.—

$$E = 2e + \frac{11}{2^4 . 3} e^3 + \frac{599}{2^{10} . 5} + \frac{17219}{2^{16} . 7} + \dots$$

$$e = \tfrac{1}{2} E - \frac{11}{2^4 . 3} E^3 - \frac{587}{2^{16} . 3 . 5} - \frac{40583}{2^{23} . 5 . 7 . 9} - \dots$$

The first equation is correct; but in the second equation the coefficient of E^3 should be $-\frac{11}{2^8 . 3}$, instead of $-\frac{11}{2^4 . 3}$. As the author appears to have followed Delambre very closely (see note, p. 229; also p. 274, line 2), it is very likely that these formulæ have been copied from Delambre's *Abrégé d'Astronomie*, p. 256, where the very same error occurs.

In pages 19 and 66 the excentricity of the earth's orbit (1 Jan. 1801) is stated to be ·016783568. The author, having remarked that it differs from that given by Laplace (·01685318), informs us that he has deduced this value from the greatest equation of the centre (1° 55′ 27″·3), by means of the formula $\frac{1}{2} E - \frac{11}{48} E^3$. The correct formula, however, as above stated, is $\frac{1}{2} E - \frac{11}{768} E^3$; and hence the excentricity corresponding to the greatest equation of the centre here assumed is 0·016791707. That this is the proper value is confirmed by Bessel's value for the same epoch (·0167918226), given in the Appendix, p. 270. With respect to Laplace's value of this quantity (·01685318), it would seem to be that corresponding to a previous epoch; but not having hitherto had the advantage of consulting any of Laplace's works, I am unable to verify the conjecture.

Again, assuming the values of greatest equation of the centre given in p. 66, I have calculated the corresponding values of the excentricities; they are inserted in the last column of the following table; from which it appears that values of these quantities, given by Mr. Baily, for *Mercury, Venus, Earth,* and *Mars,* are very inconsistent.

Planet.	Greatest Equation of Centre. Baily, p. 66. ° ′ ″	Excentricity. Baily, p. 66.	True Excentricity corresponding to Greatest Equation.
Mercury	23 39 51·0	·205514940	·20549252
Venus	0 47 15·0	·006860740	·00687220
Earth	1 55 27·3	·016783568	·016791707
Mars	10 40 50·0	·093307000	·09311252
Jupiter	5 31 13·8	·048162100	·04816264
Saturn	6 26 12·0	·056150500	·05615019
Uranus	5 20 57·0	·046679380	·04666863

The elements of the orbits of *Jupiter* and *Saturn* are very nearly consistent. The error in that of the *Earth* has been accounted for above; but in those of *Mercury, Venus,* and *Mars,* the inconsistency is so great that it is difficult to account for it without reference to the original works from which they may have been extracted or deduced.

As, however, these wrong or inconsistent excentricities have been inserted by Sir John Herschel in all the editions of his treatise on Astronomy (*Lardner's Cyclopedia,* new edition, 1842; *Outlines of Astronomy,* 3d edition, 1850), and thence into almost all our popular works on Astronomy, it seems very desirable that an authoritative synopsis of the elements of our system should be prepared, and thus prevent, if possible, the perpetuation of the erroneous elements referred to.

It may be worth while to notice, briefly, another error in the formulæ which also appears to have led the author and others into error.

In formulæ xliii. p. 116, for computing the relative length of a degree of longitude and latitude, &c. on the surface of the earth, the expression for the length of a degree of latitude is wrong; it should be multiplied by the length of a degree of latitude at the equator. This mistake makes all the numbers in the column "Degree of latitude" (Table xxxiv. p. 181) erroneous. This wrong formula also causes a similar error to exist in Table viii. appended to F. W. Simms's treatise on Mathematical Instruments (London, 1844), where the lengths in feet of a second of latitude are erroneous.

A somewhat similar error occurring in the last edition of Biot's *Traité Elémentaire d'Astronomie Physique* (Paris, 1857) may be noticed. I refer to the Table of the Elements of the Orbits of the Asteroids, given in tom. v. pp. 531, 532, of the above work, where the numbers inserted in the column "Moyen mouvements" are altogether erroneous.

Ordnance Survey Office, Southampton,
19th May, 1860.

Note on the History and Present Condition of the Problem "To find the Latitude by Observations of the Pole-star at any time." By John Riddle, Esq.

In the *Philosophical Magazine* for June 1822, the late Francis Baily, Esq. drew the attention of English astronomers to a method of deducing the latitude from altitudes of the pole-star at any time, which had been published by M. Littrow in 1817, in the *Zeitschrift für Astronomie,* vol. iii. p. 208: the formulæ being identical with those which occupy so prominent a place year after year in the preface to the *Nautical Almanac.* This paper is followed by "Further Remarks" by the same author in July 1822, in which the history of the problem is given, with the conclusion that, though it appears that a general discussion of such observations most probably originated in our own country, the first *correct* solution is that of M. Littrow.

Subsequently to these papers the problem was dignified with a very prominent place in the title-page and tables of Schumacher's *Ephemeris,* and after the change in the form of the *Nautical Almanac* (1835) Schumacher's tables were adopted in its pages.

Until 1845 the following statement relating to them is annually repeated in the preface:

"The tables for finding the latitude of a place by observations of the pole-star (α *Ursæ Minoris*) at any hour of the day *are similar to those published annually by Professor Schumacher in his Ephemerides of the Planetary distances,* and are founded, &c. &c."

In 1845 and afterwards the sentence in Italics is omitted; in all other respects the statement has remained unchanged to the present time. Whatever may have been the advantages afforded by these tables at the time that their celebrated author gave them so prominent a place, it has long been my opinion, founded on a larger experience of nautical matters than falls to the share of most people, that they do not worthily occupy the large space annually devoted to them in the *Nautical Almanac.*

The first table contains the values of

$$p \cos h,$$

calculated with mean values of the polar distance and right ascension of the star. This renders necessary Table III., which gives the corrections due to the fluctuations of the true values from the mean values; a minute or two being added to render the contents of the table positive; still further obscuring the principles of the problem.

The second table contains the value of

$$\frac{1}{2} (p \sin h)^2 \tan \text{alt.}$$

generally a comparatively small quantity, not sharing in any degree in the complements of Table III.

From their complicated and unpractical character I have long ceased to employ these tables, and I am sure that they are rarely if ever used by anybody.

In order to obtain the results given in the example of their use which is also annually furnished in the explanation of the contents of the *Nautical Almanac,* a troublesome interpolation is necessary with Table I., and, if possible, a more vexatious one with Table III.; while the *Nautical Almanac* furnishes the means of finding the correct value of $p \cos h$ by the addition of two logarithms. These two tables deserve no place in the *Nautical Almanac;* they contribute to obscure the problem, and afford very little if any facility to its solution.

The second table which gives

$$\frac{1}{2} (p \sin h)^2 \tan \text{alt.}$$

might with advantage be extended and given for shorter intervals of sidereal time, and rendered more compact by condensing the minutes and seconds into seconds only.

I should prefer the meridian distance (h) for the argument, instead of the sidereal time, as showing to students more directly the connexion between the table and the formula on which it depends.

The late Dr. Inman gave in his Tables the total mean correction with latitude and sidereal time as arguments; this, as an approximation, may not be without its value to seamen who prefer the "rough and ready" to more laborious accuracy, and, it must not be omitted, in a sketch of the present position of this interesting problem, that the Board of Admiralty have recently ordered the re-calculation of Dr. Inman's table, the tables contained in the *Nautical Almanac* for the same purpose notwithstanding.

A Memoir on the Problem of the Rotation of a Solid Body.

By A. Cayley, Esq.*

The present memoir was written for the sake of the further elaboration of the analytical theory of the rotation of a solid body, upon principles similar to those of my "Memoir on the Problem of Disturbed Elliptic Motion," *Mem. R. Ast. Soc.* vol. xxvii. pp. 1–29 (1858); the like elements are adopted, and the course of the investigation corresponds precisely to that of the memoir just referred to. The formulæ for the variations of the elements in the two problems (the motion being in each case referred to a fixed plane of reference and origin of angles therein) are found to be, as it is known they should be, identical in form; an investigation, in the present memoir, of the transformation of the system to the case of a variable plane of reference and departure-point as an origin of angles in such plane, would have been a mere repetition of that contained in the former memoir, and it was therefore unnecessary to give it. A point in the present memoir to which attention may be called is the definition of the angle g (varying uniformly with the time, but used as an element) which corresponds to the mean anomaly in elliptic motion. Besides the ultimate system of formulæ for the variations of the elements in terms of the differential coefficients of the disturbing function with respect to the elements, it appears to me that the intermediate formulæ for the variations in terms of the differential coefficients with respect to the co-ordinates (which are in the ordinary investigation altogether passed over) are not without interest, and that it is possible that they might be employed with advantage in the integration of the equations of motion for the purposes of physical astronomy.

M. Otto Struve, present at the meeting, directed the attention of the Society to the results of his measures made on artificial double stars. These researches prove that the angles of position of close double stars, as measured by M. Struve with the filar micrometer of the great Pulkowa refractor, are subject to systematic errors, depending for their greater part on the angle formed by the direction of the two stars with the vertical. The amount of these errors, expressed in linear distances, exceeds for some directions considerably that of the constant or systematic errors, as they have been found to exist in measures of distances, either by direct observation of artificial stars or by comparison of measures made by different observers. Of course these angular errors must exercise a great influence on the elements of orbits of double stars, when calculated from M. Struve's observations, and, therefore, it would be almost irrational not to take account of them in the computation of orbital motions. But there might arise the question, whether the same formula, which has been deduced by M. Struve from observations made in nearly a horizontal direction, ought to be likewise applied to observations nearer the zenith. To decide this question M. Struve has made during the last five years a series of observations on northern double stars under very different horary angles; which, though not yet completely reduced, leave him no doubt that the coefficients of the variable members of the formula of correction are pretty nearly the same for all elevations above the horizon. But these observations cannot answer for the first member of the formula, which, being independent of the angle with the vertical line, varies merely with the distance or the visual angle (the distance multiplied by power) of the two compared stars. With a view of inquiring into this matter, a series of simultaneous observations has been made on different nights in the present spring by M. Struve himself and six of his collaborators at the Observatory of Pulkowa, — MM. Döllen, Wagner, Winnecke, Schiaparelli, Oom, and Oblomievski. The results of these observations were exhibited to the Society. They show that, after application of the corrections by the given formula, the angles measured by M. Struve agreed in all instances so nearly with the mean of the measures of the other observers, that the remaining differences nowhere exceed the order of admissible accidental errors. At the same time these measures exhibit very strikingly similar systematic errors, but following different laws for several other observers. Hence it must be concluded, 1st, that the correction formula, as deduced from M. Struve's observations of artificial double stars, has its full value for all inclinations of the telescope; 2d, that, though perhaps in M. Struve's case, the systematic errors are extraordinarily great, analogous errors may be supposed to exist more or less for all other observers who follow the same rules of observation as himself.

In pointing out these results, M. Struve remarked that it could hardly be expected that every observer of double stars would have the time, and might wish to spend so much time and labour on measures of artificial double stars as he had done himself; but certainly every one would be anxious to ascertain the existence and amount of similar errors in his own observations. For this purpose he suggested that approximate values of these errors might be deduced if the observer would take care to note the sidereal time of his observation and the magnifying power, and if, at the same time, he would attend to the rule, that for all observations his head was kept as nearly as possible in a vertical position. In that way each longer series of observations would certainly offer a considerable number of measures that might be compared with others for which the correction formula is deduced from direct measures on artificial double stars; and when made for the same objects under very different horary angles, would probably contain in themselves sufficient data for an approximate deduction of the variable members of the correction formula.

From what he had said M. Struve derived the general conclusion that only when the measured angles were thus cleared from their inherent systematic errors, they really would merit, in comparison with the distances, that preference which many astronomers have been in the practice of attributing to them for the deduction of the elements of orbital motions. In his opinion many discrepancies between theory and observation that have been pointed out by several computers, would disappear when the indicated corrections were applied.

In connexion with the preparations for the solar eclipse,

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* Ordered to be printed *in extenso* in the *Memoirs* of the Society.— Ed.

M. Otto Struve availed himself of the opportunity to acquaint the English astronomers with the contents of two Reports on the observations of the eclipse of 1851, one of which never has been published; the other only in the Russian language.

The first of these Reports was from the late Major-General Wrontchenno, a man who, with extraordinary qualifications for accurate observation, joined a rare combination of different talents. Having chosen his station in the plains of Texaterinoslaw, on the shores of the Dnieper, General Wrontchenno, being also an eminent painter, directed his attention particularly to the colours of the prominences, and was enabled to ascertain, during the few minutes of total eclipse, at least in one of the prominences a gradual transition from white, through rose, to dark pink. This observation is of a particular interest, as it indicates that the reason why on different stations quite different colours have been indicated for the same prominences, is to be attributed to the nature of the prominences. Therefore M. Struve recommended this sort of observations to the attention of amateur astronomers, who, not being sufficiently provided with means for exact measures, would by a careful repetition of this kind of observation render a valuable service to science, and perhaps would essentially contribute to explain the nature of the prominences.

The other Report was from Colonel (now Major-General) Chodzko, the Surveyor-General of the Caucasian provinces, who for his station had selected an isolated mountain of the Caucasus, called Galawdour, about 11,000 feet high. At this height Mr. Chodzko saw the same prominences nearly in the same position on the moon's disk, and of the same dimensions and shape, as they have been seen on the most occidental stations of the central line (in Norway, Prussia, &c.), with only the difference that he judged their colour to be yellowish. This observation seems to contradict quite decidedly the supposition maintained by some astronomers, that the prominences are formed by clouds in the atmosphere of the earth, which certainly cannot be supposed to be of the same dimensions and figure, and therefore cannot give the same appearances, on the level of the sea and 11,000 feet above it. As to the corona, it appears from Mr. Chodzko's drawing that at such a height it was considerably less extended than has been observed at lower stations. Hence it might be concluded that the phenomena of the corona are in a great measure produced by our atmosphere.

After having exposed the principal contents of these two papers, M. Otto Struve placed a verbal translation of the original Reports at the disposition of the Society, for reference on future occasions.

Lescarbault's Planet.

Mr. Ellery, Director of the Victoria Observatory, in a letter dated 1860, April 24, states that he had half-hourly observations of the sun made throughout the period indicated, without success.

Major Tennant, Director of the Madras Observatory, states that, with the exception of times when clouds prevented observation (which he gives in detail), the sun's disk was watched every few minutes from March 27 to April 10, without success. (Date of letter, April 13, 1860.)

Mr. Scott, Director of the Observatory at Sydney, states that "he looked for Lescarbault's Planet without success." (Date of letter, April 16, 1860.)

The following letters have been received in relation to the recently observed Comet.

Mr. J. M. Stothard, in a letter dated Dublin, 25th June, writes:—

"I observed the expected comet on last Friday and Saturday nights, at about 11¼ P.M., through a chink in the clouds, the rest of the sky being dark and starless, it was situated a little to the west of north, and most probably was in the constellation of *Auriga;* it was much brighter and more readily visible to the naked eye than Donati's Comet during the first few nights of its appearance. On Saturday night I observed it through my 3-inch refractor, and for a moment saw it pretty well with a distinct tail, with a power of 60."

Colonel De Rottenburg, in a letter dated Gibraltar, 24th June, writes:—

"I observed a brilliant comet in the constellation of *Auriga* this evening, a line from β through α *Geminorum* continued about three times their distance passed through its nucleus. It was very easily seen by the naked eye, the nucleus was a little less brilliant than *Castor.* Its altitude above the western horizon about equal to that of *Venus* at the same time. I turned my 6-feet equatoreal 4½ inches aperture, by Messrs. Cooke and Sons of York, on the comet. It has a bifid tail, very like that of the year 1846, as shown in Keith Johnstone's *Atlas of Astronomy,* which is edited by Mr. Hind. I used powers of 26, 50, and 100 on the comet; the nucleus has a very sensible disk. It bore powers up to 100 very well; one portion of the tail is much longer than the other, the south preceding being the longer; I give rough drawings of the comet's appearance in the telescope.

"With 100 power the nucleus was situated within the nebulosity, and the nebulosity was more arched and prominent on the south preceding part. It was first seen by a gentleman here on Saturday evening 23d June.

"I have been examining the part of the heavens where it is situated lately, but did not see it till this evening."

Mr. Edwin Clarke, in a letter dated Observatory House, Forest Hill, 26th June, writes:—

"The comet was seen here on Sunday evening about $10^{h}\cdot25$ for a few minutes extremely well. As my large equatoreal unfortunately does not sweep north-west on account of houses, I lost my time in endeavouring to mount a 4-foot equatoreal out-of-doors. I had a brilliant view through the 4-inch for a minute or two only, when the clouds again covered it, the sky was extremely clear at this moment, and the nucleus was very sharply defined, having a large distinct disk, the brush was vertical nearly below the head. It was but slightly divergent and of great length. It was a very striking object to the naked eye, and of great brilliancy. As far as I can estimate it was in the constellation *Auriga,* about 10 degrees west of *Capella;* but this is a mere guess, as I had no time for any measurement either of the nucleus or position, and no recognisable stars were visible, as the clouds did not again clear away. I was equally unsuccessful last night, the motion is evidently extremely rapid, judging from the positions given by others, and unless visible by daylight, which is possible, it will, I fear, be only seen for a few evenings longer. The head of the disk was peculiarly bright and well defined."

MISCELLANEOUS INTELLIGENCE.

The Rev. R. Main, President of the Society, has been by the Board of Trustees of the Radcliffe Observatory at Oxford appointed Radcliffe Observer.

RECENT PUBLICATIONS.

Mr. Stanford, of No. 6 Charing Cross, has lately published a reissue of the small edition of star-maps on the gnomonic projection, as originally prepared by Sir J. Lubbock for the Society for the Diffusion of Useful Knowledge. In the present edition the advantages of this very convenient arrangement of course remain, but the maps are now rather encumbered by the introduction of the constellations, which belong rather to the heavens of the ancients than to our own times. When introduced at all, it is desirable that these fanciful figures should, as the *Uranometria* of Professor Argelander, be very lightly drawn in outline, so as not to overlie the stars themselves. The details of the Milky Way are still very imperfectly given, as in the first edition, although the Southern Catalogue of Sir J. Herschel has some years since supplied the means of a far better delineation. Mr. Stanford has also an edition of the same maps printed in lithograph, in which the trajectories of the principal comets of this century have been drawn by Mr. James Breen. Both editions are sold at the same price, 3*s.* plain and 6*s.* with the constellations coloured.

The first volume of the Catalogue intended to accompany Professor Argelander's Charts has appeared, and contains nearly 111,000 positions of stars, arranged with the greatest possible condensation. The remark chiefly called for, before the completion of this great work for the Northern Hemisphere, now so rapidly and successfully progressing, is, as Professor Argelander has himself indicated, that a similar catalogue or series of maps is highly desirable for the Southern Hemisphere.

Observaciones Astronomicas hechas en el Observatorio Nacional de Santiago de Chile en los Años de 1853, 1854, *i* 1855. Por el D^r. Carlos Guill^o. Moesta, Director del Observatorio. t. I. 4°. Santiago de Chile, 1859.

The formation of the Observatory dates from the 17th August, 1852, when the Government of the Republic purchased as national property the instruments and other articles brought to Chili by the United States Astronomical Naval Expedition under the command of Lieut. Gilliss. Other instruments have since been acquired or ordered, and a proper building for the observatory is in course of construction; but the temporary observatory consists of two wooden houses (one of them containing the meridian circle, the other the equatoreal), brought ready made from North America, and placed by Lieut. Gilliss on the Cerro de S. Lucia, where they still remain, and where all the observations published in the volume have been taken. The Cerro de S. Lucia, formerly called Cerro de Huelen, is a crag rising in the centre of the city, abruptly from the plane, and composed of a metamorphic porphyry of columnar structure, very susceptible of atmospheric influences. The eastern view of the rock, with the two houses thereon, is shown in the vignette to the volume: the situation, exposed to all the noise and dust of a crowded city, and on account of the want of stability of the instruments, is stated to be an unfavourable one.

There is an endowment for an Assistant, and for two youths to study astronomy at the observatory: these are from time to time chosen by the Government from the pupils of the National Institute who have most distinguished themselves in mathematics. For want of a proper person, the post of Assistant was not filled up during the period of the observations in the volume; and all the meridian observations, and the greater part of the reductions, were made by the Director himself. The Director is, *ex officio*, Professor at the University, and gives there alternately an annual course on the differential and integral calculus, and on practical astronomy.

A chief part of the proposed plan was to take meridian observations of the stars. In the first place, it was necessary to collect data for the exact determination of the geographical position of the observatory; and for this purpose, particular attention was paid to the culminations of the moon, and to the occultations of stars by the moon, and observations were made of the zenith distances of the fundamental stars, and of various circumpolar stars. The other meridian observations contained in the volume relate chiefly to stars selected in the following order:—

1st. The southern stars of the British Association Catalogue, the positions of which appear to be doubtful or erroneous, a long list of which is given at the end of that catalogue.

2d. The stars of the catalogue of Lacaille (*A Catalogue of* 9766 *Stars in the Southern Hemisphere*, &c. London, 1847.) not observed since the time of Lacaille, or at least those which are not to be found in the catalogues of Brisbane, Taylor, and Johnson.

3d. The stars up to the 9th magnitude situate between the parallel of 62 degrees south and the zenith.

Besides the specified stars, meridian observations were made of the following planets:—*Venus, Mars, Ceres, Juno, Pallas, Vesta, Hebe, Iris, Parthenope, Psyche, Proserpine, Amphitrite, Jupiter, Saturn, Uranus*, and *Neptune*.

The Director intends to devote special attention to the observations of *Venus*, believing that in time they will furnish a contribution to the more exact determination of the sun's parallax, regard being had to the favourable position of the observatory, as compared with those in the other hemisphere.

VOL. XX. *Supplemental Notice.* No. 9.

Observations sur un Calcul de M. Delaunay inséré dans les Additions à la Connaissance des Temps pour 1862, et relatif à la Détermination du Coefficient de l'Equation Séculaire du Moyen Mouvement de la Lune. Par M. le Comte G. de Pontécoulant.

L'auteur de la note dont il s'agit s'est proposé de déterminer par la méthode de la *variation des constantes arbitraires*, le coefficient de *l'équation séculaire* du moyen movement lunaire, en n'ayant égard qu'aux termes dépendants du carré de la force perturbatrice, et de vérifier par ce moyen la valeur attribuée à ce coefficient par M. Adams dans son mémoire bien connu de 1853. Mais M. Delaunay n'a point atteint le but qu'il s'était fixé, et nous avons démontré ailleurs,* que la *prétendue* identité qu'il avait cru exister entre le résultat auquel il était parvenu et celui de M. Adams, n'était que l'effet d'une singulière illusion, et n'avait été obtenue que par une fausse application de la méthode qu'il avait employée, jointe à une valeur erronée attribuée à la quantité m, par laquelle tous les géomètres, qui ont traité depuis Laplace la théorie de la lune, ont constamment désigné le rapport du moyen mouvement du soleil à celui de la lune, donné par l'observation. Nous nous proposons ici de montrer comment l'application de ces mêmes formules *de la variation des constantes arbitraires*, dont la première idée est due au grand Euler, et dont la théorie a été ensuite si heureusement perfectionnée par Lagrange, Laplace, et Poisson, en se conformant rigoureusement aux principes qu'ils ont posés, reproduit *identiquement* les différents termes du coefficient de *l'équation séculaire* du mouvement moyen de la lune, que nous avons présentés dans le Nombre des *Monthly Notices* du mois de Mai dernier.† Nous esperons que cette parfaite concordance entre des résultats déduits de deux méthodes si différentes, dissipera les doutes que la persistance de M. Adams à soutenir l'exactitude d'une analyse dont nous avons eu plusieurs fois l'occasion de signaler le défaut, aurait pu laisser dans quelques esprits.

Si l'on désigne comme à l'ordinaire par a, e, ε, ϖ, γ, θ, les six élements de l'orbite elliptique de la lune, ou de l'orbite qu'elle décrirait autour de la terre sans l'action du soleil, par R la fonction perturbatrice provenant de cette cause, qu'on représente de plus par n le moyen mouvement qui répond au demi-grande axe a, en sorte qu'on ait $a^3 n^2 = 1$, en supposant qu'on prend pour unité la somme des masses de la terre et de la lune, on aura pour déterminer la variation de la constante ε, on de ce que les géomètres appellent la longitude *de l'époque*, résultante de l'action des forces perturbatrices, la formule suivante :‡

$$d\varepsilon = -2a^2 n\,dt\left(\frac{d\mathrm{R}}{da}\right) + \frac{an\,dt}{2}\left(e\frac{d\mathrm{R}}{de}\right) + \frac{an\,dt}{2}\left(\gamma\frac{d\mathrm{R}}{d\gamma}\right). \quad (a)$$

Nous négligeons pour plus de simplicité les termes qui dépendraient des puissances de l'excentricité et de l'inclinaison de l'orbite lunaire, d'un ordre supérieur au carré.

Bornons d'abord les approximations aux termes d'un ordre inferieur à m^4.

En dévéloppant la fonction R, soit par la méthode ordinaire, soit par la méthode de la variation des constantes arbitraires (voir *Théorie Analytique*, liv. 7, Nos. 6 et 97), et en observant qu'on a généralement $\left(a\frac{d\mathrm{R}}{da}\right) = 2\,\mathrm{R}$, on trouvera

$$a^2\frac{d\mathrm{R}}{da} = \left(\frac{9}{8}m^2 + \frac{825}{32}m^3\right)e^2e'^2 + \left(-\frac{9}{8}m^2 + \frac{93}{32}m^3\right)e'^2\gamma^2.$$

On a d'ailleurs (*Théorie Analytique*, liv. 7, Nos. 6 et 97),

$$\left(e\frac{d\mathrm{R}}{de}\right) = \left(\frac{9}{8}m^2 + \frac{825}{16}m^3\right)e^2e'^2,$$

$$\left(\gamma\frac{d\mathrm{R}}{d\gamma}\right) = \left(\frac{9}{8}m^2 + \frac{33}{16}m^3\right)e^2\gamma^2.$$

Si l'on substitue ces différentes valeurs dans la formule (a), on aura

$$\varepsilon = \left\{\left[\left(-\frac{9}{4} + \frac{9}{16} = -\frac{27}{16}\right)m^2 + \left(-\frac{825}{16} + \frac{825}{32} = -\frac{825}{32}\right)m^3\right]e^2e'^2 \right.$$
$$\left. + \left[\left(\frac{9}{4} - \frac{9}{16} = \frac{27}{16}\right)m^2 + \left(-\frac{93}{18} + \frac{33}{32} = -\frac{153}{32}\right)m^3\right]e'^2\gamma^2\right\}n\,dt.$$

Portons maintenant l'approximation jusqu'aux termes de l'ordre m^4, en négligeant pour plus de simplicité les termes dépendants de l'excentricité et de l'inclinaison de l'orbe lunaire. Il suffira de supposer dans ce cas (*Théorie Analytique*, liv. 7, No. 102)

$$d\,.\,\delta\varepsilon = -2\,dt\,\delta\,.\,a^2n\left(\frac{d\mathrm{R}}{da}\right) + \frac{dt}{2}\delta\,.\,an\left(e\frac{d\mathrm{R}}{de}\right). \quad (a')$$

On a d'ailleurs

$$\delta\,.\,a^2n\left(\frac{d\mathrm{R}}{da}\right) = an\,\delta\,.\left(a\frac{d\mathrm{R}}{da}\right) + \delta an\,.\left(a\frac{d\mathrm{R}}{da}\right), \quad (b)$$

$$\delta.an\left(e\frac{d\mathrm{R}}{de}\right) = an\,\delta\,.\left(e\frac{d\mathrm{R}}{de}\right) + \delta an\,.\left(e\frac{d\mathrm{R}}{de}\right).$$

Le dernier terme de la seconde équation ne pouvant produire que des quantités dépendantes de l'excentricité e, que nous négligeons, on peut en faire abstraction.

L'équation ordinaire $\left(a\frac{d\mathrm{R}}{da}\right) = 2\,\mathrm{R}$ donne évidemment $\delta\left(a\frac{d\mathrm{R}}{da}\right) = 2\,\delta\mathrm{R}$; on peut calculer cette dernière valeur en employant la méthode de la variation des constantes arbitraires, mais il sera plus simple de la deduire des résultats que nous avons obtenus par le dévéloppement ordinaire de la fonction perturbatrice. Nous avons trouvé (*Théorie Analytique*, liv. 7, Nos. 6 et 45), en n'ayant égard qu' aux termes que nous voulons considérer ici

$$\mathrm{a\,R} = \frac{m^2}{4} - \frac{179}{96}m^4 + \left(\frac{3}{8}m^2 - \frac{799}{64}m^4\right)e'^2,$$

(a) dans cette formule désigne le demi-grand axe lunaire dans l'orbite troublée, a désignant ce que devient (a) relativement à l'orbite elliptique, on aura évidemment

$$a\,\mathrm{R} = \frac{a}{\mathrm{a}}(\mathrm{a\,R}).$$

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* *Comptes Rendus de l'Académie des Sciences* (Séance du Lundi, 16 Juillet, 1860).

† No. 7, vol. xx.

‡ *Théorie Analytique du Système du Monde*, livre 7, No. 93.

Mais nous avons trouvé par une première approximation (*Monthly Notices*, vol. xx. p. 272),

$$\mathrm{n} = n\left(1 - m^2 - \frac{3}{2}m^4 e'^2\right),$$

d'où, en observant que $\frac{a}{\mathrm{a}} = \left(\frac{\mathrm{n}}{n}\right)^{\frac{2}{3}}$, on conclut

$$\frac{a}{\mathrm{a}} = \left(1 - \frac{2}{3}m^2 - m^2 e'^2\right).$$

En multipliant par ce rapport la valeur précédente de a R, on trouvera la suivante

$$a\,\mathrm{R} = \frac{m^2}{4} - \frac{65}{32}m^4 + \left(\frac{3}{8}m^2 - \frac{831}{64}m^4\right)e'^2,$$

et l'on en conclura

$$an\,\delta.\left(a\frac{d\mathrm{R}}{da}\right) = -n\left[\frac{65}{16}m^4 + \frac{831}{32}m^4 e'^2\right].$$

Considérons maintenant dans le dévéloppement de la fonction R (*Théorie Analytique*, liv. 7, No. 6) les termes suivants

$$a\,\mathrm{R} = m^2\,\mathrm{F}\cos 2\xi + m^2\,\mathrm{F}'\,e'\cos(2\xi - \phi') + m^2\,\mathrm{F}''\,e'\cos(2\xi + \phi'),$$

où l'on représente, pour abréger, par ξ la longitude moyenne de la lune moins celle de la soleil, par ϕ' l'anomalie moyenne de ce dernier astre, ϕ désignant l'anomalie correspondante de la lune, en sorte qu'on a

$$\xi = (1 - m)\,nt + \epsilon - \epsilon'; \quad \phi = mt + \epsilon - \varpi; \quad \phi' = mnt + \epsilon' - \varpi'.$$

Les formules connues de la variation des constantes arbitraires, donnent pour déterminer la variation du grand axe, la formule suivante (*Théorie Analytique*, liv. 7, No. 87),

$$\delta a = 2a^2\int d'\mathrm{R}. \qquad (m)$$

En observant que l'on a $\int d'\mathrm{R} = \int\left(\frac{d\mathrm{R}}{n\,dt}\right)n\,dt$, en vertu de la valeur précédente de R on trouvera

$$a\int d'\mathrm{R} = m^2\left[\mathrm{F}\cos 2\xi + \mathrm{F}'\,e'\cos(2\xi - \phi') + \mathrm{F}''\,e'\cos(2\xi + \phi')\right].$$

Si l'on substitue cette valeur dans la formule (*m*) et qu'on combine la valeur résultante avec celle de la fonction $a\frac{d\mathrm{R}}{da}$ déduit de l'équation $\left(a\frac{d\mathrm{R}}{da}\right) = 2\,\mathrm{R}$, en n'ayant égard qu' aux termes non périodiques, on aura

$$\left(a\frac{d\mathrm{R}}{da}\right)\delta a = 2m^4\left(\mathrm{F}^2 + \mathrm{F}'^2 e'^2 + \mathrm{F}''^2 e'^2\right).$$

On a d'ailleurs (*Théorie Analytique*, liv. 7, No. 6)

$$\mathrm{F} = \left(1 - \frac{5}{2}e'^2\right); \quad \mathrm{F}' = \frac{21}{8}; \quad \mathrm{F}'' = -\frac{3}{8}.$$

Par conséquent

$$\left(a\frac{d\mathrm{R}}{da}\right)\delta a = \left[\frac{9}{8}m^4 + \left(-\frac{45}{8} + \frac{441}{32} + \frac{9}{32} = \frac{135}{16}\right)m^4 e'^2\right],$$

d'où en observant que $\delta an = -\frac{1}{2}n\,\delta a$ on conclura

$$\left(a\frac{d\mathrm{R}}{da}\right)\delta an = -n\left[\frac{9}{16}m^4 + \frac{135}{32}m^4 e'^2\right].$$

À l'aide de cette valeur et de celle de $an\delta\left(a\frac{d\mathrm{R}}{da}\right)$, trouvée plus haut, on formera la suivante

$$\delta.an\left(a\frac{d\mathrm{R}}{da}\right) = -n\left[\left(\frac{65}{16} + \frac{9}{16} = \frac{37}{8}\right)m^4 + \left(\frac{831}{32} + \frac{135}{32} = \frac{483}{16}\right)m^4 e'^2\right].$$

Occupons nous maintenant de calculer la valeur de la fonction $\delta.\left(e\frac{d\mathrm{R}}{de}\right)$. On a généralement

$$\delta.\left(e\frac{d\mathrm{R}}{de}\right) = e\,\delta.\frac{d\mathrm{R}}{de} + \frac{d\mathrm{R}}{de}\delta e.$$

Soit

$$\begin{aligned} a\,\mathrm{R} = {} & m^2\,\mathrm{G}\,e\cos\phi + m^2\,\mathrm{G}'\,e\cos(2\xi - \phi) + m^2\,\mathrm{G}''\,e\cos(2\xi + \phi) \\ & + m^2\,\mathrm{H}\,e\,e'\cos(\phi - \phi') \qquad + m^2\,\mathrm{H}'\,e\,e'\cos(\phi + \phi') \\ & + m^2\,\mathrm{K}\,e\,e'\cos(2\xi - \phi - \phi') \qquad + m^2\,\mathrm{K}'\,e\,e'\cos(2\xi - \phi + \phi') \\ & + m^2\,\mathrm{K}''\,e\,e'\cos(2\xi + \phi - \phi') \qquad + m^2\,\mathrm{K}'''\,e\,e'\cos(2\xi + \phi + \phi'), \end{aligned}$$

d'où l'on tire

$$a\left(\frac{d\mathrm{R}}{de}\right) = m^2\,\mathrm{G}\cos\phi + m^2\,\mathrm{G}'\cos(2\xi - \phi) + m^2\,\mathrm{G}''\cos(2\xi + \phi) + \text{etc.}$$

Pour former la fonction $a\,\delta.\left(\frac{d\mathrm{R}}{de}\right)$ il suffira donc de faire varier dans cette valeur la constante ϖ, qui se trouve comprise dans l'angle $\phi = cnt + \epsilon - \varpi$, mais en observant qu'en n'ayant égard qu'aux termes que nous considérons dans R, on a $e\frac{d\mathrm{R}}{de} = \mathrm{R}$, on en conclura $e\frac{d^2\mathrm{R}}{de\,d\varpi} = \frac{d\mathrm{R}}{d\varpi}$, et par conséquent

$$\delta.\left(e\frac{d\mathrm{R}}{de}\right) = \frac{d\mathrm{R}}{de}\delta e + \left(\frac{d\mathrm{R}}{e\,d\varpi}\right)e\,\delta\varpi. \qquad (n)$$

Les formules de la variation des constantes arbitraires (*Théorie Analytique*, liv. 7, No. 87), en négligeant les termes qui seraient d'un ordre supérieur, par rapport à l'excentricité et à l'inclinaison de l'orbe lunaire, à ceux que nous considérons, donnent

$$\delta e = -\int\left(\frac{d\mathrm{R}}{e\,d\varpi}\right)n\,dt; \qquad e\,\delta\varpi = \int\left(\frac{d\mathrm{R}}{de}\right)n\,dt.$$

En substituant pour $\frac{d\mathrm{R}}{e\,d\varpi}$ et $\frac{d\mathrm{R}}{de}$ les valeurs qui resultent de l'expression précédente de R et intégrant les formules résultantes, on trouve

$$\begin{aligned} \delta e = {} & m^2\,\mathrm{G}\cos\phi - m^2\,\mathrm{G}'\cos(2\xi - \phi) + \frac{1}{3}m^2\,\mathrm{G}''\cos(2\xi + \phi) \\ & + m^2\,\mathrm{H}\,e'\cos(\phi - \phi') \qquad + m^2\,\mathrm{H}'\,e'\cos(\phi + \phi') \\ & - m^2\,\mathrm{K}\,e'\cos(2\xi - \phi - \phi') \qquad - m^2\,\mathrm{K}'\,e'\cos(2\xi - \phi + \phi') \\ & + \frac{m^2}{3}\mathrm{K}''\,e'\cos(2\xi + \phi - \phi') \qquad + \frac{m^2}{3}\mathrm{K}'''\,e'\cos(2\xi + \phi + \phi'). \end{aligned}$$

On aurait la valeur de $e\,d\varpi$ en changeant simplement dans cette expression les cosinus en sinus et les signes des trois termes multipliés respectivement par G′, K, et K′.

Si l'on substitue ces valeurs et celles de $\left(\frac{dR}{de}\right)$ et de $\left(\frac{dR}{ed\varpi}\right)$ dans la formule (n), en n'ayant égard qu'aux termes non périodiques, on trouvera

$$a\delta.\left(e\frac{dR}{de}\right)=m^4\left[G^2-G'^2+\frac{1}{3}G''^2+e'^2(H^2+H'^2-K^2-K'^2+\frac{1}{3}K''^2+\frac{1}{3}K'''^2)\right].$$

Pour réduire cette formule en nombres, observons que par le dévéloppement de R (*Théorie Analytique*, liv. 7, No. 6) on a

$$G=-\frac{1}{2}\left(1+\frac{3}{2}e'^2\right);\ G'=-\frac{9}{4}\left(1-\frac{5}{2}e'^2\right);\ G''=\frac{3}{4}\left(1-\frac{5}{2}e'^2\right),$$

d'où l'on tire

$$G^2-G'^2+\frac{1}{3}G''^2=\left(\frac{1}{4}-\frac{81}{16}+\frac{3}{16}=-\frac{37}{8}\right)+\left(\frac{3}{4}+\frac{405}{16}-\frac{15}{16}=\frac{201}{8}\right)e'^2.$$

On a ensuite (même No.)

$$H=-\frac{3}{4};\ H'=-\frac{3}{4};\ K=-\frac{63}{8};\ K'=\frac{9}{8};\ K''=\frac{21}{8};\ K'''=-\frac{3}{8},$$

d'où l'on conclut

$$H^2+H'^2-K^2-K'^2+\frac{1}{3}K''^2+\frac{1}{3}K'''^2=\left(\frac{9}{16}+\frac{9}{16}-\frac{3969}{64}-\frac{81}{64}+\frac{147}{64}+\frac{3}{64}=-\frac{957}{16}\right).$$

En vertu de la deuxième des formules (b), on aura donc

$$\delta.an\left(e\frac{dR}{de}\right)=an\delta.\left(e\frac{dR}{de}\right)=$$

$$n\left[-\frac{37}{8}m^4+\left(\frac{201}{8}-\frac{957}{16}=-\frac{555}{16}\right)m^4e'^2\right].$$

Si l'on substitue maintenant à la place de $\delta.an\left(a\frac{dR}{da}\right)$ et de $\delta.an\left(e\frac{dR}{de}\right)$ leurs valeurs dans la formule (a'), on aura

$$d.\delta v=\left[\left(\frac{37}{4}-\frac{37}{16}=\frac{111}{8}\right)m^4+\left(\frac{483}{8}-\frac{555}{32}=\frac{1377}{32}\right)m^4e'^2\right]ndt.$$

En joignant à ces termes ceux qui résultent de la première approximation, et qui ont été déterminés précédemment (*Théorie Analytique*, livre 7^e, No. 94) on aura donc pour la valeur complète de $d.\delta v$,

$$d.\delta v=\left[-m^2+\frac{111}{16}m^4+\left(-\frac{3}{2}m^2+\frac{1377}{32}m^4\right)e'^2\right]ndt. \qquad (a)$$

Si l'on réprésente par ζ le moyen mouvement lunaire dans l'orbite elliptique, ou (ce qui revient au même) si l'on fait $\zeta=\int ndt$, on aura pour déterminer la variation de ζ dans l'orbite troublée, la formule (*Théorie Analytique*, liv. 7, No. 92)

$$d.\delta\zeta=-3andt\int d'.\delta R+\frac{3a^2ndt}{2}\left(\int d'R\right)^2. \qquad (c)$$

Le premier terme ne peut produire des termes périodiques dans l'ordre d'approximation auquel nous nous arrêtons, conformément au théorème de l'invariabilité des grands axes planétaires ; considérons donc uniquement le second terme de cette formule. Nous avons trouvé précédemment

$$a\int d'R=m^2[F\cos 2\xi+F'e'\cos(2\xi-\varphi')+F''e'\cos(2\xi+\varphi'],$$

et, par suite, en n'ayant égard qu'aux termes non périodiques

$$a^2\left(\int d'R\right)^2=\frac{m^4}{2}(F^2+F'^2e'^2+F''^2e'^2).$$

En substituant cette valeur dans la formule (c), on aura donc

$$d.\delta\zeta=\frac{3}{4}m^4(F^2+F'^2e'^2+F''^2e'^2)ndt,$$

ou bien, en vertu des valeurs précédentes de F, F′ et F″,

$$d.\delta\zeta=\left(\frac{27}{64}m^4+\frac{405}{125}m^4e'^2\right)ndt.$$

En joignant à cette valeur les termes correspondants de $d.\delta v$, on aura donc

$$\frac{d.\delta\zeta}{ndt}+\frac{d.\delta v}{ndt}=-m^2+\left(\frac{111}{16}+\frac{27}{64}=\frac{471}{64}\right)m^4$$

$$+\left[-\frac{3}{2}m^2+\left(\frac{1377}{32}+\frac{405}{128}=\frac{5913}{128}\right)m^4\right]e'^2.$$

En réunissant à cette expression les termes de la valeur de $d.\delta v$ qui dépendent des produits $e^2e'^2$, et $e'^2\gamma^2$, qui ont été déterminés précédemment, et en observant que v représentant la longitude vraie de la lune dans l'orbite troublée, on a $v=\int ndt+\delta\zeta+\delta v$, et par conséquent $\frac{dv}{dt}=n\left(1+\frac{d.\delta\zeta}{ndt}+\frac{d.\delta v}{ndt}\right)$, que l'on a supposé d'ailleurs que l'on avait par le résultat des observations, $\frac{dv}{dt}=\mathrm{n}$; on trouvera enfin

$$\mathrm{n}=n\left[1-m^2+\frac{471}{64}m^4-\left(\frac{3}{2}m^2-\frac{5913}{128}m^4\right)e'^2\right.$$

$$-\left(\frac{27}{16}m^2+\frac{825}{32}m^3\right)e^2e'^2$$

$$\left.+\left(\frac{27}{16}m^2-\frac{153}{32}m^3\right)\gamma^2e'^2\right],$$

valeur qui coïncide identiquement avec celle que nous avons présentée dans le No. 7, vol. xx. p. 272, des *Monthly Notices*, et dans l'ordre d'approximation auquel nous nous arrêtons. La concordance de ces deux résultats obtenus par des méthodes tout à fait différentes montre l'exactitude de nos calculs.

Elements of Comet II., 1860. By Dr. H. Seeling, Assistant at the Glasgow Observatory.

I send the results of a short computation concerning the Comet 1860 (II.), in which, for a more accurate determination of the orbit, I availed myself of the observations at

Kremsmünster, published in No. 1268 of the *Ast. Nach.*, these observations most likely extending quite as far as the comet could be observed at all.

The three positions on which I based my computation are,—

1860.	Greenwich M.T.	Geoc. Long. ° ′ ″	Lat.
Hamburg.........	April 17·42332	54 44 52·6	+30 46 44·5
Kremsmünster...	May 6·35057	74 30 22·2	+32 49 33·1
,,	,, 23·39302	91 26 19·9	+32 50 2·8

Reduced already to the Mean Equinox 1860·0, and free from aberration and parallax.

The variation of the curtate distances furnishes the following elements:—

T 1860, March 5·71694, G.M.T.

$$\left.\begin{aligned}\pi - \Omega &= 41^\circ\ 19'\ 56''\!\cdot\!4\\ \Omega &= 8\ \ 56\ \ \ 8\cdot5\end{aligned}\right\}\ \text{Mean Eq., 1860·0.}$$

$$i = 48\ \ 13\ \ \ 3\cdot8$$

$$\text{Log}\ q = 0\cdot1167062.$$

Motion direct.

The first and last positions are, of course, accurately represented. For the middle position the direct computation gives—

C. – O.

In long............. 0″·0

In lat. .... +1·2

From this it can be inferred, with some probability, that the parabola suffices for the representation of the existing observations within the limits of their faults.

I add the logarithms of the curtate distances for the first and last observations, which might perhaps be applied to a definitive determination of the orbit:—

$$\text{Log}\ \rho = 0\cdot2515281$$

$$\text{Log}\ \rho_2 = 0\cdot2955208$$

Glasgow Observatory, July 30, 1860.

Comet III., 1860.

The conspicuous comet, of which passing notice was taken at the end of the last Number of the *Notices*, appears to have been first seen by Professor Caswell from the deck of the steamship Arabia on June 20, while on his voyage to England, and first observed on the 21st of June, and announced by Mr. Tuttle, of the observatory of Harvard College, Massachussetts.

Several communications have been received relating to this comet.

Professor Bond in a letter dated Harvard College, Mass. (U.S.), June 27, 1860, writes:

"Long before this reaches you the new comet will have been seen in Europe: it was discovered at this observatory on the 21st, at 9 P.M., by Mr. H. P. Tuttle. The nucleus is as bright as a star of the fourth magnitude, with a tail of 6° or 7°, visible to the naked eye. Changes are going on in its physical aspect resembling those exhibited by the great Comet of 1858.

"On the evening of the 26th the nucleus was surrounded on its sunward side by a faint dark halo, 1″ or 2″ broad, distant 18″ at its apex. Within it, in the south preceding quadrant of the envelope, was an irregular clouded mass, while the nucleus itself presented the appearance of a streak of very bright nebulosity, curiously elongated in the direction of right ascension. On the 22d it had been round, and only 2″·5 in diameter; on the 26th it was spindle-shaped, the longer axis 10″ or 15″, the shorter 3″. The greater part of this change occurred between the 25th and the 26th.

"The dark hollow of the axis of the tail, the contrast in brightness of the two branches of nebulosity issuing from the head, the luminous jets from the nucleus, and the envelopes, afford interesting points of comparison with the great Comet of 1858."

Mr. W. Scott in a letter dated Observatory, Sydney, July 20th, 1860, writes:

"On the 6th of July a large comet was seen in different parts of Australia, setting soon after the sun. I was not aware of its discovery, the weather at Sydney being cloudy, until the 9th, when I immediately made all the preparation in my power for its observation. The only instrument at my disposal for such purposes (until the arrival of an 8-inch refractor, which has been ordered) is an old 3½-inch telescope on a Smeaton's block in a very dilapidated state, and with no serviceable micrometer. I, therefore, had a ring micrometer made of rather large dimensions, in order that I might always get the comet and some tolerably bright star to pass through the field within a short interval. The time of transit of an equatoreal star across the diameter is $99^{s}\cdot71$.

"On the 9th the evening was cloudy, but I caught sight of the comet for a few minutes not far from *Saturn*, but before I could make an observation it was obscured by clouds. The comet was then a brilliant object, and being near the horizon its tail extended upwards at an angle of about 60°.

"On the following day the comet could not be seen; on the 11th it was again visible for a very short time, but no micrometrical measures could be made; by a comparison with *α Leonis* its R.A. was $10^{h}\ 18^{m}\ 22^{s}$, and Declination 11° 24′ at $6^{h}\ 32^{m}$, Sydney mean time.

"On the 12th, 13th, 15th, 16th, 17th, 18th, a number of transits were taken, the results of which I now send. On the 18th the comet was still visible to the naked eye, guided by the telescope. Throughout the whole series of observations I have been unable to distinguish any marked peculiarity in its appearance. There was a decided brilliant nucleus, but not sufficiently well defined to form a good subject for observation, around which the light appeared diffused very uniformly. In consequence of the want of definition of the nucleus I consider all the transits to be liable to errors of at least one second, I have, therefore, in computing the declinations rejected all observations in which the time of transit exceeded 90^{s}. After this precaution has been adopted, it will be seen from the results that the determined right ascensions are more to be relied on than the corresponding declinations. On the 19th I had just succeeded in finding the comet with the telescope when it became obscured by clouds and was no more visible that night. On the 20th the comet was again hidden by clouds; rainy weather has now set in, and I fear I shall not be able to obtain any more observations."

The observations accompanying Mr. Scott's letter are as follows:—

Results of Observations of Comet and Stars.

Sydney Mean Time. d	h m s	Star.	Comet — Star R.A. m s	Comet — Star Decl. ′ ″	
July 12	6 14 1·5	48 Leonis	+2 5·25	+17 32	
	6 22 43·5	,,	+2 9·00	+16 26	
	7 18 54·5	,,	+2 35·25	Too near diameter of micrometer.	
	7 25 45·25	,,	+2 39·75		
	7 32 30·25	,,	+2 41·25		
	7 37 43·50	,,	+2 44·00		
	7 42 51·25	,,	+2 47·50	+ 4 28	
12	7 48 6·00	48 Leonis	+2 50·00	+ 3 50	
	7 53 31·75	,,	+2 53·00	+ 2 37	
	8 3 4·25	,,	+2 55·25	+ 1 20	
	8 8 50·75	,,	+2 59·00	+ 0 24	
	8 13 53·00	,,	+3 0·50	+ 0 3	
	8 19 51·50	,,	+3 4·00	− 0 46	
	8 25 53·00	,,	+3 6·00	,,	
	8 31 0·00	,,	+3 8·75	− 2 39	
13	6 23.23	34 Sextantis	+5 31·50	+14 59	
	6 32 15	,,	+5 36·00	,,	
	6 39 22	Star of 7th or 8th magnitude not in any of the catalogues in my possession. Approximate R.A. $10^h\,42^m\,12^s$ Decl. 4° 10′ N.	−1 5·00	+20 32	
	6 44 44		−1 3·00	+19 49	
	7 18 54		−0 49·25	+14 18	
	7 25 12		−0 46·00	+14 21	
	7 28 9		−0 44·25	+14 22	
	7 31 2		−0 43·25	,,	
	7 34 14		−0 41·75	,,	
	7 37 14		−0 40·25	,,	
	7 42 17		−0 37·75	,,	
	7 45 24		−0 36·50	+11 43	
	7 50 13		−0 34·25	+12 7	
	7 53 7		−0 33·00	,,	
	7 56 5		−0 32·50	,,	
	7 59 15		−0 29·75	,,	
	8 3 33		−0 29·00	,,	
15	6 26 19	61 Leonis	+7 20·40	−13 20	
	6 38 58	,,	+7 26·50	,,	
	7 20 31	,,	+7 46·25	−21 15	
	7 49 40	Star of 8th magnitude. Approximate R.A. $11^h\,1^m\,44^s$ Decl. 1° 58′ S.	+0 52·00	−11 23	Star since identified with No. 1109, Hour 10, in Bessel's Zones, Weisse 1846.
	8 4 28		+0 57·75	−13 31	
	8 20 51		+1 4·50	−14 49	
	8 24 7		+1 6·00	−15 47	
	8 27 12		+1 7·00	−16 20	
	8 31 0		+1 9·25	,,	
	8 34 37		+1 11·00	−16 56	
	8 37 44		+1 12·25	−17 35	
	8 40 57		+1 11·75	−17 48	
	8 44 25		+1 16·00	−18 35	
	8 47 25		+1 16·50	−18 46	
July 16	5 58 43	Star of 8th magnitude. Approximate R.A. $11^h\,12^m\,5^s$ Decl. 5° 10′ S.	+0 25·50	+12 56	Star of comparison believed to be No. 190, Hour 11, Weisse.
	6 6 28		+0 28·25	,,	
	6 20 49		+0 35·00	,,	
	6 23 38		+0 35·00	,,	
	6 26 55		+0 36·75	,,	
	6 30 4		+0 39·00	,,	
	7 2 42		+0 50·25	,,	
	7 10 7		+0 54·50	+ 2 6	
	7 13 11		+0 55·25	+ 1 37	
	7 16 24		+0 57·00	+ 1 18	
	7 25 49		+1 1·00	+ 0 17	
	7 30 16		+1 1·50	+ 0 35	
	7 34 26		+1 4·00	− 0 24	
	8 14 14		+1 19·50	,,	
	8 23 21		+1 23·25	,,	
	8 28 40		+1 24·50	,,	
	8 32 16		+1 27·00	,,	
	8 37 26		+1 30·50	,,	
17	6 52 3	Weisse, No. 250, Hour 11.	6 4·75	,,	
	7 48 14		6 25·00	+ 2 9	
	9 2 11		6 52·25	,,	
18	6 5 42	Weisse, No. 544, Hour 11.	−1 23·50	− 4 9	
	6 8 55		−1 22·50	,,	
	6 12 29		−1 20·15	,,	
	7 0 42		−1 4·25	,,	
	7 4 29		−1 3·50	−12 3	
	7 7 46		−1 1·40	−12 6	
	7 10 48		−0 59·50	,,	
	7 14 27		− 58·75	,,	
	7 17 37		− 58·10	−13 28	
	7 20 44		− 57·75	−13 44	
	7 23 29		− 56·60	−14 2	
	7 29 55		− 53·25	−14 25	
	7 32 52		− 52·00	−14 24	
	7 35 56		− 51·00	−15 0	
	7 38 46		− 49·75	−15 25	
	7 41 28		− 48·75	−15 48	
	7 44 9		− 48·25	−15 44	

Mr. R. J. L. Ellery, Director of the Observatory, Williamstown, in a letter dated Victoria, July 24th, 1860, writes:—

"I enclose a few places of a comet that made its appearance here on the 7th ultimo. Our only extra-meridional instrument (the 18-inch altazimuth) was unfitted for observation during its apparition, so that we could only obtain sextant observations. Such as they are, I believe the observations to be as correct as is possible with such an instrument.

"The comet was very bright at its first apparition; its tail extended over about four degrees the first night; the head appeared very nebulous throughout the time of its visibility here."

Date. 1860.	h m	Comet's R.A. h m s	Comet's Declination. ° ′ ″
July 10	6 8	10 6 5·6	14 43 57 N.
11	6 40	10 18 22·5	11 10 16 N.
12	6 6	10 29 50·4	8 3 0 N.
13	6 35	10 41 20·3	4 25 30 N.
15	7 36	11 2 54·2	2 11 40 S.
16	7 15	11 12 33·7	5 11 14 S.
18	7 43	11 31 1·7	10 47 16 S.
19	8 3	11 39 36·9	13 20 0 S.
20	7 38	11 47 38·3	15 35 36 S.

Capt. T. Spratt in a letter to Admiral Smyth, dated H.M.S. Medina, Crete, 3 July, 1860, writes:—

"I suppose the twilight in England has prevented your seeing the new comet, which promises from its first appearance to be a splendid one; but between twilight and moonlight (now advanced to nearly full) its tail has become less and less distinct to us.

"It was first noticed on board the Medina, on June 25th, and was visible to the naked eye for about 20 minutes only, the light of the moon, as it succeeded the twilight, obliterating its visibility; and as the moon's power increased every subsequent night, the comet became less and less distinct; so that now it is hardly visible, except the nucleus, which increases in size, although changing its position slowly, and appears about equal to a star of the fourth or fifth magnitude.

"On the night of the 25th, when it was first seen during the short interval of dusk between twilight and moonlight, at 8·40 P.M. apparent time of place, in lat. 35° 23′, long. 24° 28′, off the town of Retuno, Crete, the tail appeared to be about 15 degrees in length, and having a slight curve, but inclined towards the two Bears in a direction nearly midway between the Pointers and Pole-star. The following were its angular measurements:—

	°
To Pole-star............	47·18
Saturn...............	43·13

"On the 26th, at the town of Candia, in lat. 35° 21′, long. 25° 8′, its angular distances were, 48° 17′ to Pole-star; ditto to *Saturn*, 42° 5′, or 41° 43′, being indistinct for measurement with sextant.

"At Spina Longa, on July 1st, in lat. 35° 15′ and long. 25° 48′, its angle from Pole-star, at 8·30 P.M., was 54°."

The following Elements were computed by M. Powalky, from observations of June 23, 26, and 29:—

T 1860, June 16·0395, Berlin M.T.

	° ′ ″	
ϖ	161 26 23·5	App. Equinox, June 26.
☊	84 47 17	
i	79 19 6	

Log *q*...... 9·465480.

Motion direct.

Elements slightly different from the above, obtained by several computers, are published in the *Astronomische Nachrichten* and in Gould's *Astronomical Journal.*

Discovery of Minor Planet (59).

M. Leverrier's *Bulletin* of September 14 contains the observations of this new Minor Planet by M. Chacornac, at Paris, on the 12th and 13th September.

	Paris M.T. h m s	R.A. h m s	Decl. ° ′ ″	Comps.
Sept. 12	13 43 28·07	0 39 34·72	......	8
	13 46 8·68	......	+1 9 25·9	6
13	13 49 32·87	0 38 59·55	......	3
	14 3 43·64	......	+1 0 35·8	4
	14 27 54·96	0 38 58·49	......	3

Equal to a star of 9th to 10th magnitude.

Approximate Elements of Minor Planet (59), *deduced from observations extending from Sept.* 12 *to Sept.* 19. By Mr. W. Ellis, Assistant at the Royal Observatory, Greenwich.

	° ′ ″	
Mean Anom.	15 51 43	Sept. 15, 12h G.M.S.T.
Long. of Perih.	337 51 49	Mean Equinox 1860·0
" Node	167 8 47	
I	6 35 55	
φ	11 19 18	
Log *a*	0·4158610	
μ	843″·753	

Approximate Ephemeris, calculated from the Elements above.

For Greenwich Mean Midnight.

1860.	Apparent R.A. h m s	Apparent N.P.D. ° ′	Log. Δ
Sept. 12	0 39 37·9	88 49·8	0·0592
13	39 2·9	88 58·7	
14	38 26·9	89 7·7	
15	37 50·0	89 16·7	0·0567
16	37 12·2	89 25·9	
17	36 33·5	89 35·1	
18	35 53·9	89 44·3	0·0551
19	35 13·5	89 53·5	
20	34 32·5	90 2·8	
21	33 51·0	90 12·2	0·0542
22	33 8·8	90 21·5	
23	32 26·2	90 30·8	
24	31 43·5	90 40·2	0·0542
25	31 0·6	90 49·5	
26	30 17·5	90 58·9	
27	29 34·2	91 8·2	0·0551
28	28 50·5	91 17·3	
29	28 6·4	91 26·2	
30	27 22·2	91 35·0	0·0567
Oct. 1	26 38·7	91 43·9	
2	25 55·4	91 52·7	
3	25 12·6	92 1·4	0·0592
4	24 30·2	92 9·9	
5	23 47·9	92 18·3	
6	23 6·5	92 26·5	0·0625
7	22 25·7	92 34·6	
8	21 45·3	92 42·5	
9	21 5·9	92 50·3	0·0666
10	0 20 27·3	92 57·8	

1860.		Apparent R.A.	Apparent N.P.D.	Log. Δ
		h m s	° '	
Oct.	11	0 19 49·4	93 5·2	
	12	19 12·6	93 12·4	0·0715
	13	18 36·8	93 19·4	
	14	18 1·9	93 26·1	
	15	17 28·3	93 32·7	0·0771
	16	16 55·9	93 39·0	
	17	16 24·4	93 45·1	
	18	15 54·5	93 50·9	0·0834
	19	15 26·0	93 56·5	
	20	14 58·7	94 1·8	
	21	14 32·9	94 7·0	0·0902
	22	14 8·6	94 11·8	0·0902
	23	13 45·6	94 16·4	
	24	13 24·2	94 20·8	0·0976
	25	13 4·4	94 25·0	
	26	12 46·0	94 28·8	
	27	12 29·4	94 32·4	0·1056
	28	12 14·4	94 35·7	
	29	12 0·9	94 38·8	
	30	0 11 49·2	94 41·6	0·1139

By an observation on Sept. 25, the ephemeris was found to be still correct.— G. B. Airy.

Discovery of Minor Planet (80).

M. Leverrier's *Bulletin* of Sept. 23 contains the communication of this discovery by letter, dated Sept. 20, from Châtillon-sous-Bagneux.

The new star was first seen by M. Goldschmidt at 8ʰ P.M. on the 9th, at 49 seconds east of the star Lalande 44384, and recognised as a planet on the 19th, when it appeared equal to a star of the 11th magnitude.

The following approximate positions are given:—

1. For the star:—

		R.A.	Decl.
		h m s	° ' "
Lalande 43981		22 24 8	3 37 0

2. For the planet:—

Sept. 19	8ʰ 2ᵐ Par. M.T.	22 25 43·83	3 48 43

Motion in the 10 Days.

In R.A. — 10 minutes of time.

In Decl. + 24 minutes of arc.

The communications received in relation to the Solar Eclipse of 18 July last are reserved until after the meeting of the Society in November, when some further ones will probably come in.— Ed.

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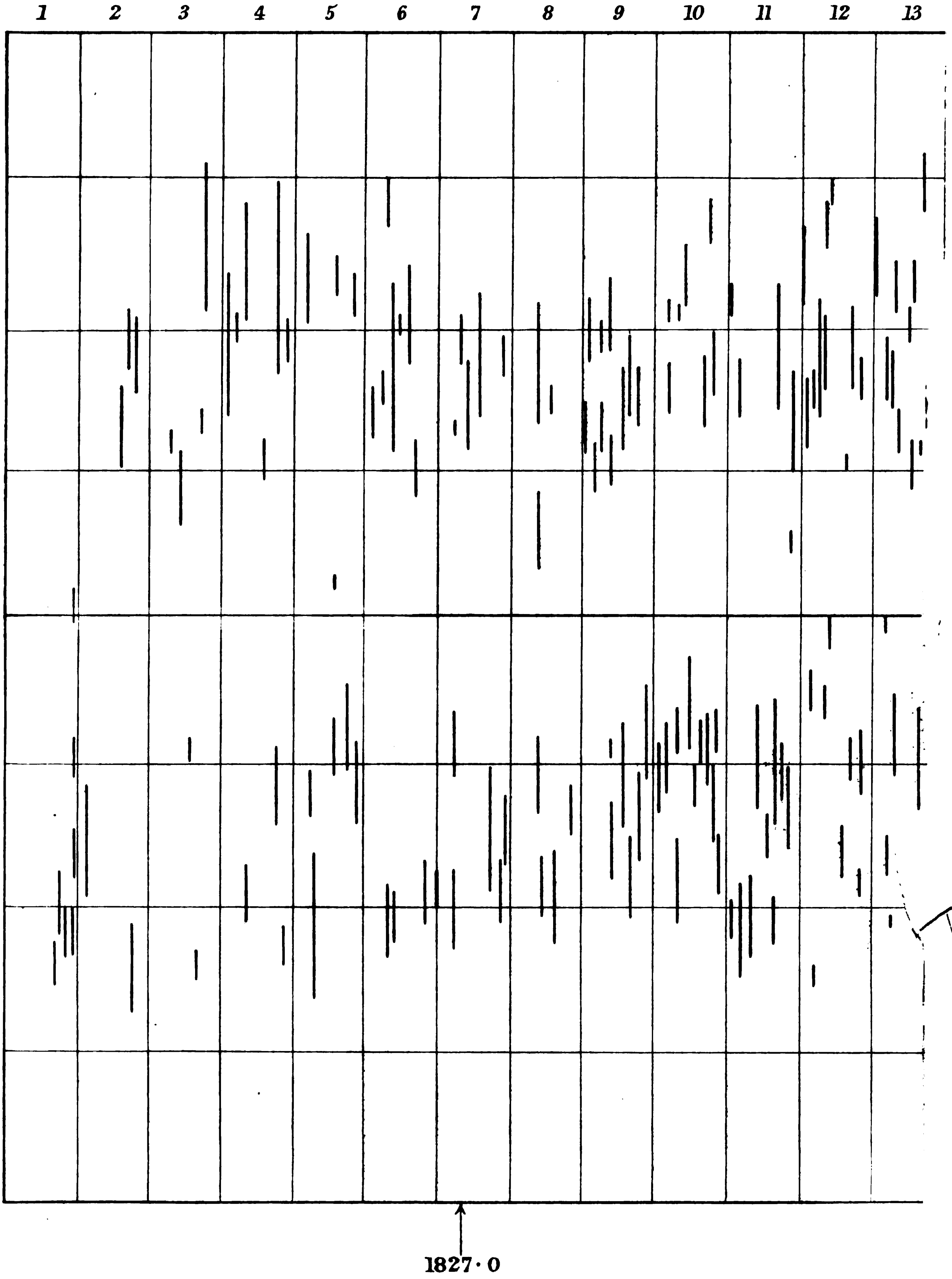
London: — Strangeways & Walden (late G. Barclay), Printers, Castle St. Leicester Sq.

A Map exhibiting the distribution in latitude of the Solar

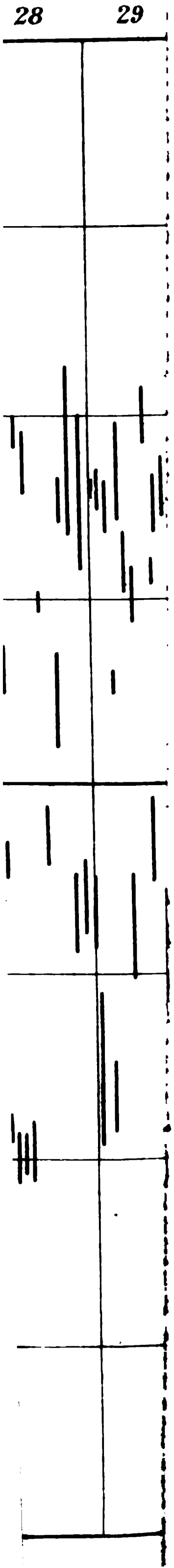
Scale in latitude 10° to 1 inch. Scale in longitude compressed to



Dr. Sœmmering appears to have discontinued his observations bet

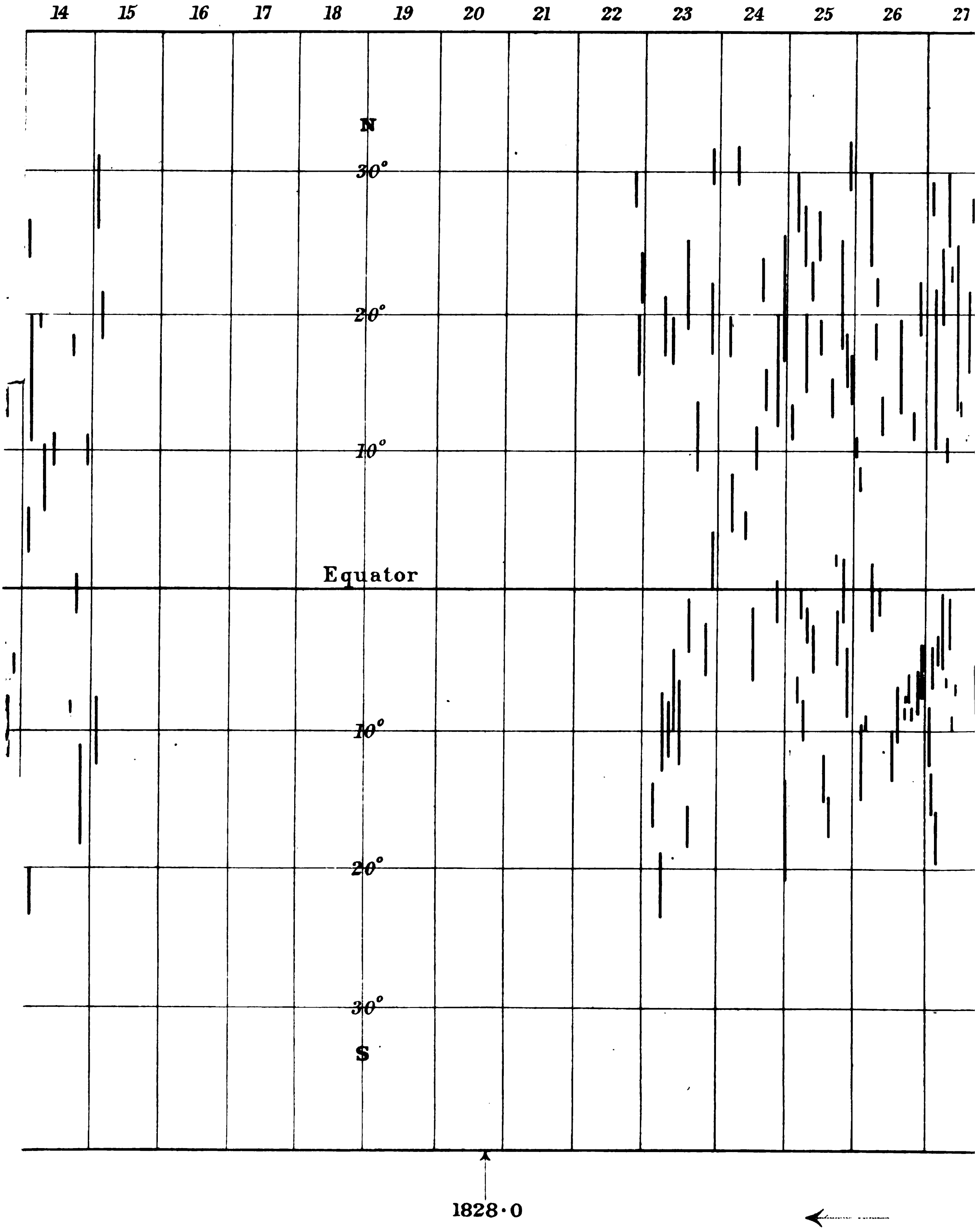
the.

by vert



Ti

360° to half an inch, whereby spots and groups of spots are repre



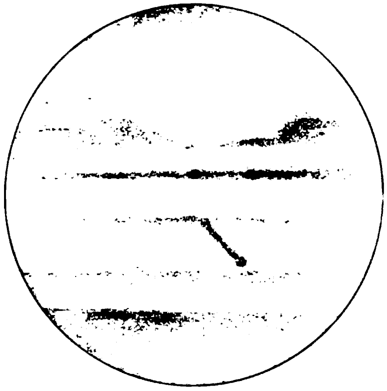
een the Months of August 1827 and February 1828 both inclusive.

Fig. 1.

1860. Feb. 29
h. m.
7 45 G.M.T.

Mr. Long. with
5 in. Achromatic
power 305

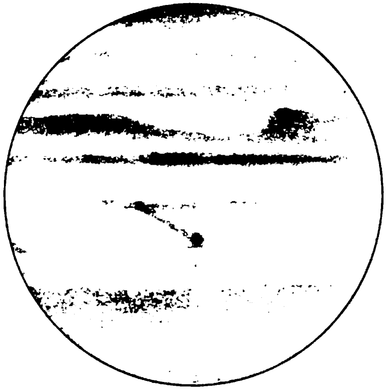
Fig. 2.



1860 March 2nd
h. m.
9 24. G.M.T

Mr. Baxendell.
with 5 in Achromatic
power 223

Fig. 3

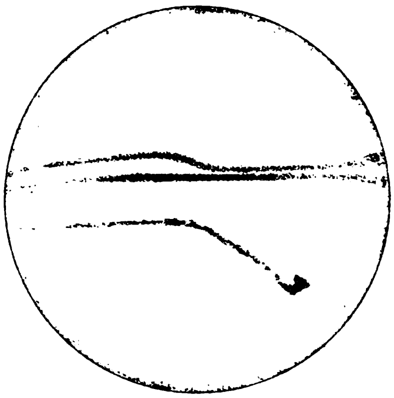


1860 March 5th
h. m.
7.18 G.M.T

Mr. Baxendell
with 13 in Reflector
power 301.

Views of Jupiter

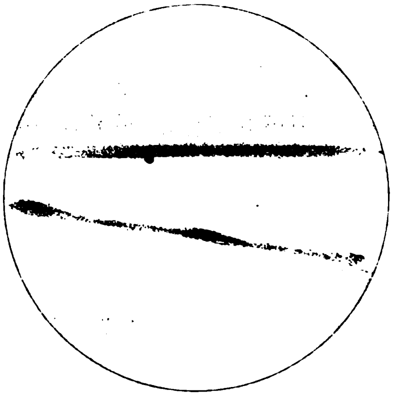
Fig. 4.



1860. March 12.
$7^h\ 40^m$ G.M.T.

As seen in Mr
Fletchers Equatorial
9.5 in. power 250.

Fig. 5.



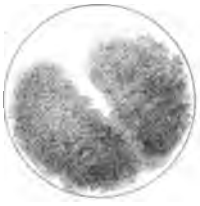
1860. April 9.
$8^h\ 0^m$ G.M.T.

Mr Baxendell
with 5 in. Achromatic
power 223.

Mr Dawes' Satellites of Jupiter.

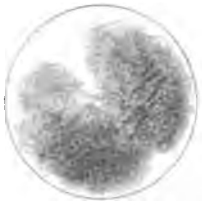
The third Satellite

Fig. 3.



1860. Jan. 31
8¼ inch refractor.
power 530.

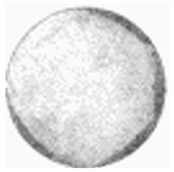
Fig. 2.



1849. Feb. 11.
6⅓ inch refractor
power 460.

Fourth Satellite

Fig. 1.



1849. Feb. 12.
6⅓ inch refractor
power 460.

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